# Irrotational field

- $\vec{\nabla} \times \vec{F} = 0$: irrotational field.
- Recall: conservative field can be written as a gradient of a scalar potential ($\phi$) i.e. $\vec{F} = \vec{\nabla} U$.
- $\vec{\nabla} \times \vec{F} = \vec{\nabla} \times (\vec{\nabla} U) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial U}{\partial x} & \frac{\partial U}{\partial y} & \frac{\partial U}{\partial z} \end{vmatrix} =$
  $\hat{x}\left(\frac{\partial^2 U}{\partial x \partial y} - \frac{\partial^2 U}{\partial x \partial y}\right) + \hat{y}\,() + \hat{z}\,()$
- An irrotational field can always be written as $\vec{\nabla} U$, $U \equiv$scalar potential

# Solenoidal field

- $\vec{\nabla}.\vec{F} = 0$: solenoidal field
- Recall divergence theorem: $\oint\limits_S \vec{F}.\vec{ds} = \int\limits_V (\vec{\nabla}.\vec{F})dv$. Thus, a solenoidal field over a volume $V$ doesn't have any source or sink in that volume.
- You can show that (try it): $\vec{\nabla}.(\vec{\nabla} \times \vec{A}) = 0$
- A solenoidal field can always be written as $\vec{\nabla} \times \vec{A}$, $\vec{A} \equiv$vector potential

# Review of some vector identities (these 'll be used frequently)

- $\vec{A} \times \left(\vec{B} \times \vec{C}\right) = \vec{B}\left(\vec{A}.\vec{C}\right) - \vec{C}\left(\vec{A}.\vec{B}\right)$ [remember, this product is not associative i.e. $\vec{A} \times \left(\vec{B} \times \vec{C}\right) \neq \left(\vec{A} \times \vec{B}\right) \times \vec{C}$]
- $\vec{\nabla}.(\vec{\nabla} \times \vec{A}) = 0$, $\vec{\nabla} \times \left(\vec{\nabla} U\right) = 0$, $\vec{\nabla}.\left(\vec{\nabla} U\right) = \nabla^2 U$
- $\vec{\nabla} \times \left(\vec{\nabla} \times \vec{A}\right) = \vec{\nabla}\left(\vec{\nabla}.\vec{A}\right) - \nabla^2 A$
- $\vec{\nabla}.\left(\vec{A} \times \vec{B}\right) = \vec{B}.\left(\vec{\nabla} \times \vec{A}\right) - \vec{A}.\left(\vec{\nabla} \times \vec{B}\right)$
- Divergence theorem: $\oint\limits_S \vec{F}.\vec{ds} = \int\limits_V (\vec{\nabla}.\vec{F}) dv$ (note the small circle in the surface integral: means the surface is closed)
- Stoke's theorem: $\int\limits_S \left(\vec{\nabla} \times \vec{A}\right).d\vec{S} = \oint\limits_C \vec{A}.\vec{dl}$

# Divergence operator in spherical coordinate system

- $\vec{\nabla}.\vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial\phi}$
- Refer to Griffiths for an exhaustive list of vector identities and expressions of vector calculus operators in different coordinate systems.