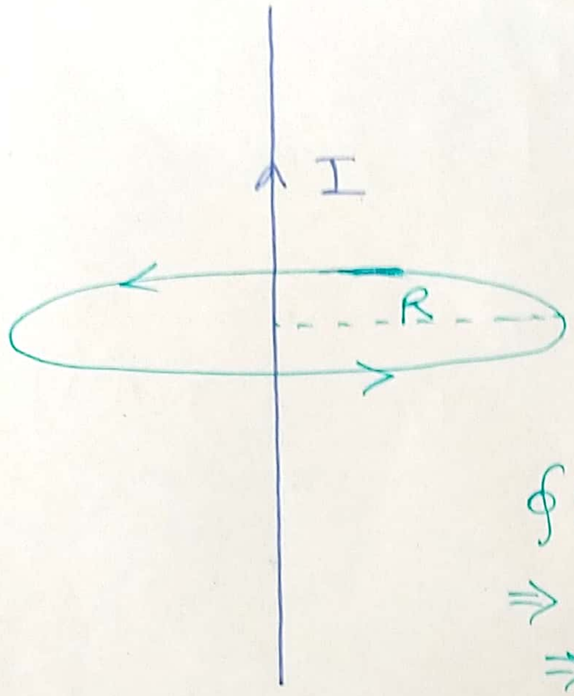
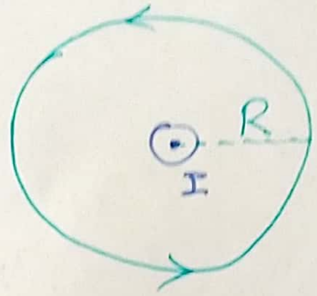
A simple application of Ampere's law:

(Magnetic field due to an infinite conducting wire carrying a current I).



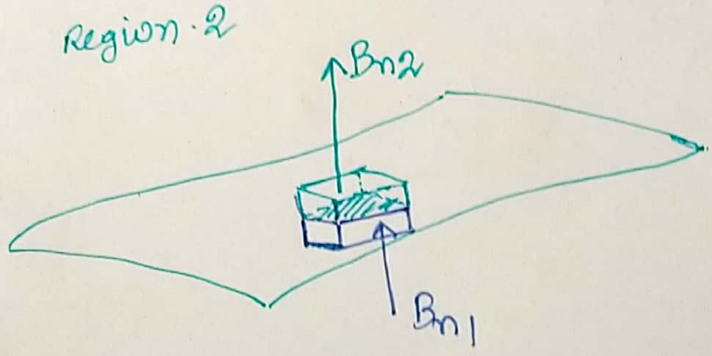


$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I$$

$$\Rightarrow B\,2\pi R = \mu_0 I.$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi R}.$$

Magnetostatic boundary conditions:



Region 1

(Gaussian pillbox is infinitely thin and has a cross-section ΔA)

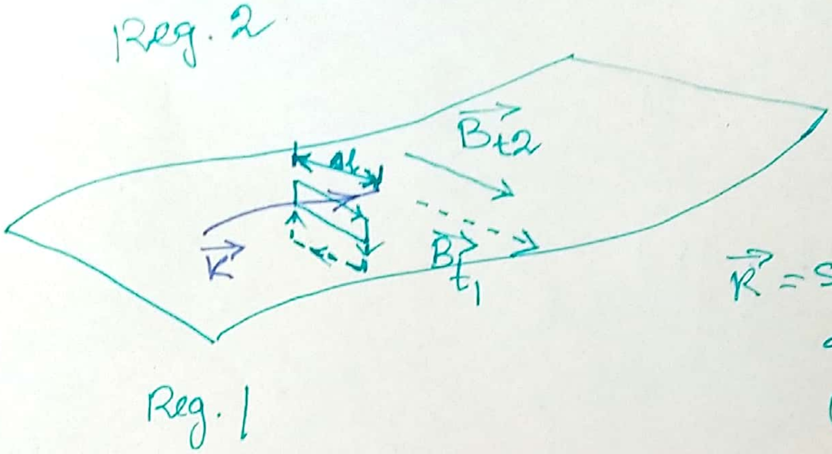
Gauss's law:

$$\oint \vec{B}\cdot\vec{dS} = 0.$$

$$\Rightarrow (B_{n2} - B_{n1})\Delta A = 0.$$

$$\Rightarrow B_{n2} = B_{n1}$$

Normal components of $\vec{B}$ is <u>always</u> continuous across an interface.



