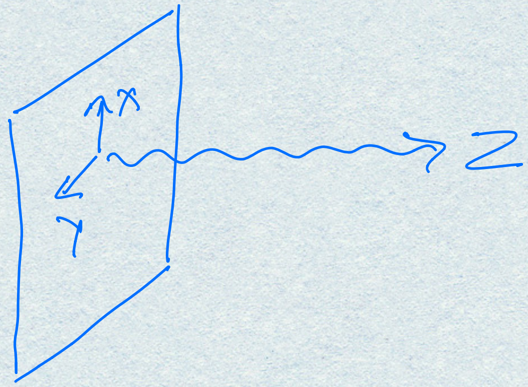
ECE545: Lecture 4

Recap:



EM wave propagation in 1D.

Assumption 1: Fields are independent of x and y and vary only along z

(i.e. constant in xy plane: uniform wave)

Helmholtz equation: $\frac{d^2\vec{E}}{dz^2} + k^2\vec{E} = 0$

$\left[k = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda},\right.$ known as: wave number$\left.\right]$

Assumption 2: The wave is polarized in x-direction. i.e. $\vec{E} = \hat{x} E_x$

$$\therefore \frac{d^2 E_x}{dz^2} + k^2 E_x = 0.$$

Soln: $E_x = \underbrace{Ae^{ikz}}_{\text{Travels along } -z} + \underbrace{Be^{-ikz}}_{\text{Travels along } +z}$

$$\therefore \mathcal{E}_x(z,t) = E_x(z)\, e^{i\omega t}$$

$$= Ae^{i(kz+\omega t)} + Be^{-i(kz-\omega t)}$$

Let's consider a purely forward travelling wave i.e.

$$\mathcal{E}_x(z,t) = Be^{-i(kz-\omega t)}$$

$$= B e^{i(\omega t - kz)}$$

This part is called amplitude part.

This part is called phase part.

We are interested to find out the condition such that at a given time t_1, the phase part is constant.

In other words,

$$\omega t_1 - kz = \theta_{constant}$$

$$\Rightarrow kz = \omega t_1 - \theta_{constant}$$

$$\Rightarrow z = \left(\frac{\omega t_1 - \theta_{constant}}{k}\right) \quad (1)$$

$$= \text{constant.}$$

So, all the points in space that have same phase at an instant $t = t_1$, are situated on a plane z = constant.

We'll call such waves as plane waves.

z = constant plane is also known as phase front.

(phase front is a surface consisting of points having equal phase)

Also, note from eqn. (1) that; as the time evolves, the equiphase surfaces/phase fronts progresses in the same direction as the direction of propagation of the wave.

Now we know the meaning of the term 'uniform plane wave'.

Apart from plane wave, one can have cylindrical wave, spherical wave.

(For example, if you drop a stone in water, the resulting wave front is cylindrical).

Transverse electromagnetic (TEM) wave:

Consider a plane wave with a wave vector $\vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$.

The electric field associated with this wave is:

$$\vec{E} = (\hat{x}E_{0x} + \hat{y}E_{0y} + \hat{z}E_{0z})\, e^{-i\vec{k}\cdot\vec{r}}$$

$$= (\hat{x}E_{0x} + \hat{y}E_{0y} + \hat{z}E_{0z})\, e^{-i(k_x x + k_y y + k_z z)} \qquad (2)$$

where, E_{0x}, E_{0y} and E_{0z} are constants.

Now, we know that the operators like $(\vec{\nabla}\times)$ and $(\vec{\nabla}\cdot)$ contain differential operators like $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$.

Suppose, we want to find out $(\vec{\nabla}\cdot\vec{E})$, where $\vec{E}$ corresponds to a plane wave (see eq (2)).

$$\therefore \vec{\nabla}\cdot\vec{E} = \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right)$$

$$\frac{\partial E_x}{\partial x} = \frac{\partial}{\partial x}\left\{E_{0x}e^{-i(k_x x + k_y y + k_z z)}\right\}$$

$$= -ik_x E_{0x} e^{-i(k_x x + k_y y + k_z z)}$$

$$= -ik_x E_x$$

operating $\frac{\partial}{\partial x}$ amounts to multiplication by $-ik_x$

Similarly, $\frac{\partial E_y}{\partial y} = -ik_y E_y$

$$\frac{\partial E_z}{\partial z} = -ik_z E_z$$

$$\therefore \vec{\nabla}\cdot\vec{E} = -i(k_x E_x + k_y E_y + k_z E_z)$$

$$= -i\vec{k}\cdot\vec{E}$$

Similarly, one can prove,

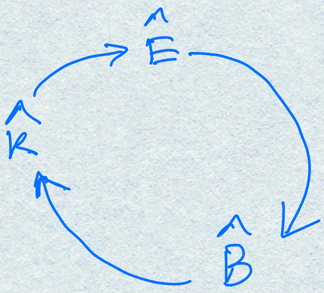
$$(\vec{\nabla}\times\ ) \equiv -i(\vec{k}\times\ )$$

Thus, the Maxwell's equations for a plane wave in a source free region becomes:

$$\vec{\nabla} \times \vec{E} = -i\omega\vec{B} \iff -i(\vec{K}\times\vec{E}) = -i\omega\vec{B}$$

$$\Rightarrow \boxed{(\vec{K}\times\vec{E}) = \omega\vec{B}} \quad \text{——(3)}$$

$$\vec{\nabla}\times\vec{B} = i\omega\mu\varepsilon\vec{E} \iff \boxed{(\vec{K}\times\vec{B}) = -\omega\mu\varepsilon\vec{E}} \quad \text{——(4)}$$



Direction of propagation is along $\hat{K}$ i.e. $(\vec{E}\times\vec{B})$

[Also $\vec{K}, \vec{E}, \vec{B}$ are normal to each other]

$$\vec{\nabla}\cdot\vec{E} = 0 \iff \boxed{\vec{K}\cdot\vec{E} = 0} \quad \text{——(5)}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \iff \boxed{\vec{K}\cdot\vec{B} = 0} \quad \text{——(6)}$$

$\therefore$ From eqn (3),

$$|\vec{K}|\,|\vec{E}| = \omega|\vec{B}|$$

$$\Rightarrow \frac{|\vec{E}|}{|\vec{B}|} = \frac{\omega}{|\vec{K}|} = c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\Rightarrow \frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\frac{\mu}{\varepsilon}}$$

TEM wave: Both $\vec{E}$ and $\vec{B}$ are perpendicular to the propagation direction $\vec{k}$.

Guided waves, TE & TM modes:

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