TUTORIAL 5

Q1) A current I flow down a wire of radius a

a) If it is uniformly distributed over the surface, what is the surface current density K?

b) If it is distributed in such a way that the volume current density is inversely proportional to the distance from the axis, what is J?

Sol. a) $\vec{K} = \frac{d\vec{I}}{dl}$ $\quad$ l is parallel to current flow

$K = \frac{I}{2\pi a}$ $\quad$ (direction is same as of current I)

b) J = $\frac{k}{x}$

$I = \int J\ da\ = \int \frac{k}{x}\ \text{da}$

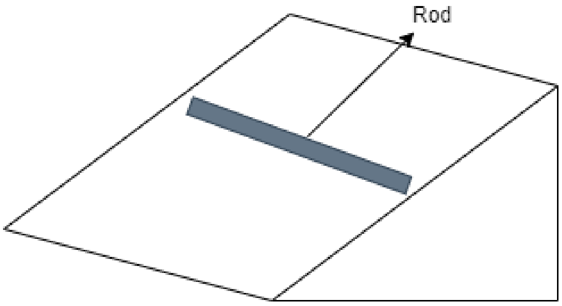
$a\ = \pi x^2$

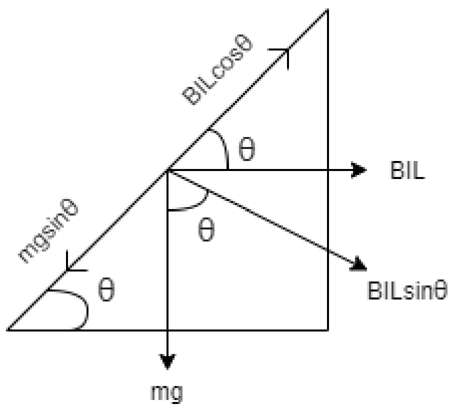
da = $2\ \pi\ x\ dx$

$I = \int_0^a \frac{k}{x}\ \ 2\ \pi\ x\ dx = 2\ \pi\ k\ a$

k = $\frac{I}{2\ \pi\ a}$ => J = $\frac{I}{2\ \pi\ a\ x}$ $\quad$ (direction is same as current)

Q2) A metallic rod of linear density 0.25kg - m^{-1} is lying horizontally on smooth inclined plane which makes an angle of 45° with the horizontal. The rod is not allowed to slide down by flowing a current through it when a magnetic field of strength 0.25T is acting on it in the vertical direction. Calculate the electric current flowing in the rod to keep it stationary.



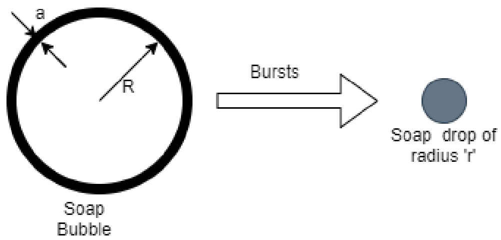


$\text{B IL}\cos\theta\ = mg\ sin\theta$

$\text{B I}\cot\theta\ = \frac{mg}{L}$

$\text{I} = \frac{mg}{BL}\ \tan\theta\ = 9.8\ A$

Q3) A thin soap bubble of radius R = 1cm, and thickness a = 3.3 um (a << R), is at a potential of 1 V with respective to a reference point at infinity. The bubble bursts and becomes a single spherical drop of soap (assuming all the soap is contained in the drop) of radius r. The volume of the soap in the thin bubble is $4\,\pi\,R^2 a$ and that of the drop is $\frac{4}{3}\pi r^3$. The potential in volts, of the resulting single spherical drop with respect to the same reference point at infinity is ___. (Give the answer up to two decimal places).



Sol. $(4\,\pi\,R^2 a)\rho \;=\; (\frac{4}{3}\pi r^3)\rho$

ρ -> $charge\ density\ (c/m^3)$

$Y \;=\; (3R^2 a)^{1/3}$

The potential of the bubble

V = $\frac{1}{4\pi\varepsilon}\,\frac{Q}{R}$

Q = $(4\pi\varepsilon R)\,V$

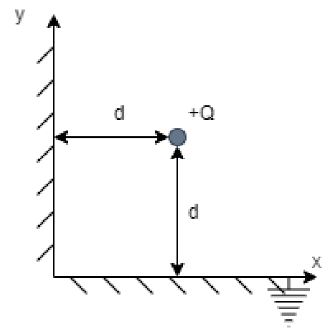
Q = $4\pi\varepsilon_o\,1 \times 10^{-2}$

