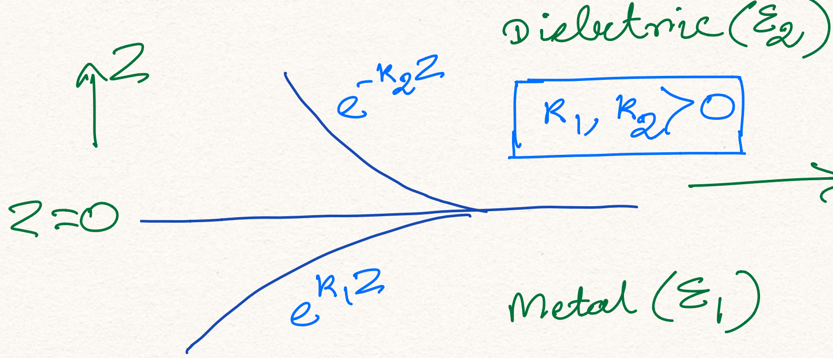
Lecture 8 (ECE545)

We have obtained dispersion relation of SPP at a metal-dielectric interface in last lecture. We did so for the TM modes. Let's investigate the possibility of SPP for TE modes.

To refresh your memory, we considered the following geometry:



Also, we saw that for TE mode the non-zero components are: E_y, H_x and H_z.

In TE mode, we also have:

$$H_x = -\frac{i}{\omega\mu_0}\left(\frac{\partial E_y}{\partial z}\right) \quad \text{——— (1)}$$

$$H_z = \frac{\beta}{\omega\mu_0} E_y \quad \text{——— (2)}$$

$$\frac{\partial^2 E_y}{\partial z^2} + \left(k_0^2\varepsilon - \beta^2\right) E_y = 0 \quad \text{——— (3)}$$

$$z>0 \begin{cases} E_y = A_2 e^{-i\beta x} e^{-k_2 z} & \text{——— (4)} \\ H_x = \dfrac{i A_2 k_2}{\omega \mu_0} e^{-i\beta x} e^{-k_2 z} & \text{——— (5)} \\ H_z = \dfrac{\beta A_2}{\omega \mu_0} e^{-i\beta x} e^{-k_2 z} & \text{——— (6)} \end{cases}$$

$$z<0 \begin{cases} E_y = A_1 e^{-i\beta x} e^{k_1 z} & \text{——— (7)} \\ H_x = -\dfrac{i A_1 k_1}{\omega \mu_0} e^{-i\beta x} e^{k_1 z} & \text{——— (8)} \\ H_z = \dfrac{\beta}{\omega \mu_0} A_2 e^{-i\beta x} e^{k_1 z} & \text{——— (9)} \end{cases}$$

Apply boundary conditions at $z=0$:

① E_y continuous across interface

$\Downarrow$

From (4) and (7),

$$\boxed{A_1 = A_2}$$

② H_x continuous across interface:

Equating (5) and (8),

$$A_2 k_2 = -A_1 k_1$$

$$\Rightarrow A_1 k_2 = -A_1 k_1$$

$$\Rightarrow A_1 (k_1 + k_2) = 0$$

Note that, for SPP to exist, $k_1, k_2 > 0$.

$\therefore A_1 = 0$

This means, all the field components of TE SPP modes are zero.

Consequently, SPP cannot exist for TE mode!!

SPP dispersion relation (TM mode)

In our last lecture, we obtained the dispersion relation of SPP:

$$\beta = k_0 \sqrt{\frac{\varepsilon_1 \varepsilon_2}{(\varepsilon_1 + \varepsilon_2)}} \quad \text{——— (10)}$$

[Here, $\varepsilon_1(\omega) < 0$ and $\varepsilon_1 + \varepsilon_2 < 0$]

Using, $\varepsilon_1(\omega) = \left(1 - \omega_p^2/\omega^2\right)$ in $\varepsilon_1 + \varepsilon_2 < 0$, we obtained:

$$\omega < \frac{\omega_p}{\sqrt{1 + \varepsilon_2}} \quad \text{——— (11)}$$

This maximum frequency for which SPP can exist, is denoted by ω_{sp}.

