# Assignment 1 - Solutions

## Q1

Using Eqs. (2.59) and (2.9), we obtain

$$y(t) = h(t) * \delta_T(t) = h(t) * \left[ \sum_{n=-\infty}^{\infty} \delta(t - nT) \right]$$

$$= \sum_{n=-\infty}^{\infty} h(t) * \delta(t - nT) = \sum_{n=-\infty}^{\infty} h(t - nT) \tag{2.69}$$

(*a*) For $T = 3$, Eq. (2.69) becomes

$$y(t) = \sum_{n=-\infty}^{\infty} h(t - 3n)$$

which is sketched in Fig. 2-11(*a*).

(*b*) For $T = 2$, Eq. (2.69) becomes

$$y(t) = \sum_{n=-\infty}^{\infty} h(t - 2n)$$

which is sketched in Fig. 2-11(*b*).

(*c*) For $T = 1.5$, Eq. (2.69) becomes

$$y(t) = \sum_{n=-\infty}^{\infty} h(t - 1.5n)$$

which is sketched in Fig. 2-11(*c*). Note that when $T < 2$, the triangular pulses are no longer separated and they overlap.

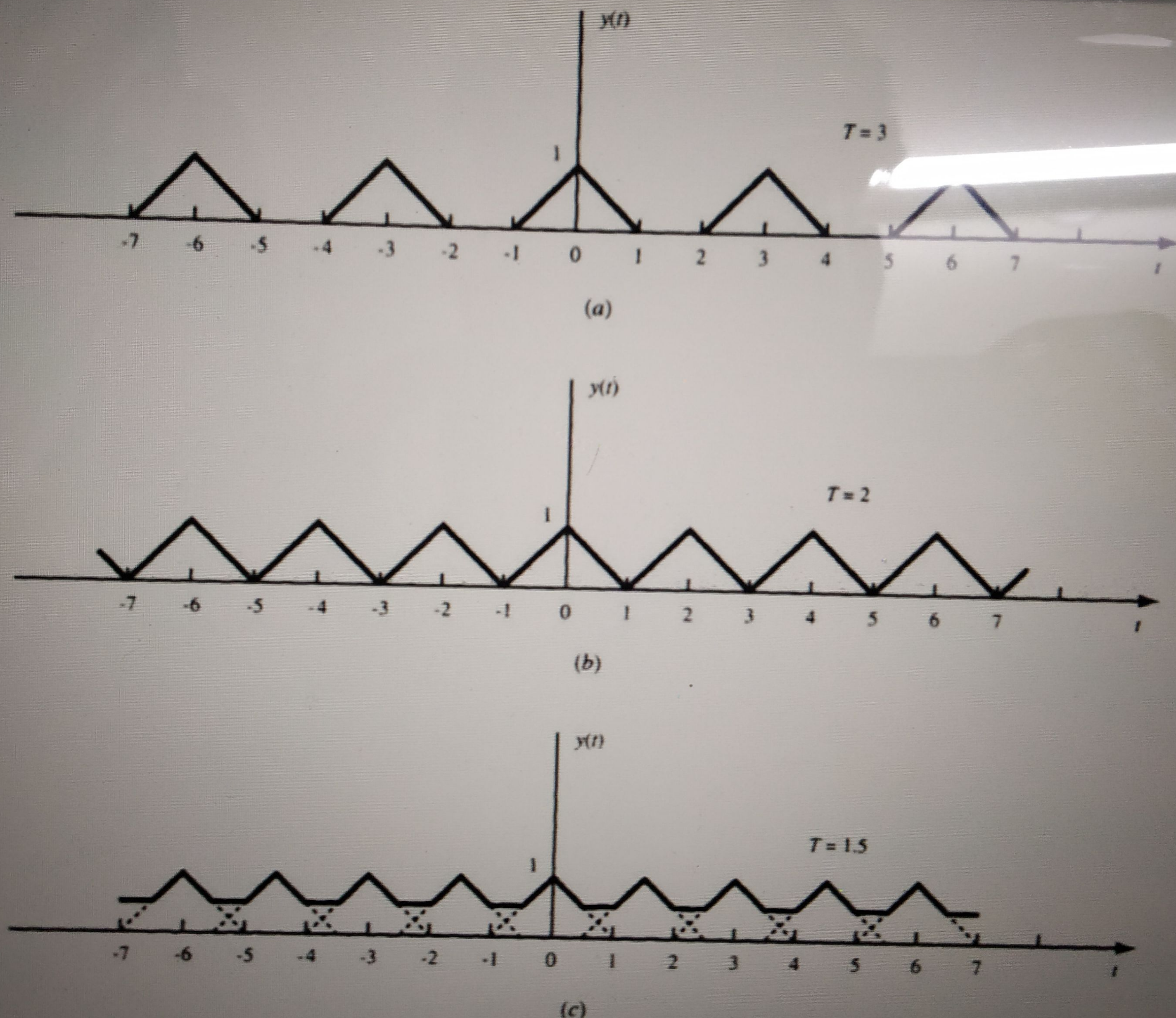

y(t)
T = 3
1
-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7
t
(a)
y(t)
T = 2
1
