## Lecture – 14

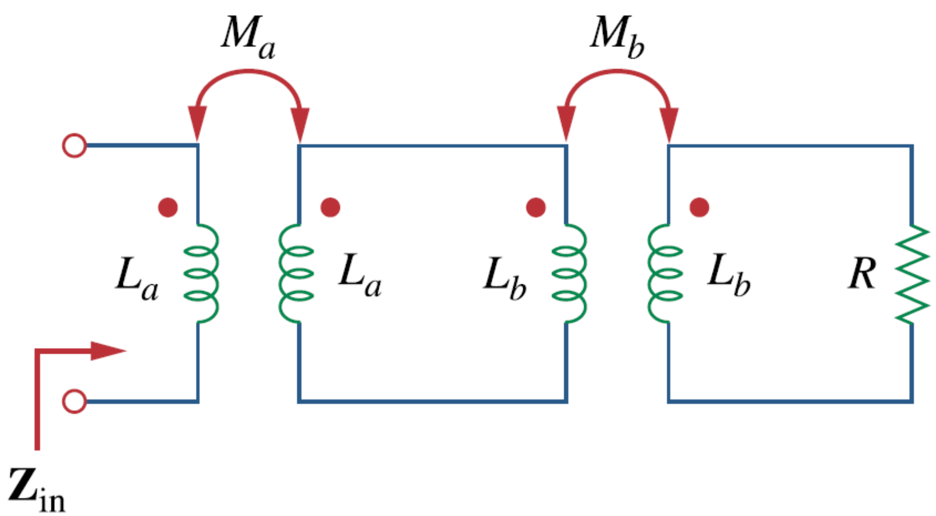
## Date: 29.09.2016

- Transformer

# Example – 1

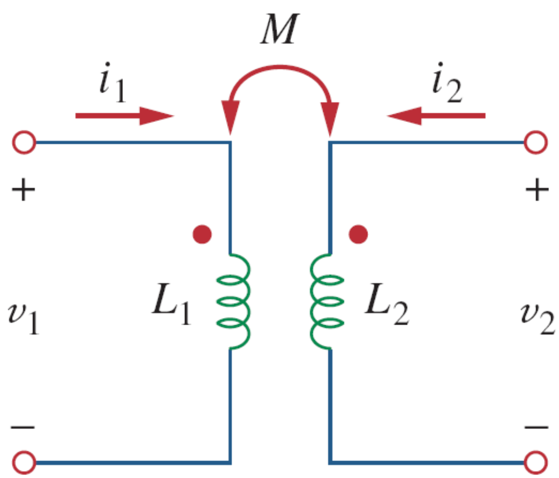
Two linear transformers are cascaded as shown below. Show:

$$\mathbf{Z}_{\text{in}} = \frac{\omega^2 R(L_a^2 + L_a L_b - M_a^2) + j\omega^3 (L_a^2 L_b + L_a L_b^2 - L_a M_b^2 - L_b M_a^2)}{\omega^2 (L_a L_b + L_b^2 - M_b^2) - j\omega R(L_a + L_b)}$$



## Ideal Transformers

- An ideal transformer exhibits perfect coupling i.e., $k = 1$.
- It consists of two (or more) coils with a large number of turns wound on a common core of high permeability. Because of this high permeability of the core, the flux links all the turns of both coils, thereby resulting in a perfect coupling.




- In the frequency domain:

$$\mathbf{V}_1 = j\omega L_1 \mathbf{I}_1 + j\omega M \mathbf{I}_2$$

$$\mathbf{V}_2 = j\omega M \mathbf{I}_1 + j\omega L_2 \mathbf{I}_2$$

$$\mathbf{V}_2 = j\omega L_2 \mathbf{I}_2 + \frac{M\mathbf{V}_1}{L_1} - \frac{j\omega M^2 \mathbf{I}_2}{L_1}$$

# Ideal Transformers (contd.)

- We know for ideal transformer $k = 1$ and therefore $M = \sqrt{L_1 L_2}$

$$\mathbf{V}_2 = j\omega L_2\mathbf{I}_2 + \frac{\sqrt{L_1L_2}\mathbf{V}_1}{L_1} - \frac{j\omega L_1L_2\mathbf{I}_2}{L_1} = \sqrt{\frac{L_2}{L_1}}\mathbf{V}_1 = n\mathbf{V}_1$$

