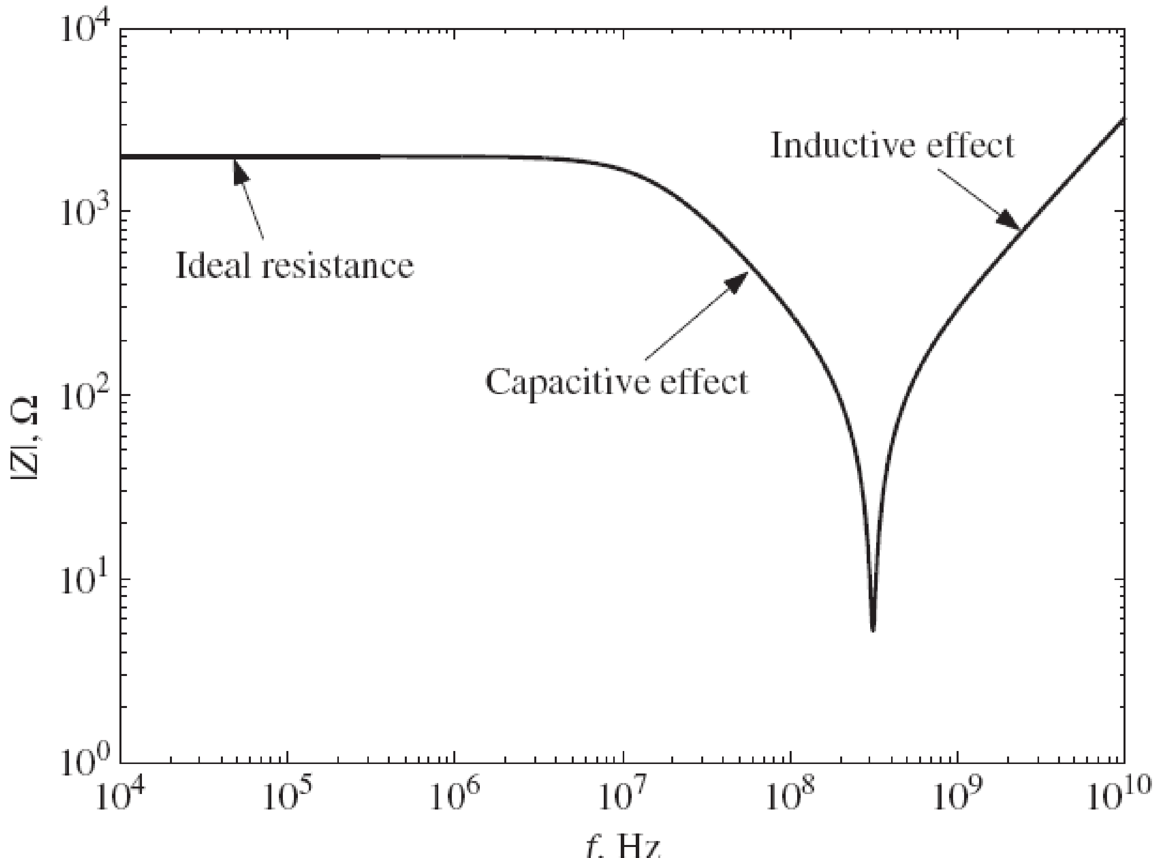
Lecture-2 Date: 07.08.2014

- Review – Lecture 1
- Transmission Lines (TL) Introduction
- TL Equivalent Circuit Representation
- Definition of Some TL Parameters
- Examples of Transmission Lines

Review – Lecture 1

What is the reason for following behavior of a 2000Ω thin-film resistor?

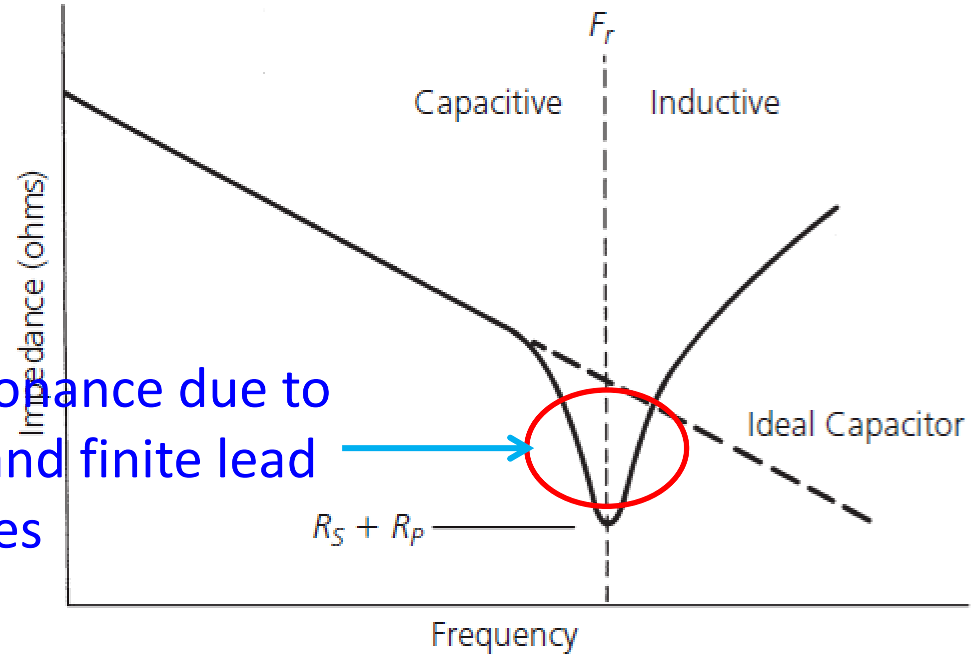


HW # 0

Demonstrate using either ADS or MATLAB

Review – Lecture 1

Capacitors at High Frequencies:



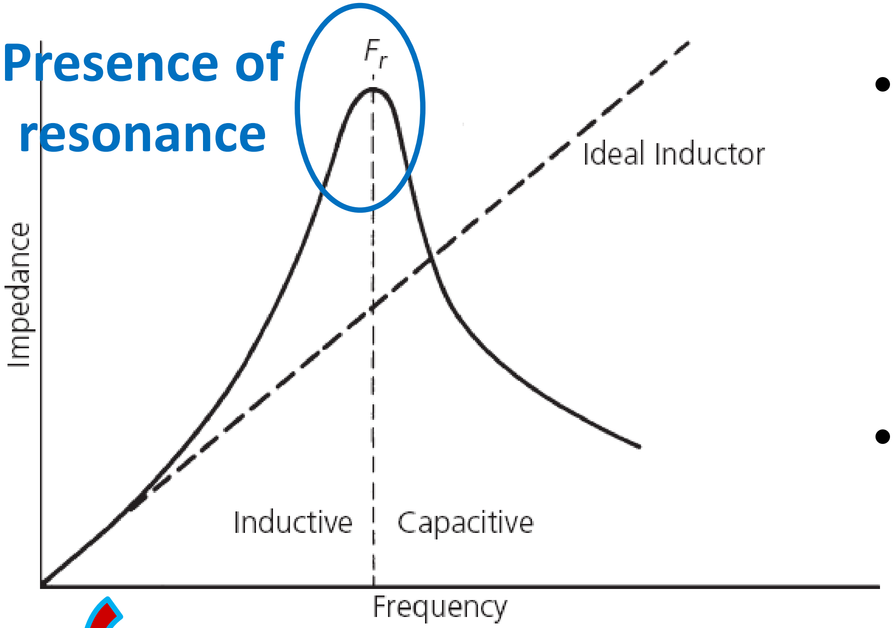
Presence of resonance due to dielectric loss and finite lead wires

