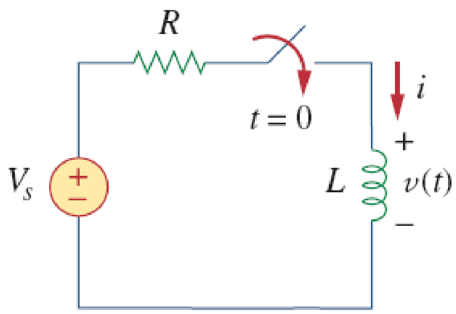
Circuit Design Research Lab

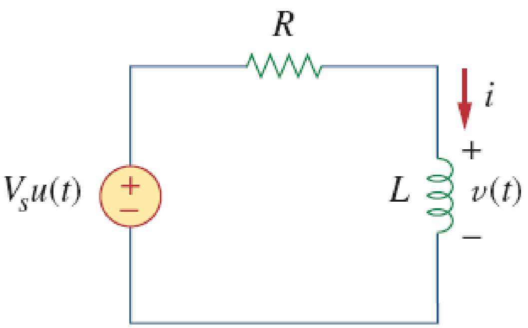
# Lecture – 3

# Date: 08.08.2016

- First-Order Circuit (contd.)

# Step Response of an RL Circuit




Complete Response: $i = i_t + i_{ss}$

$$i_t = Ae^{-t/\tau}, \qquad \tau = \frac{L}{R} \qquad\qquad i_{ss} = \frac{V_s}{R}$$

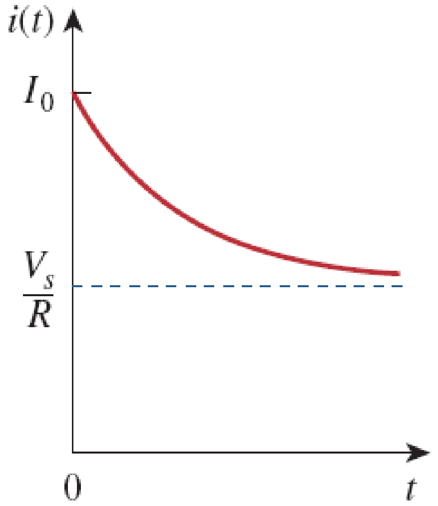
- Now the current through the inductor cannot change instantaneously:

$$i(0^+) = i(0^-) = I_0$$

# Step Response of an RL Circuit (contd.)

At t=0: $I_0 = A + \frac{V_s}{R}$ → $A = I_0 - \frac{V_s}{R}$ → $i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-t/\tau}$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$




- For $I_0=0$

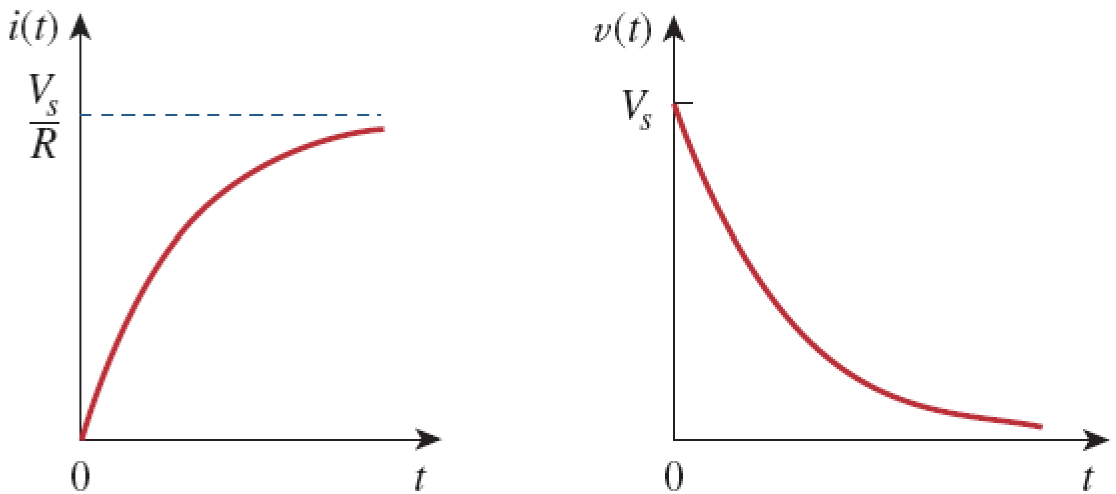
$$i(t) = \begin{cases} 0, & t < 0 \\ \frac{V_s}{R}(1 - e^{-t/\tau}), & t > 0 \end{cases}$$