-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7
t
(b)
y(t)
T = 1.5
1
-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7
t
(c)

Fig. 2-11

## Q2

Consider a continuous-time LTI system whose step response is

$$s(t) = e^{-t}u(t)$$

Determine and sketch the output of this system to the input $x(t)$ shown in Fi
2-15($a$).

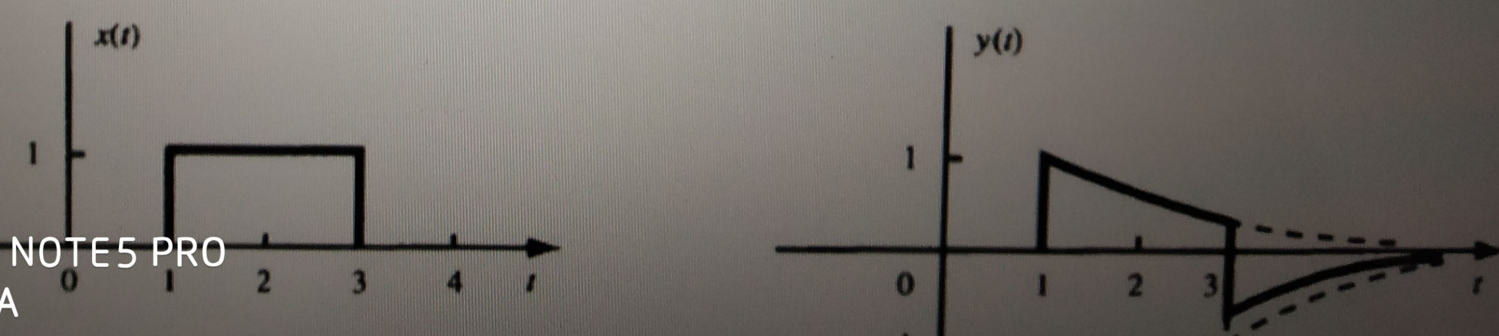
From Fig. 2-15($a$) the input $x(t)$ can be expressed as

$$x(t) = u(t-1) - u(t-3)$$

Since the system is linear and time-invariant, the output $y(t)$ is given by

$$y(t) = s(t-1) - s(t-3)$$
$$= e^{-(t-1)}u(t-1) - e^{-(t-3)}u(t-3)$$

which is sketched in Fig. 2-15($b$).






($a$)

**Q3**

From Eq. (2.41) the impulse response $h[n]$ is given by

$$h[n] = s[n] - s[n-1] = \alpha^n u[n] - \alpha^{n-1} u[n-1]$$
$$= \{\delta[n] + \alpha^n u[n-1]\} - \alpha^{n-1} u[n-1]$$
$$= \delta[n] - (1-\alpha)\alpha^{n-1} u[n-1]$$

