TUTORIAL 4

Q1. Calculate the potential and field due to the dipole of dipole moment $4.5 \times 10^{-10} C/m$ at a distance of 1m from it, (i) on its axis (ii) on its perpendicular bisection.

Sol.

As we know the potential due to dipole is –

$$V = \frac{1}{4\pi\varepsilon_o}\frac{p\cos\theta}{r^2}$$

Here $p = 4.5 \times 10^{-10}\frac{C}{m}$ $and\ r = 1m$

i. On its axis $\theta = 0$

$$V = \frac{1}{4\pi\varepsilon_o}\frac{p}{r^2} = \frac{9 \times 10^9 \times 4.5 \times 10^{-10}}{1^2} \ [\frac{1}{4\pi\varepsilon_o} = 9 \times 10^9]$$

$$= 4.05V$$

and field is given by-

$$E = \frac{p(3(\cos\theta)^2+1)^{1/2}}{4\pi\varepsilon_o r^3}$$

$$[\partial E = \frac{-\partial V}{\partial r} => E = \frac{V}{r}$$

As, $\theta = 0$

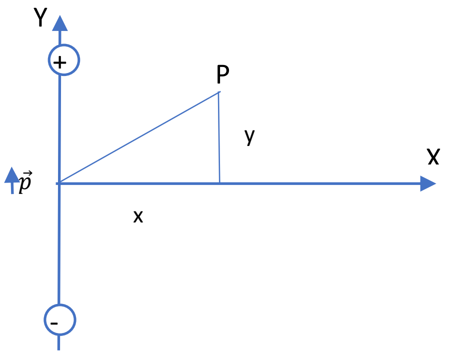
$$= \frac{2p}{4\pi\varepsilon_o r^3} = \frac{2 \times 9 \times 10^9 \times 4.5 \times 10^{-10}}{1^3} = 8.1\frac{V}{m}$$

ii. On its perpendicular bisector, $\theta = 90°$ then $\cos 90° = 0$

⇨ V = 0

⇨ $E = \frac{p}{4\pi\varepsilon_o r^3} = \frac{9 \times 10^9 \times 4.5 \times 10^{-10}}{1^3} = \frac{4.05V}{m}$

Q2. Consider a dipole, as shown in Fig, located at the origin of xy system and dipole pointing along the positive y-axis. Calculate E_x , E_y component and total electric field $\vec{E}$ at a far away point P(x,y).



We know the value of V is-

$$V(x,y,z) = \frac{1}{4\pi\varepsilon_o}\frac{p\cos\theta}{r^2}$$

Here,

$$r = \sqrt{x^2+y^2}\,, r^2 = x^2+y^2$$

And $\cos\theta = \frac{y}{\sqrt{x^2+y^2}}$, $\cos\theta = \frac{y}{r}$

$$V = \frac{p}{4\pi\varepsilon_o}\frac{y}{(x^2+y^2)^{3/2}}$$

The component of the field along y-axis

$$E_y = \frac{-\partial V}{\partial y} = \frac{-p}{4\pi\varepsilon_o}\left[\frac{-3}{2}\frac{(2y)y}{(x^2+y^2)^{5/2}} + \frac{1}{(x^2+y^2)^{3/2}}\right]$$
$$= \frac{p}{4\pi\varepsilon_o}\left[\frac{3y^2}{(x^2+y^2)^{5/2}} - \frac{1}{(x^2+y^2)^{3/2}}\right]$$
$$= \frac{p}{4\pi\varepsilon_o}\frac{1}{(x^2+y^2)^{3/2}}\left[\frac{3y^2}{(x^2+y^2)} - 1\right]$$

