Lecture – 8

Date: 01.09.2016

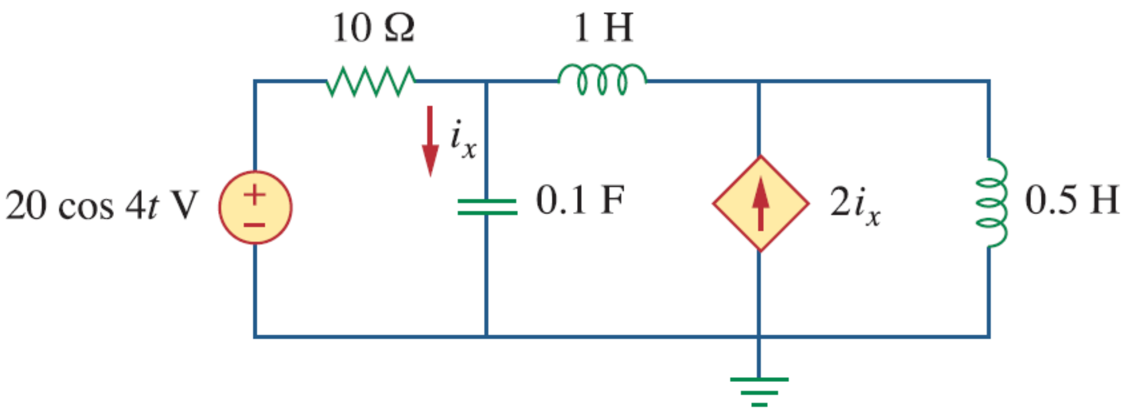
- AC Circuits: Steady State Analysis

Analysis Steps

1. Transfer the circuit to the phasor domain
2. Solve the circuit (using Mesh, Nodal techniques etc.)
3. Convert the results into time domain

Nodal Analysis

Find i_x in the following circuit.



Convert the quantities to frequency domain.

Nodal Analysis (contd.)

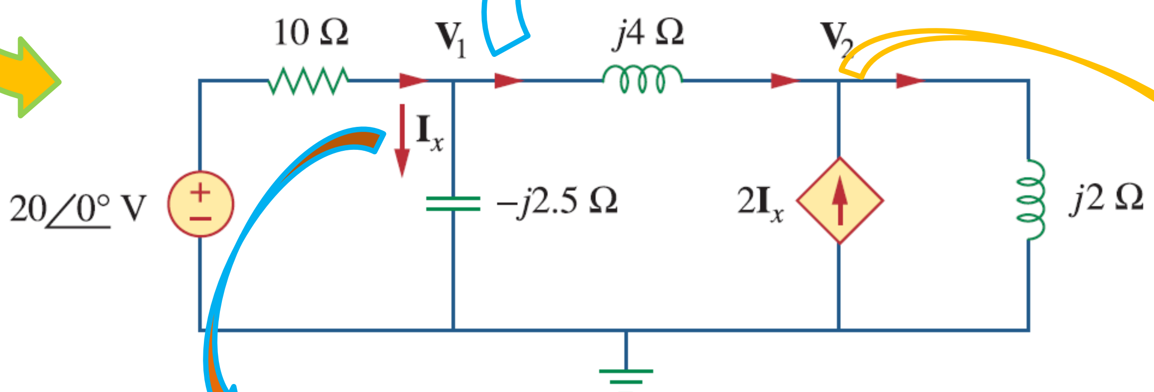
$$20\cos 4t \Rightarrow 20\angle 0^\circ, \quad \omega = 4 \text{ rad/s}$$

$$1 \text{ H} \Rightarrow j\omega L = j4$$

$$0.5 \text{ H} \Rightarrow j\omega L = j2$$

$$0.1 \text{ F} \Rightarrow \frac{1}{j\omega C} = -j2.5$$

Freq Domain



KCL here

$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20$$

KCL here

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0$$

$$\mathbf{I}_x = \mathbf{V}_1 / -j2.5$$

Nodal Analysis (contd.)

