User Mobility-Aware Time Stamp for UAV-BS Placement

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***Abstract*—Unmanned aerial vehicle (UAV) assisted communication networks have garnered considerable attention of the research community due to their key features of high mobility, fast deployment and easy access to remote areas. UAV base stations (UAV-BSs) are the indispensable component of such networks. Specifically, three-dimensional (3-D) placement of UAV-BS is pivotal in serving the ground users with the desired quality-of-service (QoS). Moreover, in a scenario where users are mobile, UAV-BS placements need to be updated while considering the user mobility. However, an equally important challenge is to optimize the time separation of the placement updates or update interval. Hence, in this work, we propose a framework for joint optimization of UAV-BS placement and the update interval while accounting for the user and UAV mobility parameters. Furthermore, we determine the analytical expressions for the coverage probability of users. We also evaluate the network service time and the number of users covered for the considered network.**

***Index Terms*—Unmanned Aerial Vehicles (UAV), Placement Timeline, Update Interval, User Mobility, Coverage Probability**

I. Introduction

Unmanned aerial vehicles (UAVs) have witnessed tremendous growth in their popularity over the last few years. UAVs have numerous applications like real-time aerial video transmissions, remote surveillance, aerial inspection and monitoring, etc. [1]. Recently, the telecom sector is exploring ways to reap the benefits from deploying UAVs. Since UAVs have key features of high mobility, fast deployment, and easy access to remote areas, researchers have proposed UAV-assisted communication networks for on-demand wireless connectivity. UAV-assisted communication networks can also facilitate other applications such as emergency communication during disasters, military communication in remote areas, etc [2].

In UAV-assisted communication networks, the UAVs can either act as aerial relays or as aerial base stations (BSs). These aerial BSs are referred to as UAV-BSs. UAV-BS placement is one of the important challenges in UAV-assisted communication networks [3]. In the existing literature, placement optimization has been studied for both static and mobile UAV-BS [4]. In the case of static deployment, the UAV-BSs are placed at a hovering location and remain static throughout their mission duration. For instance, [5] considers a heterogeneous network consisting of macro BSs and UAV-BSs. It optimizes the placement of UAV-BSs to maximize the downlink received signal strength. Similar to [5], [6] also considers a heterogeneous network. However, in [6], the authors formulate a problem to optimize 3-D UAV-BS placement in order to maximize the number of users covered by the UAV-BS. Further, [7] jointly optimizes UAV-BS placement and user association in a heterogeneous network to maximize the spectral efficiency of a hotspot area. The authors consider the spectral efficiency of the wireless access as well as the wireless backhaul links. The minimum number of UAV-BSs required to provide coverage to a set of ground users in the absence of fixed infrastructure is determined in [8]. In the case of mobile UAV-BSs, the placement of UAV-BSs changes over time. For instance, in order to collect data from the deployed sensor nodes in a wireless sensor network (WSN) framework, the UAV-BSs have to move around the region of interest [9]. In [10], an energy-efficient strategy for UAV-BS control and user coverage fairness is proposed. The UAV-BS placement is updated after fixed time intervals as the limited number of UAV-BSs are unable to cover the target region all the time.

The repeated placement optimization of UAV-BSs becomes more challenging when the ground users are mobile. For instance, in [11], UAV-BS locations are updated in accordance with the user locations over time in order to maximize the network throughput. Further, the authors in [12] propose a reinforcement learning (RL) framework to efficiently update the placement of UAV-BS in a dynamic heterogeneous network while maintaining the desired quality-of-service (QoS). In [13], an echo state network based prediction algorithm is used to predict the user positions, and then a multi-agent Q-learning based algorithm is used to design UAV-BS trajectory in advance. The problem of placement of multiple UAV-BSs is formulated for maximizing the sum mean opinion score of ground users. It may be noted that the placement optimization of mobile UAV-BSs is generally carried out at specific time instants. Moreover, we believe that determining the time separation between two consecutive UAV-BS placement updates (or update interval) is also an important parameter to optimize the performance of UAV-assisted communication networks. However, [11] and [13] assume a fixed update interval whereas [12] lacks in quantifying such an update interval.

To the best of our knowledge, no existing work investigates the relationship between user mobility and UAV placement update interval. In the following, we summarize our contributions:

- We consider a network where a UAV-BS optimizes its placement to serve the mobile ground users. We propose a joint optimization of the UAV-BS placement and the placement update interval.

- Specifically, the UAV-BS placement is optimized to maximize the number of users covered at an update instant while accounting for the user fairness as well as UAV-BS flight time.
- We propose the use of two metrics, i.e., total UAV-BS flight time and user coverage probability to optimize the update interval.
- We propose an iterative approach to solve the optimal UAV-BS placement and update interval problems jointly.
- Further, we derive the analytical expressions for user coverage probability in terms of user mobility parameter and update interval.

