ECE230: Fields & Waves (Lect. 2, Winter 2021)

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Gravitation vs. EM

- Case 1: Two point-masses (1 kg. each), separated by 1m $\left(F_{grav} = G\frac{m_1 m_2}{r^2} \sim 10^{-11}\right)$ $[G = 6.67408 \times 10^{-11} Nm^2 kg^{-2}]$
- Case 2: Let's replace the two point masses with point charges, 1 Coulomb each $\left(F_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \sim 10^{12}\right)[\varepsilon_0 = 8.85 \times 10^{-12} F/m]$

Gravitation vs. EM

- $\frac{F_E}{F_{grav}} = \frac{1}{4\pi\varepsilon_0 G} \sim \frac{10^{12}}{10^{-11}} = 10^{23}!!!$

How fast does an electron move?

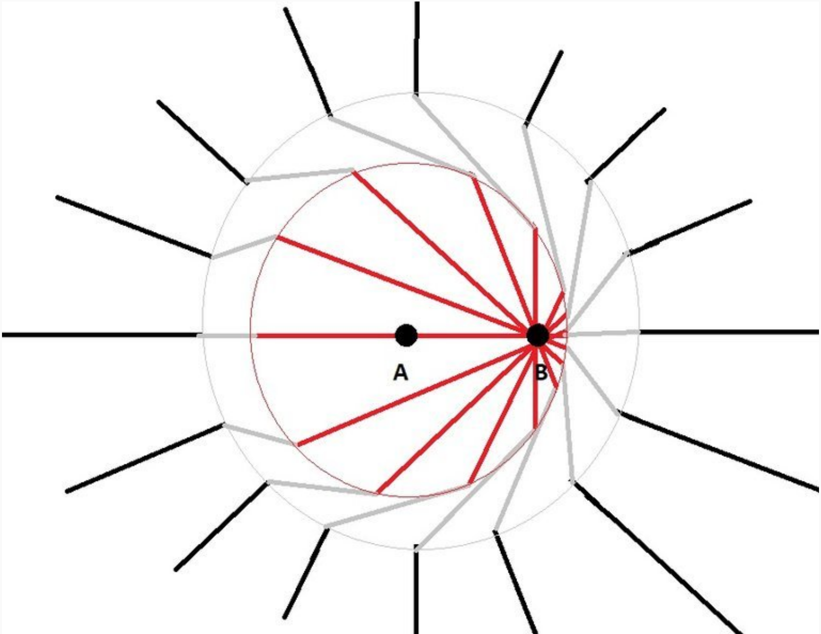
"Around the world in ~~80 days~~ 7mins"?!

- What length-scale are you interested in?
- $\sim 10^5 m/s$
- Circumference of earth $40,000 km = 40 \times 10^6 m$
- The electron could circle the earth in less than $7 min$.

How fast does an electron (really) move?

- In a conductor: about few *mm* in $1s$.
- So, just few meter in about 15 mins.

Making waves: accelerated charge



An electromagnetics lecture where we feel sad for KCL, KVL

- Let's consider a resistance connected to a AC source through a small piece of wire (assume that the distance between the resistor and the source is $30cm$)
- How does the circuit behave at $50Hz$?
- How does the circuit behave at $10GHz$?
- Open circuit?