- The two nodal equations can be expressed in matrix form:

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97\underline{/18.43^\circ} \text{ V}$$

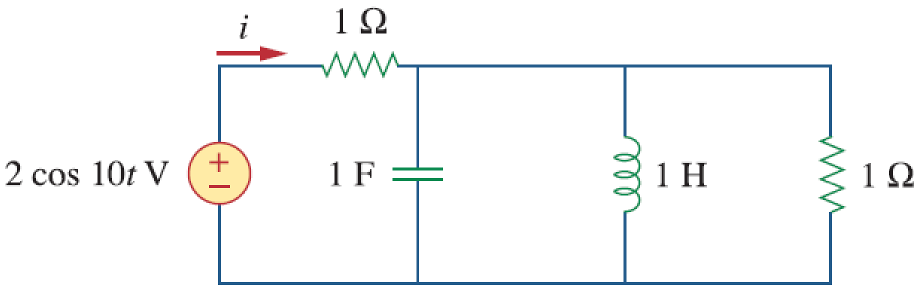
$$\Delta_2 = \begin{vmatrix} 1 + j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91\underline{/198.3^\circ} \text{ V}$$

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\underline{/18.43^\circ}}{2.5\underline{/-90^\circ}} = 7.59\underline{/108.4^\circ} \text{ A}$$

$$i_x = 7.59 \cos(4t + 108.4^\circ) \text{ A}$$

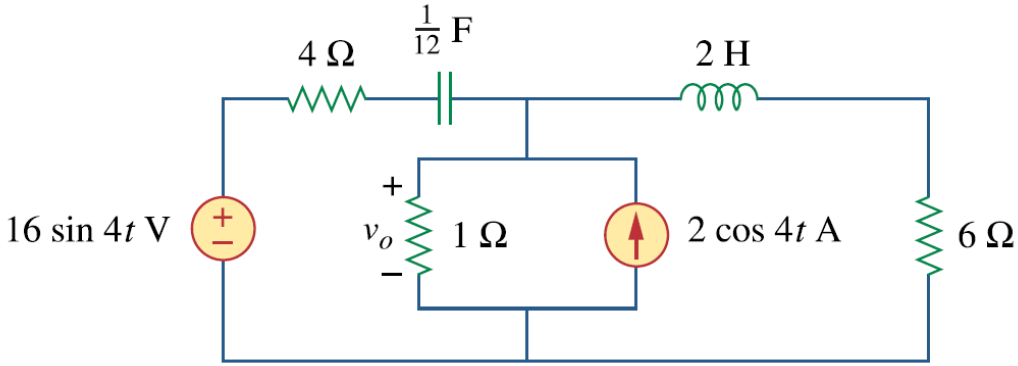
Practice: Find i in the following circuit.



$$i(t) = \underline{1.9704\cos(10t + 5.653^\circ)\ \text{A}}$$

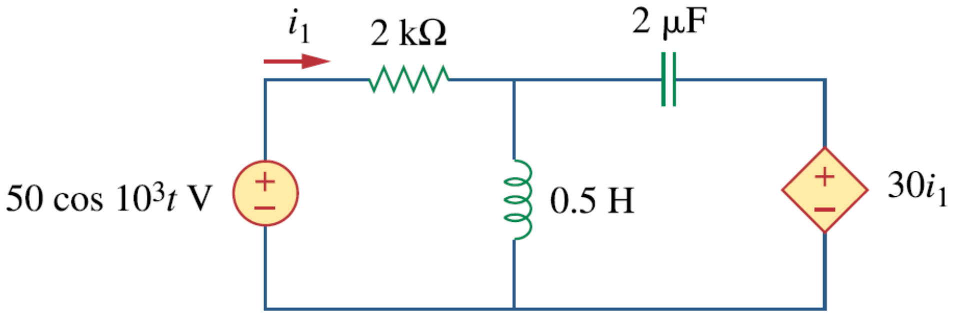
Example – 1

Determine v_0 in this circuit.



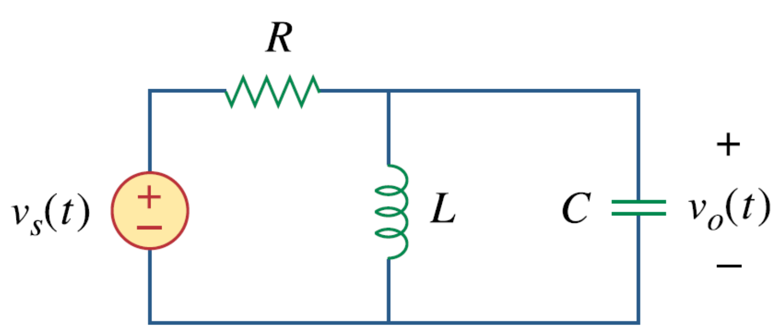
Example – 2

Determine i_1 in this circuit.