The rest of the paper is organized as follows. Section II presents the network model and the problem formulated in our work. Section III discusses the solution, and Section IV demonstrates the results obtained. Finally, Section V concludes the paper.

Notations: In the following, we mention the notations used throughout the paper. $\mathcal{N}(\mu, \sigma^2)$ is used to denote a Gaussian distribution with mean μ and variance σ^2. $\mathbb{E}\,[]$ is the expectation operator.

II. Network Model and Problem Formulation

We consider a network model with one UAV-BS and N multiple ground users, as shown in Fig. 1. The UAV-BS operates for T seconds to serve the ground users. The users are moving around following a random walk mobility model [14]. The distance traveled by a user in each transition of random walk is assumed to be Rayleigh distributed with shape parameter σ [15]. For analytical tractability, it is assumed that the velocity of the users is constant and same for all the users. The velocity of a user is denoted by v_u. Further, in line with the existing literature, it is assumed that the altitude of the UAV-BS is fixed, and it is denoted by H [16]. Hence, we focus only on the 2-D UAV-BS placement in the horizontal plane. The desired quality-of-service[1] (QoS) determines the coverage radius R of the UAV-BS. Consequently, at any time instant, due to the user mobility and fixed R, the UAV-BS may not be able to cover all the users. So, UAV-placement needs to be updated after certain time intervals.

In our work, an update instant denotes the point in time where the UAV-BS placement is updated. Fig. 2 illustrates the UAV-BS placement update timeline for T seconds. At update instant k, $t_{up}(k)$ denotes the update interval. In other words, $t_{up}(k)$ is the time interval between the k^{th} and $(k{+}1)^{th}$ update instants. In the proposed work, at update instant k, we optimize the UAV-BS placement in order to maximize the number of users covered subject to the fairness of coverage. The 2-D UAV-BS placement at the k^{th} decision instant is denoted as $[U_x(k), U_y(k)]$ where $U_x(k)$ and $U_y(k)$ are the coordinates of the projection of UAV-BS in the 2-D plane. Let $A_n(k)$ be the indicator variable. If user n is covered at the update instant k, $A_n(k) = 1$ otherwise $A_n(k) = 0$. Since the users are moving, there is a possibility that some users get covered more often as compared to others. Hence, there is a need to maintain fairness of coverage. We define fairness at update instant k as follows [16]:

$$fair(k) = \frac{\left(\sum_{n=1}^{N}\sum_{i=0}^{i=k} A_n(i)\right)^2}{N\left(\sum_{n=1}^{N}\left(\sum_{i=0}^{i=k} A_n(i)\right)^2\right)} \tag{1}$$

Further, we consider the fly-hover communicate protocol for UAV-BS operation [17], [18]. This implies that UAV-BS will only serve users when it is hovering. Let $F(k)$ be the time taken by the UAV-BS to fly to its new location at k^{th} update and can be given as:

$$F(k) = \frac{1}{v_{uav}}\sqrt{(U_x(k) - U_x(k-1))^2 + (U_y(k) - U_y(k-1))^2} \tag{2}$$

where v_{uav} is the UAV-BS velocity.

Now, the optimal UAV-BS placement problem at k^{th} update instant can be formulated as follows:

$$\text{(P1)} \qquad \max \sum_{n=1}^{N} A_n(k) \tag{3}$$

$$\text{s.t. } fair(k) > fair_{thres} \tag{4}$$

$$F(k) < t_{up}(k) \tag{5}$$

[1] In our work, path-loss is utilized as a QoS metric. We compute the path-loss using the air-to-ground (AtG) path loss model in [16].

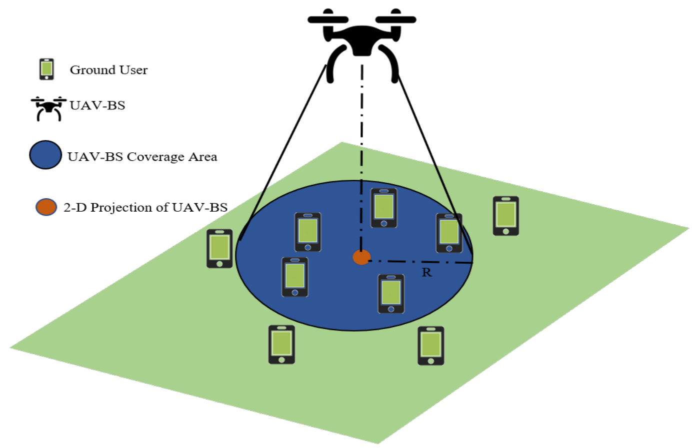

Fig. 1. Network Model

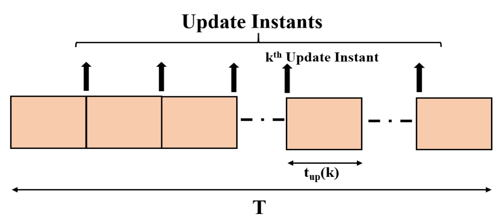

Fig. 2. UAV-BS Placement Timeline

$$D_i = \sqrt{(X_{in} + X_1 + \cdots X_i - U_x(k))^2 + (Y_{in} + Y_1 + \cdots Y_i - U_y(k))^2} \tag{6}$$

