Lecture – 24

Date: 13.04.2015

• Reflection of Plane Wave at Oblique Incidence (Snells’ Law, Brewster’s Angle, Parallel Polarization, Perpendicular Polarization etc.)
• Introduction to RF/Microwave
• Introduction to Transmission Lines
Introduction

- One can’t expect plane waves to be incident normally on a plane in all types of applications.
- Therefore one must consider the general problem of a plane wave propagating along a specified axis that is arbitrarily located relative to a rectangular coordinate system.
- The most general form of a plane wave in a lossless media is given by:

\[
\vec{E}(r,t) = \vec{E}_o \cos(\vec{\beta}.r - \omega t)
\]

Where:

\[
\vec{\beta} = \beta_x \hat{a}_x + \beta_y \hat{a}_y + \beta_z \hat{a}_z
\]

\[
r = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z
\]

\[
\beta^2 = \beta_x^2 + \beta_y^2 + \beta_z^2
\]

One can deduce Maxwell’s equations in the following form:

\[
\vec{\beta} \times \vec{E} = \omega \mu \vec{H}
\]

\[
\vec{\beta} \times \vec{H} = -\omega \varepsilon \vec{E}
\]

\[
\vec{\beta}.\vec{H} = 0
\]

\[
\vec{\beta}.\vec{E} = 0
\]

They show two things: (i) \(\vec{E}, \vec{H}\) and \(\vec{\beta}\) are orthogonal, (ii) \(\vec{E}\) and \(\vec{H}\) lie on the same plane.
**Introduction (contd.)**

- Furthermore: \( \vec{\beta} \cdot r = \beta_x x + \beta_y y + \beta_z z = \text{const} \tan t \)

- The corresponding magnetic field is:
  \[
  \vec{H} = \frac{1}{\omega \mu} \vec{\beta} \times \vec{E} = \frac{\hat{a}_\beta \times \vec{E}}{\eta}
  \]
Reflection at Oblique Incidence

- The plane defined by the propagation vector $\vec{\beta}$ and a unit normal vector $\hat{a}_n$ to the boundary is called the plane of incidence.
- The angle between $\vec{\beta}$ and $\hat{a}_n$ is the angle of incidence.

$$
\vec{E}_i = \vec{E}_{io} \cos(\beta_i x + \beta_{iy} y + \beta_{iz} z - \omega_i t)
$$

$$
\vec{E}_r = \vec{E}_{ro} \cos(\beta_{rx} x + \beta_{ry} y + \beta_{rz} z - \omega_r t)
$$

$$
\vec{E}_t = \vec{E}_{to} \cos(\beta_{tx} x + \beta_{ty} y + \beta_{tz} z - \omega_t t)
$$

Where: $\beta_i$, $\beta_r$ and $\beta_t$ will have normal and tangential components to the plane of incidence.
Reflection at Oblique Incidence (contd.)

- From boundary condition we can write: the tangential component of $\vec{E}$ must be continuous at $z = 0$.

$$\vec{E}_{i\tan}(z=0) + \vec{E}_{r\tan}(z=0) = \vec{E}_{t\tan}(z=0)$$

This boundary condition can be satisfied if:

- $\omega_i = \omega_r = \omega_t = \omega$
- $\beta_{ix} = \beta_{rx} = \beta_{tx} = \beta_x$
- $\beta_{iy} = \beta_{ry} = \beta_{ty} = \beta_y$

First condition implies that the frequency remains unchanged.
Reflection at Oblique Incidence (contd.)

- From second and third conditions we can write:
  \[ \beta_1 \sin \theta_i = \beta_1 \sin \theta_r \]
  \[ \beta_1 \sin \theta_i = \beta_2 \sin \theta_t \]

  Where, \( \theta_r \) is the angle of reflection and \( \theta_t \) is the angle of transmission.