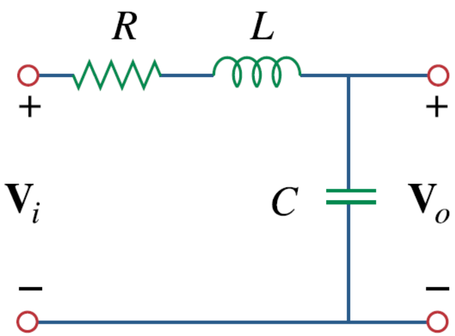
Example – 3

In this circuit if $v_s(t) = V_m Sin\omega t$ and $v_0(t) = Asin(\omega t + \varphi)$, derive the expressions for A and φ.



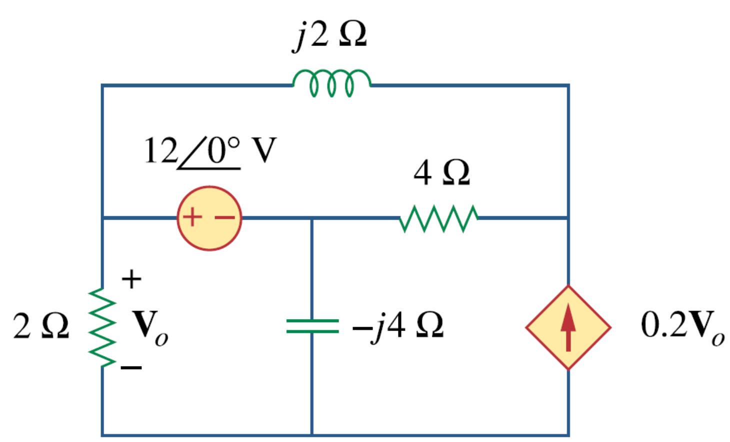
Example – 4

Find $^{V_0}/_{V_i}$ for $\omega = 0, \omega \rightarrow \infty$ and $\omega^2 = {}^{1}/_{LC}$.



Example – 5

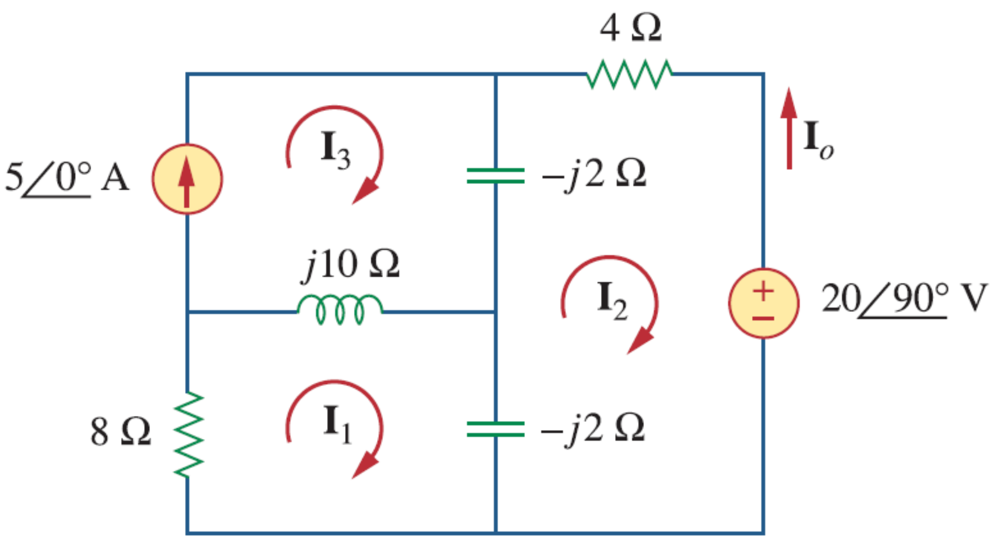
Use nodal analysis to find V_0.



Mesh Analysis

KVL is the basis for this.

