

Quality Induced Fingerprint Identification using Extended Feature Set

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Abstract—Automatic fingerprint identification systems use level-1 and level-2 features for fingerprint identification. However, forensic examiners utilize inherent level-3 details along with level-2 features. Existing level-3 feature extraction algorithms are computationally expensive to be used for identification. This paper presents a novel algorithm for fast level-3 feature extraction and identification. The algorithm starts with computing local image quality score using redundant discrete wavelet transform. A fast curve evolution algorithm is then used to extract four level-3 features namely, pores, ridge contours, dots, and incipient ridges. Along with level-1 and level-2 features, these level-3 features are used in a Delaunay triangulation based indexing algorithm. Finally, quality-based likelihood ratio is used to further improve the identification performance. Experiments conducted on a high resolution fingerprint database containing rolled, slap and latent images indicate that the algorithm offers significant benefits for fast fingerprint identification.

I. INTRODUCTION

With the advancement in sensor technology, high resolution fingerprint images (≥ 1000 dpi) provide multiple levels of features that can be used for identification. Existing automatic fingerprint identification systems (AFIS) use level-1 features for classification and level-2 minutia features for identification [14]. Forensic experts, on the other hand, use level-3 features such as pores, ridges, dots, and scars along with level-1 and level-2 features for matching latent fingerprints. It is an important research problem to identify unique and discriminating level-3 features and combine them with level-2 features for automatic identification. ANSI/NIST CDEFFS [4] and federal agencies are also investigating fingerprint features that can be used for recognition.

Researchers have proposed algorithms for level-3 feature based fingerprint verification (1:1 matching) [6], [10], [20]. These algorithms extract level-3 features and fusion with level-2 information is performed hierarchically or at match score level. However, these algorithms are computationally expensive to be used for identification. Several identification approaches have been proposed by the researchers but none of them use level-3 features [2], [3], [9], [14], [17]. Further, identifying latent fingerprints is also a major challenge for law enforcement agencies and existing algorithms do not provide satisfactory performance [11].

To address these challenges, this paper presents a novel algorithm for quality induced fast fingerprint identification using level-1, level-2 and level-3 features. The algorithm first

computes an image quality score that encodes the degree of irregularity present in the local regions. Next, using the curve evolution technique, a fast level-3 feature extraction algorithm is proposed for extracting pores, ridge contours, dots, and incipient ridges. Along with level-1 and level-2 features, the extracted level-3 features are utilized in a fingerprint indexing scheme for fast identification. Finally, quality-based likelihood ratio is used to further improve the identification performance. The performance is evaluated using a high resolution fingerprint database of 700 gallery images and 2900 probe images (rolled, slap, and latent images).

II. PROPOSED FINGERPRINT IDENTIFICATION ALGORITHM

The proposed algorithm computes the fingerprint image quality score, extracts fingerprint features and performs identification using an indexing scheme. Fig. 1 shows the steps involved in the proposed algorithm. This section presents each of these steps in detail.

A. Local Fingerprint Image Quality Assessment

In general, the performance of a fingerprint identification algorithm depends on the quality of the probe image. Image quality can vary locally in the fingerprint image and factors such as sensor noise, pressure, and wetness can degrade the quality. Therefore, it is important to compute the image quality locally and encode the edge information, smoothness and noise present in the fingerprint image. In this paper, we extend our previously proposed quality assessment algorithm [20] and design a local image quality assessment algorithm using Redundant Discrete Wavelet Transform (RDWT).

Let F be a high resolution fingerprint image. F is divided into blocks of size $n \times m$. For the k^{th} block, F^k , RDWT decomposition is computed for $j = 1, \dots, l$ levels.

$$[F_{A_j}^k, F_{H_j}^k, F_{V_j}^k, F_{D_j}^k] = RDWT(F^k) \quad (1)$$

where, $i = A, H, V, D$ represents the approximation, horizontal, vertical and diagonal subbands. For each block, the quality score is computed using (2).