- We know, for lossless media:
  \[ \beta_1 = \omega \sqrt{\mu_1 \varepsilon_1} \]
  \[ \beta_2 = \omega \sqrt{\mu_2 \varepsilon_2} \]

\[ \frac{\sin \theta_t}{\sin \theta_i} = \frac{\beta_1}{\beta_2} = \frac{u_2}{u_1} = \frac{\sqrt{\mu_1 \varepsilon_1}}{\sqrt{\mu_2 \varepsilon_2}} \]

\[ n_1 \sin \theta_i = n_2 \sin \theta_t \]

\( n_1 \) and \( n_2 \) are the refractive indices of the media

Snell’s Law
Example – 1

A dielectric slab with index of refraction \( n_2 \) is surrounded by a medium with index of refraction \( n_1 \) as shown. If \( \theta_i < \theta_c \), show that the emerging beam is parallel to the incident beam.

At the upper surface:

\[
\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1
\]

Similarly at the lower surface:

\[
\sin \theta_3 = \frac{n_2}{n_3} \sin \theta_2 \quad \Rightarrow \sin \theta_3 = \frac{n_2}{n_1} \sin \theta_2
\]

\[
\Rightarrow \sin \theta_3 = \left( \frac{n_2}{n_1} \right) \left( \frac{n_1}{n_2} \right) \sin \theta_1 = \sin \theta_1
\]

The slab displaces the beam’s position but the beam’s direction remains unchanged.
Reflection at Oblique Incidence (contd.)

• For normal incidence, the reflection and transmission coefficients $\Gamma$ and $\tau$ at a boundary between two media are independent of the polarization of the incident wave, as both the $\vec{E}$ and $\vec{H}$ of a normally incident plane wave are tangential to the boundary regardless to the wave polarization.

• This is not the case for wave travelling at an angle $\theta_i \neq 0$ with respect to the normal to the interface.

• A wave of arbitrary polarization may be described as the superposition of two orthogonally polarized waves, one with its $\vec{E}$ parallel to the plane of incidence (parallel polarization or transverse magnetic (TM) polarization) and the other with $\vec{E}$ perpendicular to the plane of incidence (perpendicular polarization or transverse electric (TE) polarization).
Parallel Polarization

- Consider this figure: $\vec{E}$ field lies in the $xz$-plane, the plane of incidence.
- It illustrates the case of “Parallel Polarization”.

- In medium 1 the incident waves are:

$$\vec{E}_{is} = \vec{E}_{io} (\hat{a}_x \cos \theta_i - \hat{a}_z \sin \theta_i) e^{-j\beta_1 (x \sin \theta_i + z \cos \theta_i)}$$

$$\vec{H}_{is} = \frac{\vec{E}_{io}}{\eta_1} e^{-j\beta_1 (x \sin \theta_i + z \cos \theta_i)} \hat{a}_y$$
Parallel Polarization (contd.)

- In medium 1 the reflected waves are:

\[ \vec{E}_{rs} = \vec{E}_{ro} (\hat{a}_x \cos \theta_r + \hat{a}_z \sin \theta_r) e^{-j \beta_1 (x \sin \theta_r - z \cos \theta_r)} \]

\[ \vec{H}_{rs} = -\frac{\vec{E}_{io}}{\eta_1} e^{-j \beta_1 (x \sin \theta_r - z \cos \theta_r)} \hat{a}_y \]

- The transmitted fields in medium 2 are given by:

\[ \vec{E}_{ts} = \vec{E}_{to} (\hat{a}_x \cos \theta_t - \hat{a}_z \sin \theta_t) e^{-j \beta_2 (x \sin \theta_t + z \cos \theta_t)} \]

\[ \vec{H}_{ts} = \frac{\vec{E}_{to}}{\eta_2} e^{-j \beta_2 (x \sin \theta_t + z \cos \theta_t)} \hat{a}_y \]

- We know: \( \theta_i = \theta_r \) and tangential components of electric and magnetic fields are continuous at the boundary \( z=0 \).
- Therefore:

\[ (E_{io} + E_{ro}) \cos \theta_i = E_{to} \cos \theta_t \]

\[ \frac{1}{\eta_1} (E_{io} - E_{ro}) = \frac{1}{\eta_2} E_{to} \]
Parallel Polarization (contd.)