→ Determine I_0 using Mesh Analysis.



in Loop 3:

$$\mathbf{I}_3 = 5$$

KVL in Loop 1: $(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0$

KVL in Loop 1: $(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\angle 90^\circ = 0$

Mesh Analysis (contd.)

$$(8 + j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j50 \qquad j2\mathbf{I}_1 + (4 - j4)\mathbf{I}_2 = -j20 - j10$$

$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{vmatrix} = 32(1 + j)(1 - j) + 4 = 68$$

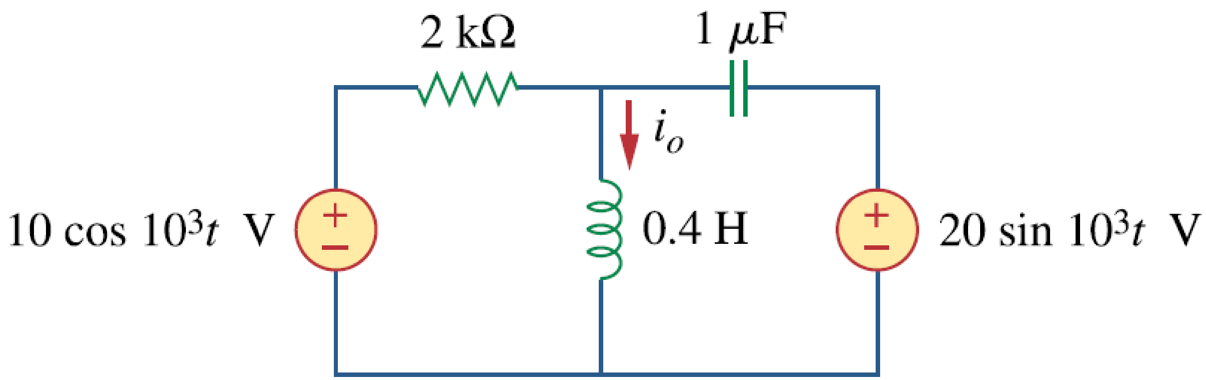
$$\Delta_2 = \begin{vmatrix} 8 + j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340 - j240 = 416.17\angle -35.22^\circ$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17\angle -35.22^\circ}{68} = 6.12\angle -35.22^\circ \text{ A}$$

$$\mathbf{I}_o = -\mathbf{I}_2 = 6.12\angle 144.78^\circ \text{ A}$$

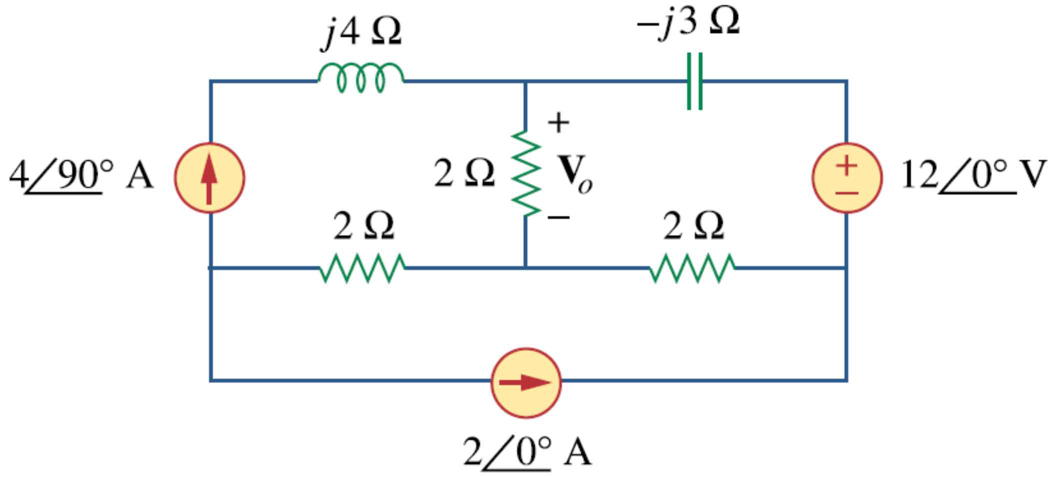
Example – 6

- Use mesh analysis to find i_0.



Example – 7

- Use mesh analysis to find v_0.