$$P_{n,t_{up}(k)} = P[D_1 < R]P[D_2 < R|D_1 < R] \cdots P[D_i < R|D_{i-1} < R] \cdots P[D_{q_{t_{up}(k)}} < R|D_{q_{t_{up}(k)}-1} < R] \tag{7}$$

$$P[D_i < R|D_{i-1} < R] = \frac{\int_{-R}^{R}\int_{-R-u}^{R-u}\int_{-C_2}^{C_2} e^{-\frac{(u-\mu_1)^2}{2\sigma^2(i-1)}} e^{-\frac{v^2}{2\sigma^2}} e^{-\frac{(w-\mu_2)^2}{2\sigma^2(i-1)}} \Phi\left(\frac{C_3}{\sqrt{2}\sigma}\right) dw\, dv\, du}{\sigma\sqrt{2\pi}\int_{-R}^{R}\int_{-C_2}^{C_2} e^{-\frac{(u-\mu_1)^2}{2\sigma^2(i-1)}} e^{-\frac{(w-\mu_2)^2}{2\sigma^2(i-1)}}\, dw\, du} \tag{11}$$

$$\min \left(\frac{\alpha\left(\sum_{j=1}^{k-1} F(j) + Q\,\mathbb{E}\left[F_{t_{up}(k)}\right]\right)}{T}\right) + \left(\frac{(1-\alpha)}{\sum_{n=1}^{N} A_{n,k} P_{n,t_{up}(k)}}\right) \tag{12}$$

Here, (3) represents the total number of users covered at update instant k. Constraint in (4) is used to maintain a fairness above a threshold of $fair_{thres} \in [0, 1]$ in the network. To ensure non-zero service time during the upcoming $t_{up}(k)$ seconds, $F(k)$ must be less than $t_{up}(k)$. This has been applied using the constraint given in (5). For a given $t_{up}(k)$, (P1) is solved exhaustively. The exhaustive search details are provided in Section III.

Now, the next problem is to find the optimal value of $t_{up}(k)$. The choice of $t_{up}(k)$ depends on two factors: 1) total UAV flight time and 2) temporal user coverage probability. As mentioned above, an increase in the UAV-BS flight time decreases its service time. Hence, minimization of total UAV-BS flight time during T seconds must be considered when optimizing $t_{up}(k)$. In the following, we will discuss in detail the temporal coverage probability metric.

A. Temporal Coverage Probability

The coverage probability of a user must capture the change in user locations over time. We define temporal coverage probability of user n, which is covered after the k^{th} update, as the probability of it being within UAV-BS coverage area for the upcoming $t_{up}(k)$ seconds. During these $t_{up}(k)$ seconds user may have multiple transitions. The number of transitions depends on σ and v_u. In other words, coverage probability can be defined as the probability that the displacement of the user from UAV-BS must be less than R for all the transitions within $t_{up}(k)$.

Let D_i denote the displacement of a user from UAV-BS after the i^{th} transition during its random walk. D_i is defined in (6) where X_{in} and Y_{in} are the initial coordinates of a user at k^{th} update instant. Further, $X_i, Y_i \sim \mathcal{N}(0, \sigma^2)$ are the change in the x and y coordinates of a user due to the i^{th} transition. The coverage probability is given in (7) where $q_{t_{up}(k)}$ denotes the average number of user transitions during $t_{up}(k)$. $q_{t_{up}(k)}$ is determined as given below:

$$q_{t_{up}(k)} = \frac{t_{up}(k) v_u}{\sigma\sqrt{\pi/2}} \tag{8}$$

Let $W_1 = X_1 + X_{in} - U_x(k)$ and $W_2 = Y_1 + Y_{in} - U_y(k)$. Hence, $W_1 \in \mathcal{N}(\mu_1, \sigma^2)$ and $W_2 \in \mathcal{N}(\mu_2, \sigma^2)$ where $\mu_1 = X_{in} - U_x(k)$ and $\mu_2 = Y_{in} - U_y(k)$. Now, the first term on the R.H.S in (7) can be evaluated as follows:

$$\begin{aligned} P[D_1 < R] &= P\left[\sqrt{W_1^2 + W_2^2} < R\right] \\ &= \int_{-R}^{R}\int_{-C_1}^{C_1} f_{W_2W_1}(w_2, w_1)\, dw_2\, dw_1 \end{aligned} \tag{9}$$