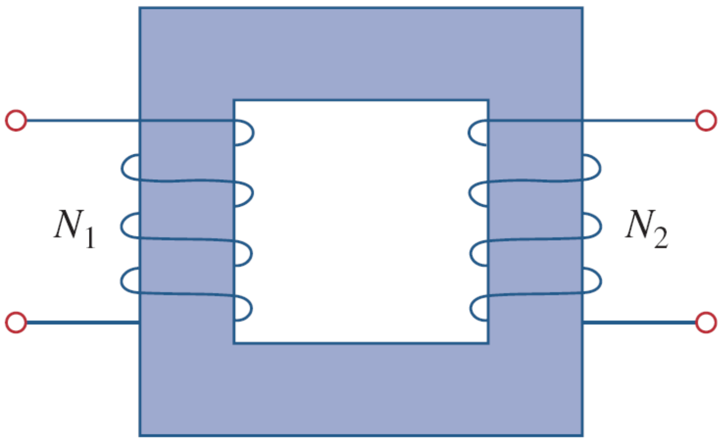
$$n = \sqrt{L_2/L_1}$$

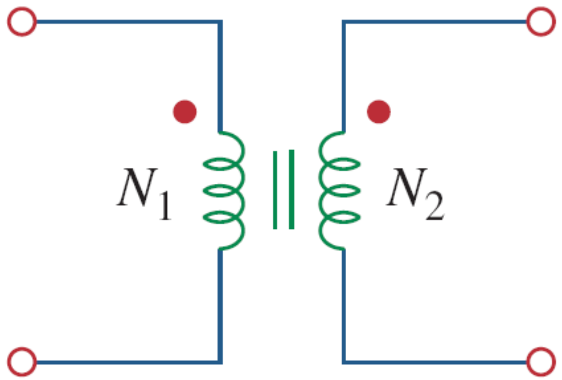
For $L_1, L_2, M \rightarrow \infty$, the turn ratio $n$ remains the same and in such a scenario the coupled coils become an ideal transformer.

- **A transformer is said to be ideal if:**
  1. Coils have very large reactances ($L_1, L_2, M \rightarrow \infty$)
  2. Coupling coefficient is equal to unity (k=1)
  3. Primary and secondary coils are lossless ($R_1 = 0 = R_2$).

**Iron-core transformers are close approximations to ideal transformers.**

# Ideal Transformers (contd.)




$$v_1 = N_1 \frac{d\phi}{dt}$$

$$v_2 = N_2 \frac{d\phi}{dt}$$

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} = n \Rightarrow \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} = n$$

- For power conservation, the energy supplied to the primary must equal the energy absorbed by the secondary (there are no losses in an ideal transformer).

$$v_1 i_1 = v_2 i_2$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = n$$

**the primary and secondary currents are related to the turns ratio in the inverse manner as the voltages.**

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2} = \frac{1}{n}$$

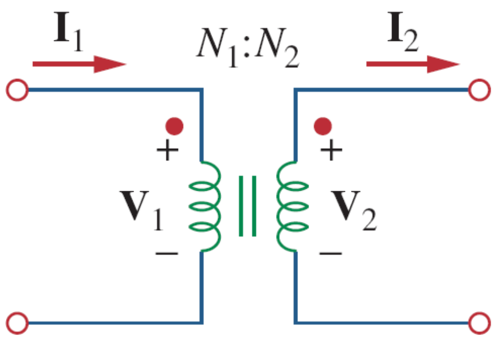
## Ideal Transformers (contd.)

- For $n > 1$, we have a ***step-up transformer***, as the voltage is increased from primary to secondary ($V_2 > V_1$).
- For $n < 1$, the transformer is a ***step-down transformer***, as the voltage is decreased from primary to secondary ($V_2 < V_1$).