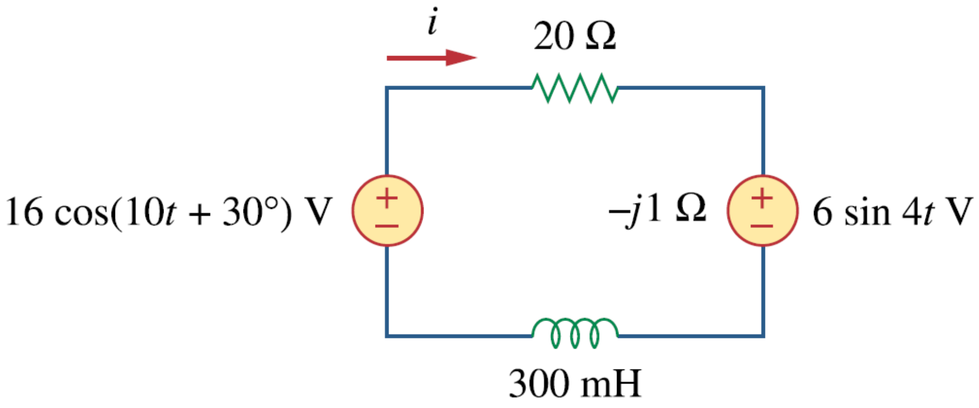


Superposition Theorem

- These circuits are linear and hence you can apply superposition theorem.
- If sources have different frequencies then individual response must be added in the time domain.
- You can't add them in phasors as they have different $e^{j\omega t}$.

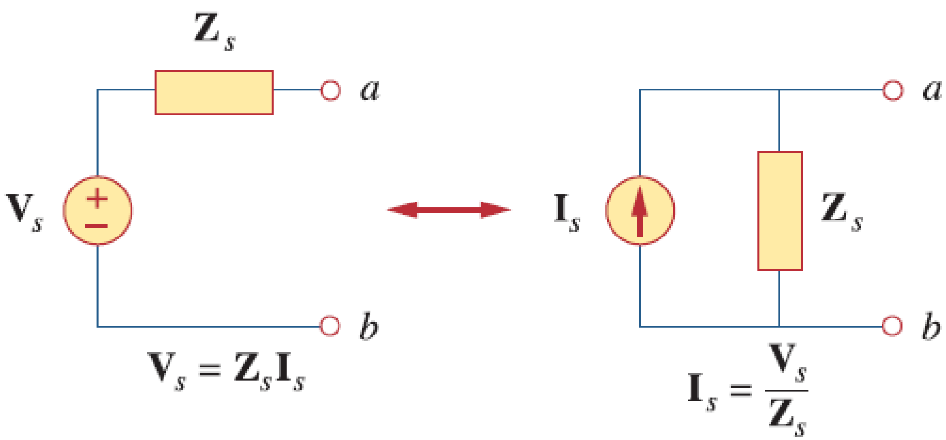
Example – 8

- Use superposition to find $i(t)$.



Source Transformation

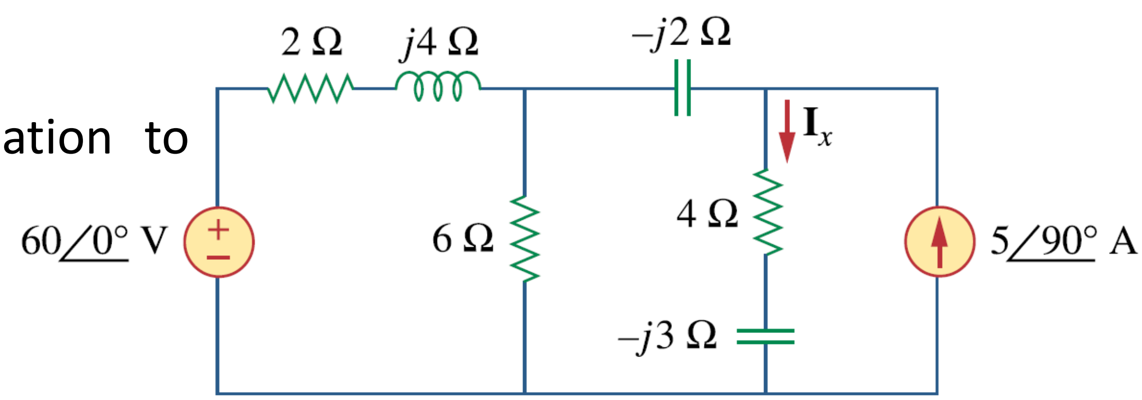
It involves transformation of **voltage source in series with an impedance** to a **current source in parallel with an impedance**, or vice versa.