$$q^k = \frac{a_A^k b_A^k + a_H^k b_H^k + a_V^k b_V^k + a_D^k b_D^k}{b_A^k + b_H^k + b_V^k + b_D^k} \quad (2)$$

where

$$a_i^k = \sum_{j=1}^l \ln \sqrt{\left(\frac{|\mu_{ij}^k - \sum_{j=1}^l \sum_{x,y=1}^{n,m} F_{ij}^k(x,y)|}{\sigma_{ij}^k} \right)^2} / nm \quad (3)$$

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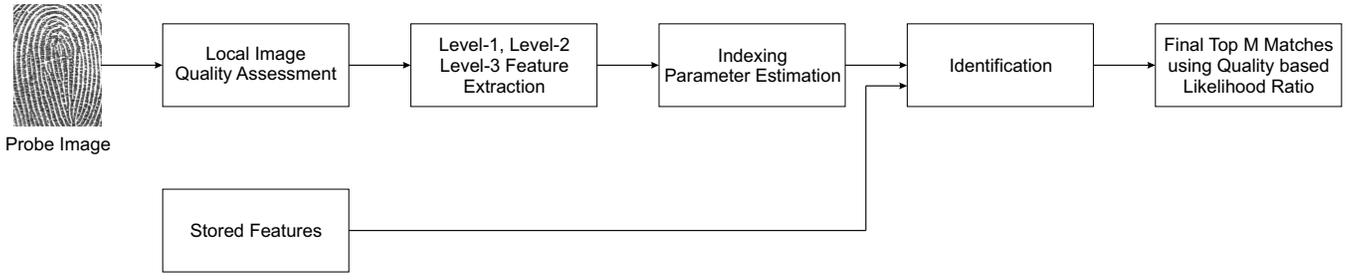


Fig. 1. Illustrating the steps involved in the proposed fingerprint identification algorithm.

and

$$b_i^k = \sum_{j=1}^l \ln \sqrt{\left(\frac{1}{1 + \sum_{x,y=1}^{n,m} \nabla F_{ij}^k(x,y)} \right)^2} / nm \quad (4)$$

Here, μ_{ij}^k and σ_{ij}^k are the mean and standard deviation of the RDWT coefficients of the i^{th} subband and the j^{th} level respectively, and ∇ denotes the gradient operator. Finally, the quality score, q^k , is normalized in the range of $[0, 1]$ using min-max normalization [18].

Multilevel RDWT decomposition provides the per-subband noise relationship and the spatial and frequency information which are helpful in computing the edge and noise information [8]. The algorithm also performs error normalization by incorporating the weight factor, b_i , and encoding the degree of irregularity in the local regions. The proposed local quality assessment algorithm yields a quality score for every local region. Thus, a quality score vector, \mathbf{q} , is computed for the complete fingerprint image and is used during identification.

B. Level-1, Level-2 and Level-3 Feature Extraction

As mentioned before, fingerprint images provide multiple levels of information that are useful for identification. Level-1 features or fingerprint patterns are extracted using the multi-channel approach [12] and are categorized into five classes: arch, tented arch, right loop, left loop, and whorl. Level-2 minutia features are extracted using the ridge tracing algorithm [13]. For level-3 features, we propose a fast feature extraction algorithm using the level-set based approach.

Level-3 feature extraction algorithm employs level-set based curve evolution [19] which begins with the energy functional [5]

$$E(C) = \int_0^1 (E_{in}C(q) + E_{out}C(q)) dq \quad (5)$$

where, $C(q) : [0, 1] \rightarrow R^2$ is a planar curve that can be applied on the image $F : [0, x] \times [0, y] \rightarrow R^+$ to detect feature boundaries. E_{in} and E_{out} are the energy functionals inside and outside the curve respectively. The energy functional can be further decomposed into (6) using the input image F .

$$E(C) = \alpha \int_{\Omega} \phi ||\bar{C}'|| dx dy + \beta \int_{in(C)} |F - C_1|^2 dx dy + \lambda \int_{out(C)} |F - C_2|^2 dx dy \quad (6)$$

where, α, β and λ are positive constants, Ω represents the image domain, ϕ is the stopping term, \bar{C} is the evolution curve, i.e., $\bar{C} = \{(x, y) : \bar{\psi}(x, y) = 0\}$, and C_1 and C_2 are the average pixel values inside and outside the curve respectively.