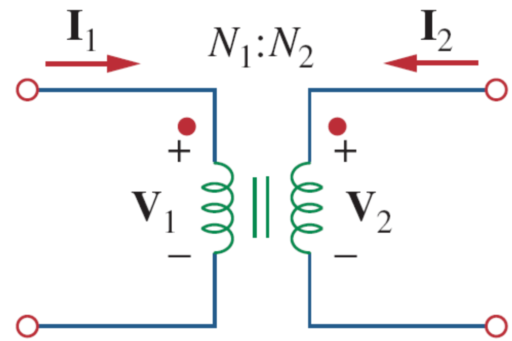
**The ratings of transformers are usually specified as $V_1$ / $V_2$. A transformer with rating 2400/120 V should have 2400 V on the primary and 120 in the secondary (i.e., a step-down transformer). Keep in mind that the voltage ratings are in rms.**

- It is important to get the proper polarity of the voltages and the direction of the currents for the transformer. The two simple rules to follow are:
  - If V1 and V2 and are *both* positive or both negative at the dotted terminals, use +n in the equations. Otherwise, use –n.
  - If I1 and I2 and *both* enter into or both leave the dotted terminals, use +n. Otherwise use –n.

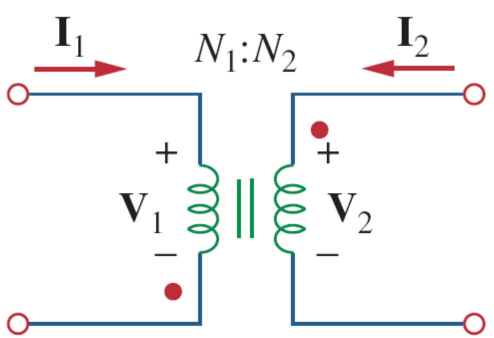
# Ideal Transformers (contd.)



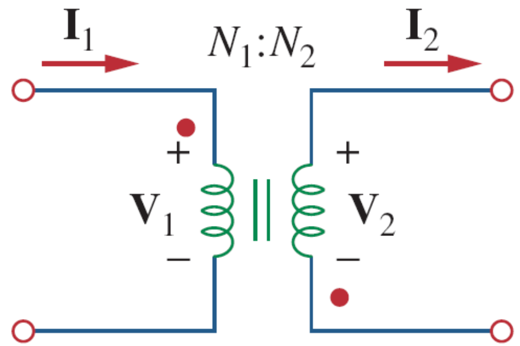

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2}$$




$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{N_1}{N_2}$$




$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = -\frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2}$$




$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = -\frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{N_1}{N_2}$$

## Ideal Transformers (contd.)

- we can always express:

$$\mathbf{V}_1 = \frac{\mathbf{V}_2}{n} \quad \text{or} \quad \mathbf{V}_2 = n\mathbf{V}_1$$

$$\mathbf{I}_1 = n\mathbf{I}_2 \quad \text{or} \quad \mathbf{I}_2 = \frac{\mathbf{I}_1}{n}$$

- The complex power in the primary winding is:

$$\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^* = \frac{\mathbf{V}_2}{n}(n\mathbf{I}_2)^* = \mathbf{V}_2\mathbf{I}_2^* = \mathbf{S}_2$$

**the complex power supplied to the primary is delivered to the secondary without loss. The transformer absorbs no power. It is expected as the ideal transformer is lossless.**

- The input impedance seen by the source:

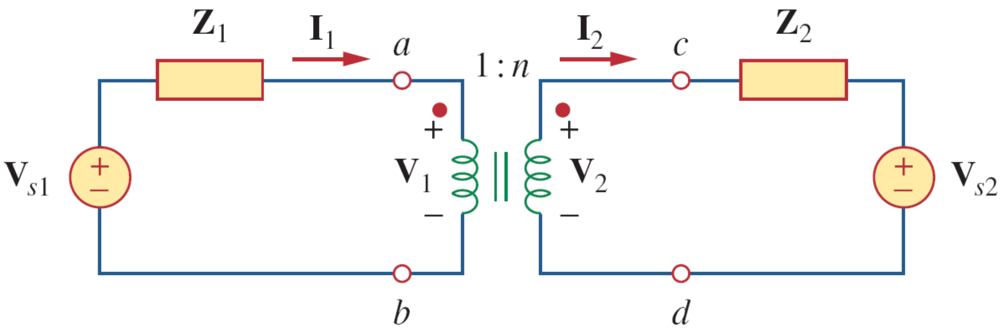
$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{1}{n^2}\frac{\mathbf{V}_2}{\mathbf{I}_2} \quad \Rightarrow \quad \mathbf{Z}_{\text{in}} = \frac{\mathbf{Z}_L}{n^2}$$

