Quick recap:

Faradays law of induction:

$$\mathcal{E} = -\frac{d\phi}{dt}, \quad \mathcal{E} = \text{e.m.f. and } \phi = \text{mag. flux.}$$

$$\Rightarrow \oint \vec{E} \cdot \vec{dl} = -\frac{d\phi}{dt}$$ —— Integral form of Faraday's law.

$$\Rightarrow \oint \vec{E} \cdot \vec{dl} = -\frac{d}{dt} \int \vec{B} \cdot \vec{ds}$$

$$\Rightarrow \int (\vec{\nabla} \times \vec{E}) \cdot \vec{ds} = -\int \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$$

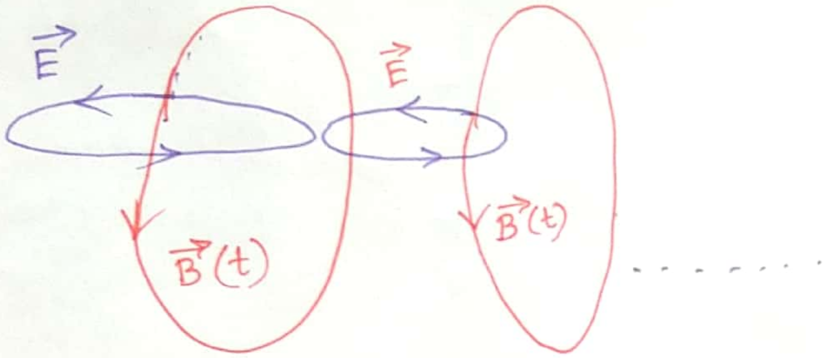
$$\Rightarrow \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ —— Differential form of Faraday's law.

A changing magnetic field produces electric field. But do we need a current loop for this law to be true? (Remember: we started our experiment with a ~~co~~ conducting loop and a moving bar magnet...)

The answer is NO. The existence of a conducting loop ensures that we provide some path for the current to flow. If we don't put the loop, conduction current wouldn't flow. But the field would exist!

So, how can the electric field exist without any charge? After all, we know that $\vec{E}$-field lines start from ~~∅~~ + charge, end on − charge or infinity. Alternatively, they may start from infinity, end on a negative charge.

But we are simply causing a change in ϕ (perhaps, moving a magnet). There are no charge anywhere! How would then $\vec{E}$-field lines exist? There's another possibility we have overlooked. $\vec{E}$-field lines can form a loop just like $\vec{B}$-field lines.



Inconsistency in Ampere's law:

$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}$$

$$\Rightarrow \vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \mu_0(\vec{\nabla}\cdot\vec{J})$$

$$\Rightarrow \vec{\nabla}\cdot\vec{J} = 0 \quad \text{—— (1)}$$

But we learnt in continuity equation that:

$$\vec{\nabla}\cdot\vec{J} = -\left(\frac{\partial \rho}{\partial t}\right) \quad \text{—— (2)}$$

(1) and (2) are consistent only when ρ is independent of time. i.e. the DC case.

Continuity equation is law of conservation of charge and must always be true. So, we must modify Ampere's law for timevarying (or, electrodynamic cases). This is what Maxwell did.

From (2),

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{D})$$

$$\Rightarrow \vec{\nabla}\cdot\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) = 0.$$

The inconsistency goes away when we re-write Ampere's law as;

$$\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J}+\frac{\partial\vec{D}}{\partial t}\right)$$

$$\Rightarrow(\vec{\nabla}\times\vec{B}) = \mu_0\left(\vec{J}+\varepsilon\frac{\partial\vec{E}}{\partial t}\right) \quad \text{——— (3)}$$

This is Ampere-Maxwell's law.

Physically, this means: $\vec{B}$ can be produced by two methods:

① A current density.

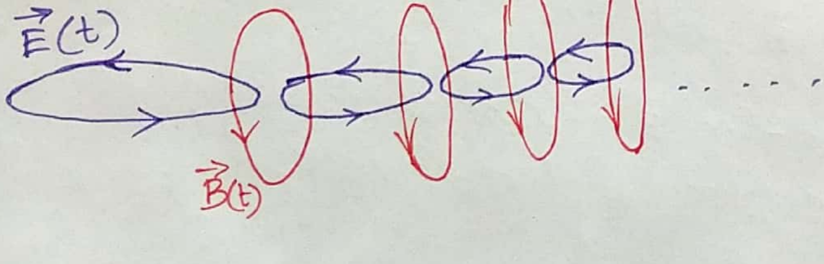
② A changing $\vec{E}$-field.

└ Look how symmetric it is!

According to Farraday's law, a $\vec{B}$ changing with time produces $\vec{E}$. Now, from modified Ampere's law, a changing $\vec{E}$ produces $\vec{B}$! We donot need a current carrying conductor for creating $\vec{B}$.

