In the last lecture, we calculated $(\vec{\nabla}\cdot\vec{E})$ and this led us to Gauss's law. ~~Let~~ Let's calculate $(\vec{\nabla}\times\vec{E})$.

$$\vec{E} = \left(\frac{1}{4\pi\varepsilon_0}\frac{q\,\hat{r}}{r^2}\right) \qquad \text{———— (1)}$$

But this field can be written as gradient of a scalar:

$$\underbrace{\vec{E} = -\vec{\nabla}\phi}_{(2)}, \text{ where, } \phi = \left(\frac{q}{4\pi\varepsilon_0 r}\right) \qquad \text{———— (3)}$$

$$\left[\vec{\nabla} \equiv \hat{r}\frac{\partial}{\partial r}\right]$$

$$\therefore \underbrace{\vec{\nabla}\times\vec{E} = 0}_{(4)}$$ [So, electrostatic field is irrotational] (and conservative)

However, we have proved eqn.(4) only in case of a single charge. What would happen if we have multiple charges/a charge distribution?

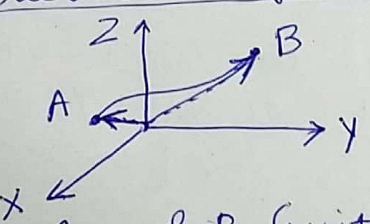
The trick is to use superposition principle.

$$\vec{E} = \sum_i \vec{E}_i$$ [$\vec{E}_i$ is field due to each charge]. [Each $\vec{E}_i$ satisfies eqn.(1)]

$$(\vec{\nabla}\times\vec{E}) = \sum_i(\vec{\nabla}\times\vec{E}_i) = 0 \longleftarrow$$ This condition holds for any electrostatic case.

So, $\vec{E} = -(\vec{\nabla}\phi)$ $\boxed{\text{always in electrostatics}}$

Reference point for electrostatic potential:



Consider two points A and B (with radius vectors $\vec{r}_A$ and $\vec{r}_B$) in an electrostatic field $\vec{E}$.

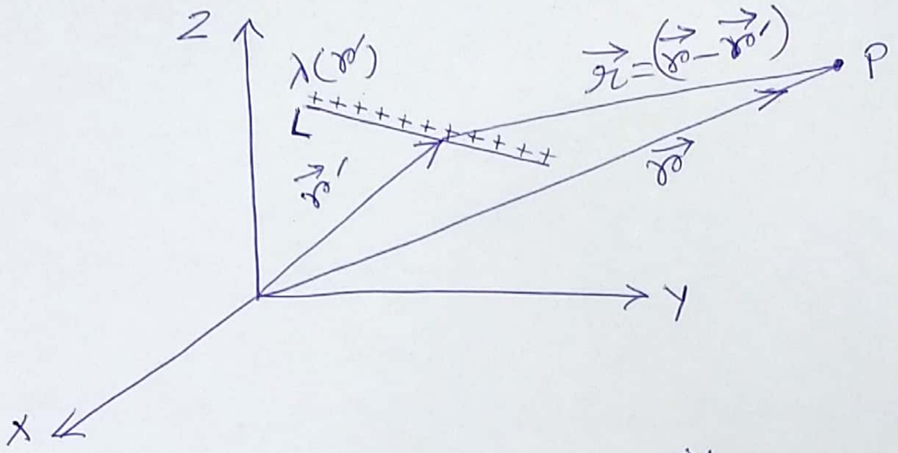
How much work do we have to do to move a unit positive charge from $\vec{A}$ to $\vec{B}$?

$$W = -\int_A^B \vec{E}\cdot\vec{dl} = \int_A^B (\vec{\nabla}\phi)\cdot\vec{dl} = \int_A^B d\phi = \phi(B) - \phi(A)$$

= Potential difference between two points.

We can choose the point A to be a reference point and measure potentials of all other points w.r.t. A. A standard convention is to choose potential of the reference to be zero. (For example, we often say that the potential of 'ground' is zero).