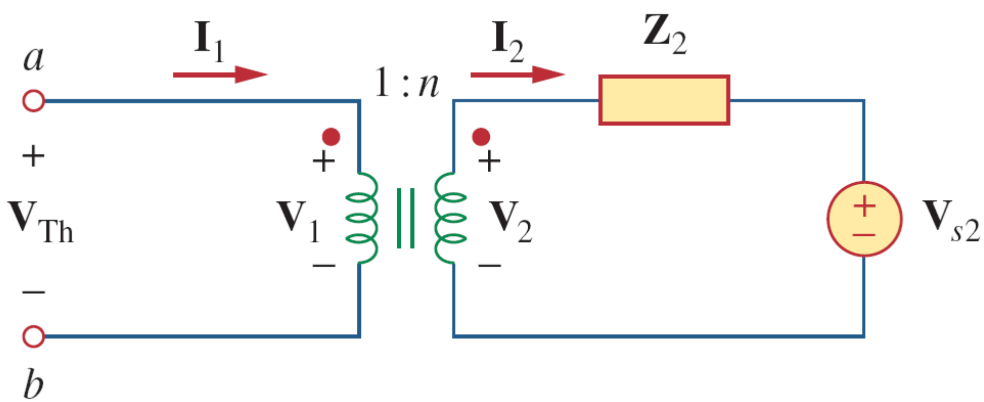
- The input impedance is also called the *reflected impedance*. The ability of the transformer to transform a given impedance into another impedance provides us a means of *impedance matching* to ensure maximum power transfer.

## Ideal Transformers (contd.)

- In analyzing a circuit containing an ideal transformer, it is common practice to eliminate the transformer by reflecting impedances and sources from one side of the transformer to the other.

- suppose we want to reflect the secondary side of the circuit to the primary side i.e, *a*-*b*.



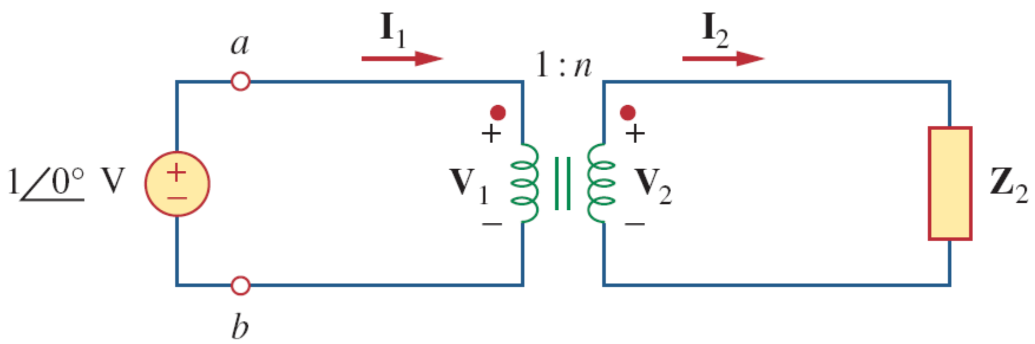

- First, obtain $V_{TH}$ as the open-circuit voltage at terminals *a*-*b*.

