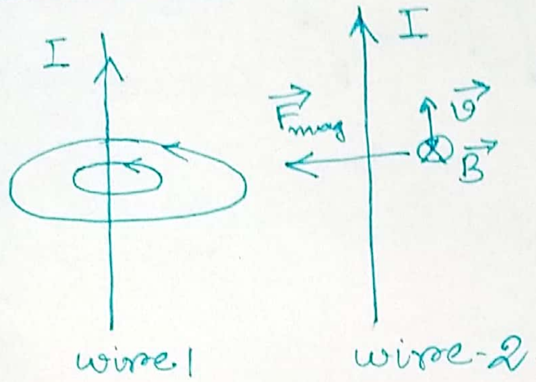
Lorentz force law:

$$\vec{F}_{mag} = Q(\vec{v} \times \vec{B})$$

In presence of both $\vec{E}$ and $\vec{B}$, $\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})$.

Parallel currents: (See demo)

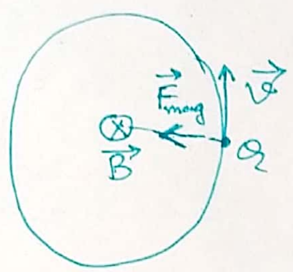


The wires carrying parallel current 'll attract each other.

Similarly, anti-parallel currents repel.

Cyclotron motion:

$\vec{F}_{mag}$ is always perpendicular to $\vec{v}$.

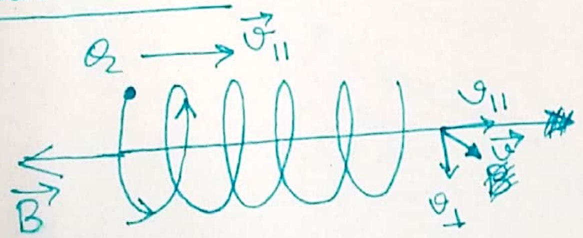


$$QvB = \frac{mv^2}{r}$$

$$\Rightarrow r = \frac{mv}{QB}$$

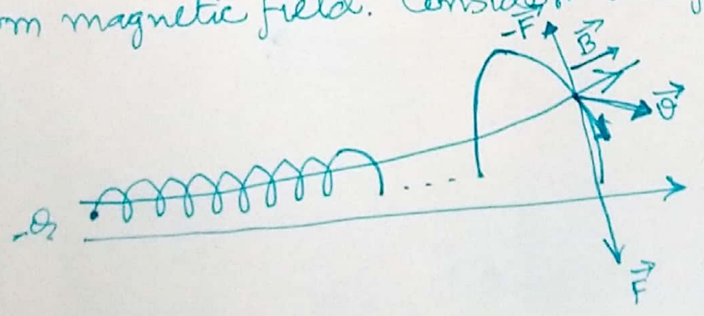
$$\text{Momentum} = mv = (QBr)$$

Helical motion:

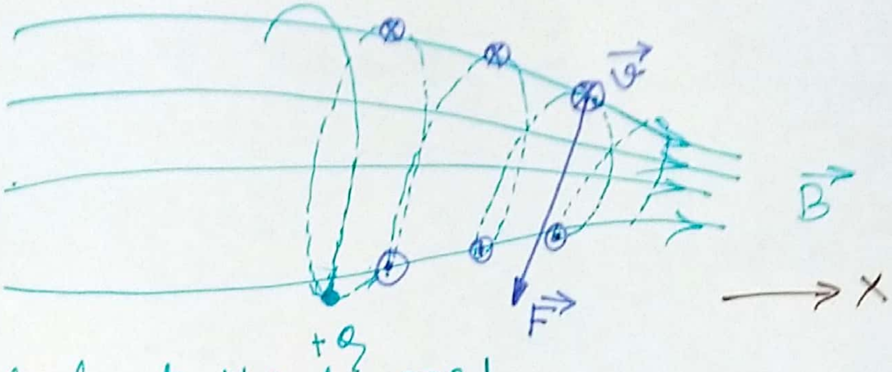


The component of $\vec{v}$ that is normal to $\vec{B}$ (let's call it $v_{\perp}$) causes the charge to go in a circle. But $v_{\parallel}$ (component of $\vec{v}$ parrallel to magnetic field) makes the charge moving along / opposite to $\vec{B}$ [$\vec{F} = Q(\vec{v} \times \vec{B})$]

So far, we have assumed that the particle is moving in a uniform magnetic field. Consider the following situation:



Suppose, the magnetic field lines are converging.



F_x tries to decelerate the charge!

--- Responsible for the greatest EM show on the earth!