$\vec{K}$ = surface current density (A/m).

$\therefore$ Total current enclosed by the Amperian loop $= (K \Delta l)$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}.$$

$$\left(B_{t2} - B_{t1}\right) \Delta l = \mu_0 K \Delta l$$

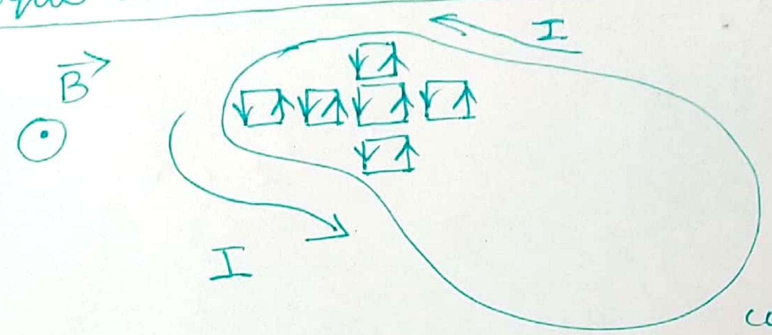
$$\Rightarrow \left(B_{t2} - B_{t1}\right) = \mu_0 K$$

[contribution from sides of the rectangle $\perp$ to the interface are zero in the limit of very thin Amperian loop].

Tangential components of the magnetic field are discontinuous across an interface by an amount that is equal to the surface current density at the interface.

Magnetic fields in matter

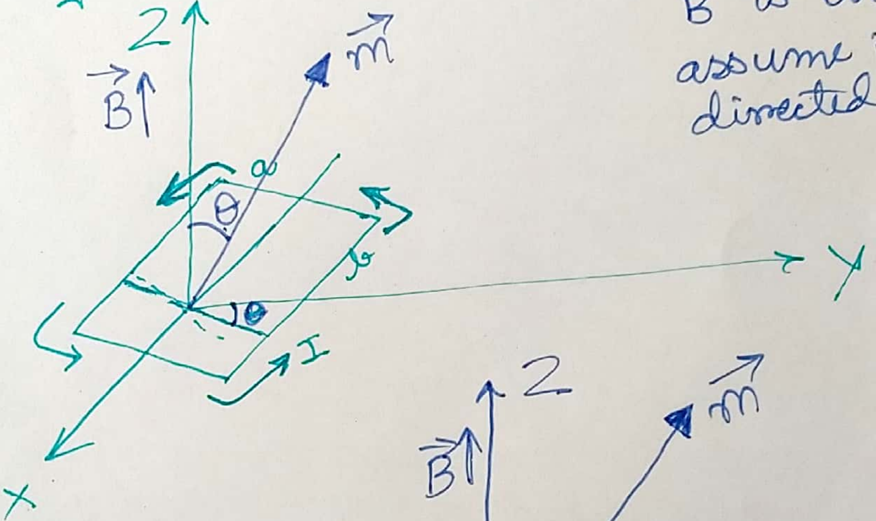
Torque on a current carrying loop:

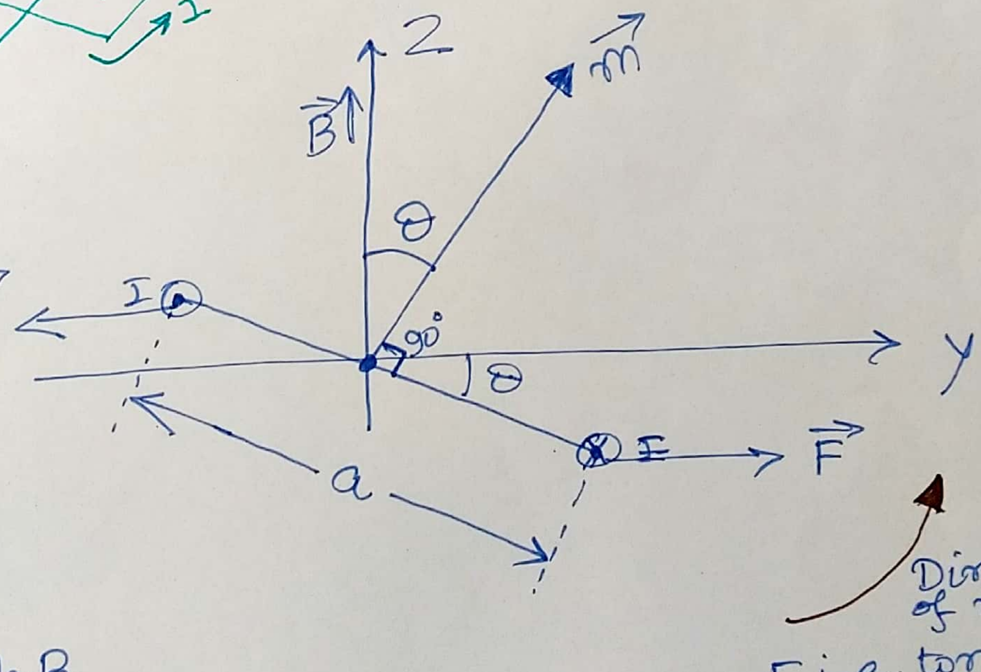


Divide the entire loop into small rectangles. The currents (and hence, the forces) on adjacent sides of two consecutive rectangles cancel each other.

We'll study the torque on a rectangular loop.

$\vec{B}$ is uniform and assume it to be directed along $\hat{z}$.





Direction of rotation

[i.e. torque is along $\hat{x}$, out of the paper]

$$|\vec{F}| = IbB$$

$$\vec{\tau} = aF\sin\theta\,\hat{x}$$

$$= IabB\sin\theta\,\hat{x}$$

$$= IAB\sin\theta\,\hat{x} \quad [A = \text{area of the loop} = ab]$$

$$= \vec{m} \times \vec{B}$$

Note: The torque tends to rotate the magnetic dipole and align it along XY-plane (i.e. the loop lies in XY-plane) but there's no net force on this dipole as long as the field is uniform.

Because: Net force $= \vec{F} = I \oint d\vec{l} \times \vec{B} = I\left(\oint d\vec{l}\right) \times \vec{B}$

[$\because \vec{B}$ is uniform]

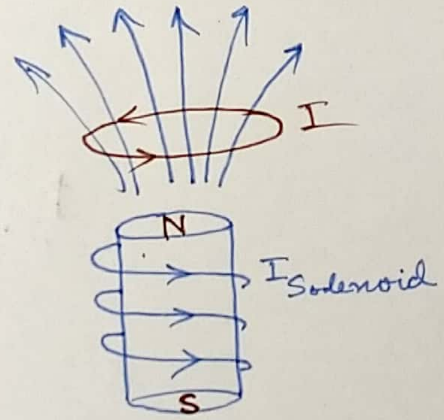
$= 0$

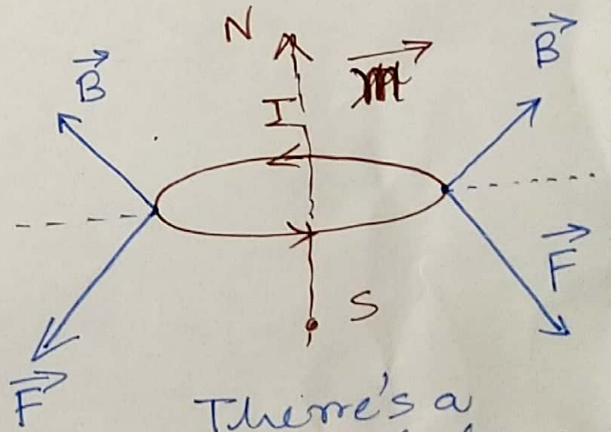
You can also see this from the figure in the last page. The forces on two arms of the rectangle are equal and opposite.

Interesting things happen when magnetic dipole is placed in a non-uniform field!

Consider a magnetic dipole that came (almost) to rotational equilibrium after ~~expseier~~ experiencing a torque/ $\vec{m}$ is along $+z$.

