Complex refractive index/ permittivity:

From the frequency-domain representation, we have found that:

$$\vec{\nabla}\times\vec{H}(\omega) = \sigma\vec{E}(\omega) + i\omega\vec{D}(\omega)$$
$$= \sigma\vec{E}(\omega) + i\omega\varepsilon\vec{E}(\omega)$$
$$= (\sigma + i\omega\varepsilon)\vec{E}(\omega) \quad \text{——— (5)}$$

If the current density was absent, this equation becomes:

$$\vec{\nabla}\times\vec{H} = i\omega\varepsilon\vec{E}(\omega) \quad \text{——— (6)} \quad [\sigma = 0]$$

Suppose, for a material $\sigma \neq 0$, but we still want to write $\vec{\nabla}\times\vec{H}$ in a form that looks like eq. (6). In order to do this, we define an equivalent permittivity or effective permittivity ε_c [the suffix 'c' stands for complex]:

$$\vec{\nabla}\times\vec{H} = i\omega\varepsilon_c\vec{E} \quad \text{——— (7)}$$

From (5), $\vec{\nabla}\times\vec{H} = i\omega\varepsilon\left(1 + \frac{\sigma}{i\omega\varepsilon}\right)\vec{E}$

$$= i\omega\varepsilon\left(1 - i\frac{\sigma}{\omega\varepsilon}\right)\vec{E} \quad \text{——— (8)}$$

Comparing eqns. (7) and (8),

$$\boxed{\varepsilon_c = \varepsilon\left(1 - i\frac{\sigma}{\omega\varepsilon}\right)} \Rightarrow \varepsilon_c \text{ is frequency-dependent.}$$

It's also possible that σ and ε are functions of frequency.

In general,

$$\varepsilon_c = \varepsilon(\omega)\left(1 - i\frac{\sigma(\omega)}{\omega\varepsilon(\omega)}\right).$$

The quantity $\sigma(\omega)/\omega\varepsilon(\omega)$ can be used to classify a given material:

$\frac{\sigma(\omega)}{\omega\varepsilon(\omega)} \gg 1 \longrightarrow$ Good conductor

$\frac{\sigma(\omega)}{\omega\varepsilon(\omega)} \sim 1$ (comparable to 1) $\longrightarrow$ Lossy dielectric

$\frac{\sigma(\omega)}{\omega\varepsilon(\omega)} \ll 1 \longrightarrow$ Good dielectric.