Problems on VECTORS

Qus.(1) Given vectors **A**= a_x+3a_z. and **B**= 5a_x+2a_y-6a_z determine

(a) |**A+B**| (b) **5A-B** (c) The component of **A** along a_y (d)A unit vector parallel to **3A + B**

Qus.(2) An airplane has a ground speed of 350 km/hr in the direction due west. If there is a wind blowing north west at 40 km/hr, calculate the true airplane speed and heading of the airplane.

Qus.(3) If **A**= 5 a_x+3 a_y +2 a_z , **B**= -a_x+4 a_y +6 a_z , **C**= 8a_x+2 a_y

determine α and β such that the α **A**+ β **B**+ **C** is parallel to the y-axis.

Qus.(4) Given vectors **A**= α a_x+ a_y +4 a_z , **B**= 3a_x+ β a_y +6 a_z , **C**= 5a_x-2 a_y + γ a_z

determine α, β and γ such that the vectors are mutually orthogonal.

Qus.(5) Given vectors **T**= 2a_x-6 a_y +3 a_z , **S**= a_x+ 2 a_y + a_z , find: (a) the scalar projection of **T** on **S**,

(b) the vector projection of **S** on **T**, (c) the smaller angle between **T** and **S**.

Qus.(6) Given vectors **A**= - a_x+ 6a_y +5 a_z , **B**= a_x+ 2 a_y +3 a_z , find: (a) the scalar projection of **A** on **B**,

(b) the vector projection of **B** on **A**, (c) the unit vector perpendicular to the plane containing **A** and **B**.

Qus.(7) Show that the dot and cross in the triple scalar product may be interchanged, i.e. ,

A• (**B** x **C**) = (**A** x **B**) • **C**

Qus.(8) If **r** is the position vector of the point (x, y, z) and **A** is a constant vector, show that:

(a) (**r-A**) • **A** =0 is the equation of a constant plane.

(b) **(r-A) • r** =0 is the equation of a sphere.

Qus.(9) (a) Prove that **P** = cos$\theta_1$$a_x$+ sin$\theta_1$$a_y$ and **Q**= cos$\theta_2$$a_x$+ sin$\theta_2$$a_y$ are unit vectors in the x y-plane respectively making angles θ_1 & θ_2 with the x-axis.

(b) By means of dot product, obtain the formula for cos (θ_2-θ_1) By similarly formulating **P** and **Q**, obtain the formula for cos (θ_2+θ_1)

(c) If θ is the angle between **P** and **Q**, find1/2(|**P-Q** |) in terms of θ.

Qus.(10) If **E** = 2x a_x+ a_y + yz a_z , **F** = xya_x-y^2 a_y +xyz a_z , find:

(a) |**E**| at (1,2,3) (b) The component of **E** along **F** at (1 ,2,3)

(c) A vector perpendicular to both **E** and **F** at (0,1,-3) whose magnitude is unity.

Problems on Co-ordinate Systems

Qus.(1) (a) Convert points P(1,3,5), T(0 ,-4 ,3), and S (-3 ,-4 ,-10) from Cartesian to cylindrical and spherical coordinates.

(b) Transform vector $\mathbf{Q} = \frac{\sqrt{x^2+y^2}\mathbf{a}_x}{\sqrt{x^2+y^2+z^2}} - \frac{yz\,\mathbf{a}_z}{\sqrt{x^2+y^2+z^2}}$ to cylindrical and spherical coordinates.

(c) Evaluate **Q** at T in the three coordinate systems.

Qus.(2) Given point P (-2,6,3) and vector **A**= y a_x+ (x +z) a_y express P and **A** in cylindrical and spherical coordinates. Evaluate **A** at P in the Cartesian, cylindrical, and spherical systems.

Qus.(3) Express vector $\mathbf{B} = \frac{10}{r}\mathbf{a}_r + r\cos\theta\,\mathbf{a}_\theta + \mathbf{a}_\phi$ in Cartesian and cylindrical coordinates. Find **B** (-3,4,0) and **B** (5,Π /2,-2).

Qus.(4) Two uniform vector fields are given by **E** = $-5a_\rho+10a_\phi+3a_z$ and **F** = $a_\rho+2a_\phi-6a_z$

(a) |**E x F**| (b) The vector component of **E** at P (5, Π /2,3) parallel to the line x=2,z=3

(c) The angle **E** makes with the surface z=3 at P

Qus.(5) Match the items in the left list with those in the right list. Each answer can be used once,more than once,or not at all.

(a) $\theta = \pi/4$	(i) infinite plane
(b) $\phi = 2\pi/3$	(ii) semiinfinite plane
(c) $x = -10$	(iii) circle
(d) $r = 1, \theta = \pi/3, \phi = \pi/2$	(iv) semicircle
(e) $\rho = 5$	(v) straight line
(f) $\rho = 3, \phi = 5\pi/3$	(vi) cone
(g) $\rho = 10, z = 1$	(vii) cylinder
(h) $r = 4, \phi = \pi/6$	(viii) sphere
(i) $r = 5, \theta = \pi/3$	(ix) cube
	(x) point

Qus.(6) (a) If V= xz-xy+yz, express V in cylindrical coordinates,

(b) If U= $x^2+2y^2+3z^2$, express U in spherical coordinates.

Qus.(7) Let $\mathbf{H} = 5\rho\sin\phi\,\mathbf{a}_\rho - \rho z\cos\phi\,\mathbf{a}_\phi + 2\rho\mathbf{a}_z$. At point $P(2, 30°, -1)$, find:

(a) a unit vector along **H** (b) the component of **H** parallel to $\mathbf{a}_x$

(c) the component of **H** normal to $\rho = 2$

(d) the component of **H** tangential to $\phi = 30°$