**Q4**

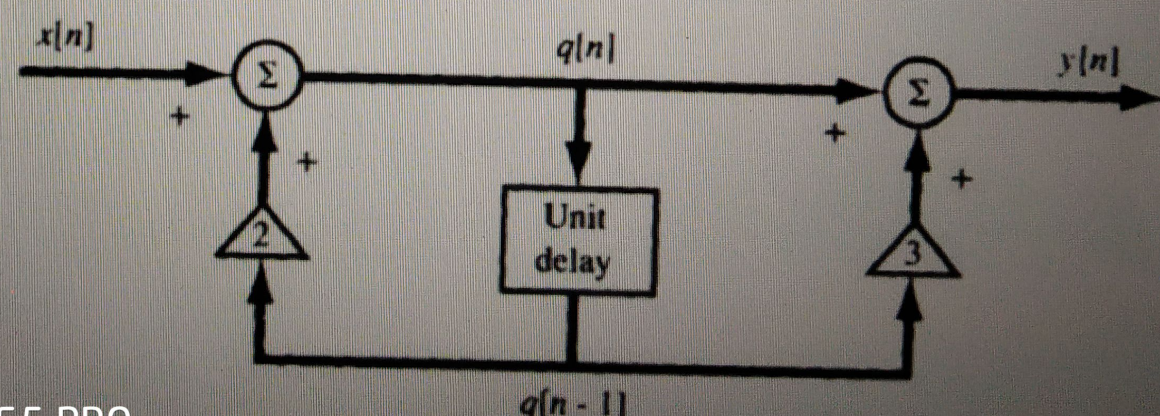


Fig. 2-30

Let the input to the unit delay element be $q[n]$. Then from Fig. 2-30 we see that

$$q[n] = 2q[n-1] + x[n] \tag{2.146a}$$

$$y[n] = q[n] + 3q[n-1] \tag{2.146b}$$

Solving Eqs. (2.146a) and (2.146b) for $q[n]$ and $q[n-1]$ in terms of $x[n]$ and $y[n]$, we obtain

$$q[n] = \tfrac{2}{5}y[n] + \tfrac{3}{5}x[n] \tag{2.147a}$$

$$q[n-1] = \tfrac{1}{5}y[n] - \tfrac{1}{5}x[n] \tag{2.147b}$$

Changing $n$ to $(n-1)$ in Eq. (2.147a), we have

$$q[n-1] = \tfrac{2}{5}y[n-1] + \tfrac{3}{5}x[n-1] \tag{2.147c}$$

Thus, equating Eq. (2.147b) and (Eq. (2.147c), we have

$$\tfrac{1}{5}y[n] - \tfrac{1}{5}x[n] = \tfrac{2}{5}y[n-1] + \tfrac{3}{5}x[n-1]$$

Multiplying both sides of the above equation by 5 and rearranging terms, we obtain

$$y[n] - 2y[n-1] = x[n] + 3x[n-1] \tag{2.148}$$

which is the required difference equation.

**1.46.** Express the signals shown in Fig. 1-41 in terms of unit step functions.

*Ans.* (*a*) $x(t) = \dfrac{t}{2}[u(t) - u(t-2)]$

(*b*) $x(t) = u(t+1) + 2u(t) - u(t-1) - u(t -$

<u>**Q8**</u>

1.31. (a) Note that $x_2(t) = x_1(t) - x_1(t-2)$. Therefore, using linearity we get $y_2(t) = y_1(t) - y_1(t-2)$. This is as shown in Figure S1.31.

(b) Note that $x_3(t) = x_1(t) + x_1(t+1)$. Therefore, using linearity we get $y_3(t) = y_1(t) + y_1(t+1)$. This is as shown in Figure S1.31.

