Lecture – 7 **Date: 29.08.2016**

- AC Circuits: Impedance and Admittance, Kirchoff's Laws, Phase Shifter, AC bridge

Impedance and Admittance

- we know: $\mathbf{V} = R\mathbf{I}, \quad \mathbf{V} = j\omega L\mathbf{I}, \quad \mathbf{V} = \frac{\mathbf{I}}{j\omega C}$

$$\frac{\mathbf{V}}{\mathbf{I}} = R, \quad \frac{\mathbf{V}}{\mathbf{I}} = j\omega L, \quad \frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{j\omega C}$$

- we express Ohm's law in phasor form: $\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \quad \text{or} \quad \mathbf{V} = \mathbf{ZI}$

where **Z** is a frequency-dependent quantity known as *impedance*, measured in ohms. It is the ratio of the phasor voltage **V** to the phasor current **I**, measured in Ω.

$$\mathbf{Z}_L = j\omega L \text{ and } \mathbf{Z}_C = -j/\omega C$$

- For $\omega = 0$(i.e., dc sources): $Z_L = 0$ and $Z_C \rightarrow \infty$.
- the inductor acts like a short circuit, while the capacitor acts like an open circuit.

- For $\omega \rightarrow \infty$(i.e., high frequencies): $Z_L \rightarrow \infty$ and $Z_C = 0$.
- the inductor acts like an open circuit, while the capacitor acts like a short circuit.

Impedance and Admittance

- complex quantity $\mathbf{Z} = R + jX \rightarrow \mathbf{Z} = |\mathbf{Z}| \angle \theta$

$$R = |\mathbf{Z}|\cos\theta, \qquad X = |\mathbf{Z}|\sin\theta \qquad |\mathbf{Z}| = \sqrt{R^2 + X^2}, \qquad \theta = \tan^{-1}\frac{X}{R}$$

- It is sometimes (parallel circuits) convenient to work with the reciprocal of impedance, known as *admittance* ***(Y)***.

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{\mathbf{I}}{\mathbf{V}} \rightarrow \mathbf{Y} = G + jB$$

$$G = \frac{R}{R^2 + X^2}$$

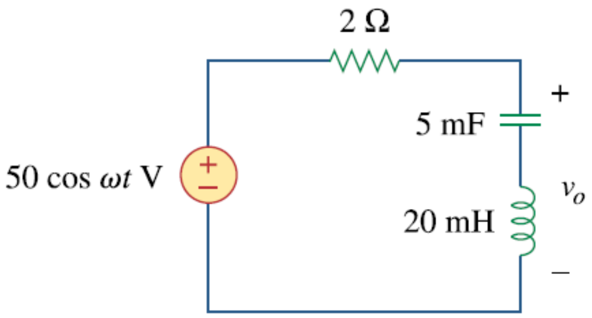
$$B = -\frac{X}{R^2 + X^2}$$

Example – 1

A linear network has a current input $4\cos(\omega t + 20°)$A and a voltage output $10\cos(\omega t + 110°)$V. Determine the associated impedance.

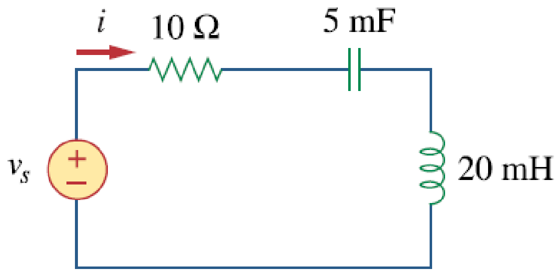
Example – 2

What value of ω will cause the forced response v_0 in this circuit to be zero?



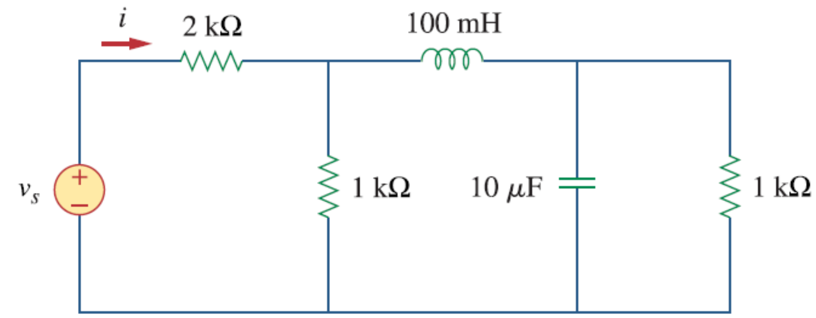
Example – 3

Find current i in this circuit, when $v_s(t) = 50cos200t$ V.