- Above F_r, the capacitor behaves as an inductor.
- In general, larger-value capacitors tend to exhibit more internal inductance than smaller-value capacitors.
- Therefore, it may happen that a $0.1\mu F$ may not be as a good as a $300pF$ capacitor in a bypass application at $250\ MHz$.
- The issue is due to significance of lead inductances at higher frequencies.

Review – Lecture 1

Inductors at High Frequencies:

Presence of resonance

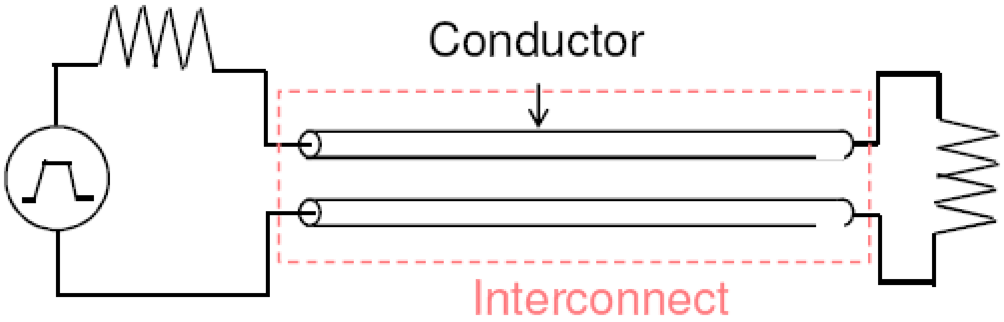


- Initially the reactance of inductor follows the ideal but soon departs from it and increases rapidly until it reaches a peak at the inductor's resonant frequency (F_r). **Why?**
- Above F_r, the inductor starts to behave as a capacitor.

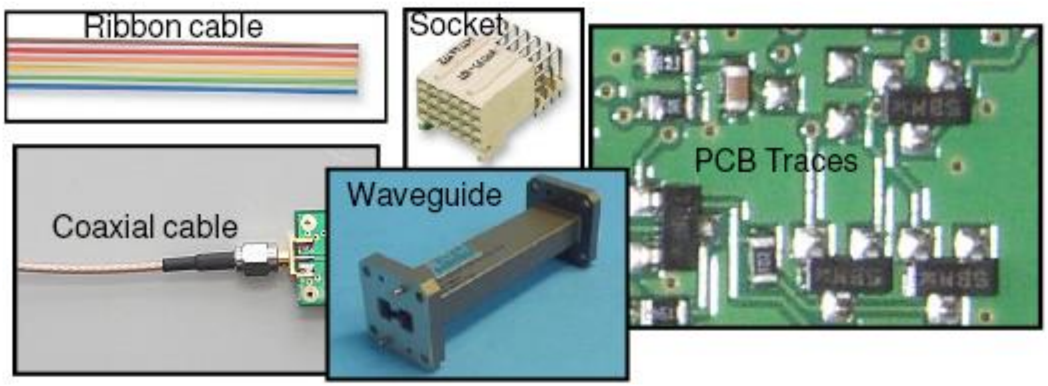
Implement this in MATLAB or ADS → **HW#0**

Transmission Line

- It is a standard practice to use **metallic conductors** for transporting electrical energy from one point of a circuit to another. These conductors are called **interconnects**.

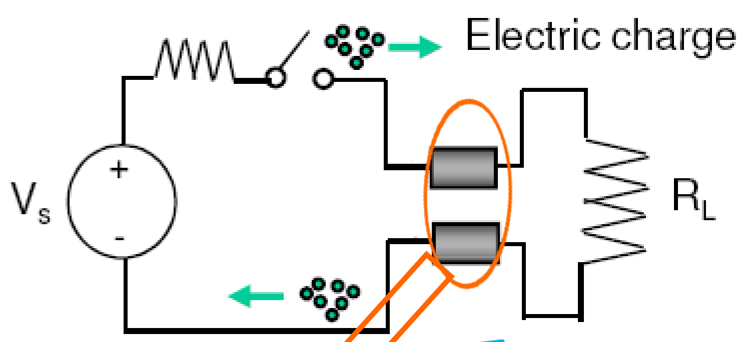


- Therefore **cables**, **wires**, **conductive tracks** on printed circuit boards (**PCBs**), **sockets**, **packaging**, **metallic tubes** etc are all examples of interconnect.



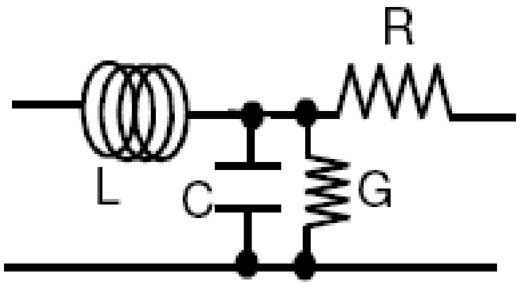
Transmission Line (contd.)

Static EM Field changes uniformly i.e, when field at one point increases, field at other location also increases



EM Field is static

Model of Interconnect

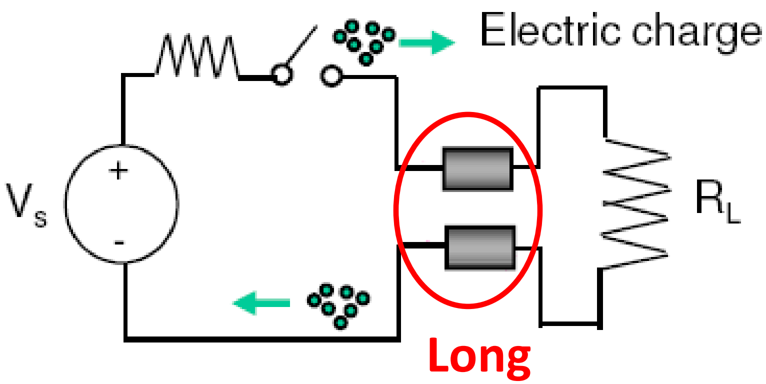


It is important to note that the values of RLCG are usually very small, at low frequencies their effects can be simply ignored

- For <u>short interconnect</u>, the moment the switch is closed, a voltage will appear across R_L as current flows through it. The effect is instantaneous.
- Voltage and current are due to electric charge movement along the interconnect.
- Associated with the electric charges are **static electromagnetic (EM) field** in the space surrounding the short interconnect.
- The short interconnect system can be modelled by lumped RLC circuit.