Potential of a charge distribution:



$$V_P = \frac{1}{4\pi\varepsilon_0} \int_L \frac{\lambda(\vec{r}')}{\mathscr{r}} \, dr'$$

Similarly, for a surface charge distribution:

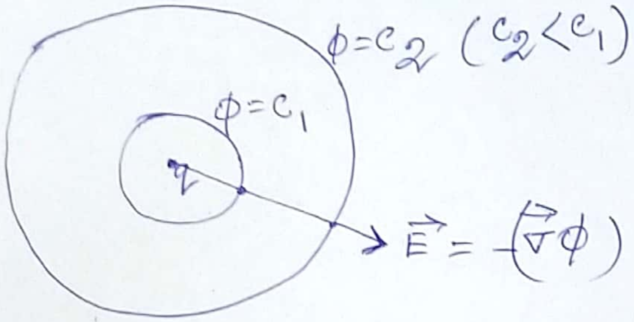
$$V_P = \frac{1}{4\pi\varepsilon_0} \int_{S'} \frac{\sigma(\vec{r}')}{\mathscr{r}} \, ds'$$

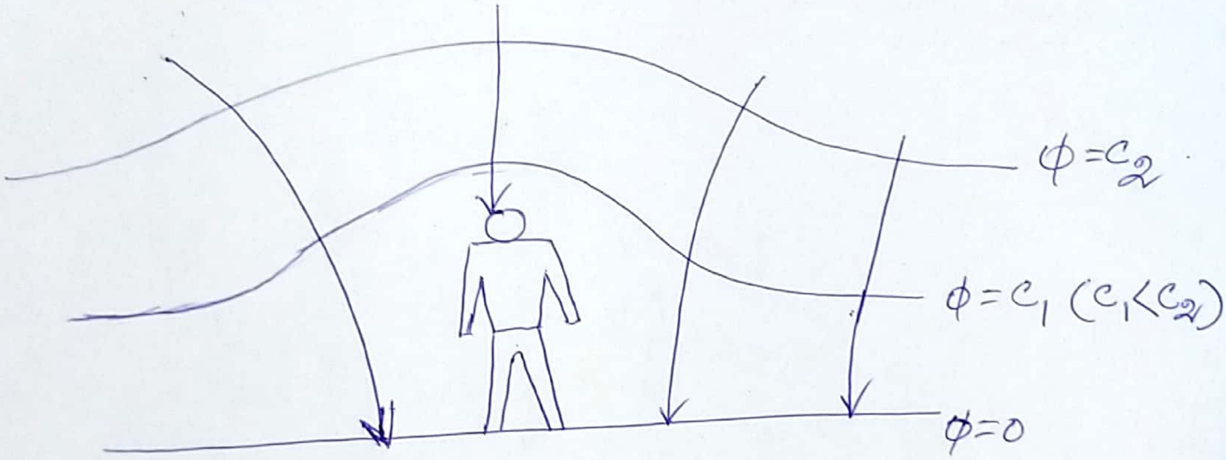
and for a volume distribution;

$$V_P = \frac{1}{4\pi\varepsilon_0} \int_{V'} \frac{\rho(\vec{r}')}{\mathscr{r}} \, dv'$$

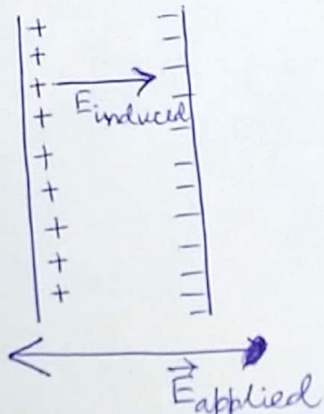
Equipotential surfaces (Link to demo 'll be posted in course webpage) $\phi(x,y,z)$ is a scalar field. Recall from our first lecture, a surface that satisfies $\phi(x,y,z) = \text{constant}$, is called a level surface. For potential, we'll call ~~that~~ them equipotential surface.

~~For~~ For a level surface, the gradients are along surface normals. So, electric fields 'll be normal to equipotential surfaces.





<u>Conductors</u>: Perfect conductor has unlimited supply of free electrons.