Substituting the value of r^2 and $\cos\theta$, we get

$$E_y = \frac{p}{4\pi\varepsilon_o}\frac{1}{r^3}[3(\cos\theta)^2 - 1]$$

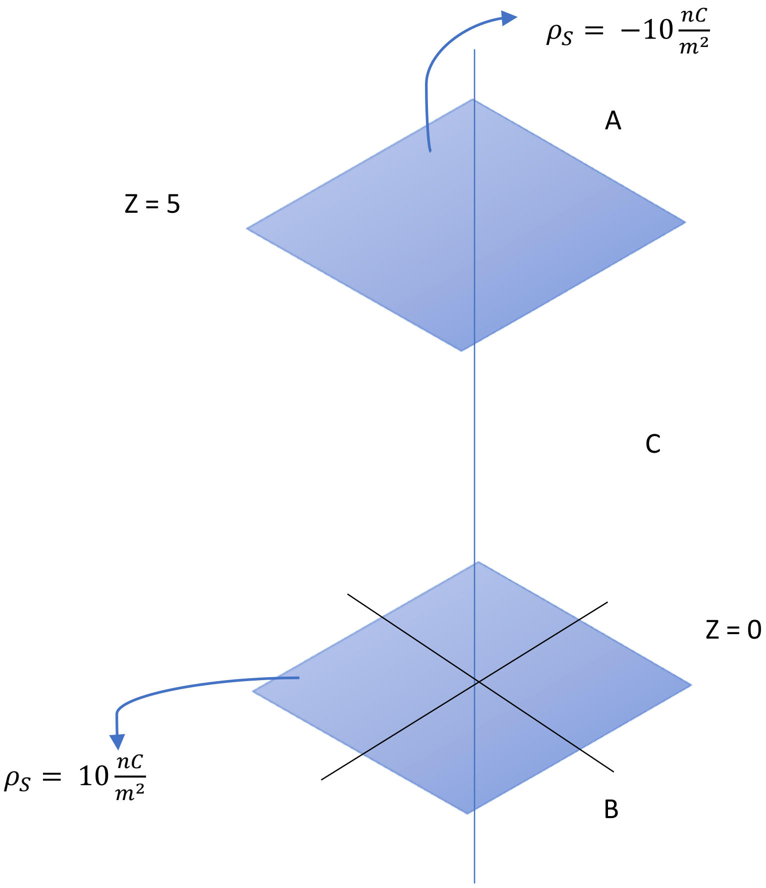
Similarly the component of field along x-axis

$$E_x = \frac{3p}{4\pi\varepsilon_o}\frac{sin\theta\ cos\theta}{r^3}$$

$\vec{E}\ at\ P = \frac{\vec{p}}{4\pi\varepsilon_o r^3}[3sin\theta\ cos\theta\ \hat{\imath} + (3(\cos\theta)^2 - 1)\hat{\jmath}]$

Q3. Planes z=0, z=5 carries a charge of $10\frac{nC}{m^2}$, $-10\frac{nC}{m^2}$ respectively. Find the electric field

i) Above the two planes.
ii) Below the two planes.
iii) In between the two planes.

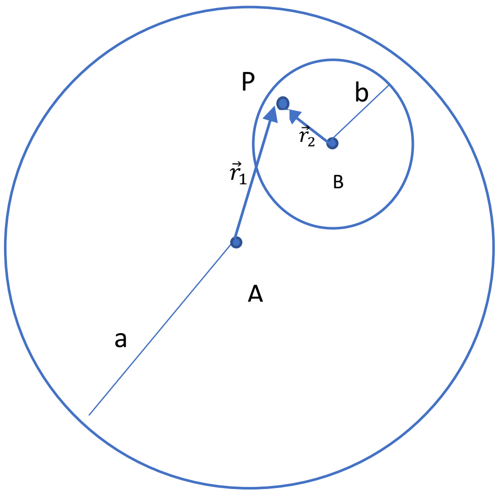


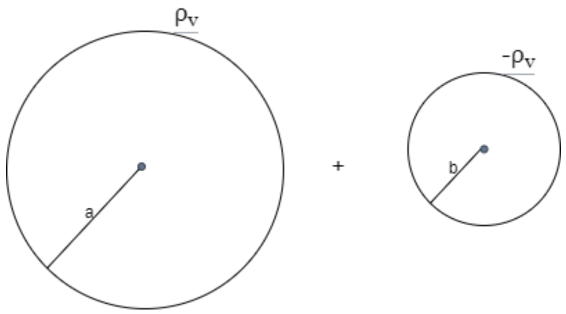
Sol. $E_A = E_1 + E_2 = \frac{10 \times 10^{-9}}{2\varepsilon}\,\hat{a}_z - \frac{10 \times 10^{-9}}{2\varepsilon}\,\hat{a}_z = 0$

$E_B = E_1 + E_2 = \frac{10 \times 10^{-9}}{2\varepsilon}\,(-\hat{a}_z) - \frac{10 \times 10^{-9}}{2\varepsilon}(-\,\hat{a}_z) = 0$

$$E_C = \frac{10\ \times 10^{-9}}{2\varepsilon}\,\hat{a}_z - \frac{10\ \times 10^{-9}}{2\varepsilon}(-\,\hat{a}_z) = 1.12\hat{a}_z\frac{nC}{m^2}$$

Q4. A sphere of radius ‘a’ having a charge of $\rho_V \frac{C}{m^3}$. A spherical cavity of radius ‘b’ is made inside the sphere. If the center of cavity is at a distance of d from center of the sphere, then electric field at any point inside the cavity is?