$$i(t) = \frac{V_s}{R}(1 - e^{-t/\tau})u(t)$$

# Step Response of an RL Circuit (contd.)

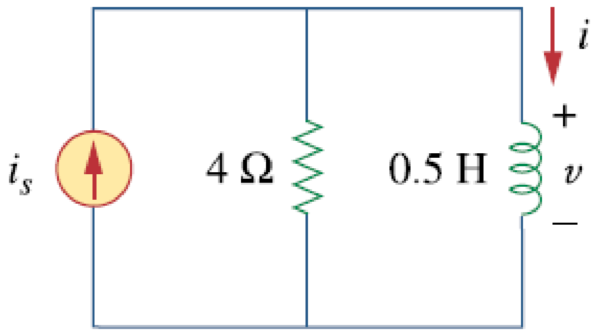
- The voltage across the inductor:

$$v(t) = L\frac{di}{dt} = V_s\frac{L}{\tau R}e^{-t/\tau}, \qquad \tau = \frac{L}{R}, \qquad t > 0 \quad \Rightarrow \quad v(t) = V_s e^{-t/\tau}u(t)$$



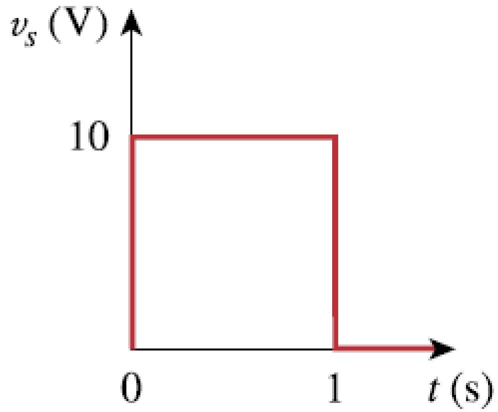
# Example – 1

In the following circuit, the current $i_s$ changes from 5A to 10A at $t$ = 0 that is, $i_s = 5u(-t) + 10u(t)$. Find $v$ *and* $i$.

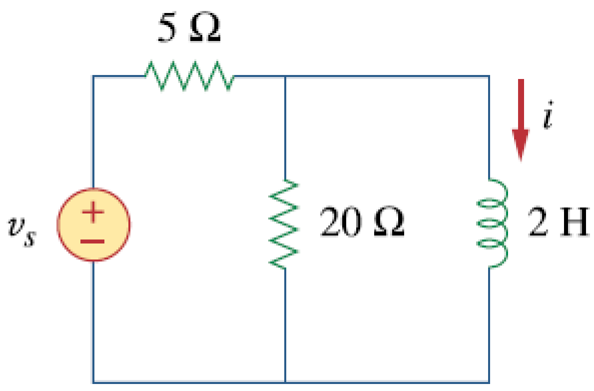


# Example – 2

If the input pulse in Fig. (a) is applied to the circuit in Fig. (b), determine the response $i(t)$.



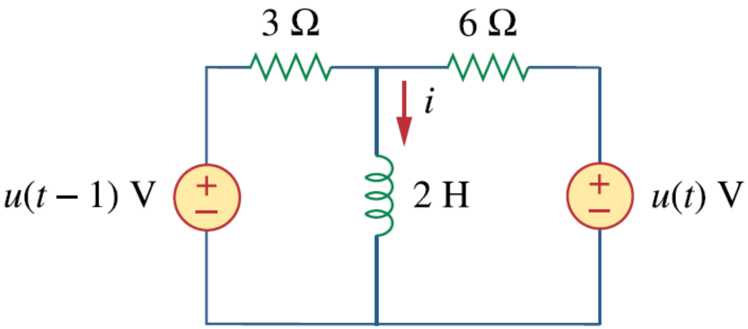

(a)




(b)