where $f_{W_2W_1}(w_2, w_1)$ is the joint pdf of W_1 and W_2, and $C_1 = \sqrt{R^2 - w_1^2}$. Since X_1 and Y_1 are independent, W_1 and W_2 are independent. Now, (9) can be written as follows:

$$\begin{aligned} P[D_1 < R] &= \int_{-R}^{R}\int_{-C_1}^{C_1} f_{W_2}(w_2) f_{W_1}(w_1)\, dw_2\, dw_1 \\ &= \frac{1}{2\pi\sigma^2}\int_{-R}^{R}\int_{-C_1}^{C_1} e^{-\frac{(w_1-\mu_1)^2}{2\sigma^2}} e^{-\frac{(w_2-\mu_2)^2}{2\sigma^2}}\, dw_2\, dw_1 \end{aligned} \tag{10}$$

For the i^{th} term on the R.H.S. in (7), where $i > 1$, substitute $X = X_{in} - U_x(k) + X_1 + \cdots X_{i-1}$, $Y = Y_{in} - U_y(k) + Y_1 + \cdots Y_{i-1}$. The i^{th} term of the product in (7) is given in (11) where $C_2 = \sqrt{R^2 - u^2}$ and $C_3 = \sqrt{R^2 - (u+v)^2} - w$. The details and proof for (11) is provided in Appendix.

The proposed coverage probability metric helps in establishing a relationship between user mobility and $t_{up}(k)$. $t_{up}(k)$ must be selected in such a manner that it maximizes the coverage probability. Further, while optimizing $t_{up}(k)$, the coverage probability of all the users covered at k^{th} instant must be considered.

Since there are two factors impacting the choice of $t_{up}(k)$, we propose the minimization of a weighted single objective function, given in (12). The first term corresponds to the fraction of T during which UAV-BS is in flight and cannot serve. Let $F_{t_{up}(k)}$ be the UAV-BS flight time during $t_{up}(k)$ with an average of $\mathbb{E}\left[F_{t_{up}(k)}\right]$. $Q = \left\lceil \frac{T - \sum_{j=0}^{k-1} t_{up}(j)}{t_{up}(k)} \right\rceil$ denotes the number of updates that may occur if update interval is fixed as $t_{up}(k)$ until completion of operation time T. The second term in (12) is the reciprocal of the sum of the coverage probability. A weight of α and $1 - \alpha$ has been assigned to the two terms, respectively. α can be tuned according to the network operator's requirement. For instance, if maximizing the coverage probability is the only requirement, the operator may set α as 0. However, if minimizing UAV-BS flight time is the only requirement, α may be set as 1.

In (12), the term $\sum_{j=1}^{k-1} F(j)$ will be same for all $t_{up}(k)$. Hence, the optimal $t_{up}(k)$ problem can be formulated as follows:

$$\text{(P2)} \qquad \min \left(\frac{\alpha Q \, \mathbb{E}\left[F_{t_{up}(k)}\right]}{T} \right) + \left(\frac{(1-\alpha)}{\sum_{n=1}^{N} A_{n,k} P_{n,t_{up}(k)}} \right) \tag{13}$$

$$\text{s.t. } t_{min} \leq t_{up}(k) \leq t_{max} \tag{14}$$

where t_{min} and t_{max} is the lower and upper limit decided for $t_{up}(k)$.

III. Problem Solution

In order to jointly optimize the UAV-BS placement and the update interval, we propose to solve (P1) and (P2) iteratively. First, at a given update instant, we solve (P1) exhaustively using Algorithm 1 to obtain the UAV-BS placement and $A_n(k)\ \forall n$. Then, $A_n(k)$ values are given as input to (P2). Using Algorithm 2, we obtain the update interval. The output of (P2) is then fed back to (P1). This goes on iteratively till the update interval value converges. In general, for the convergence of update interval, the condition $|t_{up}(k) - t_{out}(k)| < \epsilon$ must be met, where $t_{up}(k)$ and $t_{out}(k)$ are the input and output of Algorithm 1 and Algorithm 2, respectively. ϵ is the error tolerance value.

Algorithm 1: Solution to (P1) at k^{th} update instant

Input: $t_{up}(k)$, $U_x^*(k-1)$, $U_y^*(k-1)$, $fair_{thres}$
$\mathcal{X}$ - List of x coordinates of N users at k^{th} update instant
$\mathcal{Y}$ - List of y coordinates of N users at k^{th} update instant
Hov_{loc} - List of 2-D UAV-BS Hovering locations
Cov_{prev} - List of vectors containing coverage information of all users till $(k-1)^{th}$ update instant
Output: $\mathcal{A}$ - List of $A_n(k)\ \forall n$, $U_x(k)$, $U_y(k)$
Initialize: $Cov_{mat} = \emptyset$, $Fly_{time} = \emptyset$, $Fairness = \emptyset$
for $s = 1:1:|Hov_{loc}|$ **do**
    $Cov_{mat}(:,s) = \texttt{Cov}(\mathcal{X}, \mathcal{Y}, Hov_{loc})$
    $Fly_{time}(s) = \texttt{Fly}((U_x^*(k-1), U_y^*(k-1), Hov_{loc})$
    $Fairness(s) = \texttt{Fair}(Cov_{mat}, Cov_{prev})$,
end for
$[U_x(k), U_y(k),$ $Index1] = \texttt{Find}(Cov_{mat}, Fairness, Fly_{time}, fair_{thres}, t_{up}(k))$,
$Cov_{prev}(:,k) = Cov_{mat}(:, Index1)$,
$\mathcal{A} = Cov_{mat}(:, Index1)$