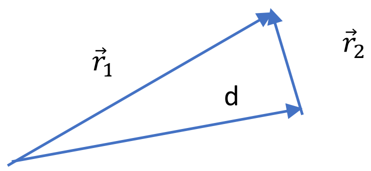


$\vec{E} = \frac{r.\rho_V}{3\varepsilon}\,\hat{a}_r = \frac{\rho_V}{3\varepsilon}\,(r\hat{a}_r)$ as, $\vec{A} = |A|\hat{a}_n$

$$\vec{E} = \frac{\rho_V}{3\varepsilon}\,\vec{r}$$

$$E_1(due\ to\ sphere) = \frac{\rho_V}{3\varepsilon}\,\vec{r}_1$$

$$E_2(due\ to\ cavity) = \frac{-\rho_V}{3\varepsilon}\,\vec{r}_2$$

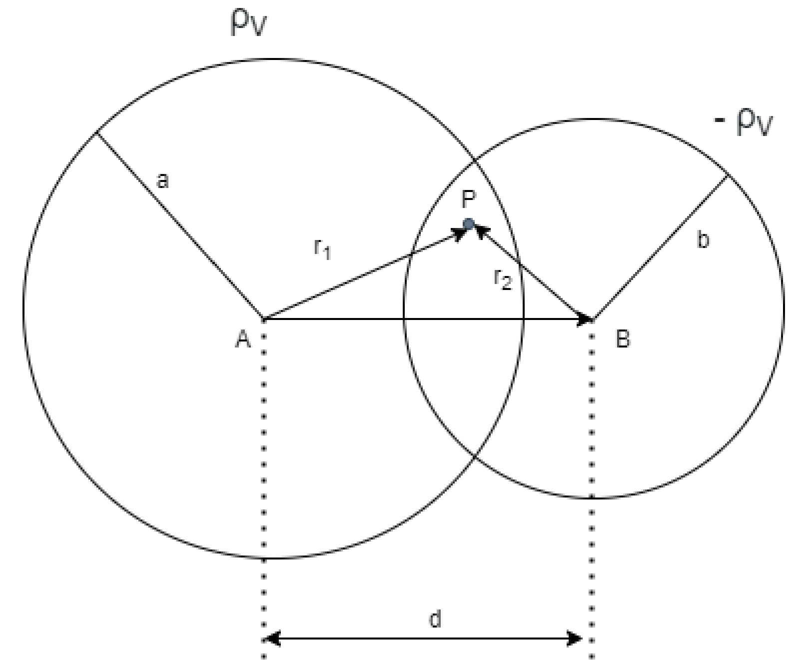


$\vec{r}_1 - \vec{r}_2 - d = 0$

⇨ $\vec{r}_1 - \vec{r}_2 = d$

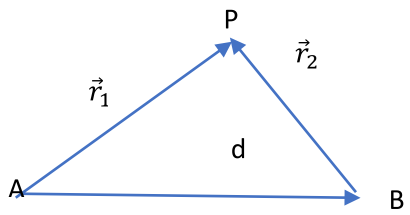
$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho_V}{3\varepsilon}\,(\vec{r}_1 - \vec{r}_2) = \frac{\rho_V d}{3\varepsilon}$$

Q5. Two spheres of radius a & b having equal amount of charge but with opposite polarity are intersecting as shown in the figure. Then find electric field at any point in common region.