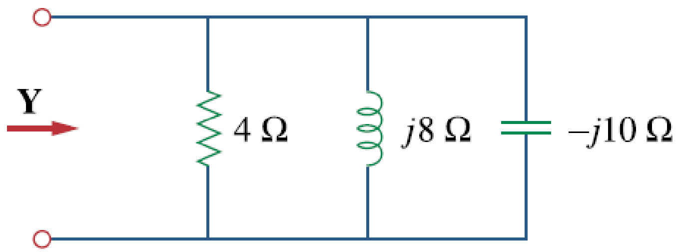
Example – 4

Find current i in this circuit, when $v_s(t) = 60\cos(200t - 10°)$ V.



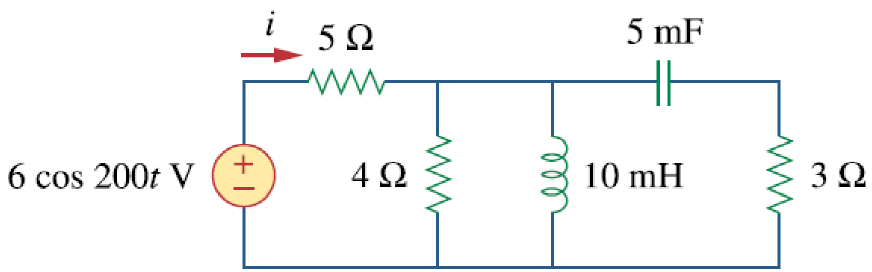
Example – 5

Determine the admittance **Y** for this circuit.



Example – 6

Find current i in this circuit.



Kirchoff's Laws

- cannot do circuit analysis in the frequency domain without Kirchhoff's current and voltage laws.
- Therefore, need to express them in the frequency domain.

<u>For KVL,</u> let $v_1, v_2, \ldots\ldots.., v_n$ are the voltages around a closed loop.

$$v_1 + v_2 + \cdots + v_n = 0 \Rightarrow V_{m1}\cos(\omega t + \theta_1) + V_{m2}\cos(\omega t + \theta_2) + \cdots + V_{mn}\cos(\omega t + \theta_n) = 0$$

$$\mathrm{Re}(V_{m1}e^{j\theta_1}e^{j\omega t}) + \mathrm{Re}(V_{m2}e^{j\theta_2}e^{j\omega t}) + \cdots + \mathrm{Re}(V_{mn}e^{j\theta_n}e^{j\omega t}) = 0$$

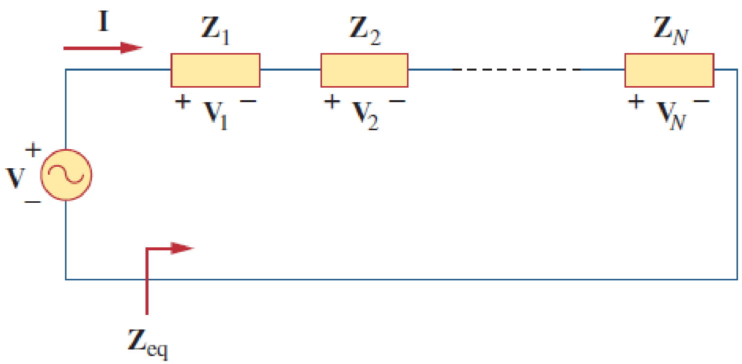
$$\mathrm{Re}[(V_{m1}e^{j\theta_1} + V_{m2}e^{j\theta_2} + \cdots + V_{mn}e^{j\theta_n})e^{j\omega t}] = 0$$

$$\Rightarrow \mathrm{Re}[(\mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n)e^{j\omega t}] = 0 \Rightarrow \mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n = 0$$

Kirchhoff's voltage law holds for phasors.