So, we have $\boxed{\omega_{sp} = \frac{\omega_p}{\sqrt{1 + \varepsilon_2}}}$ ——— (12)

and $\omega < \omega_{sp}$.

The decay constant in dielectric, k_2, is given by:

$$k_2^2 = \beta^2 - k_0^2 \varepsilon_2 \quad \text{———— (13)}$$

We have, for a bound SPP mode at the surface:

$$k_2^2 > 0$$

$$\Rightarrow \beta^2 > k_0^2 \varepsilon_2 \Rightarrow \boxed{\begin{array}{l} \beta > k_0 \sqrt{\varepsilon_2} \\ \Rightarrow \beta > k_0 n_2 \end{array}}$$

———— (14)

$n_2 = \sqrt{\varepsilon_2}$ = refractive index of the dielectric medium.

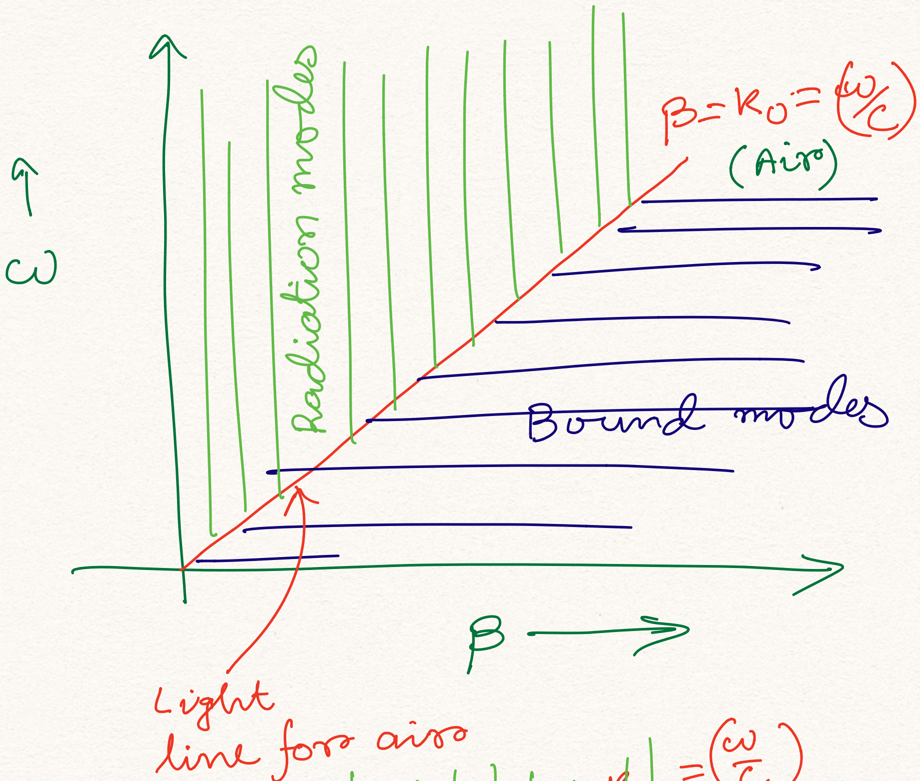
If the mode isn't bound to the surface i.e. radiating in the dielectric medium, then k_2 is imaginary.