The basic concept of feature boundary extraction using level-set curve evolution is to initialize the contour $\bar{\psi}$ over the image and it converges to the boundaries based on the stopping term ϕ . This procedure is time consuming and depends on the initial $\bar{\psi}$. On a 1000 dpi fingerprint image, if the initial contour is initialized as a grid, level-set curve evolution algorithms require around 4-30 seconds to find the feature boundaries. For real world biometric systems, such high computational complexity is not pragmatic. To address this issue, we propose a scheme that utilizes the scale multiplication based Canny edge detection [1] for finding the initial $\bar{\psi}$ and then applies a two-cycle fast curve evolution algorithm with smoothness regularization [19]. The contour extraction algorithm is as follows:

Step 1: Scale multiplication based edge detection algorithm (that multiplies filter response at adjacent scales to enhance the edge structure and detects the edges as the local maxima) [1] is applied on the input fingerprint image. The output of edge detection is a binary image in which the edges are black and non-edge regions are white. The detected regions are very close to the exact feature boundaries but may also have spurious edges due to noise. To remove these spurious edges, every non-connecting edge of 1-2 pixels are removed (morphological operation).

Step 2: In the next step, the detected edges are used as the initial contour $\bar{\psi}$ and the two-cycle curve evolution algorithm [19] is applied in which the first cycle is data dependent and the second cycle is Gaussian filter band smoothing. Since the initial contour is very close to the exact feature boundaries, using the stopping term $\phi = 1/[1 + (\nabla F)^2]$, the curve evolution algorithm converges to the feature boundaries after only few iterations. Figs. 2(a)-(d) show an example of the edge detection and contour extraction procedure.

Once the contour is obtained, curve tracing [16] is used to

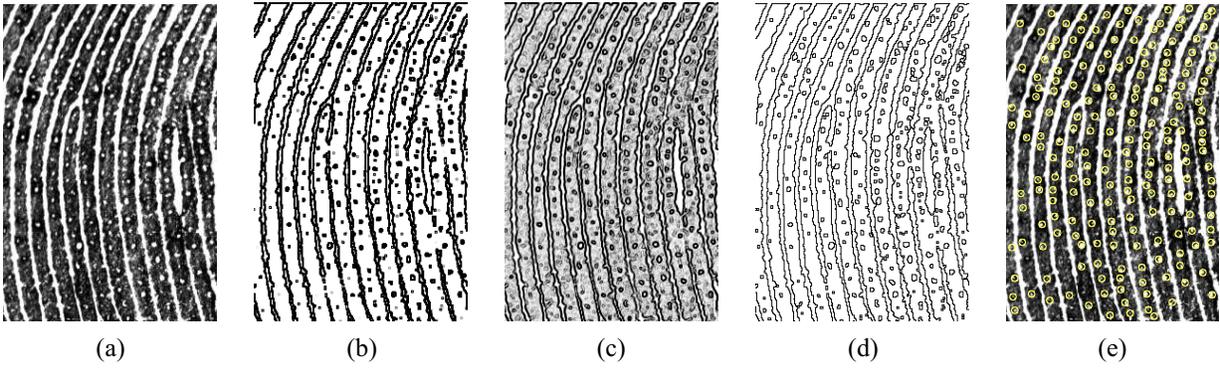


Fig. 2. Illustrating the intermediate images in the proposed feature extraction algorithm: (a) input fingerprint image, (b) edge detected image, (c) gradient image (stopping term), (d) fingerprint contour, and (e) detected pore features.

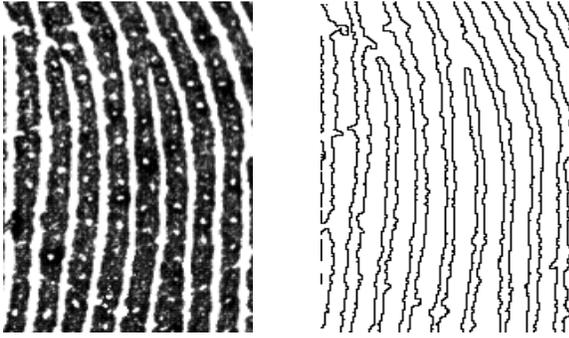


Fig. 3. Ridge contours extracted using the proposed algorithm.