Transmission Line (contd.)

- If the interconnection is **long** (in comparison to the wavelength of the signal frequency), it takes some time for the voltage and current to appear on the R_L when the switch is closed.



Electric charges move from V_S to the R_L. As the charge move, there is an associated EM field which travels along with the charges

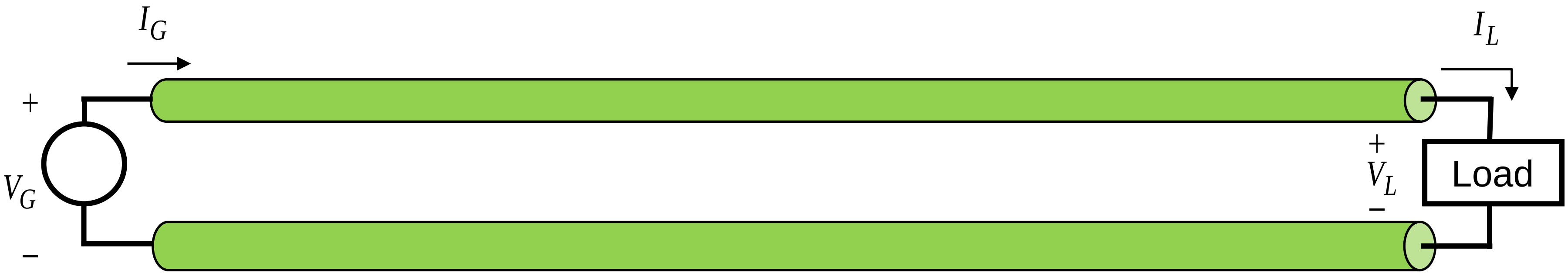
In effect, there is propagating EM field along the interconnect. The propagating EM field is called wave and the interconnect guiding the wave is called transmission line.

Why Transmission Lines Theory?

- A transmission line is a two-conductor system that is used to transmit a signal from one point to another point.

- Transmission line theory must be used instead of circuit theory for any two-conductor system if the speed-of-light travel time, T_L, across the line is a significant fraction of a signal's period T → size of the circuit dimensions are comparable to the wavelength of the traveling wave → leads to variations in current and voltage across the circuit dimensions.

Why Transmission Lines Theory?



$$L = T_L \times \textit{speed of light} = \frac{T_L}{\sqrt{\mu\varepsilon}}$$

$* \ \varepsilon_r = \mu_r = 1.0$ assumed

Frequency	Period T	$L = 3[\text{m}] \Rightarrow T_L = 10[\text{nS}]*$	$L = 30[\text{cm}] \Rightarrow T_L = 100[\text{pS}]*$	$L = 300[\mu\text{m}] \Rightarrow T_L = 1[\text{pS}]*$	$L = 3[\mu\text{m}] \Rightarrow T_L = 10[\text{fS}]*$
60 [Hz]	16.7[mS]	•	•	•	•
1 [kHz]	1[mS]	•	•	•	•
1 [MHz]	1[μS]	•	•	•	•
1 [GHz]	1[nS]	X	X	•	•
1 [THz]	1[pS]	X	X	X	•

X Transmission line theory

• Circuit theory

are you convinced now?

Why Transmission Lines Theory?

- Transmission-line theory is valid at any frequency, and for any type of waveform (assuming an ideal transmission line).
- Transmission-line theory is perfectly consistent with Maxwell's equations (although we work with voltage and current, rather than electric and magnetic fields).
- Circuit theory does not view two wires as a "transmission line": it cannot predict effects such as signal propagation, distortion, etc.

Transmission Lines (contd.)

Symbols:

Note: We use this schematic to represent a general transmission line, no matter what the actual shape of the conductors.

Transmission Lines (contd.)

- Four fundamental parameters characterize any transmission line:

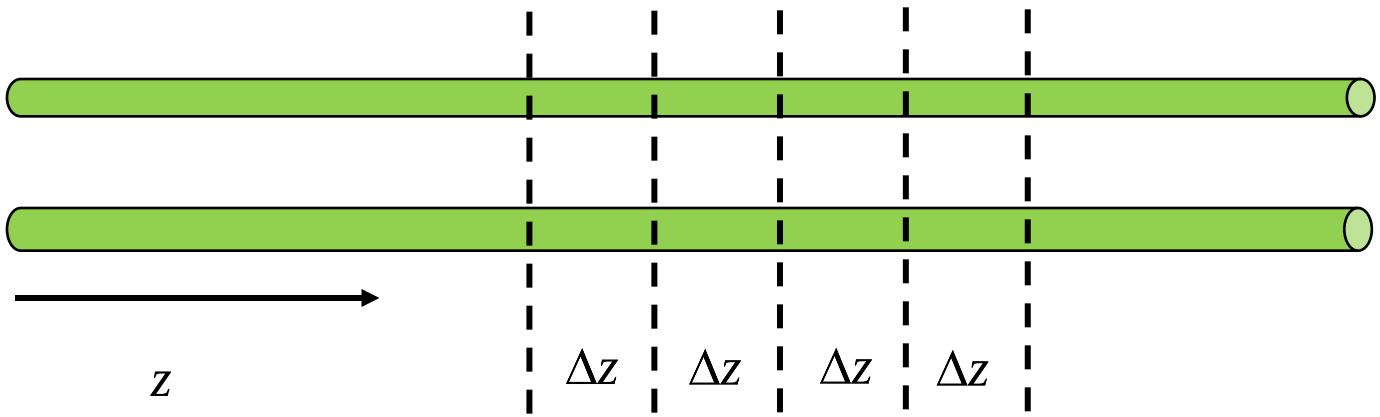
C = capacitance/length [F/m] - Capacitance/m between the two conductors
L = inductance/length [H/m] - Inductance/m due to stored magnetic energy
R = resistance/length [Ω/m] - Resistance/m due to the conductors
G = conductance/length [S/m] - Conductance/m due to the filling material between the conductors

It is usually assumed that these parameters are constant along the transmission line (ie, the transmission line is uniform)