In Algorithm 1, at $k = 1$, $t_{up}(1)$ is initialized as t_{min}. For $k > 1$, $t_{up}(k)$ is initialized as $t_{up}^*(k-1)$. Here, $t_{up}^*(k-1)$ is the optimal update interval at the $(k-1)^{th}$ update instant. $U_x^*(k-1)$ and $U_y^*(k-1)$ are the optimal coordinates at the $(k-1)^{th}$ update instant. Moreover, we consider that the 2-D hovering space of UAV-BS is discretized with a resolution of 20 m along the horizontal axis. Cov_{prev} is a list of $(k-1)$ vectors of length N. An element in a vector will be '1' if user is covered otherwise '0'. For each of the hovering locations in Hov_{loc}, function `Cov` () determines which of the N users may get covered. `Cov` () assigns '1' if a user is covered else '0'. The output of `Cov` () is stored in Cov_{mat}. Then, the flight time of the UAV-BS is computed using function `Fly` (). Further, the fairness value corresponding to each hovering location is computed using `Fair` () and stored in $Fairness$. Finally, function `Find` () outputs $U_x(k)$, $U_y(k)$ that maximize the number of users covered while satisfying the constraints (4) and (5). Also, we obtain the values of $A_n(k)\ \forall n$.

Algorithm 2: Solution to (P2) at k^{th} update instant

Input: $\mathcal{A}$ - List of $A_n(k)\ \forall n$
t_{list} - List of update interval values,
Output: $t_{up}(k)$, **Initialize**: $r = 1$
for $t = t_{min} : 5 : t_{max}$ **do**
    $Factor(r) = \frac{\alpha Q\, \mathbb{E}[F_t]}{T} + \frac{(1-\alpha)}{\sum_{n=1}^{N} A_{n,k} P_{n,t}}$
    r=r+1
end for
$Index2 = \texttt{Min}(Factor)$
$t_{out}(k) = t_{list}(Index2)$

In Algorithm 2, $\mathbb{E}\left[F_{t_{up}(k)}\right]$ and $P_{n,t_{up}(k)}$ are utilized. The details on obtaining $\mathbb{E}\left[F_{t_{up}(k)}\right]$ are presented in Section IV. Moreover, $P_{n,t_{up}(k)}$ can be computed by substituting (10) and (11) in (7). We discretize the update interval values with a resolution of 5 seconds. For each of the update interval values, the value of the function in (13) is computed and stored in $Factor$. `Min` () finds the index of the minimum value in $Factor$. Finally, $t_{out}(k)$ is obtained. In our work, we consider $\epsilon = 0$. Further, the update interval converges to $t_{up}^*(k)$ within 2 iterations[2].

IV. Results and Discussions

In the following, we present the results for the iterative solution to problems (P1) and (P2). For our study, we consider that all the users are initially inside the UAV-BS's coverage area. The initial distribution of the users and UAV-BS placement for a specific UAV-BS operation period of T seconds is given in Fig. 3. Further, we assume that the first UAV-BS placement update occurs after t_{min} seconds. On solving (P1) and (P2) iteratively, we obtain the optimal UAV-BS placement and $t_{up}(1)$. After this, the UAV-BS placement is updated at $(t_{min} + t_{up}(1))$ seconds. This process goes on till T seconds. In our study, we set the maximum allowable path-loss of 95 dB in a dense urban environment as a QoS metric. Based on the above, using the AtG path-loss model and $H = 50$ m, we obtain $R = 100$ m. The other simulation parameters are mentioned in Table I. Further, the results are averaged over 100 UAV-BS operation periods. As mentioned before, for any update interval t_{up}, the average UAV-BS flight time during t_{up} will be $\mathbb{E}\left[F_{t_{up}}\right]$ and can be written as follows:

$$\mathbb{E}\left[F_{t_{up}}\right] = \frac{\mathbb{E}\left[\sum_{i=1}^{T/t_{up}-1} F(i)\right]}{(T/t_{up}) - 1} \tag{15}$$

[2]The number of iterations depend on the choice of ϵ as well as the resolution of update interval. With an increase in ϵ number of iterations will decrease. However, with an increase in resolution the number of iterations will also increase.