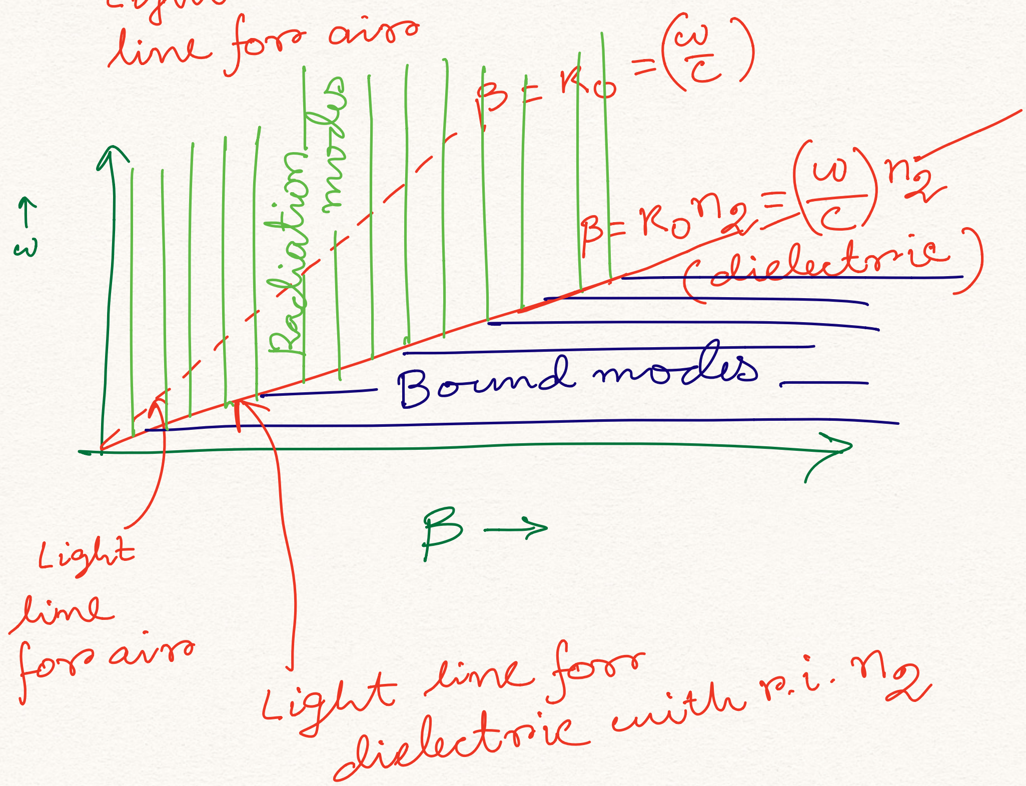
So, $k_2^2 < 0$ i.e. $\beta^2 < k_0^2 \varepsilon_2$

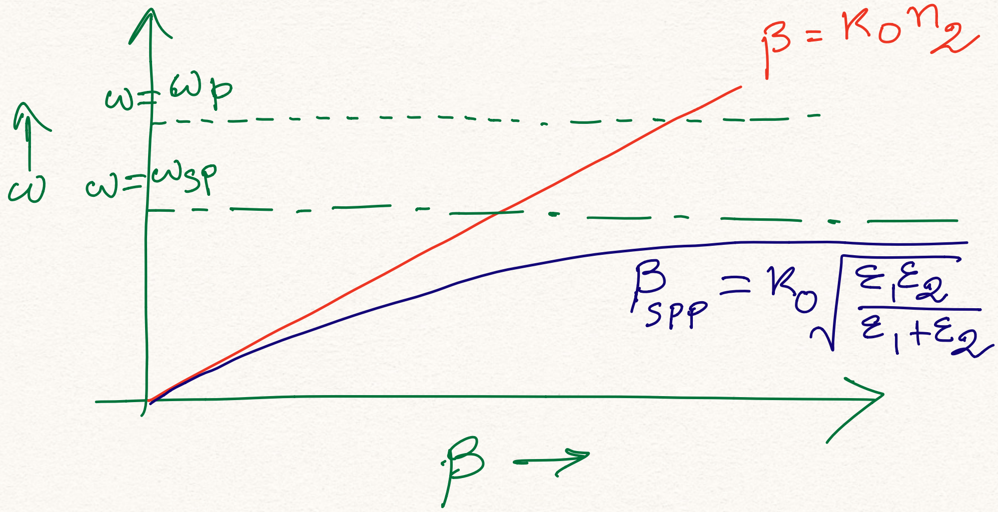
$$\boxed{\begin{array}{l} \Rightarrow \beta < k_0 \sqrt{\varepsilon_2} \\ \Rightarrow \beta < k_0 n_2 \end{array}}$$