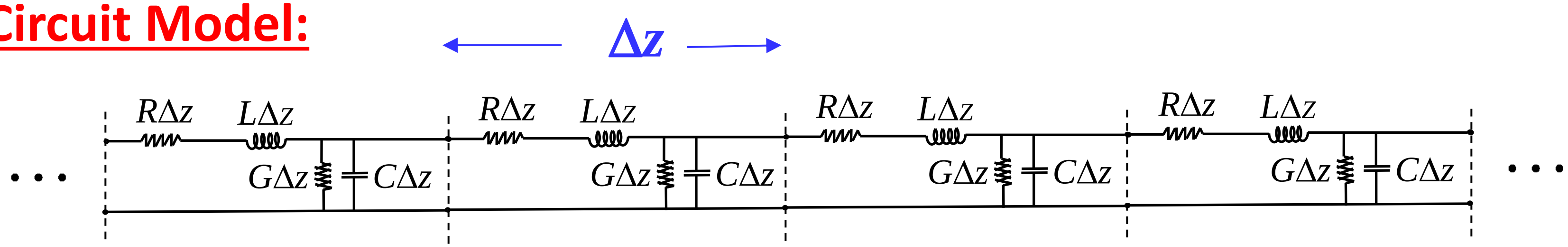
Note: most of the times these are "per unit length" parameters.

Transmission Lines (contd.)

- Variations in current and voltage across the circuit dimensions → KCL and KVL can't be directly applied → **This anomaly can be remedied** if the line is subdivided into elements of small (infinitesimal) length over which the current and voltage do not vary

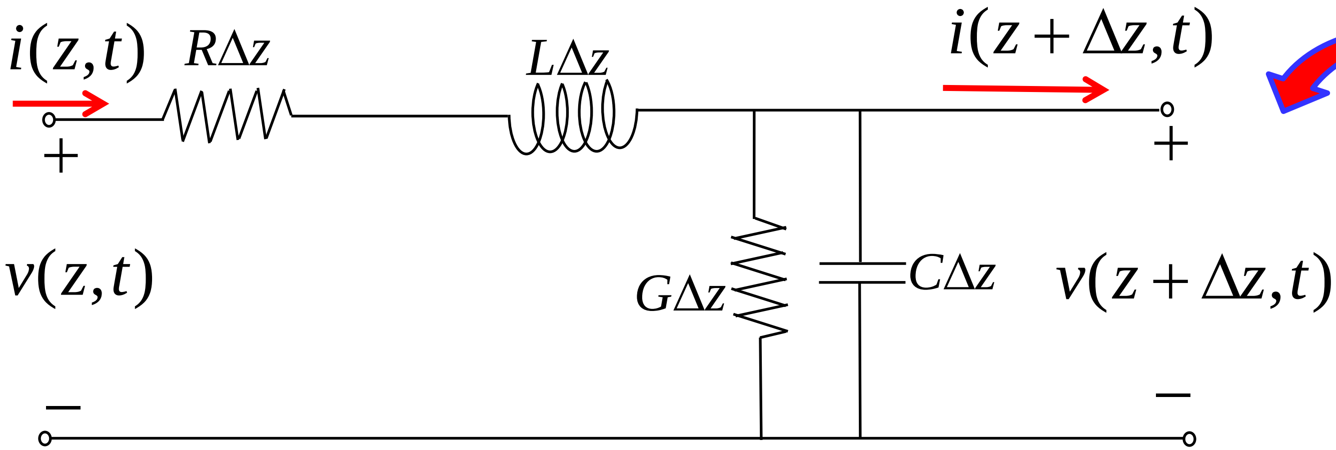


Circuit Model:

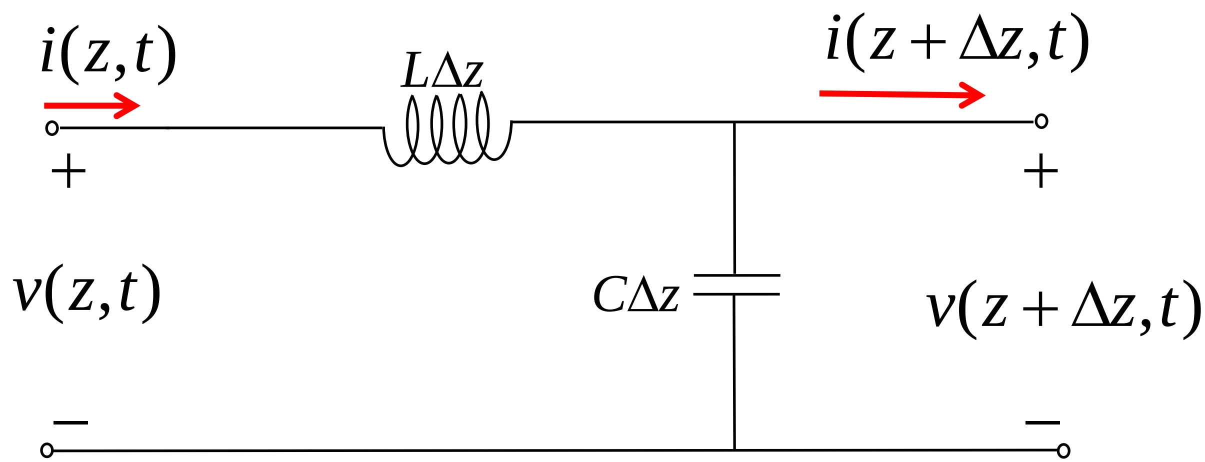


$\lim_{\Delta z \to 0} \Rightarrow$ Infinite number of infinitesimal sections

Transmission Lines (contd.)

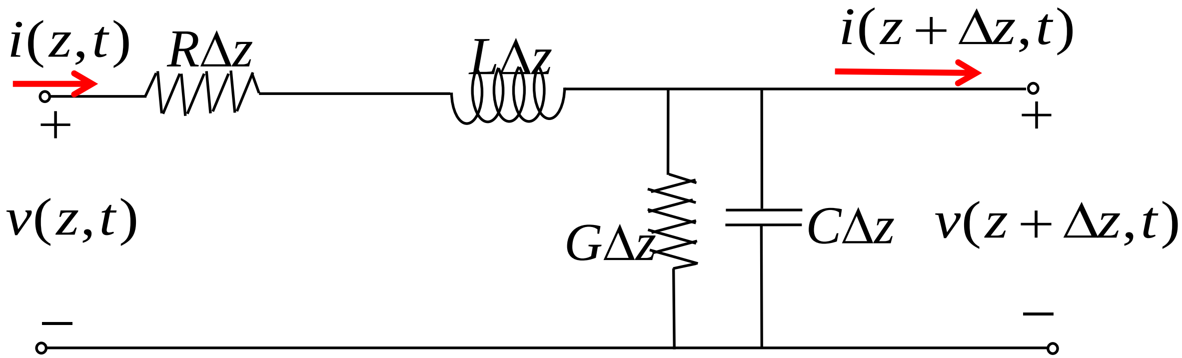


Lossy Transmission Line Circuit Model



Lossless Line Circuit Model i.e, line with negligible R and G

Transmission Lines (contd.)