• Simplification gives:

\[ \Gamma_\parallel = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 \cos \theta_t - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i} \]

\[ \tau_\parallel = \frac{E_{io}}{E_{io}} = \frac{2\eta_2 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i} \]

Fresnel’s Equations for parallel polarization

• For \( \theta_i = \theta_t = 0 \), we get:

\[ \Gamma_\parallel = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \Gamma \]

\[ \tau_\parallel = \frac{E_{io}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1} = \tau \]

• Furthermore, the expressions for reflection coefficient and transmission coefficient can be written in terms of angle of incidence.

\[ \cos \theta_t = \sqrt{1 - \sin^2 \theta_t} = \sqrt{1 - \left( \frac{u_1^2}{u_2^2} \right) \sin^2 \theta_i} \]

• In addition:

\[ 1 + \Gamma_\parallel = \tau_\parallel \left( \frac{\cos \theta_i}{\cos \theta_t} \right) \]
Parallel Polarization (contd.)

• The reflection coefficient $\Gamma_{||}$ equals zero when there is no reflection (only the parallel component is not reflected), and the incident angle at which this happens is called Brewster’s Angle $\theta_B$.  
• The Brewster’s Angle is also known as polarizing angle. 
• At this angle, the perpendicular component of $\vec{E}$ will be reflected. 
• Brewster’s concept is utilized in laser tube used in surgical procedures.

• For Brewster’s Angle, set $\Gamma_{||} = 0$:

\[
\eta_2 \cos \theta_t = \eta_1 \cos \theta_B
\]

\[
\eta_2^2 (1 - \sin^2 \theta_t) = \eta_1^2 (1 - \sin^2 \theta_B)
\]

For a lossless and nonmagnetic medium:

\[
\sin^2 \theta_B = \frac{1 - \frac{\mu_2 \varepsilon_1}{\mu_1 \varepsilon_2}}{1 - \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2}
\]

\[
\sin \theta_B = \sqrt{\frac{\varepsilon_2}{\varepsilon_2 + \varepsilon_1}}
\]

\[
\sin^2 \theta_B = \frac{1}{1 + \frac{\varepsilon_1}{\varepsilon_2}}
\]

There is a Brewster Angle for any combination of $\varepsilon_1$ and $\varepsilon_2$. 

Perpendicular Polarization

- The $\vec{E}$ field is perpendicular to the plane of incidence (the xz-plane).
- In this situation we get “Perpendicular Polarization”.
- Here, $\vec{H}$ field is parallel to the plane of incidence.

$$\vec{E}_{is} = \vec{E}_{io} e^{-j \beta_1 (x \sin \theta_i + z \cos \theta_i)} \hat{a}_y$$

$$\vec{E}_{rs} = \vec{E}_{io} e^{-j \beta_1 (x \sin \theta_r - z \cos \theta_r)} \hat{a}_y$$

$$\vec{H}_{is} = \frac{\vec{E}_{io}}{\eta_1} (-\hat{a}_x \cos \theta_i + \hat{a}_z \sin \theta_i) e^{-j \beta_1 (x \sin \theta_i + z \cos \theta_i)}$$

$$\vec{H}_{rs} = \frac{\vec{E}_{ro}}{\eta_2} (\hat{a}_x \cos \theta_r + \hat{a}_z \sin \theta_r) e^{-j \beta_1 (x \sin \theta_r - z \cos \theta_r)}$$
Perpendicular Polarization (contd.)