- **Now, a-b is open: so $V_2 = V_{s2}$:**

$$\mathbf{V}_{\mathrm{Th}} = \mathbf{V}_1 = \frac{\mathbf{V}_2}{n} = \frac{\mathbf{V}_{s2}}{n}$$

# Ideal Transformers (contd.)

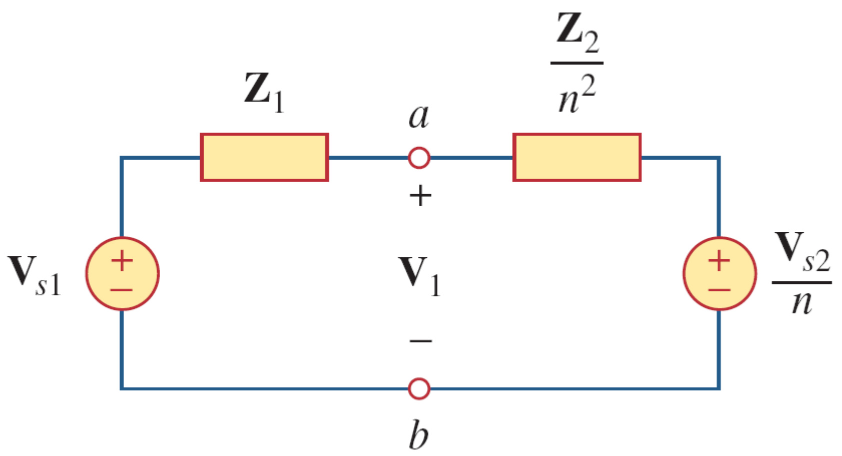
- For $Z_{TH}$, remove voltage source in secondary and excite the primary with a unit source.




- Now, $I_1 = nI_2$ and $V_1 = \frac{V_2}{n}$ and therefore:

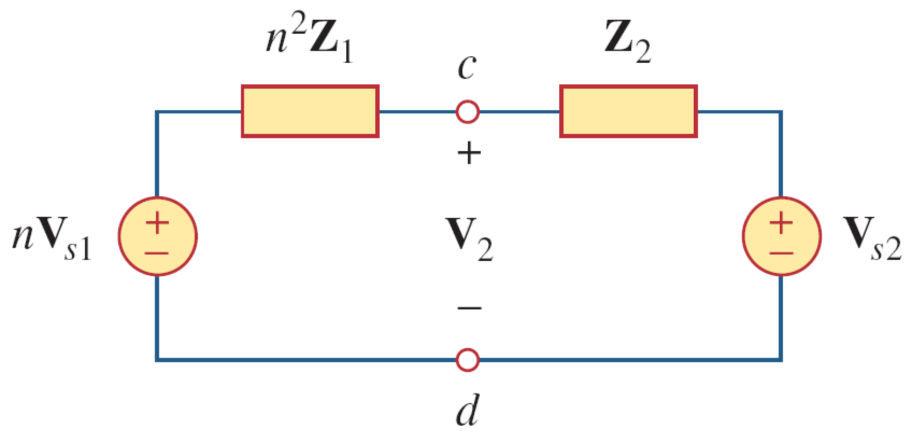
$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{\mathbf{V}_2/n}{n\mathbf{I}_2} = \frac{\mathbf{Z}_2}{n^2}, \qquad \mathbf{V}_2 = \mathbf{Z}_2\mathbf{I}_2$$

- Once we have $Z_{TH}$ and $V_{TH}$, we get the equivalent circuit as:



# Ideal Transformers (contd.)

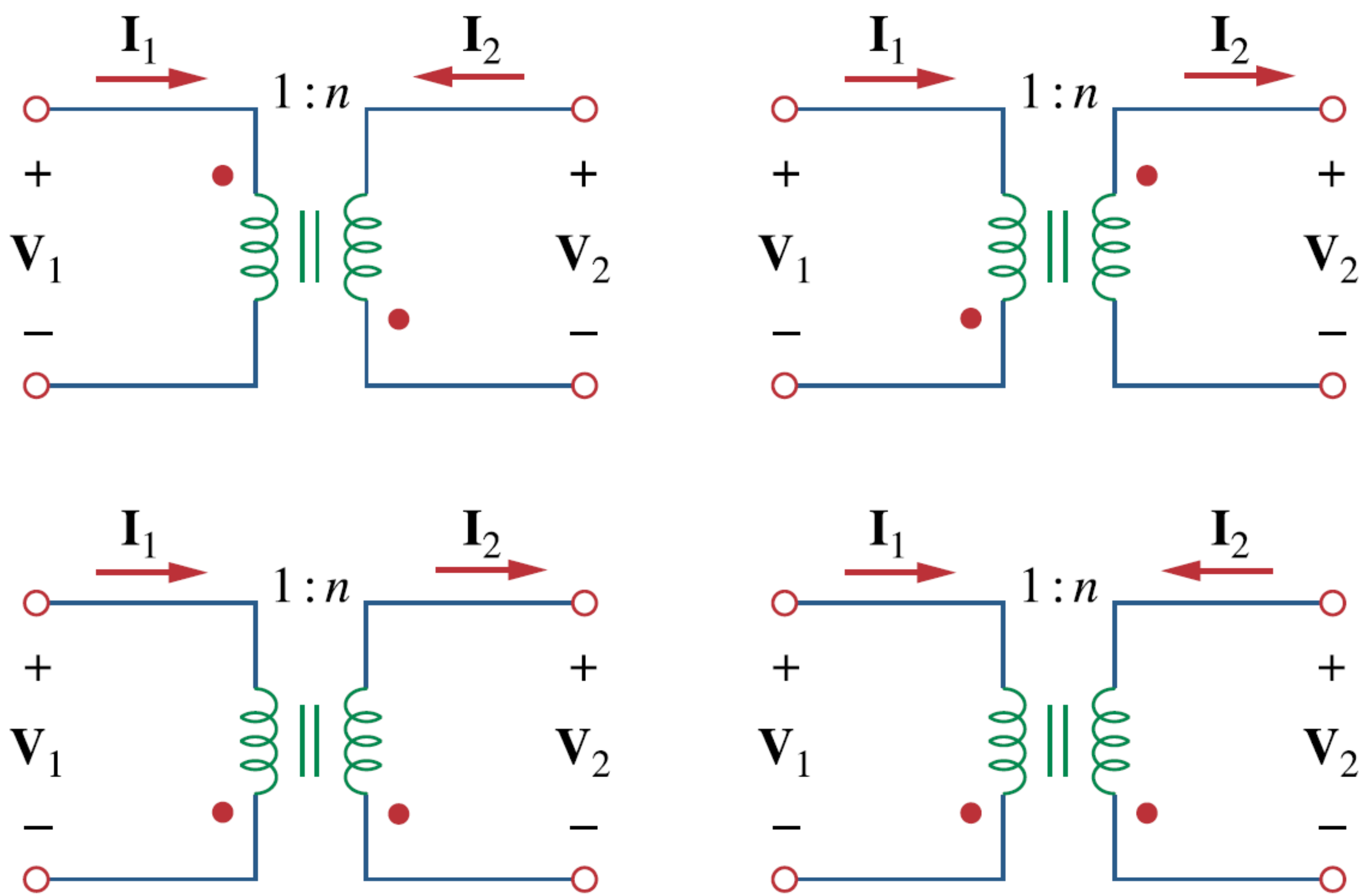
- We can also reflect the primary side of the circuit as:




- the power remains the same, whether calculated on the primary or the secondary side.
- However, this reflection approach only applies if there are no external connections between the primary and secondary windings.
- When we have external connections between the primary and secondary windings, we simply use regular mesh and nodal analysis.

# Example – 2

- obtain the relationships between terminal voltages and currents for each of the ideal transformers given below