While $\vec{J}$ is known as conduction current density, $\left(\frac{\partial\vec{D}}{\partial t}\right)$ is called displacement current density.

The interplay between $\frac{\partial\vec{D}}{\partial t}$ and $\frac{\partial\vec{B}}{\partial t}$ is the key-mechanism responsible for propagation of Electromagnetic waves.



So, we have learnt 4 equations so far that rules Electromagnetics;

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon}$$ — Gauss's law

$$\vec{\nabla}.\vec{B} = 0$$ — Non-existence of magnetic monopoles.

$$\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right)$$ — Ampere-Maxwell's law.

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$ — Faraday's law of induction.

Maxwell's eqn. in differential form.

$$\oint\vec{E}.\vec{ds} = \frac{Q_{enc.}}{\varepsilon}$$ [Q_{enc} is free charge]

$$\oint\vec{B}.\vec{ds} = 0$$

$$\oint\vec{B}.\vec{dl} = \mu_0\int\left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right).\vec{ds}$$

$$\oint\vec{E}.\vec{dl} = -\int\frac{\partial\vec{B}}{\partial t}.\vec{ds}$$

Maxwell's eqns. in integral form

It's verry important to note the last equation.

$$\oint\vec{E}.\vec{dl} \neq 0$$ in electrodynamics.

If $\vec{B}$ is time-independent (i.e. static field), so that no flux change is involved, only then we have $\oint\vec{E}.\vec{dl} = 0$.

Thus, static $\vec{E}$-field is conservative.
But time-varying $\vec{E}$-field is not!

Let's fiddle with Ampere-Maxwell's law a little more:

$$\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$$

If we have a conducting medium with a conductivity σ,

$$\vec{\nabla}\times\left(\vec{B}/\mu_0\right) = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\Rightarrow \vec{\nabla}\times\vec{H} = \left(\sigma\vec{E} + \varepsilon\frac{\partial \vec{E}}{\partial t}\right) \quad \text{———— (4)}$$

Maxwell's equations in fourier domain

Just for this part of the lecture, let's introduce some notations so that we can differentiate between time-domain and frequency domain quantities.

$$\vec{\mathcal{E}}(t) \rightleftharpoons \vec{E}(\omega)$$
$$\vec{\mathcal{B}}(t) \rightleftharpoons \vec{B}(\omega)$$
$$\rho(t) \rightleftharpoons \rho(\omega)$$
$$\vec{\mathcal{J}}(t) \rightleftharpoons \vec{J}(\omega) \qquad \vec{\mathcal{D}}(t) \rightleftharpoons \vec{D}(\omega)$$

we know from fourier transform properties that

$$\mathcal{F}\left\{\frac{d}{dt}x(t)\right\} = i\omega X(\omega)$$

Maxwell's eqns. in time domain and frequency domain:

$$\vec{\nabla}\cdot\vec{\mathcal{E}}(t) = \frac{\rho(t)}{\varepsilon} \longrightarrow \vec{\nabla}\cdot\vec{E}(\omega) = \frac{\rho(\omega)}{\varepsilon}$$

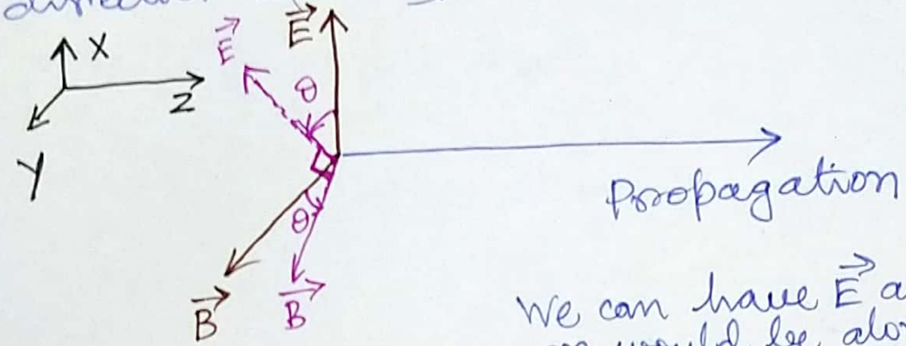
$$\vec{\nabla}\cdot\vec{\mathcal{B}}(t) = 0 \longrightarrow \vec{\nabla}\cdot\vec{B}(\omega) = 0$$

$$\vec{\nabla}\times\vec{\mathcal{B}}(t) = \mu_0\left(\vec{\mathcal{J}}(t) + \frac{\partial\vec{\mathcal{D}}}{\partial t}\right) \rightarrow \vec{\nabla}\times\vec{B}(\omega) = \mu_0\left[\vec{J}(\omega) + i\omega D(\omega)\right]$$

$$\vec{\nabla}\times\vec{\mathcal{E}}(t) = -\frac{\partial\vec{\mathcal{B}}(t)}{\partial t} \longrightarrow \vec{\nabla}\times\vec{E}(\omega) = -i\omega\vec{B}(\omega)$$