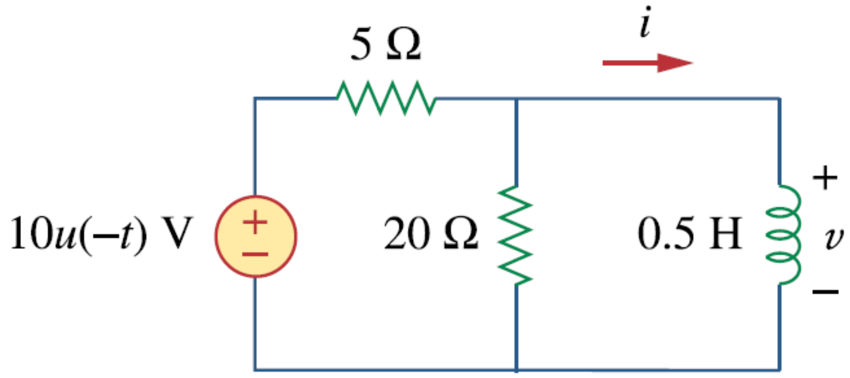
# Example – 3

For the following circuit, calculate $i(t)$ if $i(0) = 0$.

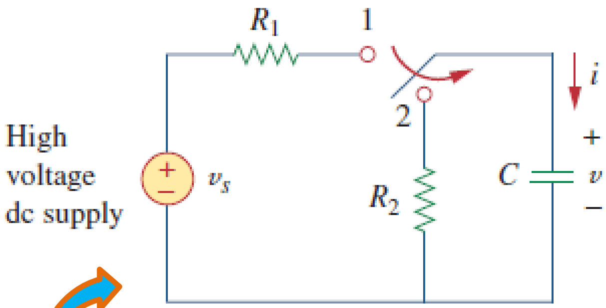


# Example – 4

Obtain $v(t)$ and $i(t)$ in the following circuit:



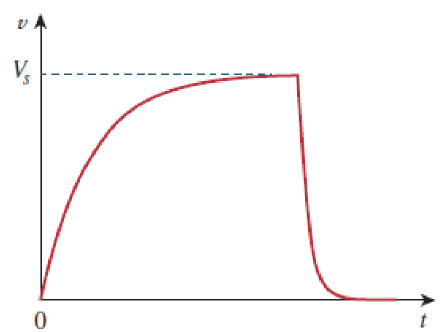
# Photo Flash Unit

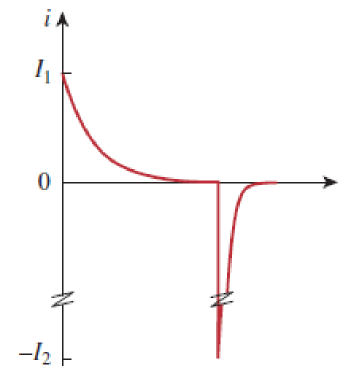



- An electronic flash unit is a common example of an *RC* circuit.
- This exploits the ability of the capacitor to oppose any abrupt change in voltage.

It consists of a high-voltage dc supply, a current-limiting large resistor $R_1$ and a capacitor *C* in parallel with the flash lamp of low Resistance $R_2$.

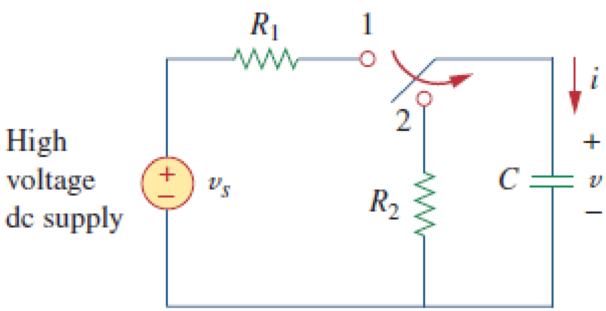
- When the switch is in position 1, the capacitor charges slowly due to the large time constant ($R_1C$ ).
- the capacitor voltage rises gradually from zero to $V_s$ while its current decreases gradually from ($I_1 = V_s/R$) to zero.



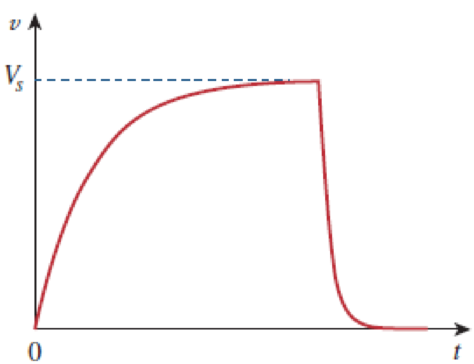



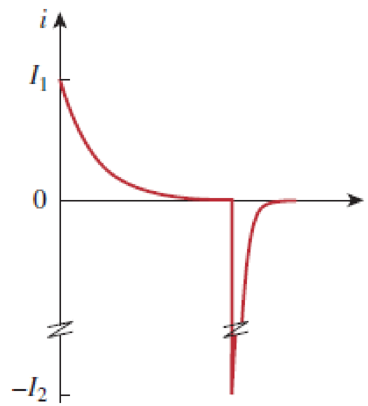

The charging time is approximately five times the time constant: $t_{charge} = 5R_1C$