$$\mathbf{V}_s = \mathbf{Z}_s\mathbf{I}_s \quad \Leftrightarrow \quad \mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_s}$$

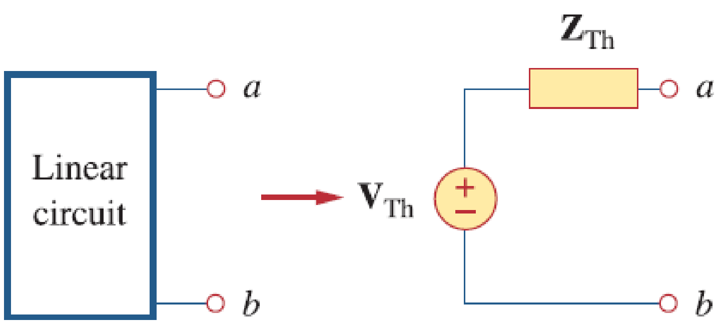
Example – 9

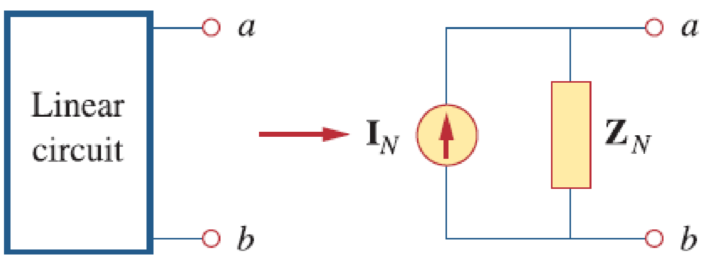
- Use source transformation to to find I_x.



Thevenin and Norton Equivalent Circuits

- These theorems are applied to AC circuits similar to the way it is applied to DC circuits.
- You need to work with complex numbers in AC circuits.
- For sources with different frequencies, you will have different equivalent circuit for each frequency.

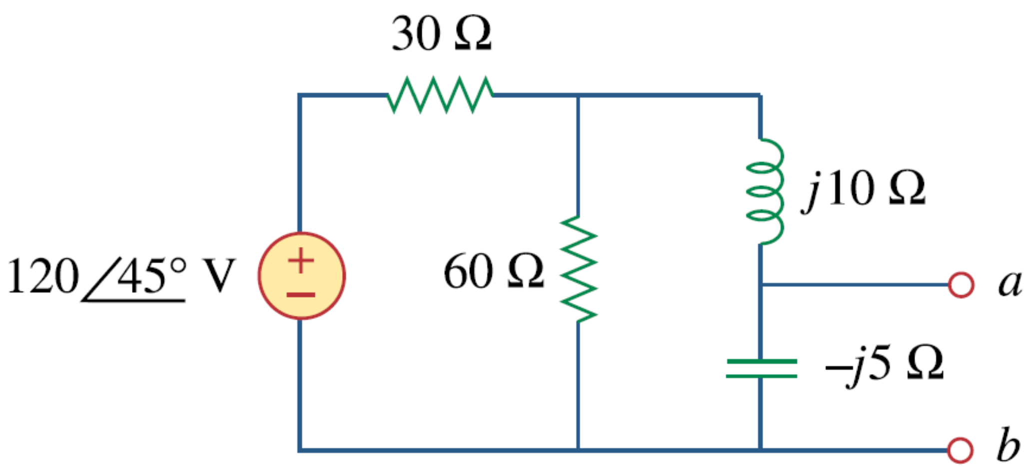




$$\mathbf{V}_{Th} = \mathbf{Z}_N \mathbf{I}_N, \qquad \mathbf{Z}_{Th} = \mathbf{Z}_N$$

Example – 10

- Obtain Thevenin and Norton equivalent circuits at terminal a-b.



Example – 11

- Using Thevenin Theorem, determine $v_o(t)$.