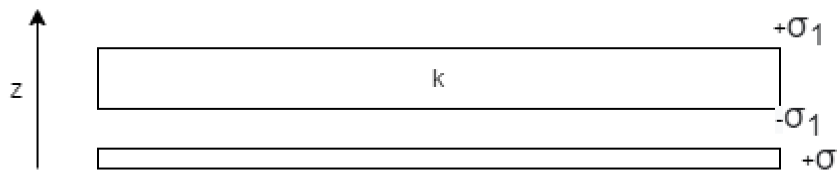
$$\vec{r}_1 - \vec{r}_2 - d = 0$$

$$\Rightarrow \quad \vec{r}_1 - \vec{r}_2 = d$$



$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho_V}{3\varepsilon}\,\vec{r}_1 + \frac{-\rho_V}{3\varepsilon}\,\vec{r}_2 = \frac{\rho_V}{3\varepsilon}\,(\vec{r}_1 - \vec{r}_2) = \frac{\rho_V d}{3\varepsilon}$$

Q6. An infinite conducting slab kept in a horizontal plane carries a uniform charge density σ. Another infinite slab of thickness t, made of a linear dielectric material of k (dielectric constant), is kept along the conducting slab. The boundary charge density on the upper surface of dielectric slab is?

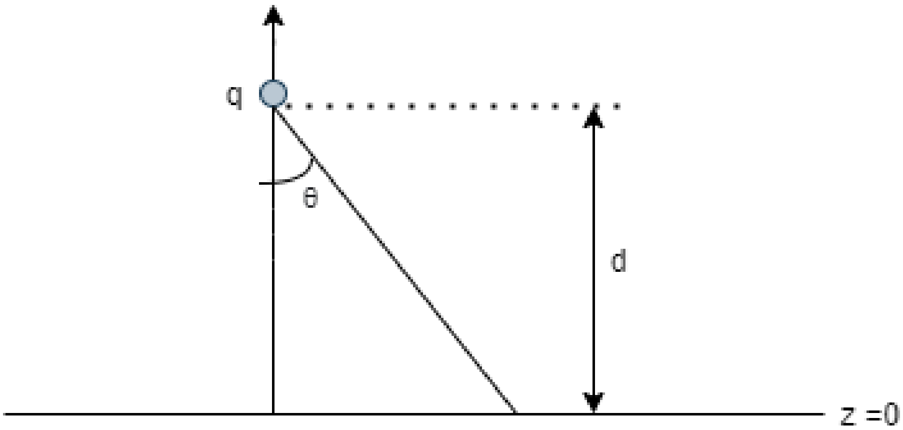


$$\vec{E} = \frac{\sigma}{\varepsilon}\,\hat{z} = \frac{\sigma}{k\varepsilon_o}\,\hat{z}$$

$$\vec{p} = \varepsilon_o X_e \vec{E} = \varepsilon_o X_e \frac{\sigma}{k\varepsilon_o}\,\hat{z} = \frac{\varepsilon_o(k-1)\sigma}{k\varepsilon_o}\,\hat{z} = \frac{(k-1)\sigma}{k}\,\hat{z}$$

$$\sigma_1 = \vec{p}.\hat{n} = \vec{p}.\hat{z} = \frac{(k-1)\sigma}{k}$$

Q7. Suppose the entire region below the plane z=0 is filled with uniform linear dielectric material of susceptibility X_e . Calculate the force on a point charge q situated a distance d above the origin.



$\sigma_b = \vec{p}.\hat{n} = p_z = \varepsilon_o X_e E_z$

$E_z = \frac{-q\cos\theta}{4\pi\varepsilon_o(r^2 + d^2)} = \frac{-qd}{4\pi\varepsilon_o(r^2+d^2)^{3/2}}$

$$\sigma_b = \varepsilon_o X_e \left[\frac{-qd}{4\pi\varepsilon_o(r^2 + d^2)^{\frac{3}{2}}} - \frac{\sigma_b}{2\varepsilon_o}\right]$$

$$\sigma_b = \frac{-1}{2\pi}\left(\frac{X_e}{X_e+2}\right)\frac{qd}{(r^2 + d^2)^{\frac{3}{2}}}$$

$$q_b = -\left(\frac{X_e}{X_e+2}\right)q$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\int \frac{\hat{r}}{r^2}\,\sigma_b\,da$$

$$\vec{F} = \frac{q\,q_b\,\hat{z}}{4\pi\varepsilon_o(2d)^2} = \frac{-1}{4\pi\varepsilon_o}\left(\frac{X_e}{X_e+2}\right)\frac{q^2}{4d^2}\hat{z}$$