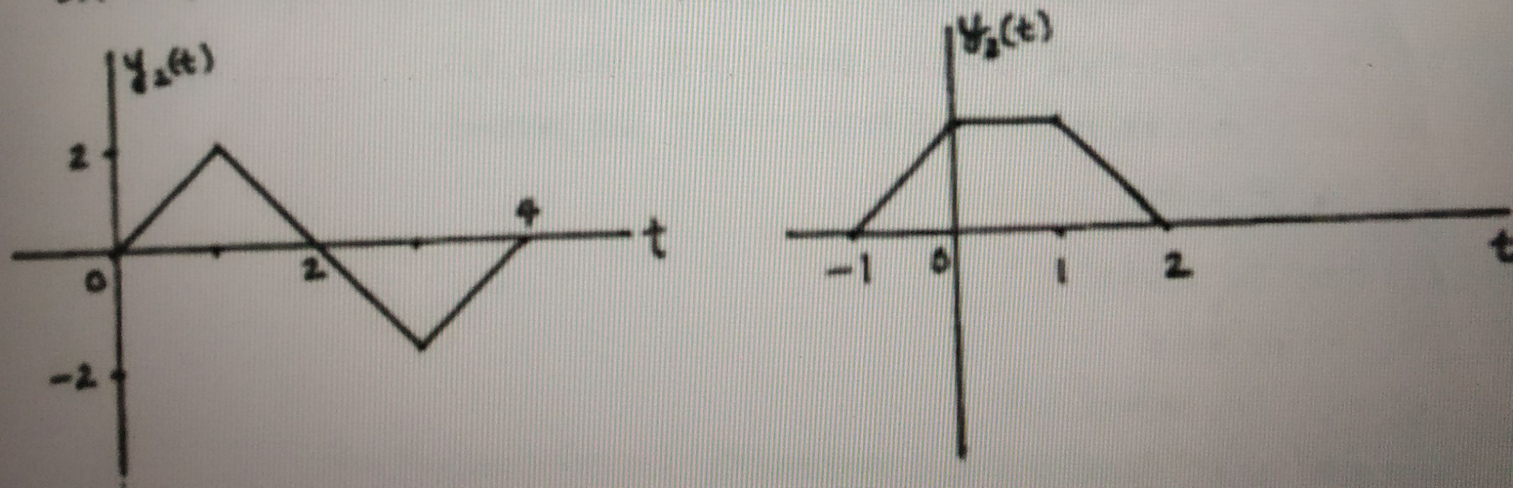



**Figure S1.31**

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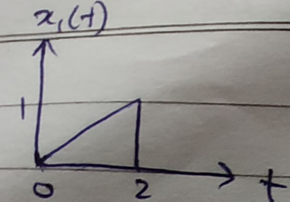
**A.6 a)**




$$x_1(t) = \begin{cases} t/2 & 0 \le t \le 2 \\ 0 & o/w \end{cases}$$

Let $x_1(t) \rightarrow y_1(t)$

$$y_1(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_1(t-\tau)\, d\tau \qquad \text{(Convolution with itself)}$$

Now, the following cases arise

① $t < 0$

There is no overlap b/w $x_1(\tau)$ and $x_1(t-\tau)$.

Hence, $y_1(t) = 0$

② $0 \le t \le 2$

$$y_1(t) = \int_0^t \left(\frac{\tau}{2}\right)\left(\frac{t-\tau}{2}\right) d\tau$$

$$= \frac{1}{4}\left[\frac{t\tau^2}{2} - \frac{\tau^3}{3}\right]_0^t$$

$$= \frac{1}{4}\left[\frac{t^3}{2} - \frac{t^3}{3}\right]$$

$$= \frac{1}{4} \times \frac{t^3}{6}$$

$$y_1(t) = \frac{t^3}{24}$$

③ $2 < t \le 4$

$$y_1(t) = \int_{t-2}^{2} \left(\frac{\tau}{2}\right)\left(\frac{t-\tau}{2}\right) d\tau$$

$$= \frac{1}{4}\left[\frac{t\tau^2}{2} - \frac{\tau^3}{3}\right]_{t-2}^{2}$$

$$= \frac{1}{4}\left[\frac{8}{3} - \frac{t(t-2)^2}{2} + \frac{(t-2)^3}{3}\right]$$

$$= \frac{1}{4}\left[2t - \frac{8}{3} - \frac{t(t^2+4-4t)}{2} + \frac{t^3+12t-6t^2-8}{3}\right]$$

$$= \frac{1}{4}\left[2t - \frac{8}{3} - \frac{t^3}{2} - 2t + 2t^2 + \frac{t^3}{3} + 4t - 2t^2 - \frac{8}{3}\right]$$

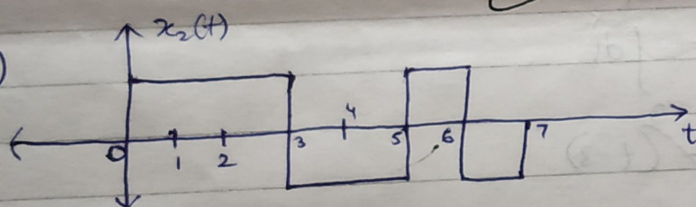
$$= \frac{1}{4}\left[-\frac{16}{3} + \frac{t^3}{6} + 4t\right]$$

$$y_1(t) = -\frac{4}{3} - \frac{t^3}{24} + t$$

(4) $t>4 \Rightarrow$ no overlap, $y_1(t)=0$

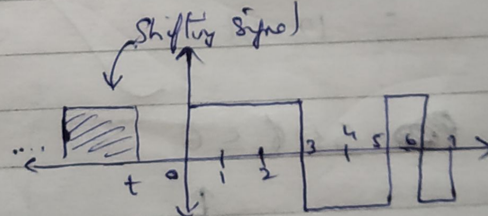
$$\therefore \quad y_1(t) = \begin{cases} \frac{t^3}{24} & 0 \le t \le 2 \\ -\frac{4}{3} - \frac{t^3}{24} + t & 2 \le t \le 4 \\ 0 & \text{o/w} \end{cases}$$