Potential of the soap drop

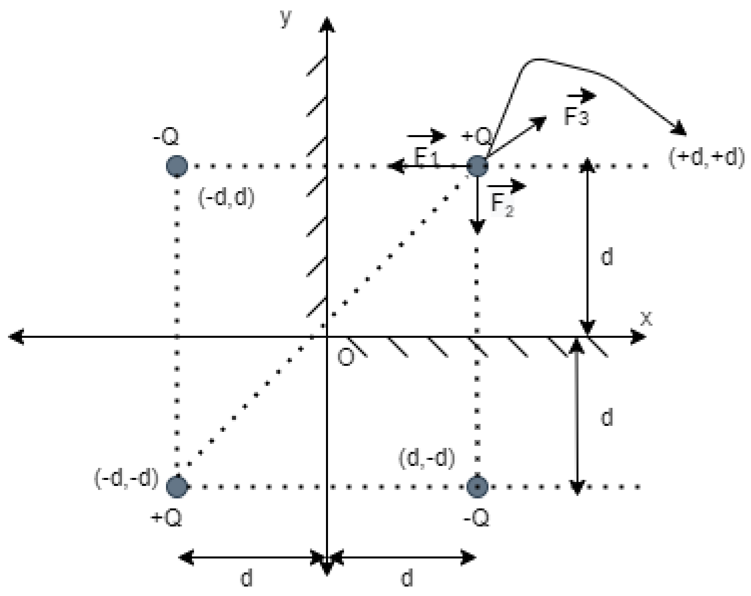
$V' = \frac{1}{4\pi\varepsilon}\,\frac{Q}{R}$

$V' \;= \frac{1}{4\pi\varepsilon_o}\,\frac{4\pi\varepsilon_o\;1\times10^{-2}}{(3\times1\times10^{-4}\times3.3\times10^{-6})^{1/3}}$ = 10.03

Q4) Two semi-infinite conducting sheets are placed at right angles to each other as shown in figure. A point charge of +Q is placed at a distance of d from both sheets. The net force on the charge is $\frac{Q^2}{4\pi\varepsilon_o}\frac{\vec{k}}{d^2}$ where $\vec{k}$ *is given* by



Sol.



The net force due to remaining charges is given by:

$$\vec{F} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3}$$

$$\vec{F} = \frac{-Q^2}{4\pi\varepsilon_o(2d)^2}\left(\frac{2d\widehat{a_x}}{2d}\right) - \frac{Q^2}{4\pi\varepsilon_o(2d)^2}\left(\frac{2d\widehat{a_y}}{2d}\right) + \frac{Q^2}{4\pi\varepsilon_o(\sqrt{8}d)^2}\left(\frac{2d\widehat{a_x} + 2d\widehat{a_y}}{2\sqrt{2}d}\right)$$

$$\vec{F} = \frac{Q^2}{4\pi\varepsilon_o(d)^2}\left[-\frac{1}{4}\widehat{a_x} - \frac{1}{4}\widehat{a_y} + \frac{1}{8\sqrt{2}}\widehat{a_x} + \frac{1}{8\sqrt{2}}\widehat{a_y}\right]$$

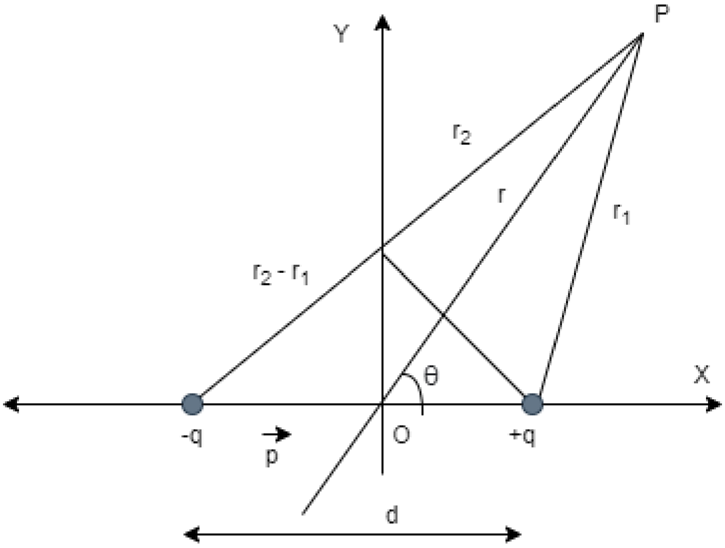
$$\vec{F} = \frac{Q^2}{4\pi\varepsilon_o(d)^2}\left[\frac{(1-2\sqrt{2})}{8\sqrt{2}}\widehat{a_x} + \frac{(1-2\sqrt{2})}{8\sqrt{2}}\widehat{a_y}\right]$$

The given net force on 'Q'

$$\vec{F} = \frac{Q^2}{4\pi\varepsilon_o (d)^2}\,\vec{K} = \frac{Q^2}{4\pi\varepsilon_o (d)^2}\left[\left(\frac{(1-2\sqrt{2})}{8\sqrt{2}}\right)\hat{\imath} + \left(\frac{(1-2\sqrt{2})}{8\sqrt{2}}\right)\hat{\jmath}\right]$$

$$\vec{K} = \frac{(1-2\sqrt{2})}{8\sqrt{2}}\hat{\imath} + \left(\frac{(1-2\sqrt{2})}{8\sqrt{2}}\right)\hat{\jmath}$$

Q5) Derive an expression for electric field due to a short dipole at a far away point P in cartesian coordinate.