Similarly, one can prove that Kirchhoff's current law holds in the frequency domain

Impedance Combinations

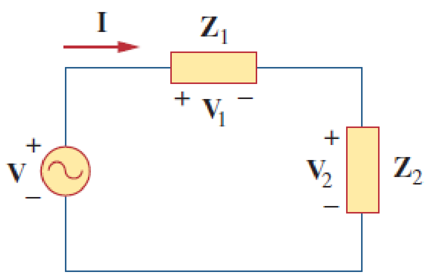


$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_N$$

$$= \mathbf{I}(\mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N)$$

$$\mathbf{Z}_{eq} = \frac{\mathbf{V}}{\mathbf{I}} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N$$

This is similar to the series connection of resistances.

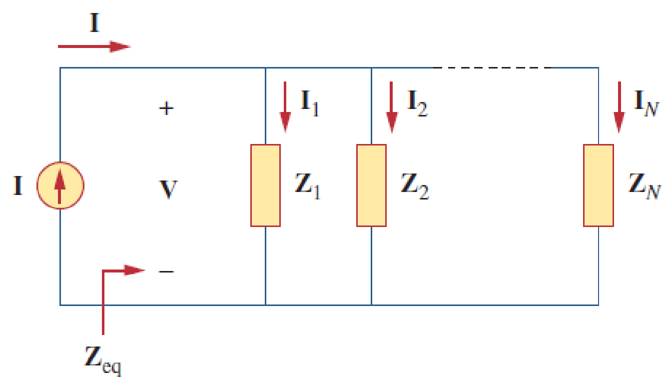


$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

$$\mathbf{V}_1 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}, \qquad \mathbf{V}_2 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}$$

voltage-division relationship

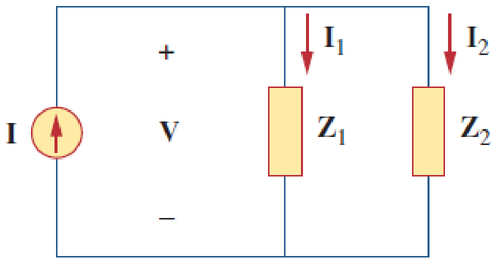
Impedance Combinations (contd.)



$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_N = \mathbf{V}\left(\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots + \frac{1}{\mathbf{Z}_N}\right)$$

$$\frac{1}{\mathbf{Z}_{eq}} = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots + \frac{1}{\mathbf{Z}_N}$$

$$\mathbf{Y}_{eq} = \mathbf{Y}_1 + \mathbf{Y}_2 + \cdots + \mathbf{Y}_N$$



$$\mathbf{Z}_{eq} = \frac{1}{\mathbf{Y}_{eq}} = \frac{1}{\mathbf{Y}_1 + \mathbf{Y}_2} = \frac{1}{1/\mathbf{Z}_1 + 1/\mathbf{Z}_2} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

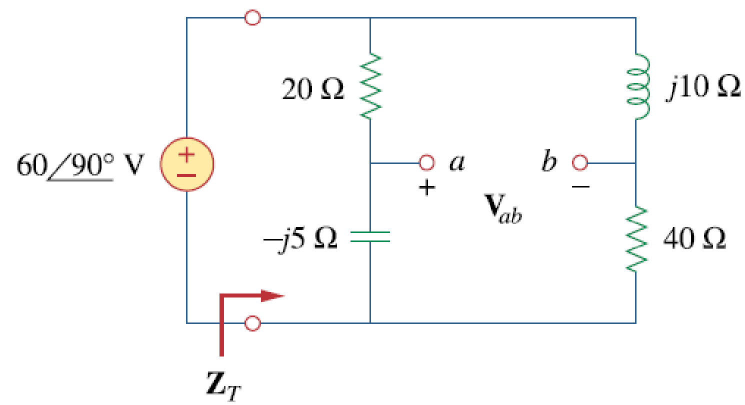
$$\mathbf{V} = \mathbf{I}\mathbf{Z}_{eq} = \mathbf{I}_1\mathbf{Z}_1 = \mathbf{I}_2\mathbf{Z}_2$$

$$\mathbf{I}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}, \qquad \mathbf{I}_2 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}$$

the *current-division* principle.

