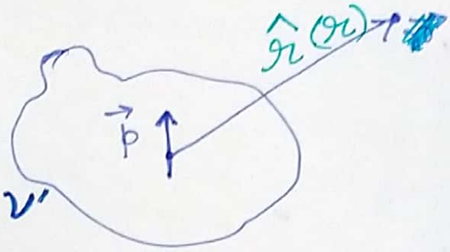
So far, we see that there can be two sources of resultant dipole in a material placed in $\vec{E}$:

① Induced dipole (if the material is not composed of polar ~~atom~~ molecules)

② ~~All~~ Most of the dipoles of polar molecules in a material aligns along the direction of $\vec{E}$.

$$\boxed{\text{Polarization } (\vec{P}) = \text{dipole moment per unit volume}}$$

Field of a polarized object:



$$f(\vec{r}) = f(x-x', y-y', z-z')$$

$$\frac{\partial f}{\partial x'} = \frac{\partial f}{\partial (x-x')} \cdot \frac{\partial (x-x')}{\partial x'} = -\frac{\partial f}{\partial (x-x')}$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial (x-x')} \cdot \frac{\partial (x-x')}{\partial x} = \frac{\partial f}{\partial (x-x')}$$

$$\boxed{\vec{\nabla}' \equiv -\vec{\nabla}}$$

Recall, $V(r) = \dfrac{1}{4\pi\varepsilon_0} \dfrac{\vec{p}\cdot\hat{r}}{r^2}$

$$= \frac{1}{4\pi\varepsilon_0} \int_{v'} \frac{\vec{P}(r')\cdot\hat{r}}{r^2}\, dv'$$

$$= \frac{1}{4\pi\varepsilon_0} \int_{v'} \vec{P}(\vec{r}')\cdot\vec{\nabla}'\left(\frac{1}{r}\right) dv'$$

Once again,

$$\int f(\vec{\nabla}\cdot\vec{A})\,dv = -\int \vec{A}\cdot(\vec{\nabla}f)\,dv + \oint (f\vec{A})\cdot d\vec{S}$$

$$\Rightarrow \int \vec{A}\cdot(\vec{\nabla}f)\,dv = \oint f\vec{A}\cdot d\vec{S} - \int f(\vec{\nabla}\cdot\vec{A})\,dv.$$

$$\therefore V(r) = \frac{1}{4\pi\varepsilon_0}\left[\oint_{S'} \frac{1}{r}\,\vec{P}\cdot d\vec{S'} - \int_{V'} \frac{1}{r}(\vec{\nabla}\cdot\vec{P})\,dv'\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\oint_{S'} \frac{\sigma_b\,dS'}{r} + \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{\rho_b}{r}\,dv'$$

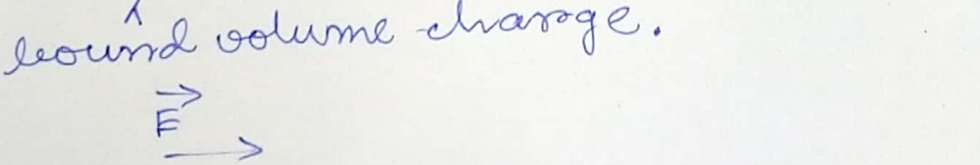
$$\left[\sigma_b = \vec{P}\cdot\hat{n},\ \rho_b = -(\vec{\nabla}\cdot\vec{P})\right]$$

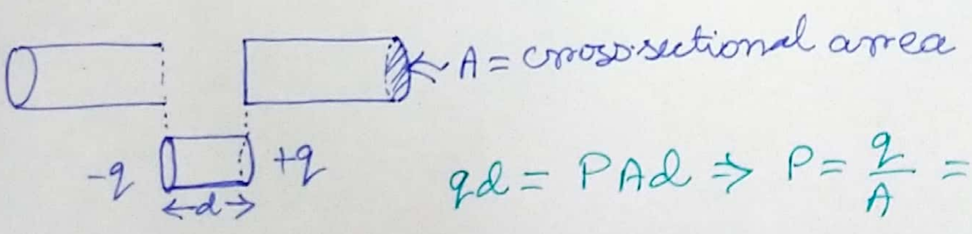
↑ Surface and volume bound charges.

$$\int_{V'} \rho_b\,dv' = -\int_{V'}(\vec{\nabla}\cdot\vec{P})\,dv' = -\oint_{S'}\vec{P}\cdot d\vec{S'} = -\oint_{S'}\sigma_b\,\hat{n}\cdot d\vec{S'}$$

$$= -\oint_{S'}\sigma_b\,dS'$$

So, the total bound surface charge is equal and opposite to the bound volume charge.





$$qd = PAd \Rightarrow P = \frac{q}{A} = \sigma_b$$

Gauss's law:

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \varepsilon_0 \vec{\nabla}\cdot\vec{E} = \rho_{free} + \rho_{bound}$$

$$\Rightarrow \varepsilon_0 \vec{\nabla}\cdot\vec{E} = \rho_{free} - (\vec{\nabla}\cdot\vec{P})$$

$$\Rightarrow \vec{\nabla}\cdot(\varepsilon_0\vec{E} + \vec{P}) = \rho_{free}.$$

Linear dielectric: $\vec{P} = \varepsilon_0 \chi_e \vec{E}$

χ_e = electrical susceptibility.

$\therefore$ Gauss's law can be written as; (for linear dielectric).

$$\vec{\nabla}\cdot(\varepsilon_0\vec{E} + \varepsilon_0\chi_e\vec{E}) = \rho_{free}$$

$$\Rightarrow \vec{\nabla}\cdot\varepsilon_0(1+\chi_e)\vec{E} = \rho_{free}$$

$$\Rightarrow \vec{\nabla}\cdot\vec{E} = \frac{\rho_{free}}{\varepsilon_0(1+\chi_e)} = \frac{\rho_{free}}{\varepsilon}.$$

ε = permittivity of the material

$\varepsilon/\varepsilon_0 = (1+\chi_e)$ dielectric constant.