classify the fingerprint features as pores, ridges and dots. The tracing technique uses the following model-based approach:

- 1) Every blob of size 2-40 pixels is classified as a pore and the center point of the blob is used as the pore feature.
- 2) The ridge contour (edge of a ridge) features are the x, y coordinates of the pixel and direction of the contour at that pixel.
- 3) Any blob of size less than $0.02''$ that does not lie on a ridge is marked as a dot and the corresponding x, y coordinates are stored.
- 4) If a blob or a ridge structure whose width is substantially thinner (less than 40% of average local ridge width) and size is greater than $0.02''$, then it is marked as an incipient ridge. Further, if the incipient ridge is a series of clearly separated dots, then these are marked as separate incipient ridges. The x, y coordinate of the endpoints and the distance are stored as the incipient ridge features.

These features and tracing procedure follow the standards defined by the ANSI/NIST CDEFFS [4]. As shown in Figs. 2-5, the tracing algorithm classifies the fine fingerprint features into pores, ridges, dots and incipient ridges depending on the shape and attributes.

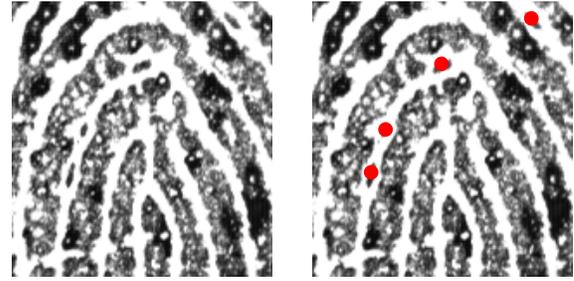


Fig. 4. x, y coordinates of the dots (red) are stored as dot features.

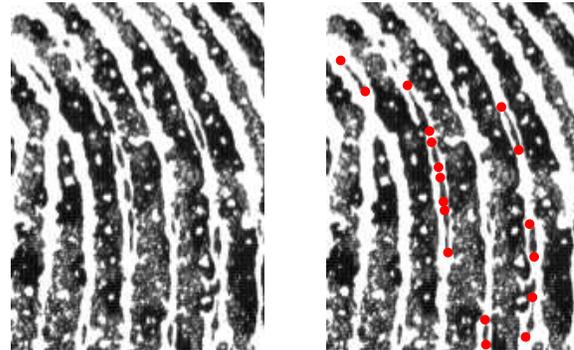


Fig. 5. End points of incipient ridges (red) are marked after tracing the contour.

C. Fingerprint Identification using Level-1, Level-2 and Level-3 Features

In literature, researchers have proposed several algorithms for fingerprint identification using level-2 features [14]. Recent identification algorithms utilize minutiae triplets and compute indexing parameters for fast and robust identification [2], [3], [17]. The challenge in incorporating level-3 features for identification lies in handling the huge amount of information. For example, there are hundreds of pores present in a rolled fingerprint and it is computationally very expensive to perform identification using the complete pore information. In this research, we propose a minutiae triplet based fingerprint identification algorithm that combines level-1, level-2, and level-3 features. The algorithm

starts with computing a Delaunay triangle using the minutia points. Each triangle in the Delaunay triangulation is used as a minutiae triplet. Since Delaunay triangulation can sustain the deformation and is not affected by local changes (due to noise or additional features), it can effectively encode fingerprint features for identification. After the triangulation, fingerprint features are encoded as the indexing parameters. The procedure for computing the indexing parameters is as follows:

- 1) Level-1 Features (**L**): The first indexing parameter is level-1 features that are used for broad classification. If a level-1 feature is present, then the parameter includes '1' followed by the type of feature; i.e., arch = 1, tented arch = 2, right loop = 3, left loop = 4, and whorl = 5. If no level-1 features are present, then '0' is stored.
- 2) Average Cosine Angle (**A**): Let θ_{min} and θ_{max} be the minimum and maximum angles in a minutiae triplet. Average cosine angle of a triplet is $A = \cos((\theta_{min} + \theta_{max})/3)$. It is computed for all the minutiae triplets and the average cosine angle vector, **A**, is used as the second indexing parameter.
- 3) Triangle Orientation Vector (**O**): Triangle orientation [3] is defined as $\theta = \text{sign}(z_{21} \times z_{32})$ where, $z_{21} = z_2 - z_1$ and $z_{32} = z_3 - z_2$, $z_i = x_i - jy_i$, x, y are the minutia coordinates and ($i = 1, 2, 3$). Triangle orientation is computed for all the triplets and **O** is used as the triangle orientation vector.
- 4) Ridge Curve Parameters (**R_C**): Each ridge can be parameterized as $y = ax^2 + bx + c$ where a, b , and c are the parameterized coefficients. Ridge curve parameter for a ridge is $\left[\frac{a}{a+b+c}, \frac{b}{a+b+c}, \frac{c}{a+b+c}\right]$. In a minutiae triplet, each minutiae is associated with a ridge. Therefore the ridge curve parameters of a minutiae triplet are $\left[\frac{a_i}{a_i+b_i+c_i}, \frac{b_i}{a_i+b_i+c_i}, \frac{c_i}{a_i+b_i+c_i}\right]$ where $i = 1, 2$, and 3 . Ridge curve parameter, R_C , is similarly computed for all the minutiae triplets.
- 5) Average Ridge Width (**R_W**): Average ridge width vector is computed by taking the average of the ridge width of each minutiae triplet.
- 6) Average Distance of k -Nearest Neighbor Pores (**P_D**): Average distance vector of the k -nearest neighboring pores is formed by computing the average distance of k -nearest neighboring pores of all the minutiae in a triplet.
- 7) Min-Max Distance between Minutia Points and k -Nearest Neighbor Pores (**P_M**): For every minutia point, the distances between minutiae and its k -nearest neighboring pores which are on the same ridge are computed. Out of these k distances, the minimum and maximum distances are used as the indexing parameters. Min-max distance vector (**P_M**) is then generated from all the minutiae present in the fingerprint image.
- 8) Dot (**D**): If dots are present in the fingerprint image, then the position of each dot is stored in the vector **D**.
- 9) Incipient Ridges (**I**): If incipient ridges are present in the fingerprint image, then the incipient ridge features

are stored in the vector **I**.

The indexing parameters (**L, A, O, R_C, R_W, P_D, P_M, D, I**) are used for identification. During search, first the level-1 feature **L** is used for broad classification, then the remaining parameters are used to find the top M matches. For matching the remaining indexing parameters, Mahalanobis distance between each indexing parameter of the gallery and probe images is computed. Mahalanobis distance between two vectors is defined as,

$$d(\mathbf{v}_1, \mathbf{v}_2) = \sqrt{(\mathbf{v}_1 - \mathbf{v}_2)^t S^{-1} (\mathbf{v}_1 - \mathbf{v}_2)} \quad (7)$$

where, S is the positive definite covariance matrix of \mathbf{v}_1 and \mathbf{v}_2 . Let $d(i)$ be the Mahalanobis distance associated with the individual indexing parameters and $i = 1, \dots, 8$ (for parameters **A, O, R_C, R_W, P_D, P_M, D**, and **I**). The indexing score, s , is computed by applying Sum rule to the octuple vector **d**, i.e,

$$s = \sum_{i=1}^8 d(i) \quad (8)$$

The indexing scores are computed by matching the probe image to all the gallery images. Finally, the indexing scores are sorted in ascending order and top M matches (in our experiments, $M = 50$) are retrieved as the possible matches.

D. Incorporating Quality Score to Attune Top M Matches

We next propose the use of quality-based likelihood ratio [15] to attune the top M matches. Quality score vector, **q**, for the probe image and indexing score, s , are given as input to the Gaussian Mixture Model based density estimation algorithm [21]. For a gallery-probe pair, let $g_{gen}(s, \mathbf{q})$ and $g_{imp}(s, \mathbf{q})$ be the genuine and impostor joint marginal densities respectively. The quality-based likelihood ratio for a gallery-probe pair is computed using (9),

$$S = \frac{g_{gen}(s, \mathbf{q})}{g_{imp}(s, \mathbf{q})} \quad (9)$$

Quality-based likelihood ratio is computed for the top M matches (gallery-probe pairs are generated by pairing the probe image with the top M gallery matches). Finally, these M values are sorted in the descending order.