Applying KVL:

$$v(z,t) - v(z+\Delta z,t) = R\Delta z\, i(z,t) + L\Delta z \frac{\partial i(z,t)}{\partial t} \Rightarrow \frac{v(z,t) - v(z+\Delta z,t)}{\Delta z} = Ri(z,t) + L\frac{\partial i(z,t)}{\partial t}$$

Describes the voltage along the transmission lines

$$\therefore -\frac{\partial v(z,t)}{\partial z} = Ri(z,t) + L\frac{\partial i(z,t)}{\partial t}$$

For $\Delta z \to 0$

KCL on this line segment gives: $i(z,t) - i(z+\Delta z,t) = G\Delta z\, v(z+\Delta z,t) + C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t}$

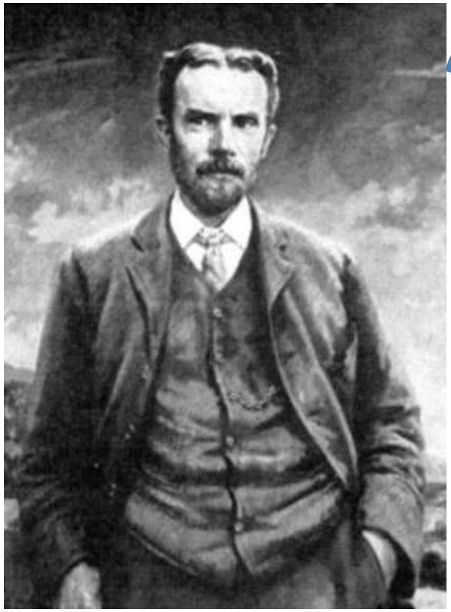
Transmission Lines (contd.)

Simplification results in:

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

For $\Delta z \to 0$

Describes the current along the transmission lines



These differential equations for current/voltages were derived by Oliver Heavyside. These equations are known as Telegrapher's Equations.

Transmission Lines (contd.)

Solution for Voltage and Current:

- For a sinusoidal excitation [i.e, $V_s(t) = V_s\{cos(\omega t)\}$], the steady state voltages and currents along the transmission line are also sinusoidal functions of time whose dependence on position and time can be expressed as:

$$v(z,t) = f(z)\cos(\omega t + \varphi(z)) \qquad i(z,t) = g(z)\cos(\omega t + \eta(z))$$

- $f(z)$ and $g(z)$ are real functions of position and $\varphi(z)$ and $\eta(z)$ describe the positional dependence of the phase.
- **Alternatively,**

$$v(z,t) = f(z)\cos(\omega t + \varphi(z)) = \mathrm{Re}\left[f(z)e^{j\varphi(z)}e^{j\omega t} \right]$$

$$i(z,t) = g(z)\cos(\omega t + \eta(z)) = \mathrm{Re}\left[g(z)e^{j\eta(z)}e^{j\omega t} \right]$$

- **Let us define these phasors:** $V(z) = f(z)e^{j\varphi(z)} \qquad I(z) = g(z)e^{j\eta(z)}$

The phasors $I(z)$ and $V(z)$ are complex functions of position and express the variations of current/voltage as a function of position along the transmission line.

Transmission Lines (contd.)

- **Therefore the current and voltage functions can be expressed as:** $v(z,t) = \mathrm{Re}\left[V(z)e^{j\omega t}\right] \quad i(z,t) = \mathrm{Re}\left[I(z)e^{j\omega t}\right]$
- The time-harmonic form of the telegrapher equations are:

$$\mathrm{Re}\frac{\partial\left(f(z)e^{j\varphi(z)}e^{j\omega t}\right)}{\partial z} = -Re\left(R.g(z)e^{j\eta(z)}e^{j\omega t} + j\omega L.g(z)e^{j\eta(z)}e^{j\omega t}\right)$$

$$\mathrm{Re}\frac{\partial\left(g(z)e^{j\eta(z)}e^{j\omega t}\right)}{\partial z} = -Re\left(G.f(z)e^{j\varphi(z)}e^{j\omega t} + j\omega C.f(z)e^{j\varphi(z)}e^{j\omega t}\right)$$

- With the substitution of phasors, the equations of voltage and current wave result in:

$$\mathrm{Re}\frac{\partial\left(V(z)e^{j\omega t}\right)}{\partial z} = -Re\left(RI(z)e^{j\omega t} + j\omega LI(z)e^{j\omega t}\right)$$

$$\mathrm{Re}\frac{\partial\left(I(z)e^{j\omega t}\right)}{\partial z} = -Re\left(GV(z)e^{j\omega t} + j\omega CV(z)e^{j\omega t}\right)$$

Transmission Lines (contd.)

- The differential equations for current and voltage along the transmission line can be expressed in phasor form as:

$$Re\frac{d\left(V(z)e^{j\omega t}\right)}{dz} = -Re\left(RI(z)e^{j\omega t} + j\omega LI(z)e^{j\omega t}\right)$$

$$Re\frac{d\left(I(z)e^{j\omega t}\right)}{dz} = -Re\left(GV(z)e^{j\omega t} + j\omega CV(z)e^{j\omega t}\right)$$

As $I(z)$ and $V(z)$ are function of only position

$$\frac{\partial V(z)}{\partial z} = \frac{dV(z)}{dz}$$

$$\frac{\partial I(z)}{\partial z} = \frac{dI(z)}{dz}$$

Transmission Lines (contd.)

- The equations can be simplified as:

$$\Rightarrow Re\left[\left(\frac{d\left(V(z)\right)}{dz} + RI(z) + j\omega LI(z)\right)e^{j\omega t}\right] = 0$$

$$\Rightarrow Re\left[\left(\frac{d\left(I(z)\right)}{dz} + GV(z) + j\omega CV(z)\right)e^{j\omega t}\right] = 0$$

For further simplification

At $\omega t=0$, $e^{j\omega t}=1$:

$$\Rightarrow Re\left[\left(\frac{d\left(V(z)\right)}{dz} + RI(z) + j\omega LI(z)\right)\right] = 0$$

At $\omega t=\pi/2$, $e^{j\omega t}=j$:

$$\Rightarrow Re\left[\left(\frac{d\left(V(z)\right)}{dz} + RI(z) + j\omega LI(z)\right)j\right] = 0$$

Transmission Lines (contd.)