# Photo Flash Unit (contd.)




- With the switch in position 2, the capacitor voltage is discharged.
- The low resistance $R_2$ of the photo lamp permits a high discharge current with peak [$I_2 = V_S/R_2$] in a short duration



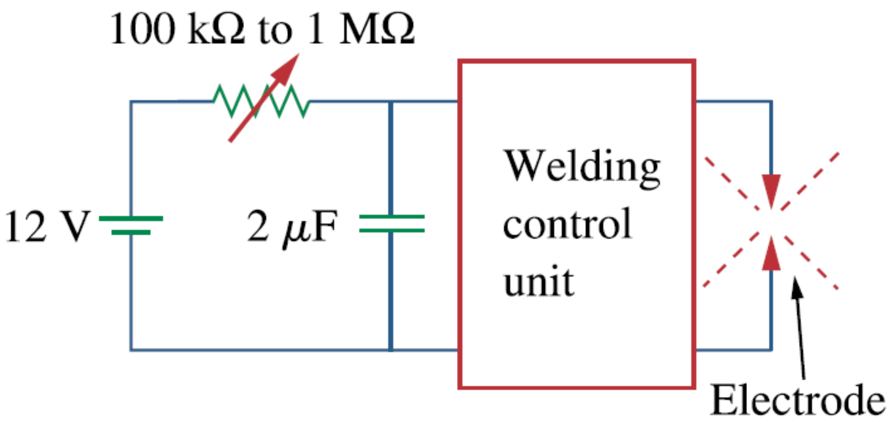




Discharging takes place in approximately five times the time constant: $t_{discharge} = 5R_2C$

- This simple *RC* circuit provides a short-duration and high current pulse.
- Such a circuit also finds applications in electric spot welding and the radar transmitter tube.

# Example – 5

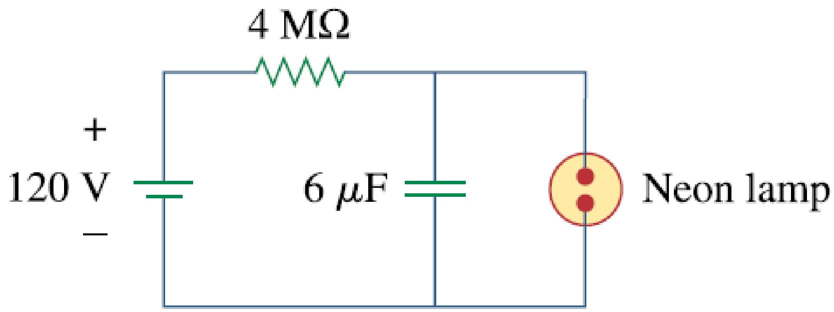
Following Figure shows a circuit for setting the length of time voltage is applied to the electrodes of a welding machine. The time is taken as how long it takes the capacitor to charge from 0 to 8 V. What is the time range covered by the variable resistor?



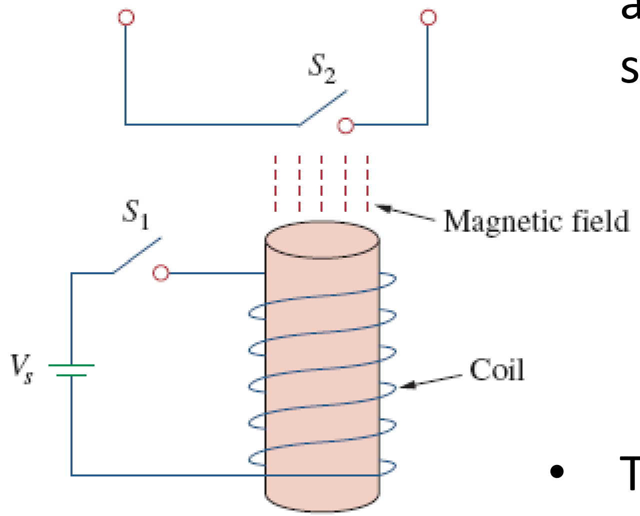
# Example – 6

A simple relaxation oscillator circuit is shown. The neon lamp fires when its voltage reaches 75V and turns off when its voltage drops to 30V. Its resistance is 120Ω when on and infinitely high when off.