• The transmitted fields in medium 2 are given by:

\[
\begin{align*}
\overrightarrow{E}_{ts} &= \overrightarrow{E}_{to} e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)} \hat{a}_y \\
\overrightarrow{H}_{ts} &= \frac{\overrightarrow{E}_{to}}{\eta_2} (-\hat{a}_x \cos\theta_t + \hat{a}_z \sin\theta_t) e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)}
\end{align*}
\]

• Again, \( \theta_i = \theta_r \) and tangential components of electric and magnetic fields are continuous at the boundary \( z=0 \).

• Therefore:

\[
(E_{io} + E_{ro}) = E_{to}
\]

\[
\frac{1}{\eta_1} (E_{io} - E_{ro}) \cos\theta_i = \frac{1}{\eta_2} E_{to} \cos\theta_t
\]

• Simplification gives:

\[
\Gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_i}
\]

\[
\frac{1}{\eta_1} (E_{io} - E_{ro}) \cos\theta_i = \frac{1}{\eta_2} E_{to} \cos\theta_t
\]

Fresnel’s Equations for perpendicular polarization

• For \( \theta_i = \theta_t = 0 \), we get:

\[
\Gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \Gamma
\]

\[
\tau_\perp = \frac{E_{io}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1} = \tau
\]
Perpendicular Polarization (contd.)

- Simplification for Brewster’s Angle in Perpendicular Polarization gives:

\[ \eta_2 \cos \theta_{B\perp} = \eta_1 \cos \theta_t \]

\[ \eta_2^2 \left(1 - \sin^2 \theta_{B\perp}\right) = \eta_1^2 \left(1 - \sin^2 \theta_t\right) \]

\[ \sin^2 \theta_{B\perp} = \frac{1 - \frac{\mu_1 \varepsilon_2}{\mu_2 \varepsilon_1}}{1 - \left(\frac{\mu_1}{\mu_2}\right)^2} \]

For nonmagnetic media, \( \mu_1 = \mu_2 = \mu_0 \) and therefore:

\[ \sin^2 \theta_{B\perp} \rightarrow \infty \]

Brewster’s Angle doesn’t exist as sine of an angle is never greater than unity

- If \( \mu_1 \neq \mu_2 \) and \( \varepsilon_1 = \varepsilon_2 \) then:

\[ \sin^2 \theta_{B\perp} = \frac{1}{1 + \frac{\mu_1}{\mu_2}} \]

\[ \sin \theta_{B\perp} = \sqrt{\frac{\mu_2}{\mu_2 + \mu_1}} \]

Theoretically possible but rarely occurs in practice
Reflection at Oblique Incidence (contd.)

- The Brewster’s Angle is also called Polarizing Angle.
- This is because if a wave composed of both the perpendicular and parallel polarization components is incident on a nonmagnetic surface at the Brewster angle $\theta_{B\parallel}$, the parallel polarized component totally transmitted into the second medium and only the perpendicularly polarized component is reflected by the surface.
- Natural light, including sunlight and light generated by most manufactured sources, is *unpolarized* because it consists of equal parallel and perpendicular rays. When they are incident upon a surface at the Brewster angle, the reflected wave is strictly perpendicularly polarized. Hence the surface acts as a polarizer.
Example – 2

- A wave in air is incident upon a soil surface at $\theta_i = 50^\circ$. If soil has $\varepsilon_r = 4$ and $\mu_r = 1$, determine the following:

\[
\begin{align*}
\Gamma_\perp & \quad \tau_\perp & \quad \Gamma_\parallel & \quad \tau_\parallel & \quad \text{The Brewster angle}
\end{align*}
\]
Applications of RF/Microwaves

• The use of RF/microwaves has greatly expanded.
• Examples include telecommunications, radio astronomy, land surveying, radar, meteorology, UHF television, terrestrial microwave links, solid-state devices, heating, medicine, and identification systems.
• Features that make microwaves attractive for communications include wide available bandwidths (capacities to carry information) and directive properties for short wavelengths.
• Currently, there are three main techniques to carry energy over long distances: (a) microwave links, (b) coaxial cables, and (c) fibre optics.
• A microwave system normally consists of a transmitter (including a microwave oscillator, waveguides, amplifiers, and transmitting antenna) and a receiver subsystem (including a receiver antenna, transmission line or waveguide, and amplifiers).
• A microwave network is usually an interconnection of various microwave components and devices.
Applications of RF/Microwaves (contd.)