- Finally we can write:

$$\frac{d\,\mathrm{V(z)}}{dz} = -\left(R + j\omega L\right)\mathrm{I(z)}$$

$$\frac{d\,\mathrm{I(z)}}{dz} = -\left(G + j\omega C\right)\mathrm{V(z)}$$

These differential equations can be solved for the phasors along the transmission line

Differentiating with respect to ***z*** gives

$$\frac{d^2\,\mathrm{V(z)}}{dz^2} - \gamma^2 V(z) = 0$$

$$\frac{d^2\,\mathrm{I(z)}}{dz^2} - \gamma^2 I(z) = 0$$

Transmission Line Wave Equations

Here

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Complex Propagation Constant

Transmission Lines (contd.)

- Complex Propagation Constant:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \alpha + j\beta$$

Attenuation Constant (nepers/m)

Phase Constant (radians/m)

- For lossless transmission line (i.e, transmission line where R and G are negligible) - most common scenario in our transmission line based circuit design:

$$\gamma = j\beta = j\omega\sqrt{LC}$$

No Attenuation

Phase Constant is also Propagation Constant for a Lossless Line

Transmission Lines (contd.)

Oh please, continue wasting my valuable time. We both know that a **lossless transmission** line is a **physical impossibility**.

True! However, a **low-loss** line **is** possible – in fact it is **typical!** **If $R \ll \omega L$ and $G \ll \omega C$,** we find that the lossless transmission line equations are excellent **approximations!!!**

Transmission Lines (contd.)

- For a lossless transmission line the second order differential equation for phasors are:

$$\frac{d^2 V(z)}{dz^2} + \beta^2 V(z) = 0$$

$$\frac{d^2 I(z)}{dz^2} + \beta^2 I(z) = 0$$

$$\beta = \omega\sqrt{LC}$$

General Solution For Voltage Phasor

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

V_0^+ **and** V_0^- **are complex constants**

- Similarly the current phasor for a lossless line can be described:

$$I(z) = -\frac{1}{j\omega L}\frac{dV(z)}{dz} = -\frac{1}{j\omega L}\frac{d}{dz}\left[V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}\right]$$

Transmission Lines (contd.)

$$\Rightarrow I(z) = \frac{\beta}{\omega L}\left[V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}\right]$$

Gives the Definition of Characteristic Impedance

$$Z_0 = \frac{\omega L}{\beta} = \frac{\omega L}{\omega\sqrt{LC}} = \sqrt{\frac{L}{C}}$$

Completely Dependent on L and C

Characteristic Impedance for a Lossless Line is Real

$$\therefore I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$$

Opposite Signs in these Terms Gives a Clue about Current Flow in Two Different Directions

Transmission Lines (contd.)

- The time dependent form of the voltage and current along the transmission line can be derived from phasors as:

$$v(z,t) = \mathrm{Re}\left[V(z)e^{j\omega t}\right] = \mathrm{Re}\left[V_0^+ e^{-j(\beta z - \omega t)} + V_0^- e^{j(\beta z + \omega t)}\right]$$

$$i(z,t) = \mathrm{Re}\left[I(z)e^{j\omega t}\right] = \mathrm{Re}\left[\frac{V_0^+}{Z_0} e^{-j(\beta z - \omega t)} - \frac{V_0^-}{Z_0} e^{j(\beta z + \omega t)}\right]$$

- For the simple case of $\boldsymbol{V_0}^+$ **and** $\boldsymbol{V_0}^-$ being real, the voltage and current along the transmission line can be expressed as:

$$v(z,t) = V_0^+ \cos(\omega t - \beta z) + V_0^- \cos(\omega t + \beta z)$$

$$i(z,t) = \frac{V_0^+}{Z_0} \cos(\omega t - \beta z) - \frac{V_0^-}{Z_0} \cos(\omega t + \beta z)$$

Transmission Lines (contd.)

$$V_0^+ \cos(\omega t - \beta z) \qquad v_1(z,t)$$

Wave Functions

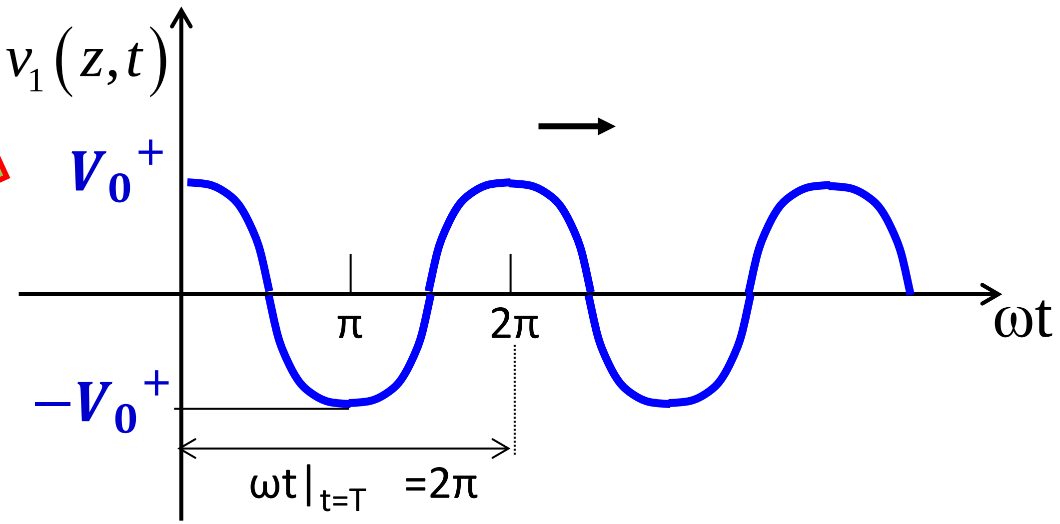
- Let us examine the wave characteristics of $v_1(z,t) = V_0^+ \cos(\omega t - \beta z)$
- **For fixed position z and variable t**