TABLE I
SIMULATION PARAMETERS

Parameter	Value
v_u	1.5 m/s
v_{uav}	25 m/s
σ	4 m
H	50 m
R	100 m
T	900 s
Simulated Area	400 $\times$ 400 sq. m
N	20
$fair_{thres}$	0.7
t_{min}	5 s
t_{max}	150 s

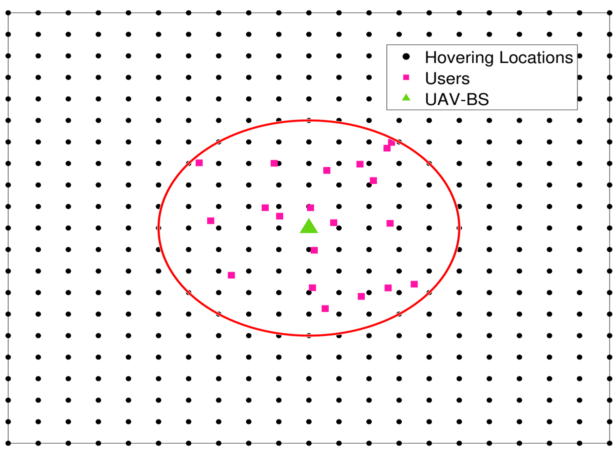

Fig. 3. Initial Deployment

where $\mathbb{E}\left[\sum_{i=1}^{T/t_{up}-1} F(i)\right]$ is the average total UAV-BS flight time if updates are done after every t_{up} seconds. First, the term $\sum_{i=1}^{T/t_{up}-1} F(i)$ can be determined in an offline manner by solving (P1) for $\left(\frac{T}{t_{up}}-1\right)$ update instants. Consequently, $\mathbb{E}\left[\sum_{i=1}^{T/t_{up}-1} F(i)\right]$ is the average of the above summation. A typical plot of average total UAV-BS flight time is shown in Fig. 4. It can be observed that as t_{up} increases the total UAV-BS flight time decreases. This is because lower update interval corresponds to frequent UAV-BS placement updates or vice versa. Consequently, the UAV-BS flies more frequently resulting in higher total UAV-BS flight time.

Next, we study the coverage probability for any update interval $t_{up} \in [t_{min}, t_{max}]$. Fig. 5 shows the coverage probability of users 1, 2 and 3 at an initial position of [56.8, -38.1] m, [-4.1, -40.1] m and [-0.3, 60.2] m respectively when the UAV-BS is located at [0, 0] m. At lower update interval, UAV-BS updates its placement frequently that results in fewer occurrences of a user moving outside the UAV-BS coverage. However, at higher update interval, the probability of the user moving outside the UAV-BS coverage area increases. Consequently, for each user, coverage probability decreases with increase in t_{up}.

Figs. 6 and 7 show the average number of update instants and average update interval at $\sigma = 4$ m. With an increase in