• Common microwave components include:
• Coaxial cables, which are transmission lines for interconnecting microwave components.
• Waveguide sections, which may be straight, curved, or twisted.
• Antenna, which transmit or receive EM waves efficiently.
• Terminators, which are designed to absorb the input power and therefore acts as one port network.
• Attenuators, which are designed to absorb some of the EM power passing through the device, thereby decreasing the power level of the microwave signal.
• Directional couplers, with a mechanism to couple between different ports.
• Isolators, which allow energy flow in only one direction.
• Circulators, which are designed to establish various entry/exit points where power can be either fed or extracted.
• Filters, which suppress unwanted signals and/or separate signals of different frequencies.
RF/Microwave Circuit

• A microwave circuit consists of microwave components such as sources, transmission lines, waveguides, attenuators, circulators, and filters.
• One way of analyzing, such circuits, are to relate the input and output variables of each component.
• At RF/microwave frequencies, where current and voltage are not well defined, it is a common practice to use S-parameters for analysis.
• S-parameters are defined in terms of wave variables which are more easily measurable at high frequencies than voltage and current.
RF/Microwave Circuit (contd.)

• Let us consider following 2-port network:

![2-port network diagram]

\[
\begin{align*}
  b_1 &= S_{11}a_1 + S_{12}a_2 \\
  b_2 &= S_{21}a_1 + S_{22}a_2
\end{align*}
\]

where, \(a_1\) and \(a_2\) are incident waves at port 1 and 2 respectively; while \(b_1\) and \(b_2\) represent the reflected waves.
RF/Microwave Circuit (contd.)

• In matrix form:

\[
\begin{bmatrix}
  b_1 \\
  b_2 \\
\end{bmatrix} =
\begin{bmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22} \\
\end{bmatrix}
\begin{bmatrix}
  a_1 \\
  a_2 \\
\end{bmatrix}
\]

The off-diagonal terms represent transmission coefficients, while the diagonal terms represent reflection coefficients.

• If the network is reciprocal, it will have the same transmission characteristic in either direction.

\[ S_{12} = S_{21} \]

• If the network is symmetric, then:

\[ S_{11} = S_{22} \]

• For matched two port network:

\[ S_{11} = S_{22} = 0 \]
RF/Microwave Circuit (contd.)

- The input reflection coefficient in terms of the S-parameters and the load $Z_L$:

$$\Gamma_i = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$
Similarly, the output reflection coefficient (with $V_g = 0$) can be expressed in terms of the generator impedance $Z_g$:

\[
\Gamma_o = \frac{b_2}{a_2} = S_{22} + \frac{S_{12}S_{21}\Gamma_g}{1 - S_{11}\Gamma_g}
\]

\[
\Gamma_g = \frac{Z_g - Z_{in}}{Z_g + Z_{in}}
\]
Example – 3

- S-parameters are obtained for a microwave transistor operating at 2.5 GHz: $S_{11} = 0.85 < -30^\circ, S_{12} = 0.07 < 56^\circ, S_{21} = 1.68 < 120^\circ, S_{22} = 0.85 < -40^\circ$. Determine the input reflection coefficient when $Z_L = Z_o = 75\Omega$.

\[
\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = 0
\]

\[
\Gamma_i = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = S_{11} = 0.85 \angle -30^\circ
\]
Waveguiding Structure

- A waveguiding structure is one that carries a wave (or power) from one point to another.
- There are three common types:
  - Fiber-optic guides
  - Waveguides
  - Transmission lines

Note: An alternative to waveguiding structures is wireless transmission using antennas.
Fiber-Optic Guides