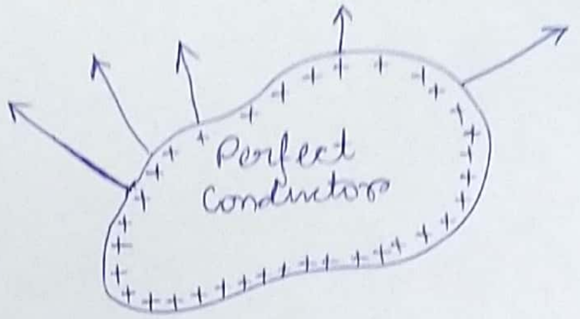


$$\vec{E}_{Total} = \vec{E}_{applied} + \vec{E}_{induced} = 0.$$

$$\boxed{\text{Electric field is zero inside a perfect conductor}}$$

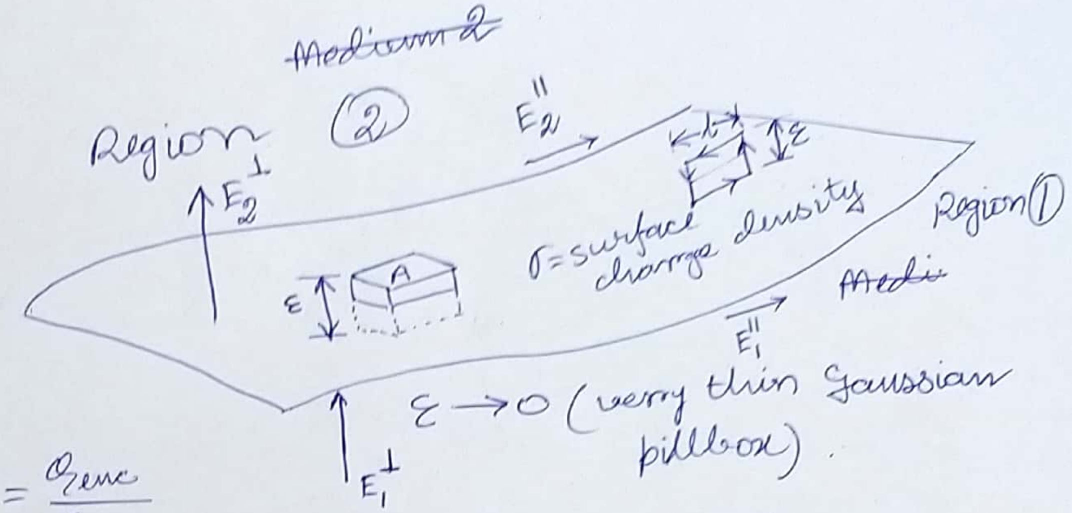
- What about $\vec{E}$ on the surface of a perfect conductor?
(zero, by same logic) → $\vec{E} = -\vec{\nabla}\phi$. Since $\vec{E} = 0$ on the surface of a perfect conductor, ϕ = constant too!
∴ Surface of a perfect conductor is equipotential.

- Volume charge within a perfect conductor:
$\vec{\nabla} \cdot \vec{E} = 0 \Rightarrow \boxed{\rho = 0}$



Field lines must be normal to a conductor (perfect) surface.

Boundary condition:



$$\oint_S \vec{E} \cdot d\vec{S} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\Rightarrow E_2^{\perp} A - E_1^{\perp} A = \sigma A/\varepsilon_0 \Rightarrow \boxed{\left(E_2^{\perp} - E_1^{\perp}\right) = \sigma/\varepsilon_0}$$

Normal component of electric field is discontinuous across a boundary by an amount $\left(\frac{\sigma}{\varepsilon_0}\right)$.

~~accor~~

For electrostatics, $\oint \vec{E} \cdot \vec{dl} = 0$

$$\left(E_1^{\parallel} - E_2^{\parallel}\right) l = 0 \quad [\text{assume } \varepsilon \to 0].$$

$$\Rightarrow \boxed{E_1^{\parallel} = E_2^{\parallel}} \quad \text{——— (5)}$$

Tangential components of electric field is continuous across any boundary (we would revisit this when we do electrodynamics. In electrodynamics, $\vec{\nabla} \times \vec{E} \neq 0$

So, $\oint \vec{E} \cdot \vec{dl} \neq 0$ in electrodynamics. However, we'll show that for ~~an infini~~ a very thin pillbox, $E_1^{\parallel} = E_2^{\parallel}$)

So, condition (5) is true in general.