Force on a current carrying wire:

$$d\vec{F}_{mag} = dq(\vec{v}\times\vec{B})$$

$$\vec{F}_{mag} = \int(\vec{v}\times\vec{B})\,dq = \int\left(\frac{d\vec{l}}{dt}\times\vec{B}\right)dq$$

$$= \int\left(\frac{dq}{dt}\right)(d\vec{l}\times\vec{B})$$

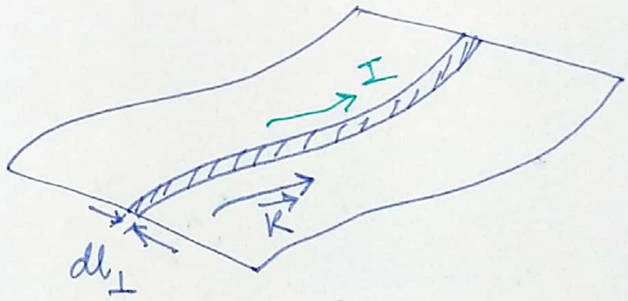
$$= \int I\,(d\vec{l}\times\vec{B}) \quad\text{———— (1)}$$

Surface current density:

$$\vec{K} = \frac{d\vec{I}}{dl_\perp}$$

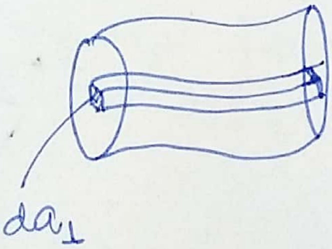
(current / unit width $\perp$ to the flow)

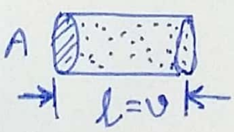
unit: A/m



Volume current density:

$$\vec{J} = \frac{d\vec{I}}{da_\perp}$$





$I = \rho A v$

$\Rightarrow J = \rho v.$

$(\vec{J} = \rho\vec{v})$

$\vec{F}_{mag}$ in terms of $\vec{J}$:

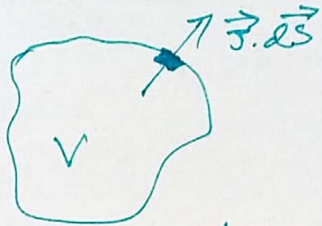
$$\vec{F}_{mag} = \int I(\vec{dl} \times \vec{B}) = \int (\vec{v} \times \vec{B}) dq$$

$$\cancel{= \int\int \vec{J}.} \qquad = \int (\vec{v} \times \vec{B}) \rho d\tau$$

$$= \int (\vec{J} \times \vec{B}) d\tau$$

Equation of continuity:

$$I = \oint_S \vec{J}.\vec{dS}$$



——— Net outgoing flux from volume V.

This must be equal to the rate of change (decrease) of charge from the volume.

[Law of conservation of charge]

$$\therefore \oint \vec{J}.\vec{dS} = -\frac{dQ}{dt} = -\frac{d}{dt}\int \rho dV$$

$$= -\int \left(\frac{\partial \rho}{\partial t}\right) dV$$

$$\Rightarrow \int (\vec{\nabla}.\vec{J}) dV = -\int \left(\frac{\partial \rho}{\partial t}\right) dV$$

$$\Rightarrow \boxed{\vec{\nabla}.\vec{J} + \left(\frac{\partial \rho}{\partial t}\right) = 0} \quad \text{———} \quad (2)$$

Eqn. of continuity (physically: law of conservation of charge).

Relaxation time:

We have seen, $\vec{J} = \rho \vec{v}$

$\vec{v}$ is proportional to $\vec{E}$ (unless $\vec{E}$ is very high).

$$\vec{v} = \mu \vec{E}.$$

$$\therefore \vec{J} = \rho \mu \vec{E} = \sigma \vec{E} \text{ — } \boxed{\text{Ohm's law}} \quad [\sigma = \text{conductivity}]$$

$$\therefore (\vec{\nabla}.\vec{J}) = \sigma(\vec{\nabla}.\vec{E}) = \sigma \frac{\rho}{\epsilon}. \quad \text{———} \quad (3)$$

Use expression (3) into the continuity equation of eq. (2):

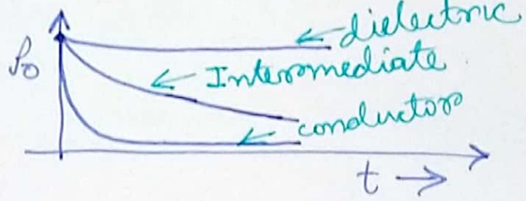
$$\frac{\sigma \rho}{\varepsilon} + \frac{\partial \rho}{\partial t} = 0$$

$$\Rightarrow \frac{\partial \rho}{\rho} = -\frac{\sigma}{\varepsilon}\partial t$$

Assume, we put a volume charge density $\rho = \rho_0$ to $t=0$.

$$\therefore \ln\left(\frac{\rho}{\rho_0}\right) = -\frac{\sigma}{\varepsilon} t$$

$$\Rightarrow \boxed{\rho = \rho_0 e^{-\frac{\sigma}{\varepsilon} t}} \quad \text{——— (4)}$$



This tells us, with time the volume charge 'll decay inside any material at a rate depending on the ratio of $\left(\frac{\sigma}{\varepsilon}\right)$!

If the volume charge decays, where does it go?
└ comes to the surface.

For good conductors, σ is very high ($\sim 10^7$ S/m)
The charge created/put in the bulk of the conductor 'll make its way to the surface pretty fast.
For a perfect conductor, $\sigma \rightarrow \infty$. So, this relaxation process is instantaneous (recall: we learnt that charge cannot reside <u>inside</u> a perfect conductor).

For good dielectrics, σ is very very low ($< 10^{-10}$ S/m)
So, the decay constant is small and the volume charge 'll dwell inside for a long long time. (forever in a perfect dielectric, $\sigma = 0$).

The time $\varepsilon/\sigma = t_0$ is called relaxation time.

[Time taken by volume charge to decay to 36.8% of its initial value].

$$\therefore \rho = \rho_0 e^{-(t/t_0)}$$