We can deduce:

$$\omega t \,|_{t=T} = 2\pi$$

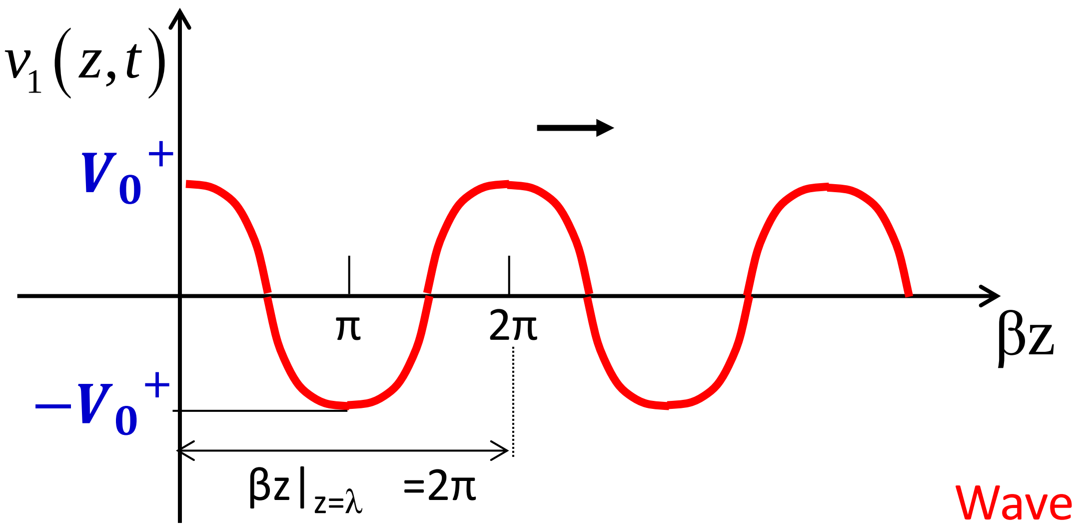
$$\Rightarrow T = \frac{2\pi}{\omega} = \frac{1}{f}$$

Time Period of Wave



Transmission Lines (contd.)

- **For fixed time t and variable position z**



We can deduce:

$$\beta z|_{z=\lambda} = 2\pi$$

$$\Rightarrow \lambda = \frac{2\pi}{\beta}$$

Wavelength

Distance between two consecutive equal values of the signal is defined as the wavelength

Transmission Lines (contd.)

- **What is the physical meaning of β**

ϕ

Let us consider once again: $V_0^+\cos(\omega t - \beta z)$

Apparently β represents the relative phase of this wave function in space (ie, function of transmission line position)

In principle, the value of β must have units of (ϕ/z) → **Radians/meter**

Therefore, if the values of β is small, we will need to move a significant distance Δz down the transmission line in order to observe a change in the relative phase of the oscillation

Conversely, if the value of β is large, a significant change in relative phase can be observed if traveling a short distance Δz down the transmission line

Transmission Lines (contd.)

- **For example**, in order to observe a change in relative phase of 2π, the distance Δ*z* is:

$$2\pi = \phi(z + \Delta z_{2\pi}) - \phi(z) = \beta \Delta z_{2\pi}$$

$$\beta = \frac{2\pi}{\Delta z_{2\pi}} \quad \Longleftrightarrow \quad \Delta z_{2\pi} = \frac{2\pi}{\beta}$$

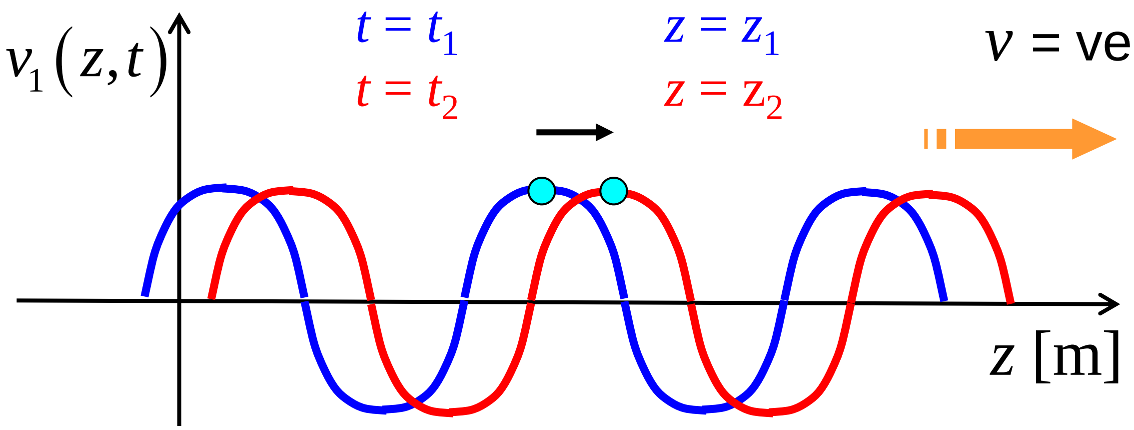
λ: Wave Length

$$\beta = \frac{2\pi}{\lambda}$$

Can't we call it spatial frequency?

Transmission Lines (contd.)

- **For variable position *z* and variable time *t***



- It is apparent that the phase of both these are identical and hence:

$$v_1(z_1,t_1) = v_2(z_2,t_2) \Rightarrow \cos(\beta z_1 - \omega t_1) = \cos(\beta z_2 - \omega t_2)$$

$$\beta z_1 - \omega t_1 = \beta z_2 - \omega t_2$$

$$\frac{z_2 - z_1}{t_2 - t_1} = \frac{\omega}{\beta}$$

Speed of Propagation → $\frac{z_2 - z_1}{t_2 - t_1}$

Phase Velocity (v_p)

Transmission Lines (contd.)

$$\Rightarrow v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

- Simplified Expression for Wavelength: $$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} = \frac{v_p}{f} = v_p T$$

i.e, the wavelength is the distance traveled by the wave in a time interval equal to one period

Transmission Lines (contd.)

Let us examine this expression:

$$\frac{z_2 - z_1}{t_2 - t_1} = \frac{\omega}{\beta}$$

- $t_2 > t_1$ and ${}^{\omega}/_{\beta}$ is a positive quantity → this implies that $z_2 - z_1$ must be positive or $z_2 > z_1$
- It ensures that the point of constant phase moves towards right (i.e, toward the load in the transmission line)
- In other words, the wave function $V_0^{+} cos(\omega t - \beta z)$ represents a traveling wave moving at a velocity v_p towards the load
- This wave is called outgoing wave when seen from the source and incident wave when viewed from the load

Transmission Lines (contd.)

- Similarly, the analysis of $V_0^{-}cos(\omega t+\beta z)$ will show that this function represents a traveling wave at a velocity v_p to the left (i.e, towards the source in a transmission line)
- This wave is called incoming wave when seen from the source and reflected wave when viewed from the load
- $V_0^{+}e^{-j\beta z}$ is called incident wave (phasor form) **and** $V_0^{-}e^{j\beta z}$ is called reflected wave (phasor form)
- In general, the voltage and current on a transmission line is composed of incident and reflected wave
- The quantity βz is known as electrical length of the line
- Therefore:

$$V(z)=V^{+}(z)+V^{-}(z)=V_0^{+}e^{-j\beta z}+V_0^{-}e^{j\beta z}$$

$$I(z)=\frac{V_0^{+}}{Z_0}e^{-j\beta z}-\frac{V_0^{-}}{Z_0}e^{j\beta z}=\frac{V^{+}(z)-V^{-}(z)}{Z_0}$$

Characteristic Impedance (Z_0)

- The characteristic impedance is defined as :

  Z_0 = (incoming voltage wave) / (incoming current wave)

  = (outgoing voltage wave) / (outgoing current wave)

- For a generic transmission line:

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

The incoming and outgoing voltage and current waves are position dependent → the ratio of voltage and current waves are independent of position → actually is a constant → an important characteristic of a transmission line → called as Characteristic Impedance

- Z_0 is not an impedance in a conventional circuit sense
- Its definition is based on the incident and reflected voltage and current waves
- As such, this definition has nothing in common with the total voltage and current expressions used to define a conventional circuit impedance
- Its importance will be apparent during the course of this COURSE!!!