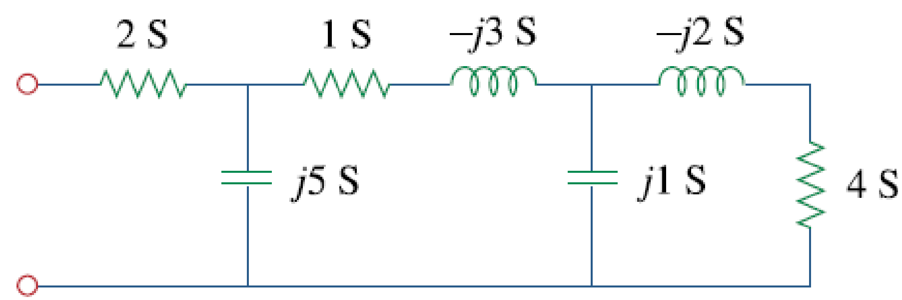
Example – 7

For this circuit, calculate $\mathbf{Z}_T$ and $\mathbf{V}_{ab}$.



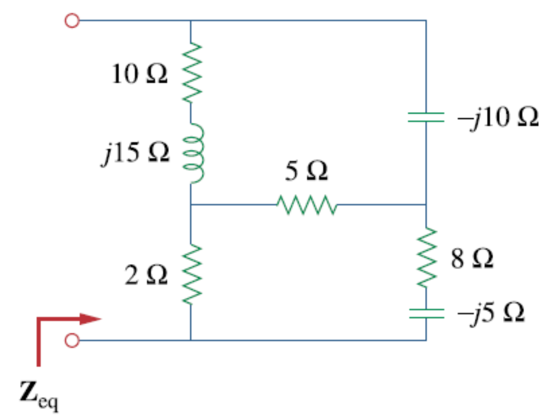
Example – 8

Find the equivalent admittance $\mathbf{Y}_{eq}$ of this circuit.



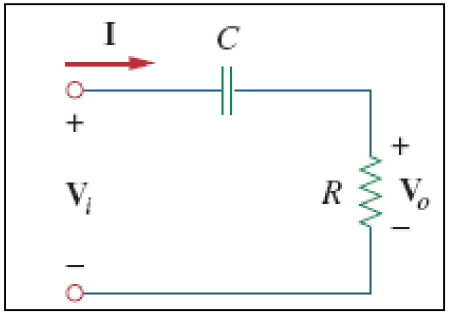
Example – 9

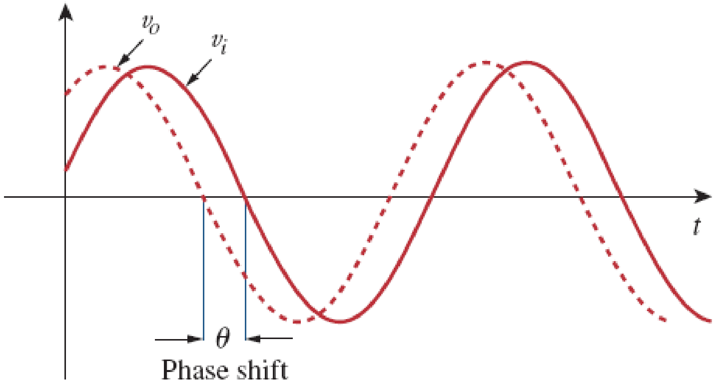
Find the equivalent impedance of this circuit.



Phase Shifter

- A phase-shifting circuit is used for correcting undesirable phase shift present in a circuit.
- It is also used for the creation of desired phase shifts.
- *RC and RL circuits are extremely useful for this purpose.*



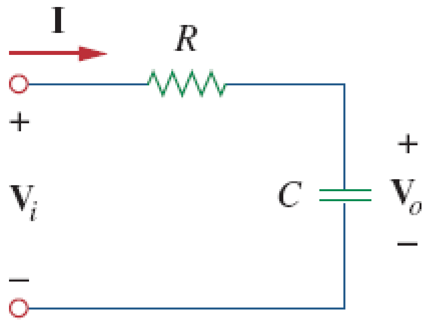


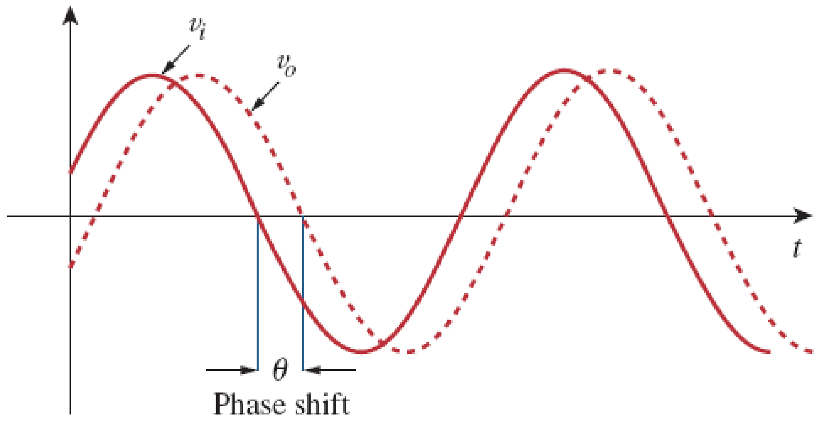
the circuit current **I** leads the applied voltage by some phase angle θ, where $0 < \theta < 90^o$ depending on the values of *R* and *C*.