A.6 b)




b

$$x_2(t) = \begin{cases} 1 & 0 \le t \le 3 \\ 1 & 5 \le t \le 6 \\ 0 & 3 < t < 5 \\ 0 & 6 < t < 7 \end{cases}$$




$$y_2(t) = \int_{-\infty}^{\infty} x_2(\tau)\, x_2(t-\tau)\, d\tau$$

(Convolution with itself)

c) The following cases arise:-

(1) $t<0$

There is no overlap b/w the two signals, hence $Y(t)=0$.

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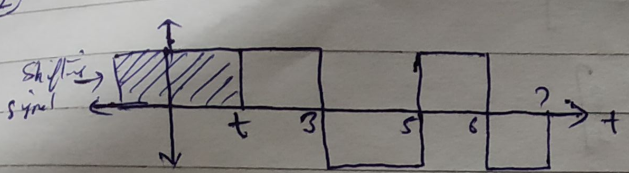
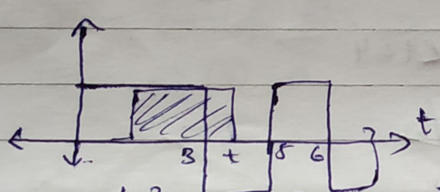
(2) $0 \le t \le 3$




We consider overlap till t.

$\Rightarrow y(t) = \int_0^t dt = t$

(3) $3 \le t \le 5$

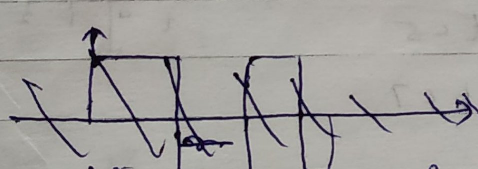



$y(t) = -\int_0^{t-3} dt + \int_{t-3}^{3} dt - \int_3^t dt$

$= -(t-3) + 3-(t-3) - (t-3)$

$= 12-3t.$

(4) $5 \le t \le 6$



$y(t) = \int_0^{t-5} dt - \int_{t-5}^{t-3} dt + \int_{t-3}^{3} dt + \int_3^5 dt + \int_5^t dt$

$= t-5 - (2) + (6-t) + 2 + t-5$

$= t-8$

$6 \le t \le 7$

$$y(t) = -\int_0^{t-6} dt + \int_{t-6}^{t-5} dt - \int_{t-8}^{3} dt + \int_3^{t-3} dt - \int_{t-3}^{5} dt + \int_5^{6} dt - \int_6^{t} dt$$

$$\cancel{-(t-6)} \cancel{+(1)} - (-t) + \cancel{(t-8)} - \cancel{(8-t)} + 1 - \cancel{(t-8)}$$

$$= \cancel{6-t} + 1$$

$$= -(t-6) + 1 - (8-t) + (t-6) - (8-t) + 1 - (t-6)$$

$$= t-8$$

(6) $7 \le t \le 8$

$$y(t) = -\int_{t-7}^{t-6} dt + \int_{t-6}^{t-5} dt - \int_{t-5}^{3} dt + \int_3^{t-3} dt - \int_{t-3}^{5} dt + \int_5^{6} dt - \int_6^{7} dt$$

$$= \cancel{1} + \cancel{1} - (8-t) + (t-6) - (8-t) + \cancel{1} - \cancel{1}$$

$$= 3t - 22$$

(7) $8 \le t \le 9$

$$y(t) = -\int_{t-7}^{t-6} dt + \int_{t-6}^{3} dt - \int_{3}^{t-5} dt + \int_{t-5}^{5} dt - \int_5^{t-3} dt + \int_{t-3}^{6} dt - \int_6^{7} dt$$

$$= -1 + 9 - t + 8 - t + 10 - t + \cancel{8-t} + 9 - t - 1$$

$$= -5t + 42.$$

(8) $9 \le t \le 10$

$$y(t) = -\int_{t-7}^{3} dt + \int_3^{t-6} dt - \int_{t-6}^{t-5} dt + \int_{t-5}^{5} dt - \int_5^{6} dt + \int_6^{t-3} dt - \int_{t-3}^{7} dt$$