EM-wave propagation:

We have discussed an intuitive model of EM wave propagation in free-space. Let's build on this intuitive understanding further. We said that $\vec{E}$ and $\vec{B}$ are perpendicular and the wave propagates in a direction perpendicular and $\vec{E}$ and $\vec{B}$, both. [In fact, the propagation direction is $(\vec{E} \times \vec{B})$]



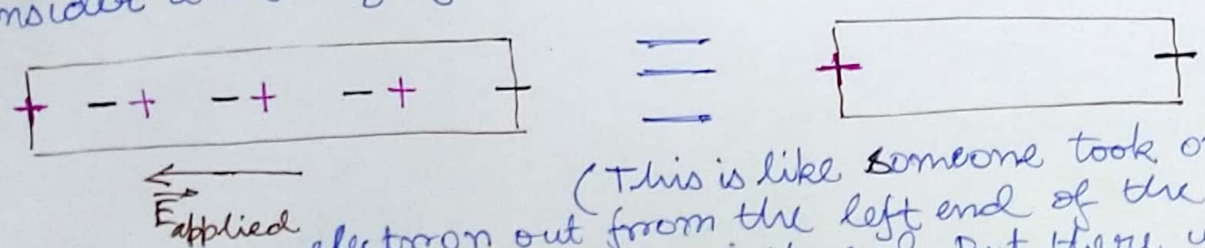
We can have $\vec{E}$ along X, $\vec{B}$ along Y. So, propagation would be along z.

Alternatively, we can rotate both $\vec{E}$ and $\vec{B}$ by an angle θ in the XY-plane and still have the propagation along Z. Thus, the propagation direction alone doesn't fix the $\vec{E}$ and $\vec{B}$ direction uniquely. In order to do so, we introduce polarization of wave (this is different from polarization of a dielectric). Wave polarization is defined as the direction of $\vec{E}$-field vector.

Displacement current:

In Maxwell's equation, we termed $\frac{\partial \vec{D}}{\partial t}$ as displacement current ($\vec{J}_D$). But what does it mean physically? Let's go back to our old picture of dielectric polarization (we applied a field $\vec{E}$ and either the induced dipoles or the molecules with inherent dipole moments arranged themselves along $\vec{E}$).

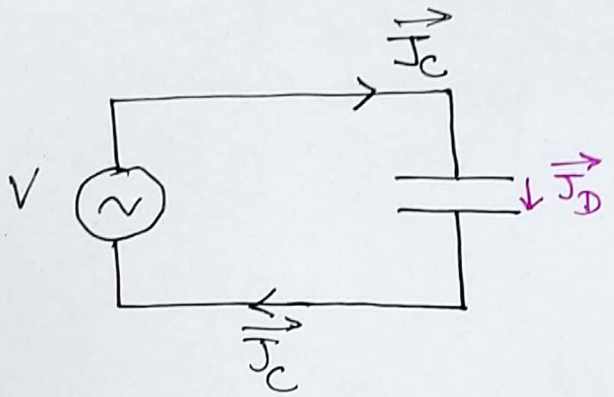
Consider a string of such dipoles:



(This is like someone took one electron out from the left end of the dielectric and put it on the rigth end. But there was no actual flow of electron!!)

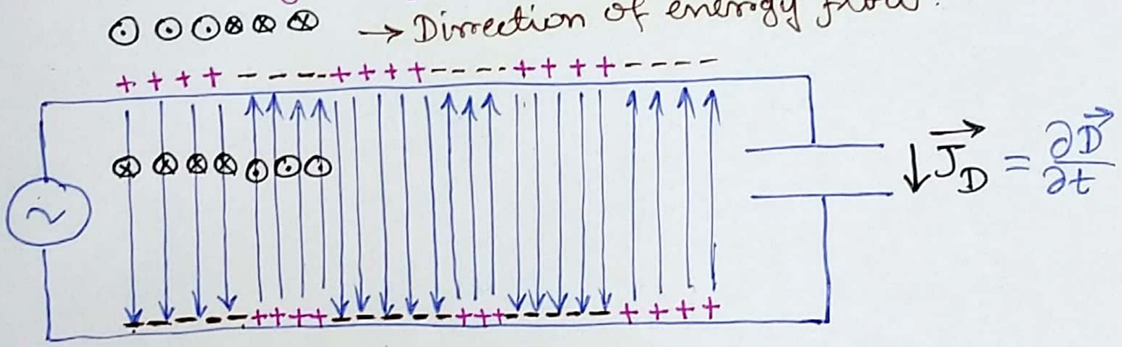
Now, imagine that the $\vec{E}$ is an alternating field and changes direction periodically. So, the + and − charges at the end will also flip signs. As if an A.C. current is flowing but there is no actual movement of electrons!!