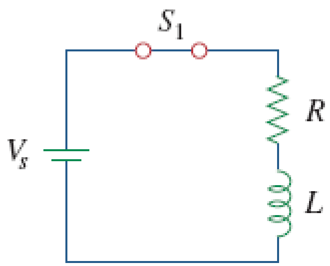
(a) For how long is the lamp on each time the capacitor discharges?
(b) What is the time interval between light flashes?



## Relay Circuit




- Relay, a magnetically controlled switch, is essentially an electromagnetic device used to open or close a switch that controls another circuit.

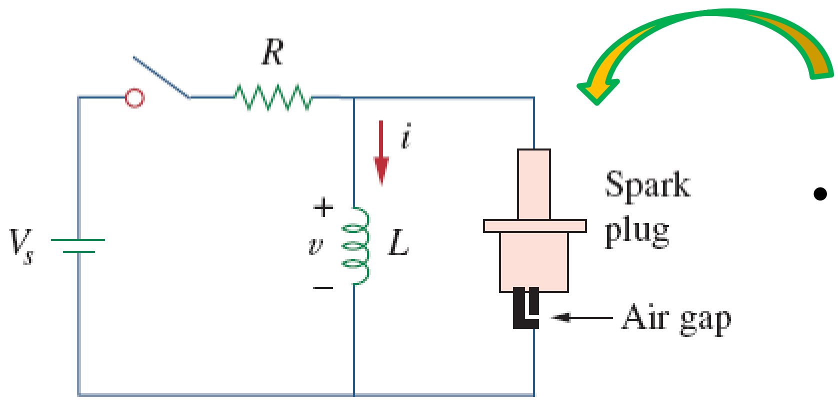



- The coil circuit is an *RL* circuit, where *R* and *L* are the resistance and inductance of the coil.

- the coil circuit is energized once switch $S_1$ is closed.
- The coil current gradually increases and produces a magnetic field. Eventually the magnetic field is sufficiently strong to pull the movable contact in the other circuit and close switch $S_2$.

Relays were used in the earliest digital circuits and are still used in high-power circuits.

## Automobile Ignition Circuit

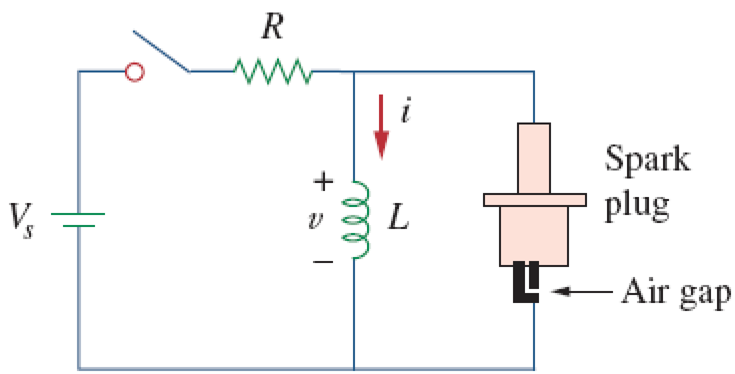



- It makes use of the ability of inductors to oppose rapid change in current as it makes them useful for spark generation.
- The ignition of fuel-air mixture in each cylinder at proper times is achieved by a spark plug, which consists of a pair of electrodes separated by an air gap.

- By creating a large voltage (thousands of volts) between the electrodes, a spark is formed across the air gap, thereby igniting the fuel. But how can such a large voltage be obtained from the car battery, which supplies only 12 V?

- This is achieved by means of an inductor (the spark coil) *L*. The voltage across the inductor $v = L\,{}^{di}/_{dt}$ can be made large by creating a large change in current in a very short time.