$= t-10+t-9-1+10-t-1+t-9+t-10$

$= 3t-30$

(9) $10 \le t \le 11$

$$y(t) = \int_{t-7}^{t-6} dt - \int_{t-6}^{5} dt + \int_{5}^{t-5} dt - \int_{t-6}^{6} dt + \int_{6}^{7} dt$$

$= 1+t-11+t-10+t-11+1$

$= 3t-30$

(10) $11 \le t \le 12$

$$y(t) = \int_{t-7}^{5} dt - \int_{5}^{t-6} dt + \int_{t-6}^{6} dt - \int_{6}^{t-5} dt + \int_{t-3}^{7} dt$$

$= 12-t+11-t+12-t+11-t+12-t$

$= -5t+58$

(11) $12 \le t \le 13$

$$y(t) = -\int_{t-7}^{6} dt + \int_{6}^{t-6} dt - \int_{t-6}^{7} dt$$

$= t-13+t-12+t-13$

$= 3t-38$

(12) $13 \le t \le 14$

$$y(t) = \int_{t-7}^{7} dt = 14-t.$$

7

(i) $t > 14$

No overlap, $y(t) = 0$.

Hence,

$$y(t) = \begin{cases} t & 0 \le t \le 3 \\ 12 - 3t & 3 < t \le 5 \\ t - 8 & 5 < t \le 7 \\ 3t - 22 & 7 < t \le 8 \\ -5t + 42 & 8 < t \le 9 \\ 3t - 30 & 9 < t \le 11 \\ -5t + 58 & 11 < t \le 12 \\ 3t - 38 & 12 < t \le 13 \\ -t + 14 & 13 < t \le 14 \\ 0 & o/w \end{cases}$$

(1).

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System 1 : $y[n] = \begin{cases} x[n/2] & : n \text{ is even} \\ 0 & : n \text{ is odd} \end{cases}$

System 2 : $y[n] = x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2] \quad \forall n$

System 3 : $y[n] = x[2n] \quad \forall n$

$x[n] \rightarrow \boxed{S1} \rightarrow \boxed{S2} \rightarrow \boxed{S3} \rightarrow Y[n]$

Let us assume that input of system 2 is $x_2[n]$ (output of S1)
input of system 3 is $x_3[n]$ (output of $S_2$)

Now, we can set up the following equations

$Y[n] = x_3[2n]$ — (1)

$x_3[n] = x_2[n] + \frac{1}{2}x_2[n-1] + \frac{1}{4}x_2[n-2]$

$\Rightarrow x_3[2n] = x_2[2n] + \frac{1}{2}x_2[2n-1] + \frac{1}{4}x_2[2n-2]$ — (2)

$x_2[n] = \begin{cases} x[n/2] & : n \text{ is even} \\ 0 & : n \text{ is odd} \end{cases}$

$\Rightarrow x_2[2n] = x[n]$ — (3)

$x_2[2n-1] = x[n-1]$ — (4)

$x_2[2n-2] = x[n-2]$ — (5)

Using equations (2), (3), (4), (5)

$\Rightarrow x_3[2n] = x[n] + \frac{1}{4}x[n-2] + \frac{1}{2}x[n-1]$ — (6)

Using equations (1), (6)

$Y[n] = x[n] + \frac{1}{4}x[n-2] + \frac{1}{2}x[n-1]$

Now, let us verify whether this system is LTI or not.

$Y_1[n] = x_1[n] + \frac{1}{4}x_1[n-2] + \frac{1}{2}x_1[n-1]$

$Y_2[n] = x_2[n] + \frac{1}{4}x_2[n-2] + \frac{1}{2}x_2[n-1]$

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Let $x_3(t) = a x_1(t) + b x_2(t)$

