Lecture 1

IIIT Delhi

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Course Information: PRP

- *Class Hours* : Wed: 11:30 to 12:45, Fri: 10:00 to 11:15
- *Office Hours*: Thursday: 1:00 to 2:00 P.M
- *TAs*: TBD
- *Website*: `www.iiitd.edu.in/~praveshb/teaching.html`
- *Textbooks*:
  1. *An Introduction to Probability Models* by Sheldon Ross
  2. *Probability and Stochastic Processes : a friendly introduction for electrical and computer engineers*, by Roy Yates and David Goodman
  3. *An Introduction to Probability*, Volume 1, by William Feller.

Grading Scheme

- *Quiz*: 4×10
  2 before Mid sem.
- *Mid Sem*: 25
- *End Sem*: 35
- *Assignments*: Ungraded
  Assignments will be checked by peers. Those questions not solved by anybody will be solved by me.
- *Effort*: 9 hrs per week. 3 (lectures) + 4 (exercises) + 2 (discussion)

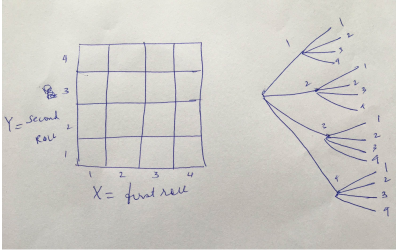
Maximising effort can lead to maximisation of grades.

Sample Space

- Set of possible outcomes during an experiment
- Lists (Set):
  - Mutually Exclusive
  - Collectively Exhaustive
- Examples:
  - Discrete: Coin, Dice
  - Continuous:
- Choice of sample space is the key – depends on the problem.

Example of a Discrete Sample Space

- Two rolls of a tetrahedral dice
- Two ways of looking at the sample space



Example of a Continuous Sample Space

A dart board !

$$\Omega = \{x, y \,|\, 0 < x < 1,\, 0 < y < 1\}$$

Event Space

- **Event**: A subset of a sample space
- Which outcome is more likely ?
- Assign probabilities to events
- What is the probability of a dart hitting a certain point in the board?

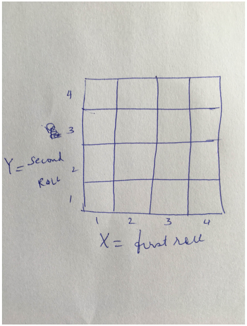
Sample Space and Event Space: Examples

- A family with 4 children. Event: The first and the fourth are boys.
- An elevator carries two persons and stops at three floors. Event: they get off at different floors.
- Experiment: Coin tosses. Sample space of encountering heads for the first time.
- Temperature at 11:00 A.M today! Event: Hot (temp above 30 deg).

Probability Axioms

- Assign probability to subsets of sample space
- Axioms:
  1. **Non negativity**: $P(A) \geq 0$
  2. **Normalisation**: $P(\Omega) = 1$
  3. **Additivity**: $P(A \cup B) = P(A) + P(B)$, if, $A \cap B = \emptyset$
- Do we need an extra axiom?
- What about the probability of union of three events?
- Can we take the argument to any set?

Probability Axioms: Example



- Let every possible outcome have probability of $\frac{1}{6}$
  1. $P(X, Y)$ is $(1,1)$ or $(1,2)$
  2. $P(X = 1)$ is
  3. $P(X + Y)$ is odd is
  4. $P(\min(X, Y) = 2)$ is

Probability Axioms: Examples

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Hence, $P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$

Discrete Uniform Law

- Let all outcomes are equally likely
- Then, $P(A) = \frac{\text{number of elements of A}}{\text{total number of sample points}}$
- Computing probabilities is the same as counting
- Example: Coints, fair dice etc.

Probability Law:Example with infinite sample space

- Sample space: $\{1, 2, \ldots\}$
- Given $P(n) = \frac{1}{2^n}$
- Find $P(\text{outcome is even})$
- $P(2, 4, 6, \ldots) = P(2) + P(4) + \cdots = \frac{1}{2^2} + \frac{1}{2^4} \ldots = \frac{1}{3}$

Probability Law: Continuous Space

A dart board !

$$\Omega = \{x, y \,|\, 0 < x < 1,\, 0 < y < 1\}$$

- What is the probability that the dart was hit at the lower half of the square?
- Counting the points not enough !
- Uniform Law == Area !
- Notion of measure of an event!