Sol. We know the potential at P due to its dipole at a far away point is given as-

V(x,y,z) = $\frac{1}{4\pi\varepsilon_o}\frac{p\cos\theta}{r^2}$

Where r = $\sqrt{(x^2+y^2+z^2)}$ is the distance of point p

cosθ = $\frac{x}{r}$

Also, $\vec{E}$ = -∇V

$$E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k} = -\left(\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k}\right)$$

Now,

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left(\frac{p\cos\theta}{4\pi\varepsilon_o r^2}\right) = -\frac{p}{4\pi\varepsilon_o}\frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) \quad \text{as } \cos\theta = \frac{x}{r}$$

$$E_x = -\frac{p}{4\pi\varepsilon_o}\left[\frac{1}{r^3} + x\frac{\partial}{\partial x}\left(\frac{1}{(x^2+y^2+z^2)^{3/2}}\right)\right]$$

$$E_x = -\frac{p}{4\pi\varepsilon_o}\left[\frac{1}{r^3} - x\,\frac{3}{2}\left(\frac{2x}{(x^2+y^2+z^2)^{5/2}}\right)\right]$$

$$E_x = -\frac{p}{4\pi\varepsilon_o}\left[\frac{1}{r^3} - \frac{3x^2}{r^5}\right]$$

$$E_x = -\frac{p}{4\pi\varepsilon_o}\left[\frac{1}{r^3} - \frac{3\cos^2\theta}{r^5}\right]$$

$$E_x = -\frac{p}{4\pi\varepsilon_o}\left[\frac{3cos^2\theta - 1}{r^3}\right]$$

Similarly,

$$E_y = -\frac{\partial V}{\partial y} = -\frac{p}{4\pi\varepsilon_o}\frac{3yx}{r^5}$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{p}{4\pi\varepsilon_o}\frac{3zx}{r^5}$$

The resultant of y and z component of field gives the component of field in y-z plane which is directed perpendicular to the x-axis i.e, the axis of the dipole.

Then, the transverse component of field is –

$$E_T = \sqrt{(E_y{}^2 + E_z{}^2)}$$

On substitution

$$E_T = \frac{3px}{4\pi\varepsilon_o r^5}\sqrt{(y^2 + z^2)} = \frac{3px}{4\pi\varepsilon_o r^5}\sqrt{(r^2 - x^2)} = \frac{3px}{4\pi\varepsilon_o r^5}\sqrt{(r^2 - r^2cos^2\theta)}$$

$$E_T = \frac{3px\, sin\theta}{4\pi\varepsilon_o r^5}$$

$$E_T = \frac{p}{4\pi\varepsilon_o}\frac{3cos\theta sin\theta}{r^3} \qquad (\text{ x} = \text{r} \cos\theta)$$

Total Field $E = \sqrt{(E_x{}^2 + E_y{}^2 + E_z{}^2)} = \sqrt{(E_r{}^2 + E_y{}^2)}$

Substituting $(E_r \ and \ E_x)$

$$\vec{E} = \frac{\vec{p}}{4\pi\varepsilon_o r^3}\left[(3cos^2\theta - 1)^2 + (3cos\theta sin\theta)^2\right]^{1/2}$$

$$\vec{E} = \frac{\vec{p}}{4\pi\varepsilon_o r^3}\left[9cos^4\theta - 6cos^2\theta + 1 + 9cos^2\theta\, sin^2\theta\right]^{1/2}$$

$$\vec{E} = \frac{\vec{p}}{4\pi\varepsilon_o r^3}\left[9cos^4\theta - 6cos^2\theta + 1 + 9cos^2\theta\,(1 - cos^2\theta)\right]^{1/2}$$

$$\vec{E} = \frac{\vec{p}}{4\pi\varepsilon_o r^3}\left[(3cos^2\theta + 1)\right]^{1/2}$$

$$\vec{E} = \frac{\vec{p}}{4\pi\varepsilon_o r^3}\left[\sqrt{(3cos^2\theta + 1)}\right]$$