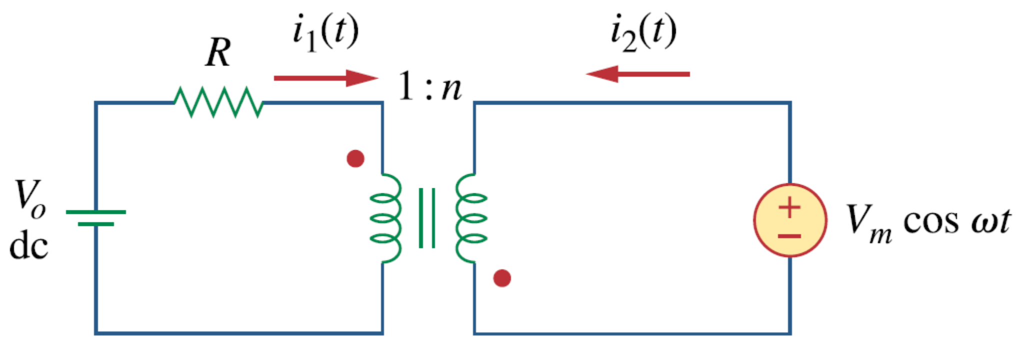


## Example – 3

The primary of an ideal transformer with a turns ratio of 5 is connected to a voltage source with Thevenin parameters $v_{TH} = 10cos2000t$ V and $R_{TH} = 100\Omega$. Determine the average power delivered to a 200Ω load connected across the secondary winding.

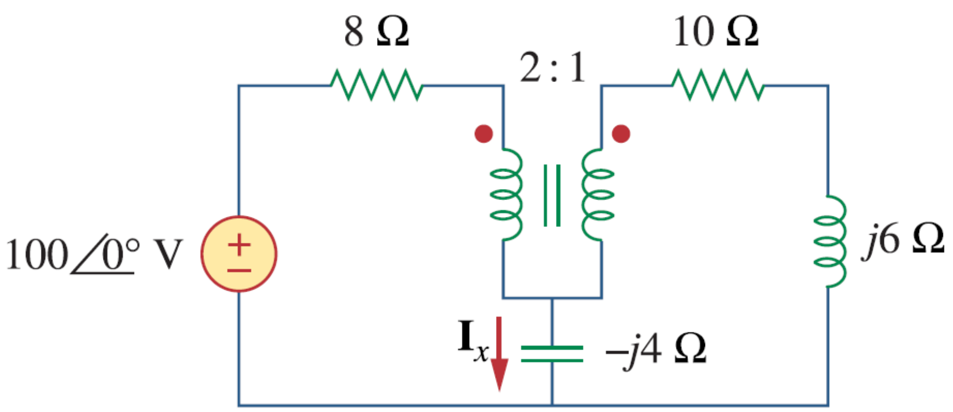
## Example – 4

- In this ideal transformer circuit, find $i_1(t)$ and $i_2(t)$.



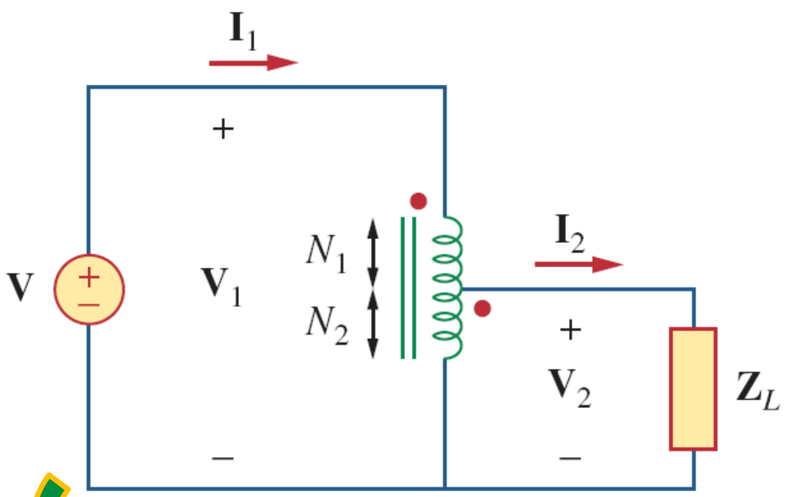

## Example – 5

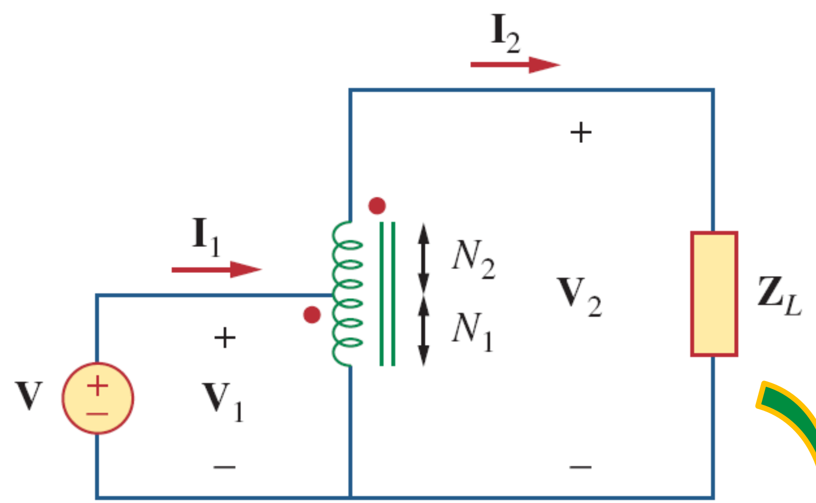
Find $I_x$ in this ideal transformer circuit