III. EXPERIMENTAL EVALUATION

To evaluate the performance of the proposed fingerprint indexing algorithm, a high resolution fingerprint database is prepared which contains 1000 dpi images from 700 subjects. The database comprises following types of images:

- 1) Four rolled and four slap (dap) fingerprints pertaining to 550 subjects, captured using an optical scanner.
- 2) One high resolution rolled fingerprint and one high resolution latent fingerprint (1000 dpi) for 150 users are obtained for evaluating the performance on latent images. The latent fingerprint images are developed using powder method, iodine foaming, and ninhydrin and silver nitrate method such that level-3 features can be marked by the latent fingerprint examiners.

One high quality rolled fingerprint image of each subject is used as the gallery (gallery database thus consists of 700 images from 700 subjects). Further, one rolled and one slap fingerprint pertaining to 550 subjects are randomly selected for training and parameter estimation, and the remaining images are used as the probe. The train-test partitioning is performed 20 times for cross-validation (note that the gallery images and the latent probe images are fixed for every cross validation trial). Finally, the identification accuracies are computed for these trials and CMC plots are generated for top 20 matches. There are 1100 probe rolled fingerprints and 1650 probe slap fingerprints pertaining to 550 subjects and 150 probe latent images pertaining to 150 subjects. Rolled and slap images are automatically processed using the algorithms described in Section II. On the other hand, for latent images, three levels of features are marked by professional fingerprint examiners and feature coordinates are stored for computing indexing parameters.

Three sets of experiments are performed: 1. Matching rolled fingerprints, 2. Matching rolled fingerprints with slap fingerprints, and 3. Matching rolled fingerprints with latent fingerprints. In each of these experiments, identification is performed against the rolled fingerprint gallery database of 700 subjects. A comparison is performed with the Delaunay triangulation and clustering based fingerprint indexing algorithm [17] that uses level-2 minutiae and ridge features. This comparison is performed for only rolled to rolled and rolled to slap matching, and not rolled to latent matching because this algorithm [17] does not work with manually marked latent images. Further, to show the effectiveness of integrating image quality scores in identification, we compare the identification performance with and without quality scores i.e., identification using quality-based likelihood ratio (Equation 9) and with indexing scores only (Equation 8). Fig. 6 shows the CMC plots and Table I summarizes the identification accuracies of the proposed and existing algorithms. The key results and analysis of our experiments are summarized below.

- 1) The results for matching rolled to rolled fingerprints show that the rank 20 accuracy for the proposed algorithm is 99.35% which is 0.8% better than the proposed algorithm without integrating quality score and around 1.3% better than the existing identification algorithm. The high performance of all three approaches is due to the fact that rolled fingerprints provide ample amount of information that can be used for identification.
- 2) The experiments with rolled to slap fingerprint matching show a decrease of around 3.6% in the identification accuracy of existing algorithm. On the other hand, the proposed approach, both with and without quality scores, exhibit consistent performance. This shows that incorporating level-3 features in an identification system makes it stable and enables it to sustain variations due to missing level-2 information.
- 3) Experiments with latent fingerprints show the main advantage of the proposed algorithm. The algorithm com-

putes indexing parameters from the manually marked level-1, level-2, and level-3 fingerprint features, and matches it with the gallery images. Since there is a significant difference between the quality of latent and rolled images (obtained using an optical scanner), the rank 20 accuracy of 95% is considerably very high. This also shows that level-3 features (for rolled fingerprint) computed using the proposed algorithm and latent fingerprint features that are manually marked by the fingerprint examiners are in accordance, thereby providing a high identification accuracy.

- 4) It is also observed that level-3 features extracted using the proposed algorithm, generally, match with the visual ground truth for both rolled and slap images.
- 5) The experiments with quality score and without quality score show that incorporating the quality factor and computing the likelihood ratio to selectively adjust the indexing results improve the identification accuracy. This improvement is because the quality-based likelihood ratio achieves optimal performance by satisfying the Neyman-Pearson theorem [7].
- 6) We observe that among the level-3 features, dots are the most stable and pores (specifically pore shape and size) are the least stable features.
- 7) Computationally¹, the proposed curve evolution based level-3 feature extraction algorithm is fast and requires around 3 seconds for contour extraction and tracing. The algorithm with quality score requires 33-38 seconds for rank 20 identification which is around 6 seconds slower than the level-2 features based algorithm. However, the improvement in identification accuracy is notably high with a small computational overhead.