$$\mathbf{Z} = R + jX_C, \quad \theta = \tan^{-1}\frac{X_C}{R}$$

the amount of phase shift depends on the values of *R*, *C*, and the operating frequency.

Phase Shifter (contd.)

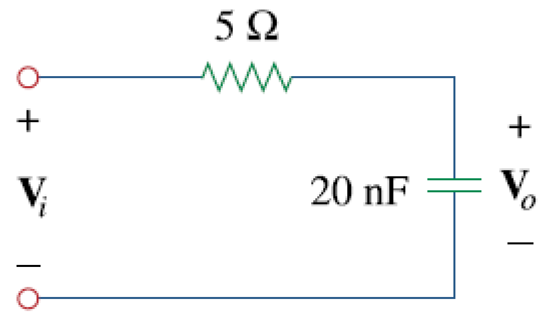




- These simple single stage RC circuits are generally not used in practice.
- These RC circuits also work as voltage dividers. Therefore, as the phase shift approaches 90^o the output voltage approaches zero. For this reason, these simple *RC* circuits are used only when small amounts of phase shift are required.
- For large phase shifts, the *RC* networks are cascaded. This provides a total phase shift equal to the sum of the individual phase shifts.

Example – 10

For this RC circuit:
(a) Calculate the phase shift at 2 MHz.
(b) Find the frequency where the phase shift is 45^o.



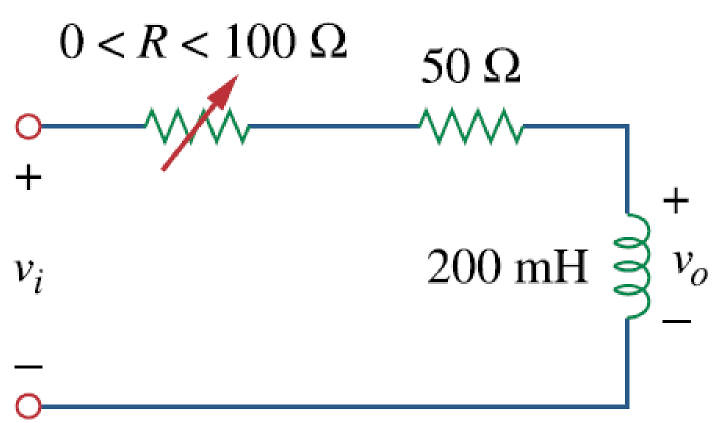
Example – 11

A coil with impedance 8 + *j*6 Ω is connected in series with a capacitive reactance *X*. The series combination is connected in parallel with a resistor *R*. Given that the equivalent impedance of the resulting circuit is $5\angle 0^o$ Ω, find the value of *R* and *X*.

Example – 12

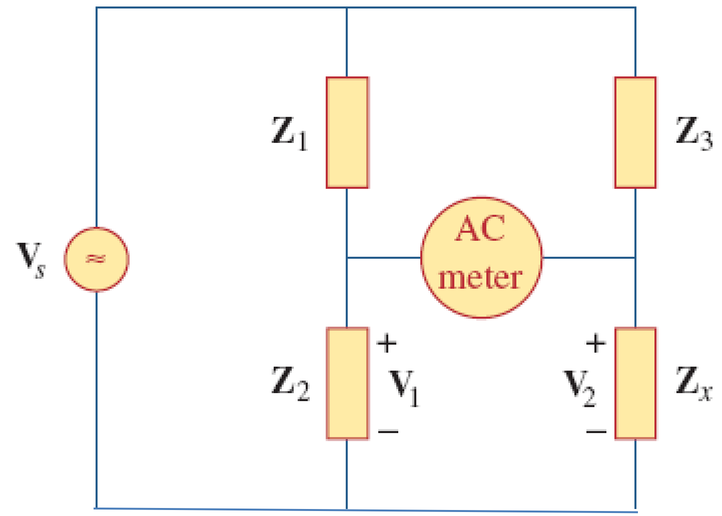
Consider this phase-shifting circuit.