Properties

• Can propagate a signal at any frequency (in theory)
• Can be made very low loss
• Has minimal signal distortion
• Very immune to interference
• Not suitable for high power
• Has both $E_z$ and $H_z$ components of the fields
Waveguides

Properties

- Has a single hollow metal pipe
- Can propagate a signal only at high frequency: $\omega > \omega_c$
- The width must be at least one-half of a wavelength
- Has signal distortion, even in the lossless case
- Immune to interference
- Can handle large amounts of power
- Has low loss (compared with a transmission line)
- Has either $E_z$ or $H_z$ component of the fields ($\text{TM}_z$ or $\text{TE}_z$)
Transmission Line

Properties

- Has two conductors running in parallel
- Can propagate a signal at any frequency (in theory)
- Becomes lossy at high frequency
- Can handle low or moderate amounts of power
- Does not have signal distortion, unless there is loss
- May or may not be immune to interference
- Does not have $E_z$ or $H_z$ components of the fields (TEM$_z$)
Transmission Line (contd.)

The two wires of the transmission line are twisted to reduce interference and radiation from discontinuities.
Transmission Line (contd.)

Transmission lines commonly used on printed-circuit boards

- **Microstrip**
- **Stripline**
- **Coplanar strips**
- **Coplanar waveguide (CPW)**
Transmission Line (contd.)

Transmission lines commonly used on printed-circuit boards

A microwave integrated circuit
Transmission Line Theory

- **Lumped circuits:** resistors, capacitors, inductors
  - Neglect time delays (phase)

- **Distributed circuit elements:** transmission lines
  - Account for propagation and time delays (phase change)

We need transmission-line theory whenever the length of a line is significant compared with a wavelength.
Transmission Line Theory (contd.)

2 conductors

4 per-unit-length parameters:

\[ C = \text{capacitance/length } [\text{F/m}] \]

\[ L = \text{inductance/length } [\text{H/m}] \]

\[ R = \text{resistance/length } [\Omega/\text{m}] \]

\[ G = \text{conductance/length } [\text{S/m}] \]
Transmission Line Theory (contd.)

\[ i(z,t) \]

\[ v(z,t) \]

\[ \Delta z \]

\[ i(z,t) \rightarrow R\Delta z \rightarrow L\Delta z \rightarrow i(z+\Delta z,t) \]

\[ v(z,t) \rightarrow G\Delta z \rightarrow C\Delta z \rightarrow v(z+\Delta z,t) \]
Using Transmission Lines to Synthesize Impedances

• This is very useful in RF/microwave engineering.

A microwave filter constructed from microstrip.
Using Transmission Lines to Synthesize Impedances (contd.)

• A lossless transmission line terminated in load impedance $Z_L$

\[ I(-\ell) = \frac{V_0^+}{Z_0} e^{+j\beta\ell} \]

\[ V(-\ell) = V_0^+ e^{+j\beta\ell} \]

\[ Z(-\ell) = Z_0 \]

\[ Z_0, \beta \]

\[ Z_L \]

\[ \ell \]

Matched load: ($Z_L = Z_0$)

\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0 \]

\[ \Rightarrow Z(-\ell) = Z_0 \]

\[ \Rightarrow Z_0 = Z_0 \]

\[ \text{For any } \ell \]

No reflection from the load
Using Transmission Lines to Synthesize Impedances (contd.)

B) Short circuit load: \( Z_L = 0 \)

\[
\Gamma_L = \frac{0 - Z_0}{0 + Z_0} = -1
\]

\[
\Rightarrow Z(-\ell) = jZ_0 \tan(\beta\ell)
\]

Always imaginary!

\[
\Rightarrow Z(-\ell) = jX_{sc}
\]
\[
X_{sc} = Z_0 \tan(\beta\ell)
\]

Note: \( \beta\ell = 2\pi \frac{\ell}{\lambda} \)

S.C. can become an O.C. with a \( \lambda/4 \) trans. line