Example – 1

- A plane wave propagating in a lossless dielectric medium has an electric field given as $E_x = E_0\cos(\omega t - \beta z)$ with a frequency of 5.0 GHz and a wavelength of 3.0 cm in the material. Determine the propagation constant, the phase velocity, the relative permittivity of the medium, and the intrinsic impedance of the wave.

The propagation constant:

$$\beta = \frac{2\pi}{\lambda} \Rightarrow \beta = \frac{2\pi}{0.03} \Rightarrow \therefore \beta = 209.4 m^{-1}$$

The phase velocity:

$$v_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \lambda f \Rightarrow v_p = 0.03\times 5\times 10^9 = 1.5\times 10^8 m/\sec$$

Lower than the speed of light in free medium

Example – 1 (contd.)

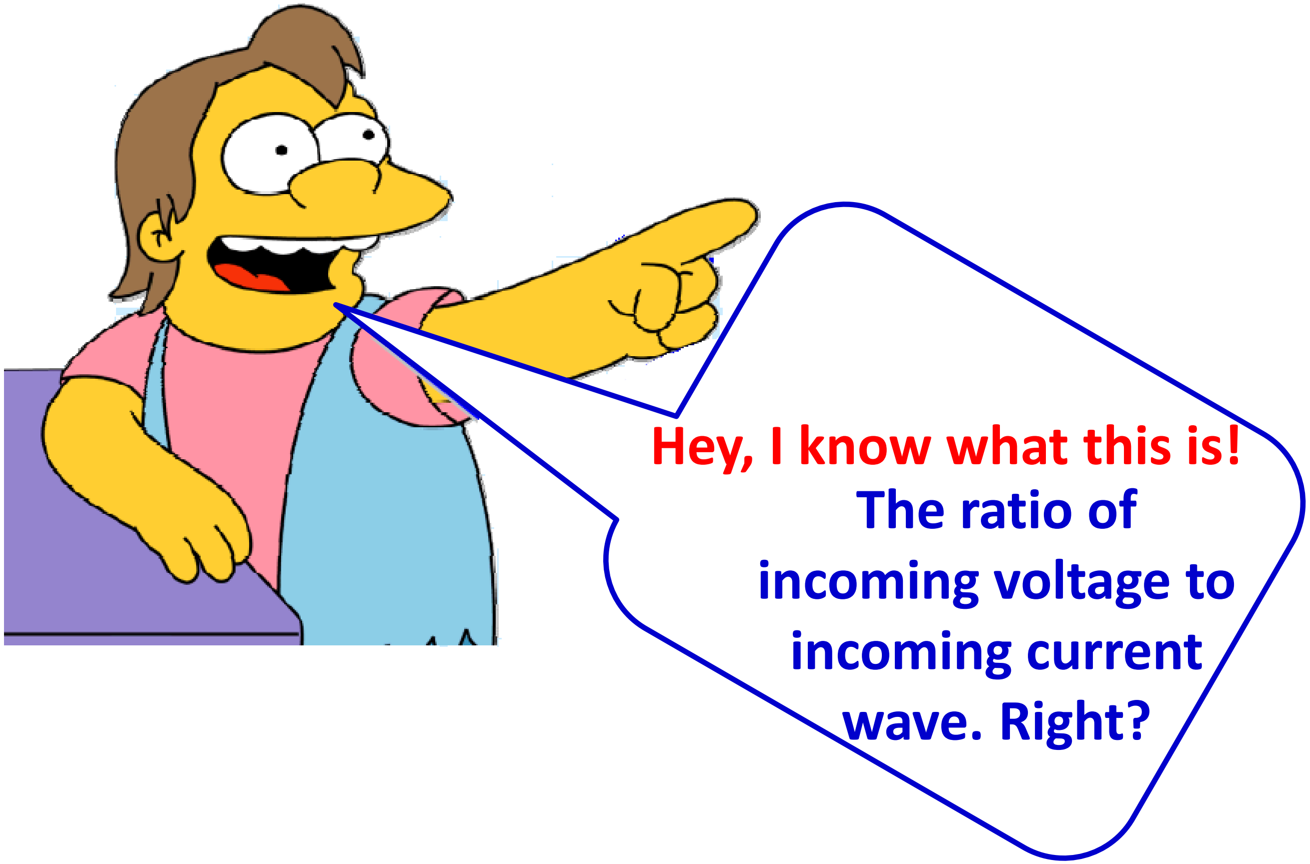
Relative permittivity of the medium:

$$v_p = \frac{c}{\sqrt{\varepsilon_r}} \Rightarrow \varepsilon_r = \left(\frac{c}{v_p}\right)^2 \Rightarrow \varepsilon_r = \left(\frac{3\times10^8}{1.5\times10^8}\right)^2 = 4.0$$

Characteristic impedance of the wave:

$$\eta_{wave} = \frac{\eta_0}{\sqrt{\varepsilon_r}} \Rightarrow \eta_{wave} = \frac{377}{\sqrt{4}} = 188.5\Omega$$

Line Impedance (Z)



NO!

Line Impedance (Z) – contd.

- Actually, line impedance is the ratio of total complex voltage (incoming + outgoing) wave to the total complex current voltage wave

$$Z(z)=\frac{V(z)}{I(z)} \Rightarrow =\frac{V^{+}(z)+V^{-}(z)}{\left(\left(V^{+}(z)-V^{-}(z)\right)/Z_0\right)} \Rightarrow \neq Z_0$$

In most of the cases

- However, the line and characteristic impedance can be equal if either the incoming or outgoing voltage wave equals ZERO!

- Say, if $V^{-}(z)=0$ then:

$$Z(z)=\frac{V^{+}(z)+V^{-}(z)}{\left(\left(V^{+}(z)-V^{-}(z)\right)/Z_0\right)}=Z_0$$

Line Impedance (Z) – contd.

It appears to me that Z_0 is a transmission line parameter, depending only on the transmission line values R, L, C and G.

Whereas, $Z(z)$ depends on the magnitude and the phase of the two propagating waves $V^{+}(z)$ and $V^{-}(z)$ → values that depend not only on the transmission line, but also on the two things attached to either end of the transmission line.

Right?

Exactly!!!