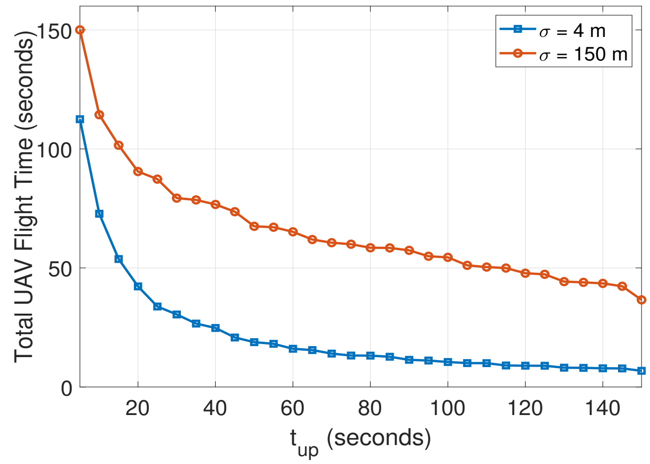

Fig. 4. Total UAV-BS Flight time

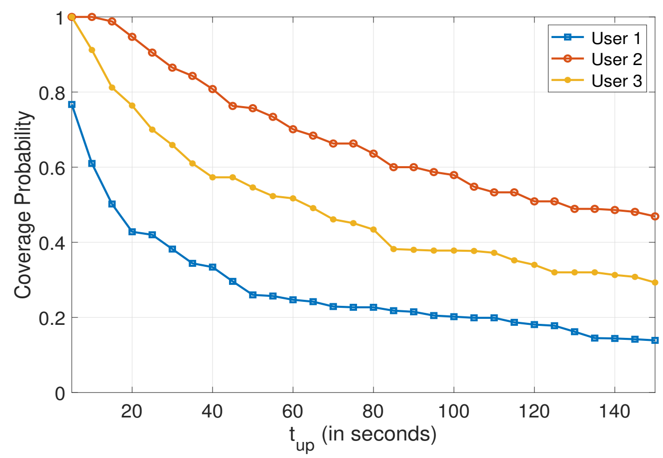

Fig. 5. Coverage probability of users

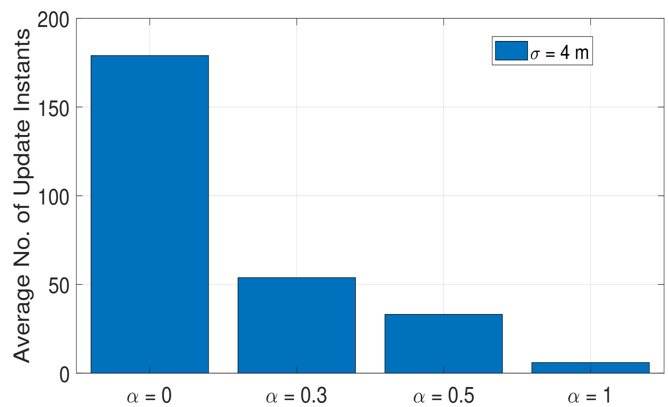

Fig. 6. Average Number of update instants during $T = 900$ seconds

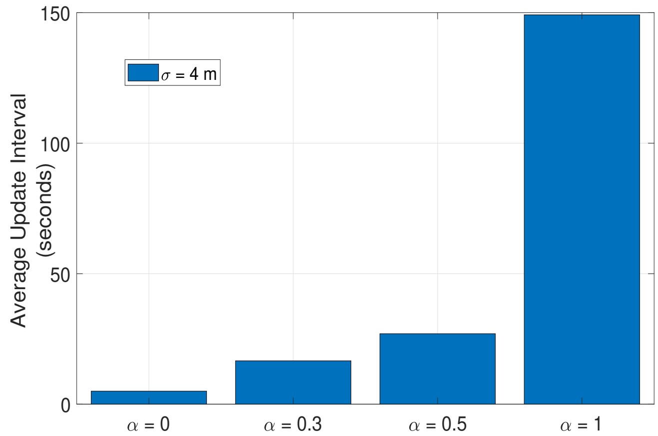

Fig. 7. Average update interval at $T = 900$ seconds

TABLE II
STANDARD DEVIATION

Parameter \ Weight, α	$\alpha = 0$	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 1$
Std. Deviation of Update Instants	0 s	15.05 s	25.85 s	1.86 s
Std. Deviation of No. of update instants	0	9.35	5.76	0

α the number of update instants reduces and update intervals increases. As evident from (13), it is because an increase in α results in a higher weight to reducing the total UAV-BS flight time. This means there would be less frequent UAV-BS placement updates. Further, Table II presents the standard deviation of update interval and number of update instants at α = 0, 0.3, 0.5 and 1.

The average number of users covered are determined by

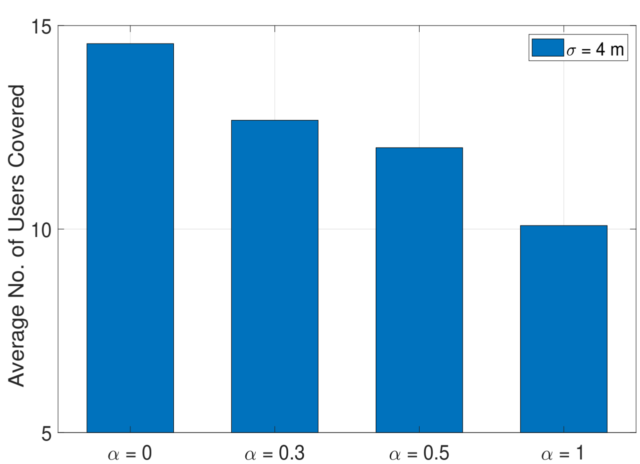

Fig. 8. Average number of users covered at $T = 900$ seconds at $N = 20$

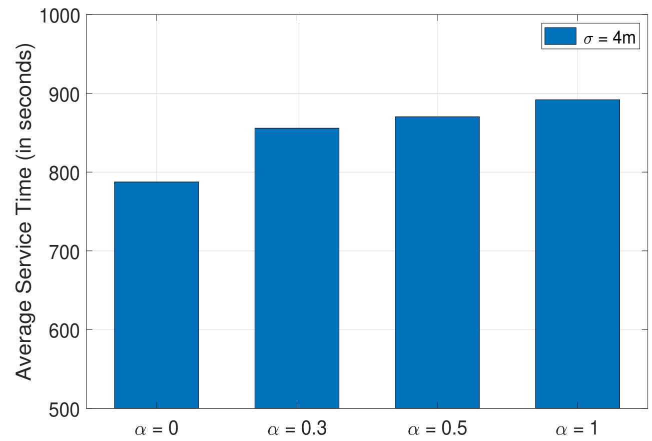

Fig. 9. Average service time at $T = 900$ seconds

$A_n(k)$ as well as coverage probability and can be written as:

$$\bar{U} = \mathop{\mathbb{E}}_{l} \left[\frac{1}{K_l} \sum_{k=1}^{K_l} \sum_{N}^{n=1} A_{n,k} P_{n,t_{up}(k)} \right] \tag{16}$$

where K_l denotes the number of update instants in l^{th} UAV-BS operation period. Fig. 8 plots the average number of users covered in the network for $T = 900$ s. As α increases, the number of users covered is decreasing. This is due to the fact that less frequent UAV-BS placement updates result in lower coverage probability. Further, it can be observed that, on average, with one UAV-BS it is possible to provide coverage to around 14 users with the desired QoS.

Further, the service time of UAV-BS can be written as:

$$\mathcal{S} = T - \mathop{\mathbb{E}}_{l} \left[\sum_{k=1}^{K_l} F(k) \right] \tag{17}$$

It can be observed from Fig. 9 that with an increase in α average service time increases. Hence, α can be tuned by the operator depending on whether more number of users should be covered or larger service time is desired.

V. CONCLUSION

The proposed work formulated two optimization problems to optimize the UAV-BS placement and update interval, respectively. Specifically, the UAV-BS placement is optimized to maximize the number of users covered at an update instant while accounting for the user fairness as well as UAV-BS flight time. Further, we introduced a weighted objective function depending on user coverage probability and total UAV-BS flight time for update interval optimization. Consequently, we proposed an iterative approach to solve the above two problems jointly. We evaluated the network service time and number of users covered for different weights in the optimal update interval problem formulation. Additionally, we derived the analytical expressions for the user coverage probability. In future work, we will perform similar optimizations and analysis for a multiple UAV-BS network model.

$$P[D_i < R|D_{i-1} < R] = \frac{\int_{-R}^{R}\int_{-R-u}^{R-u}\int_{-C_2}^{C_2}\int_{-C_3}^{C_3} e^{-\frac{(u-\mu_1)^2}{2\sigma^2(i-1)}} e^{-\frac{v^2}{2\sigma^2}} e^{-\frac{(w-\mu_2)^2}{2\sigma^2(i-1)}} e^{-\frac{y^2}{2\sigma^2}}\, dy\, dw\, dv\, du}{(\sigma\sqrt{2\pi})^2 \int_{-R}^{R}\int_{-C_2}^{C_2} e^{-\frac{(u-\mu_1)^2}{2\sigma^2(i-1)}} e^{-\frac{(w-\mu_2)^2}{2\sigma^2(i-1)}}\, dw\, du} \tag{20}$$

APPENDIX

Proof for Equation (11)

As mentioned above, $X = X_{in} - U_x(k) + X_1 + \cdots X_{i-1}$ and $Y = Y_{in} - U_y(k) + Y_1 + \cdots Y_{i-1}$. Hence, $X \in \mathcal{N}(\mu_1, \sigma^2(i-1))$ and $Y \in \mathcal{N}(\mu_2, \sigma^2(i-1))$ [15]. The i^{th} term of the product on R.H.S of (7) can be written as:

$$\begin{aligned} &P[D_i < R|D_{i-1} < R] \\ &= P\left[\sqrt{(X+X_i)^2 + (Y+Y_i)^2} < R|\sqrt{X^2+Y^2} < R\right] \\ &= \frac{\int_{-R}^{R}\int_{-R-u}^{R-u}\int_{-C_2}^{C_2}\int_{-C_3}^{C_3} f_{XX_iYY_i}(u,v,w,y)\, dy\, dw\, dv\, du}{\int_{-R}^{R}\int_{-C_2}^{C_2} f_{XY}(u,w)\, dw\, du} \end{aligned} \tag{18}$$

where $C_2 = \sqrt{R^2 - u^2}$ and $C_3 = \sqrt{R^2 - (u+v)^2} - w$. Since X_i's and Y_i's are independent, X and Y are also independent. Now, (18) can be written as follows:

$$\begin{aligned} &P[D_i < R|D_{i-1} < R] \\ &= \frac{\int_{-R}^{R}\int_{-R-u}^{R-u}\int_{-C_2}^{C_2}\int_{-C_3}^{C_3} f_X(u) f_{X_i}(v) f_Y(w) f_{Y_i}(y)\, dy\, dw\, dv\, du}{\int_{-R}^{R}\int_{-C_2}^{C_2} f_X(u) f_Y(w)\, dw\, du} \end{aligned} \tag{19}$$

We also know the following relationship from [19]:

$$\int_0^x e^{-q^2y^2} dy = \frac{\sqrt{\pi}}{2q}\Phi(qx) \tag{21}$$

where $\Phi()$ is the error function. Finally, using (20) and symmetric pdf property of normal distribution, we obtain (11).

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