———— (15)

Let's represent the bound and radiating modes pictorially:







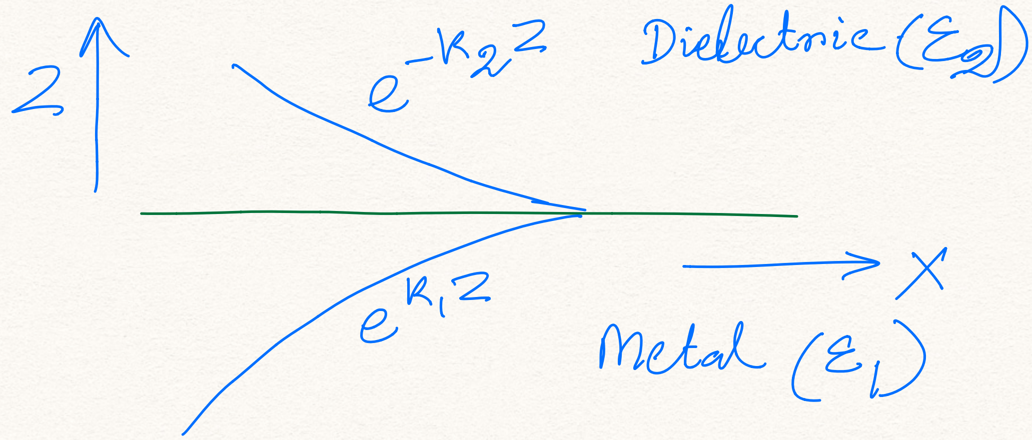
Note that, as $\omega \rightarrow \omega_{sp}$, i.e. $(\varepsilon_1 + \varepsilon_2) \rightarrow 0$, $\beta_{spp} \rightarrow \infty$.

ω_{sp} is known as the surface plasmon resonance (SPR) frequency.

The group velocity (v_g) i.e. the velocity of energy propagation becomes zero at SPR.

$$v_g = \left.\frac{\partial \omega}{\partial \beta}\right|_{\omega = \omega_{sp}} = 0$$

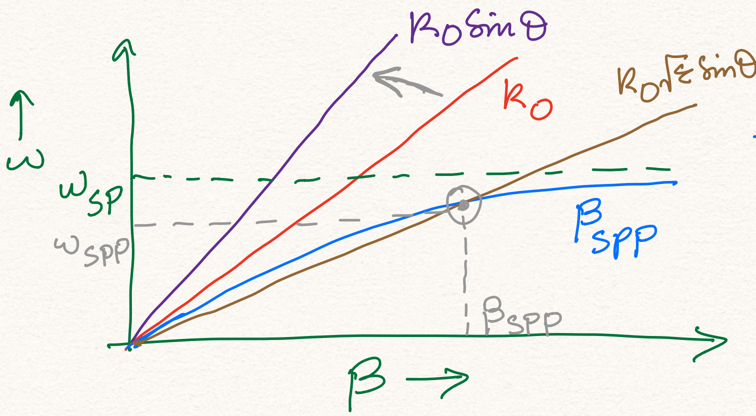
[this can be seen from the sharp rise in β_{spp} as $\omega = \omega_{sp}$ is approached].



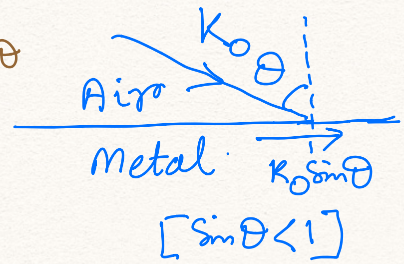
The factors $|z_2| = \frac{1}{k_2}$ and $|z_1| = \frac{1}{k_1}$ are known as the confinement lengths of SPP in dielectric and metal, respectively. These confinement lengths typically ranges from tens to few hundreds nano-meters.