(a) $\mathbf{V}_o$ when *R* is maximum
(b) $\mathbf{V}_o$ when *R* is minimum
(c) the value of *R* that will produce a phase shift of 45^o.



AC Bridges

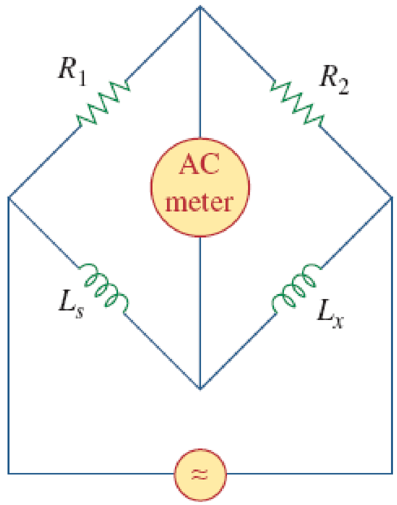
- An ac bridge circuit is used for measuring the inductance *L* of an inductor or the capacitance *C* of a capacitor.
- Similar to the Wheatstone bridge used for measuring an unknown resistance and follows the same principle.
- To measure *L* and *C*, however, an ac source is needed as well as an ac meter instead of the galvanometer.
- The ac meter may be a sensitive ac ammeter or voltmeter.

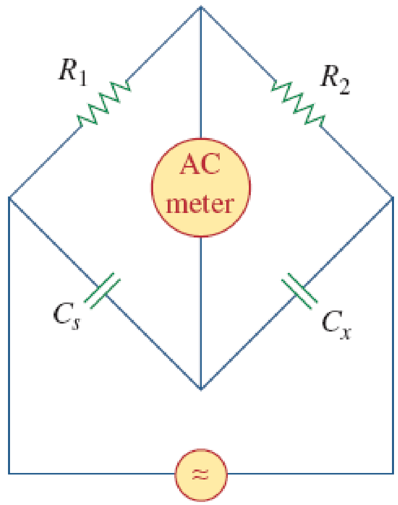


bridge is *balanced* when no current flows through the meter i.e., V1 = V2.

$$\mathbf{V}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}_s = \mathbf{V}_2 = \frac{\mathbf{Z}_x}{\mathbf{Z}_3 + \mathbf{Z}_x}\mathbf{V}_s \Rightarrow \mathbf{Z}_x = \frac{\mathbf{Z}_3}{\mathbf{Z}_1}\mathbf{Z}_2$$

AC Bridges (contd.)



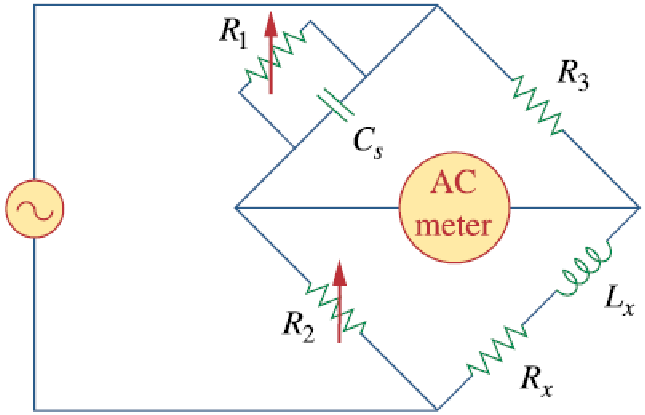


$$L_x = \frac{R_2}{R_1} L_s$$

$$C_x = \frac{R_1}{R_2} C_s$$

Example – 13

This ac bridge is known as a *Maxwell bridge* and is used for accurate measurement of inductance and resistance of a coil in terms of a standard capacitance S_s. Show that when the ridge is balanced:



$$L_x = R_2 R_3 C_s \qquad R_x = \frac{R_2}{R_1} R_3$$

Example – 14

This ac bridge is called a *Wien bridge*. It is used for measuring the frequency of a source. Show that when the bridge is balanced:

$$f = \frac{1}{2\pi\sqrt{R_2 R_4 C_2 C_4}}$$