$y_3(t) = x_3$

Let $x_3[n] = a x_1[n] + b x_2[n]$

$$Y_3[n] = x_3[n] + \frac{1}{4} x_3[n-2] + \frac{1}{2} x_3[n-1]$$

$$= a x_1[n] + b x_2[n] + \frac{1}{4} x_1[n-2] \cdot a + \frac{1}{4} b x_2[n-2] + \frac{1}{2} a x_1[n-1] + \frac{1}{2} b x_2[n-1]$$

$$Y_3[n] = a Y_1[n] + b Y_2[n]$$

$\therefore$ $Y[n]$ is linear

Also, $y_1[n] = x_1[n] + \frac{1}{4} x_1[n-2] + \frac{1}{2} x_1[n-1]$

$$x_2[n] = x_1[n-n_0]$$

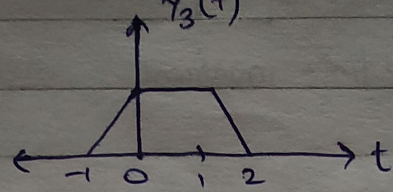
$$Y_2[n] = x_2[n] + \frac{1}{4} x_2[n-2] + \frac{1}{2} x_2[n-1]$$

$$Y_2[n] = x_1[n-n_0] + \frac{1}{4} x_1[n-n_0-2] + \frac{1}{2} x_1[n-n_0-1]$$

$$Y_2[n] = Y_1[n-n_0]$$

$\therefore$ Shift invariant

1




1



2.9 a) $(2+\sin t)\, x(t) = y(t)$.

i) For linearity

$$y_1(t) = (2+\sin t)\, x_1(t)$$
$$y_2(t) = (2+\sin t)\, x_2(t)$$

Now, let $(2+\sin t)\, x_3(t) = (2+\sin t)\, a x_1(t) + (2+\sin t)\, b x_2(t)$

$$x_3(t) = a x_1(t) + b x_2(t)$$
$$y_3(t) = (2+\sin t)\, x_3(t)$$
$$= (2+\sin t)\left[a x_1(t) + b x_2(t)\right]$$
$$= (2+\sin t)\, a x_1(t) + (2+\sin t)\, b x_2(t)$$
$$y_3(t) = a y_1(t) + b y_2(t)$$

$\therefore$ $y(t)$ is linear

ii) Time invariance

$$y_1(t) = (2+\sin t)\, x_1(t)$$
$$x_2(t) = x_1(t-t_0)$$
$$y_2(t) = (2+\sin t)\, x_2(t)$$
$$= (2+\sin t)\, x_1(t-t_0).$$

However, $y_1(t-t_0) = (2+\sin(t-t_0))\, x_1(t-t_0)$

$\therefore$ $y_2(t) \neq y_1(t-t_0)$

Hence not time invariant

Spiral

9 b). $Y(t) = x(2t)$

i) Linearity

$Y_1(t) = x_1(2t)$

$Y_2(t) = x_2(2t)$.

let $x_3(2t) = a x_1(2t) + b x_2(2t)$

$Y_3(t) = x_3(2t)$

$= a x_1(2t) + b x_2(2t)$

$Y_3(t) = a Y_1(t) + b Y_2(t)$

$\therefore$ $Y(t)$ is linear

ii) Time invariance

$Y_1(t) = x_1(2t)$

$x_2(2t) = x_1(2(t)-t_0)$

$Y_2(t) = x_2(2t)$

$= x_1(2t - t_0)$.

$Y_1(t-t_0) = x_1(2(t-t_0))$

$= x_1(2t - 2t_0)$.

$\therefore Y_2(t) \neq Y_1(t-t_0)$

Hence, not time invariant

c) $Y[n] = \sum_{n=-\infty}^{\infty} x(k)$

i) Linearity

$Y_1[k] = \sum_{n=-\infty}^{\infty} x_1[k]$

$Y_2[k] = \sum_{k=-\infty}^{\infty} x_2[k]$

$Y_3[n] = a Y_1[n] + b Y_2[n] = \sum_{k=-\infty}^{\infty} (a x_1[k] + b x_2[k])$

$= \sum_{k=-\infty}^{\infty} x_3[k]$

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Now, $\sum_{k=-\infty}^{\infty} x[k] = Y[k]$

$$\sum_{k=-\infty}^{\infty} x_1[k-k_0] = \cancel{\Sigma}\, x_2[k] \Rightarrow Y_2(k]$$

$$= \sum_{k=-\infty}^{\infty} x_2[k]$$

$$= \sum_{k=-\infty}^{\infty} x_1(k-k_0]$$

$$= y[k-k_0]$$

$\therefore$ Time invariant