We placed it in a way so that





There's a net downward force on the magnetic dipole!!

The force 'll try to attract the magnetic dipole towards the stronger field region!

Electron spin in an atom gives rise to dipole (magnetic). Usually, when no. of electrons in an atom is odd, there's a residual dipole moment. This is the origin of paramag-netism.

However, paramagnetism is extremely weak. So, the force felt by paramagnetic material is very small. If you move a magnet near to a paramagnetic material, it'll be impossible to pull it up as gravity is stronger than the force between magnet and paramagnetic material.

However, liquids are lighter (compared to solid) and at low temperature there are many dipoles aligned to the magnetic field due to lesser thermal agitation. Liquid oxygen is paramagnetic and can be suspended from a magnet!! Let's look at the illustration (link will be shared in course webpage).

Magnetization:

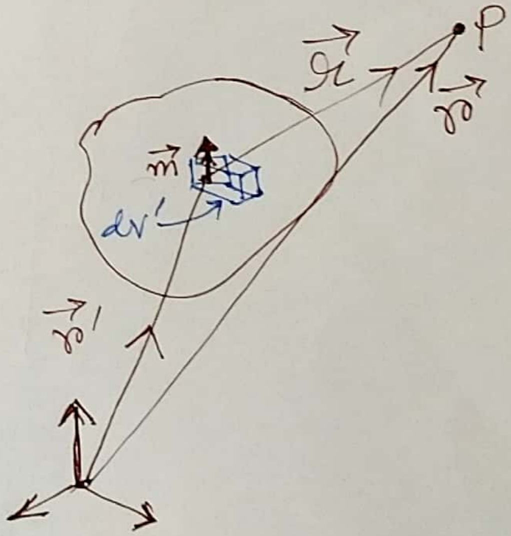
$\vec{M}$ = magnetic dipole moment per unit volume

(analogous to polarization $\vec{P}$ of electrostatics)

Magnetic vector potential of a single dipole:

(we'll not prove this)

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$



For a distribution of magnetic dipoles:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(\vec{r}') \times \hat{\mathscr{r}}}{\mathscr{r}^2} dv' \quad (1)$$

Assignment problem:

Just like bound volume and surface charge of electrostatics, eq. (1) can be written as:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_{V'} \frac{\vec{J}_b(\vec{r}')}{\mathscr{r}} dv' + \frac{\mu_0}{4\pi} \oint_{S'} \frac{\vec{K}_b(\vec{r}')}{\mathscr{r}} ds'$$

$\vec{J}_b(\vec{r}') = (\vec{\nabla} \times \vec{M}) \rightarrow$ Bound volume current

$\vec{K}_b(\vec{r}') = (\vec{M} \times \hat{n}) \rightarrow$ Bound surface current.

Ampere's law in magnetized materials:

(Recall, electrostatic analog: Gauss's law inside a polarized dielectric)

$$\frac{1}{\mu_0}(\vec{\nabla}\times\vec{B}) = \vec{J}_{total} = \underbrace{\vec{J}_f}_{\text{free current}} + \underbrace{\vec{J}_b}_{\text{bound current}} = \vec{J}_f + (\vec{\nabla}\times\vec{M})$$

$$\Rightarrow \vec{\nabla}\times\left(\frac{\vec{B}}{\mu_0} - \vec{M}\right) = \vec{J}_f$$

$$\Rightarrow \vec{\nabla}\times\vec{H} = \vec{J}_f \qquad \left[\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}\right] \Rightarrow \boxed{\vec{B} = \mu_0(\vec{H}+\vec{M})}$$

Differential form

$$\Rightarrow \oint \vec{H}\cdot d\vec{l} = I_{f\,enc}$$

Integral form.

Magnetic susceptibility and permeability:

Linear medium $\Rightarrow$ $\vec{M} = \chi_m \vec{H}$ (i.e. magnetization is proportional to $\vec{H}$)

χ_m: magnetic susceptibility.

$$\therefore \vec{B} = \mu_0(\vec{H} + \chi_m\vec{H}) = \mu_0(1+\chi_m)\vec{H}$$
$$= \mu\vec{H}.$$

$$\boxed{\mu = \mu_0(1+\chi_m)}$$

[redacted]