IV. CONCLUSIONS AND FUTURE WORK

This paper presents a level-3 feature extraction algorithm and subsequently incorporates it in a quality induced fingerprint identification scheme. First, RDWT-based local image quality assessment algorithm is proposed followed by the level-3 feature (pores, ridges and dots) extraction algorithm using a fast level-set curve evolution technique. These extracted level-3 features are combined with level-1 and level-2 features in a fingerprint indexing based identification scheme. Finally, quality-based likelihood ratio is used to further improve the top matches. The experiments are conducted on a high resolution database that contains rolled, slap and latent fingerprint images. The results demonstrate the effectiveness of the proposed algorithm with respect to both accuracy and speed. Indeed, the algorithm yields significantly high accuracy for latent images by utilizing three levels of fingerprint information. Currently, we are investigating ways to incorporate the quality score during indexing parameter computation.

¹Time is computed on a 2 GHz Pentium Duo Core processor with 2 GB RAM under MATLAB environment.

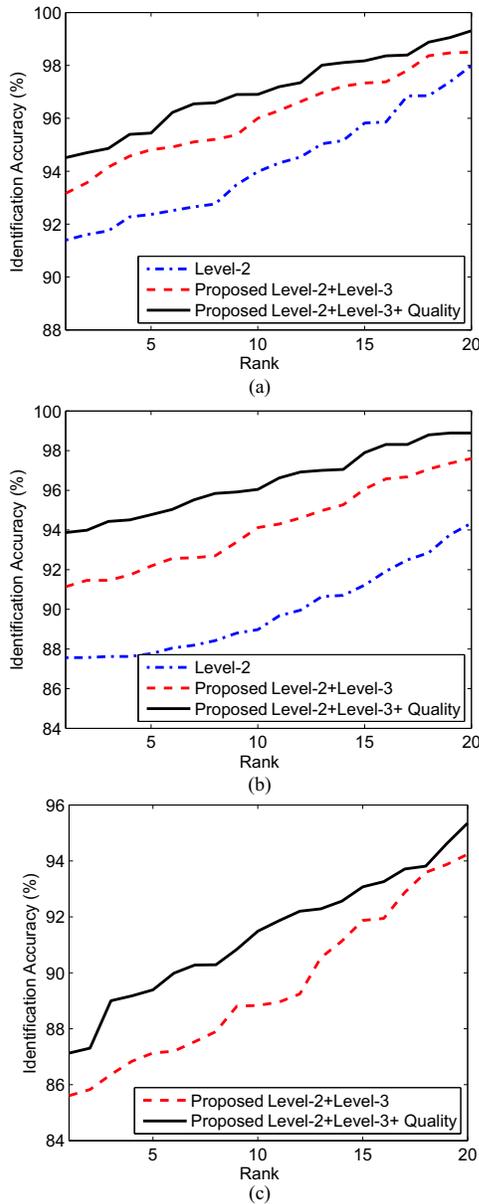


Fig. 6. CMC plots to evaluate the performance of the proposed algorithm for the three experiments. (a) Rolled to rolled, (b) Rolled to slap, and (c) Rolled to latent.

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TABLE I

IDENTIFICATION PERFORMANCE OF THE PROPOSED AND EXISTING ALGORITHMS FOR ALL THREE EXPERIMENTS.

Experiment	Algorithm	Rank 20 Accuracy (%)	Average Time (Seconds)
Rolled with rolled	Level-2 [17]	97.99	32.4
	Proposed algorithm level-2 + level-3	98.50	35.1
	Proposed algorithm level-2 + level-3 + quality	99.35	38.7
Rolled with slap	Level-2 [17]	94.34	30.2
	Proposed algorithm level-2 + level-3	97.60	33.8
	Proposed algorithm level-2 + level-3 + quality	98.89	36.5
Rolled with latent	Proposed algorithm level-2 + level-3	94.24	30.9
	Proposed algorithm level-2 + level-3 + quality	95.35	33.7

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