# Automobile Ignition Circuit (contd.)

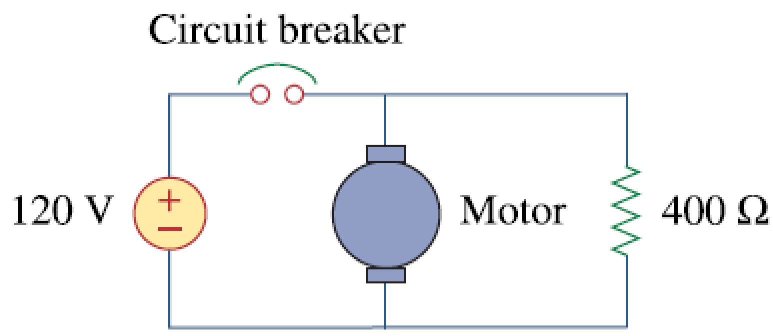



- When the switch is closed, the current through the inductor increases gradually and reaches the final value of $^{Vs}/_{R}$, where $Vs$ is 12V.

- At steady state $^{di}/_{dt} = 0$ and hence the inductor voltage $v = 0$.
- When the switch suddenly opens, a large voltage is developed across the inductor (due to the rapid change in $^{di}/_{dt}$) causing a spark in the air gap.
- The spark continues until the energy stored in the inductor is dissipated in the spark discharge.

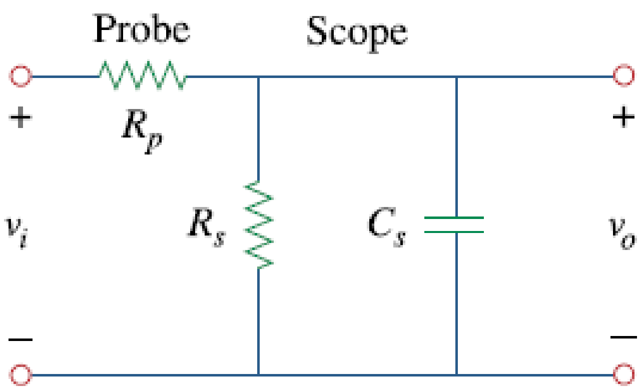
## Example – 7

A 120-V dc generator energizes a motor whose coil has an inductance of 50H and a resistance of 100Ω. A field discharge resistor of 400Ω is connected in parallel with the motor to avoid damage to the motor. The system is at steady state. Find the current through the discharge resistor 100ms after the breaker is tripped.



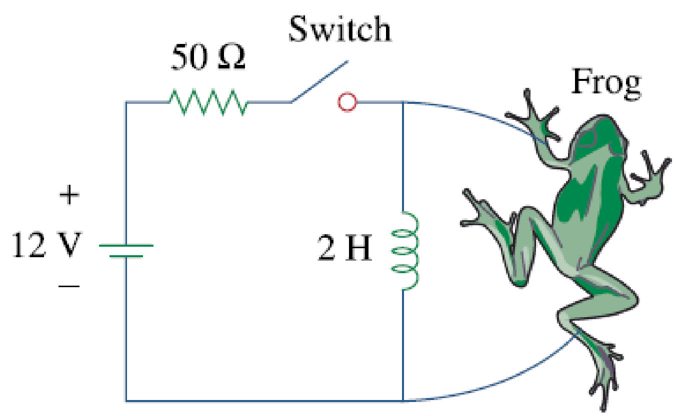
# Example – 8

An attenuator probe employed with oscilloscopes was designed to reduce the magnitude of the input voltage $v_i$ by a factor of 10. As shown in Fig, the oscilloscope has internal resistance R*s* and capacitance C*s* while the probe has an internal resistance R*p*. If R *p* is fixed at 6MΩ find R*s* and C*s* for the circuit to have a time constant of 15 μs.



# Example – 9

Following circuit is used by a biology student to study "frog kick." She noticed that the frog kicked a little when the switch was closed but kicked violently for 5s when the switch was opened. Model the frog as a resistor and calculate its resistance. Assume that it takes 10 mA for the frog to kick violently.



## Second-Order Circuits

- A second-order consists of resistors and the equivalent of two energy storage elements and is characterized by a second-order differential equation.

**Analysis of such circuits require understanding of $v(0), i(0), v(\infty), i(\infty), {}^{dv(0)}/_{dt}$, and ${}^{di(0)}/_{dt}$**

- **Two key points:**
  - carefully handle the polarity of voltage $v(t)$ across the capacitor and the direction of the current $i(t)$ through the inductor.
  - keep in mind that the capacitor voltage is always continuous $[v(0^+) = v(0^-)]$ and the inductor current is always continuous $[i(0^+) = i(0^-)]$.