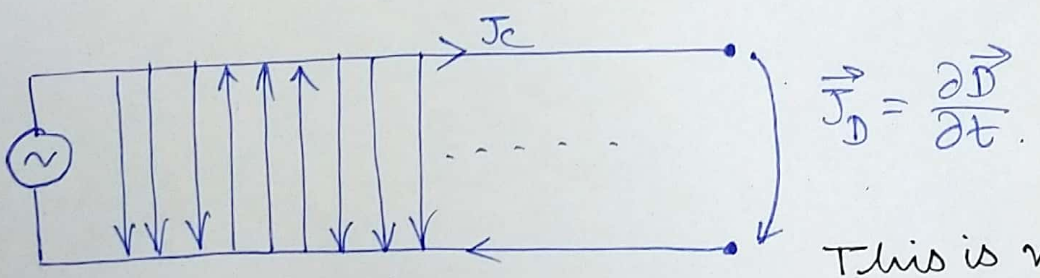
This is what happens in a capacitor:



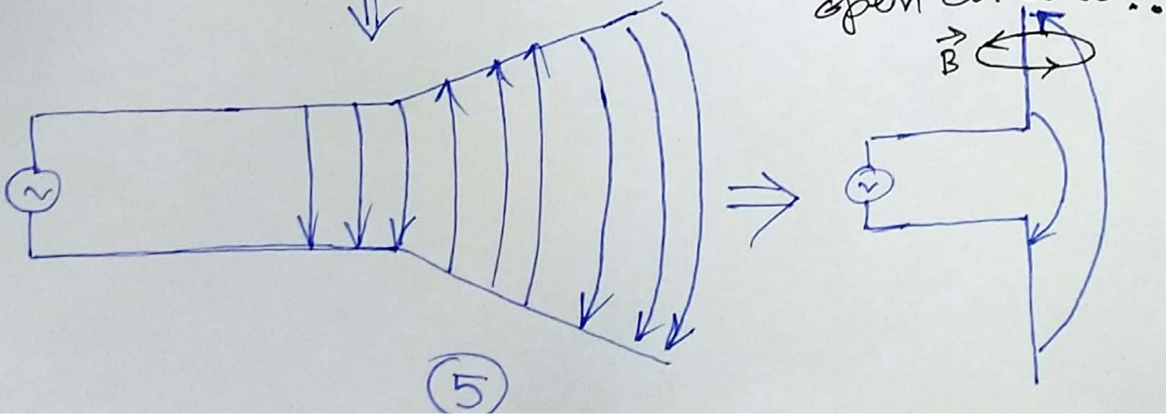
So, Maxwell's eqns. are at the heart of current flow in any circuit.

A really really long circuit: (much longer than the wavelength of the applied AC signal).





This is not an open circuit!!



Basic concept of a receiving antenna: if the conducting rod is oriented along the polarization of the incoming wave, $\vec{E}$ will make the electrons oscillate back and forth the conductor.

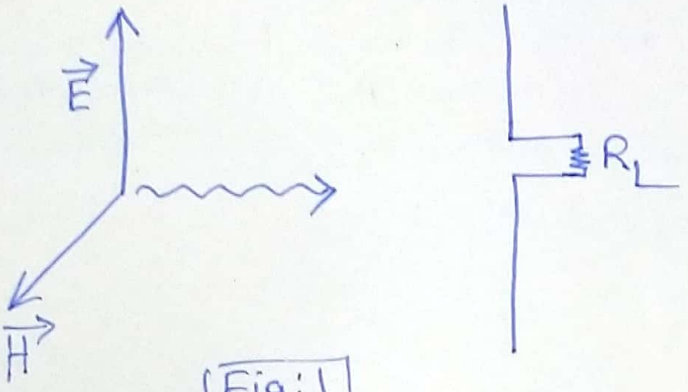


Fig: 1

But if the conductor is placed along a direction $\perp$ to $\vec{E}$ (shown in fig. 2), no current 'll be produced.

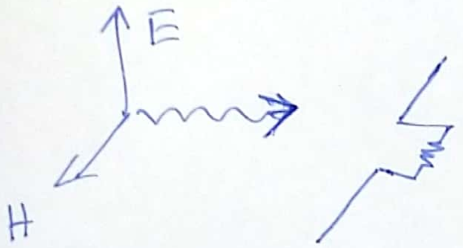


Fig. 2