Fields

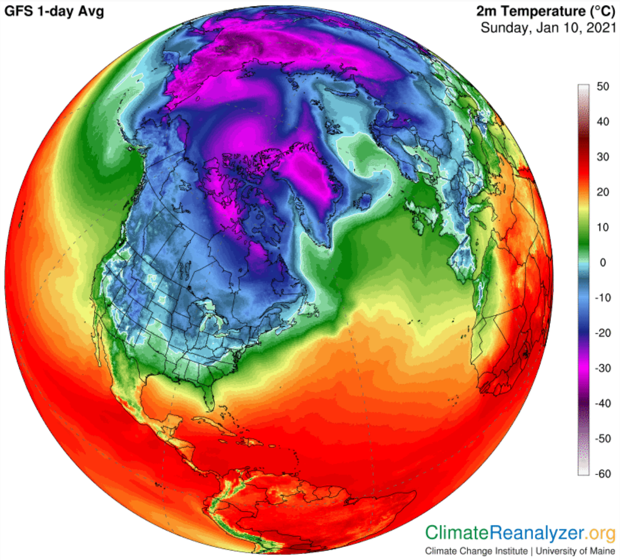


Figure 1: Scalar field

Fields

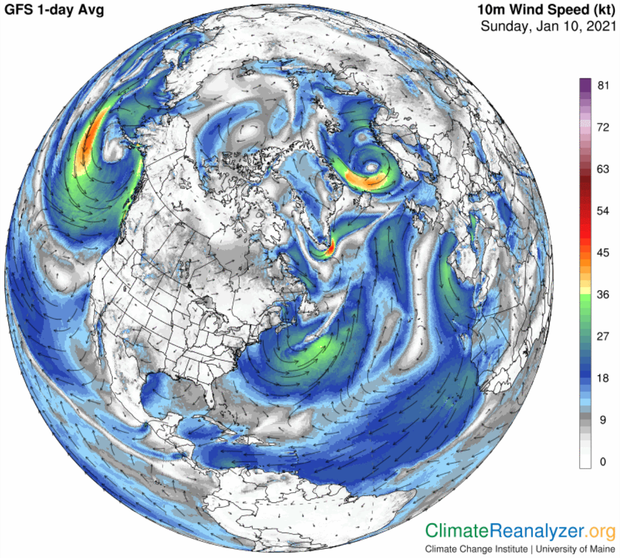


Figure 2: Vector field

Unified theory of electricity and magnetism: bit of a history

- 1785 : Charles-Augustin de Coulomb reports inverse square law for charges
- 1800 : Alessandro Volta invents battery
- 1820 : Hans Christian Ørsted shows deflection of compass needle brought in the proximity of a current carrying wire
- 1820 : Ampere shows two parallel current-carrying wire attracts/repel depending on the direction of the current
- 1831 : Michael Faraday discovers electromagnetic induction

Unified theory of electricity and magnetism: Maxwell's equations

Figure 3: James Clerk Maxwell (1831 – 1879)

Unified theory of electricity and magnetism: Maxwell's equations

$$\left.\begin{aligned}
&\mathrm{PX} + (p+h)x + (k+l)y = \int \mathrm{A}dt - \int \mathrm{D}dt,\\
&\mathrm{Q}(\mathrm{X}-\mathrm{Z}) + (h+q)x + (m+n)y = \int \mathrm{D}dt - \int \mathrm{C}dt,\\
&\mathrm{RY} + (k+m)x + (r+o)y = \int \mathrm{A}dt - \int \mathrm{E}dt,\\
&\mathrm{S}(\mathrm{Y}+\mathrm{Z}) + (l+n)x + (o+s)y = \int \mathrm{E}dt - \int \mathrm{C}dt,\\
&\mathrm{GZ} = \int \mathrm{D}td - \int \mathrm{E}dt.
\end{aligned}\right\} \quad (24)$$

Solving these equations for Z, we find

$$\left.\begin{aligned}
&\mathrm{Z}\left\{\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{Q}}+\frac{1}{\mathrm{R}}+\frac{1}{\mathrm{S}}+\mathrm{B}\left(\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{R}}\right)\left(\frac{1}{\mathrm{Q}}+\frac{1}{\mathrm{S}}\right)+\mathrm{G}\left(\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{Q}}\right)\left(\frac{1}{\mathrm{R}}+\frac{1}{\mathrm{S}}\right)+\frac{\mathrm{BG}}{\mathrm{PQRS}}(\mathrm{P}+\mathrm{Q}+\mathrm{R}+\mathrm{S})\right\}\\
&= -\mathrm{F}\frac{1}{\mathrm{PS}}\left\{\frac{p}{\mathrm{P}}-\frac{q}{\mathrm{Q}}-\frac{r}{\mathrm{R}}+\frac{s}{\mathrm{S}}+h\left(\frac{1}{\mathrm{P}}-\frac{1}{\mathrm{Q}}\right)+k\left(\frac{1}{\mathrm{R}}-\frac{1}{\mathrm{P}}\right)+l\left(\frac{1}{\mathrm{R}}+\frac{1}{\mathrm{Q}}\right)-m\left(\frac{1}{\mathrm{P}}+\frac{1}{\mathrm{S}}\right)\right.\\
&\qquad \left. +n\left(\frac{1}{\mathrm{Q}}-\frac{1}{\mathrm{S}}\right)+o\left(\frac{1}{\mathrm{S}}-\frac{1}{\mathrm{R}}\right)\right\}.
\end{aligned}\right\} \quad (25)$$

Figure 4: James Clerk Maxwell, A Dynamical Theory of the Electromagnetic Field, Royal Society Publishing (1865)

Oliver Heaviside: condensed form of Maxwell's equations (1885)

- $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$
- $\vec{\nabla}.\vec{B} = 0$
- $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
- $\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right)$

Questions so far?