Excitation of SPP

(Although we'll consider air-metal interface in this section, but the procedures illustrated here are general enough to be applied at any dielectric-metal interface).



(a)



(b)

When an EM wave is incident on an air-metal interface at an angle θ, the wave vector component parallel to the interface is $k_0\sin\theta$. Now, $\sin\theta < 1$. As a result, the $\beta = k_0\sin\theta$ line is shifted inside the light cone. In contrast, the β_{spp} lies outside the light-cone.

Thus, a simple configuration described above (and shown in fig. [b]), is insufficient to excite SPP at the interface.

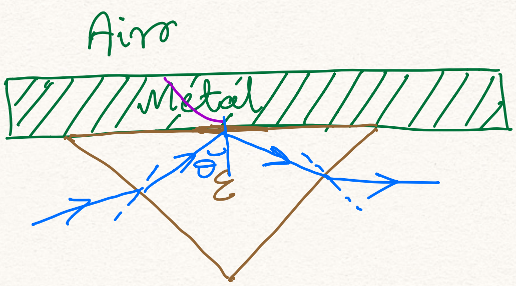
The trick is to use additional arrangements that are capable of producing wave vector $> k_0$ (i.e. shifts the dispersion outside the light-cone). To this end, we'll explore two mechanisms:

(a) prism coupling

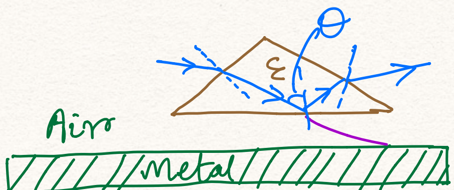
(b) grating coupling.

(a) <u>Prism coupling</u> –

Suppose, the prism has a dielectric constant ε (shown in fig. [c] and [d]).



(c): Kretschmann configuration.

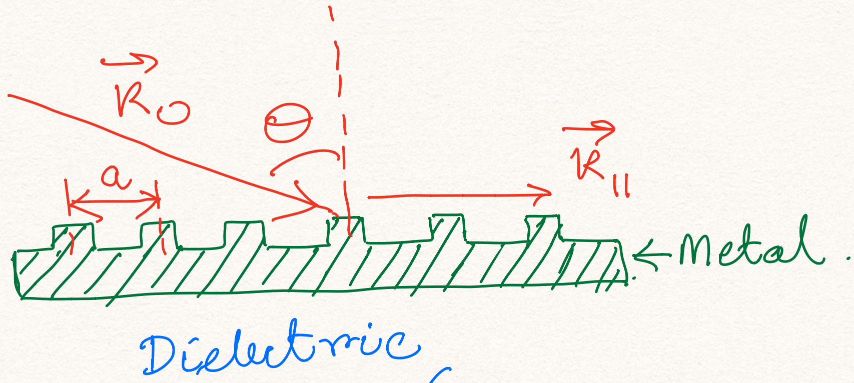


(d): Otto configuration.

In both (c) and (d), the prism produces a wavevector $k_0\sqrt{\varepsilon}\sin\theta$ after total internal reflection. Since $\sqrt{\varepsilon} > 1$, the dispersion is shifted outside to light-cone (shown in fig. [a]). This can then have an intersecti-on with the β_{SPP} and excite SPP at the interface at this point of intersection.

(b) Grating coupling

a = pitch of the grating.



Here $K_{\parallel} = K_0 \sin\theta + m \cdot \left(\frac{2\pi}{a}\right)$

$[m = \pm 1, \pm 2, \pm 3, ---]$

$\therefore K_{\parallel} > K_0$

The grating pitch can be chosen in a way so that $K_{\parallel} = \beta_{SPP}$ at the frequency of the incident wave.

If this condition is satisfied, the grating will excite SPP at the metal-dielectric interface.