# Ideal Autotransformers

- An *autotransformer* has a single continuous winding with a connection point called a *tap* between the primary and secondary sides.
- The tap is often adjustable so as to provide the desired turns ratio for stepping up or stepping down the voltage. This way, a variable voltage can be provided to the load connected to the autotransformer.




Step-down

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1 + N_2}{N_2} = 1 + \frac{N_1}{N_2}$$

Step-up

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_1 + N_2}$$

# Ideal Autotransformers

- an ideal autotransformer, there are no losses, so the complex power remains the same in the primary and secondary:

$$\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^* = \mathbf{S}_2 = \mathbf{V}_2\mathbf{I}_2^*$$

$$V_1 I_1 = V_2 I_2 \quad \Rightarrow \quad \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

Step-down

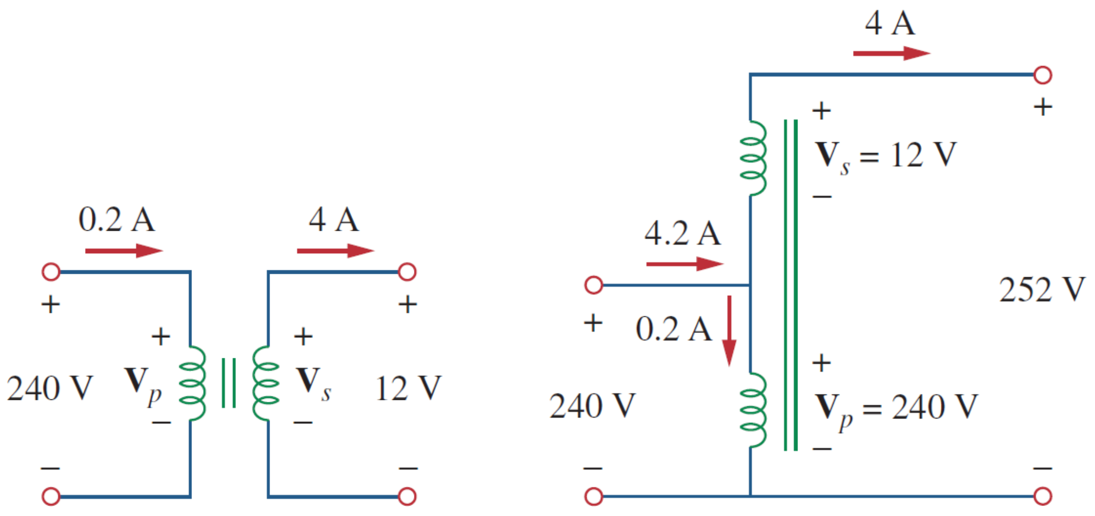
$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_2}{N_1 + N_2}$$

Step-up

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_1 + N_2}{N_1} = 1 + \frac{N_2}{N_1}$$

- A major difference between conventional transformers and autotransformers is that the primary and secondary sides of the autotransformer are not only coupled magnetically but also coupled conductively.
- The autotransformer can be used in place of a conventional transformer when electrical isolation is not required.

# Example – 6



0.2 A
4 A
+
+
+
+
240 V
$\mathbf{V}_p$
$\mathbf{V}_s$
12 V
–
–
–
–
4 A
+
+
$\mathbf{V}_s = 12$ V
–
4.2 A
252 V
+
0.2 A
+
240 V
$\mathbf{V}_p = 240$ V
–
–
–