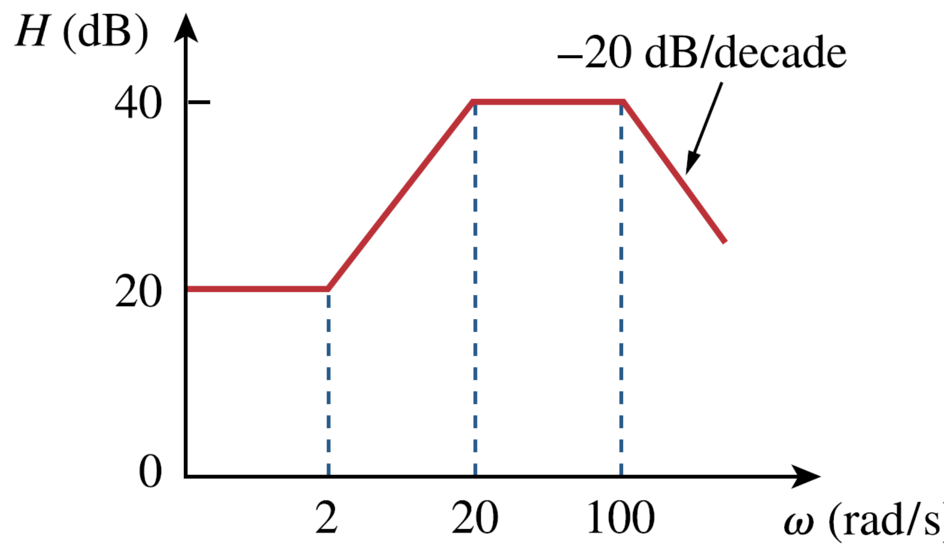
Lecture – 16

Date: 20.10.2016

- Bode Plot (contd.)
- Series and Parallel Resonance

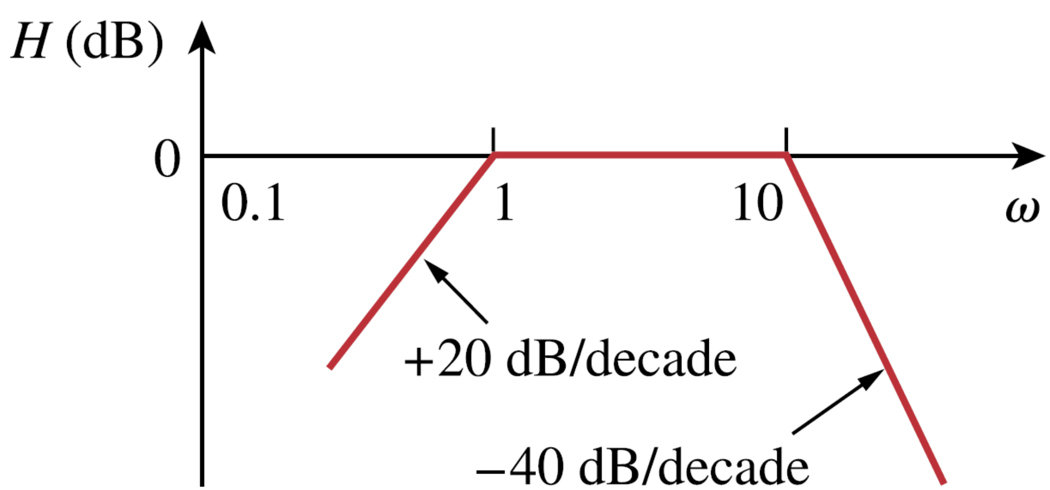
Example – 1

- Find the transfer function **H**(ω) with this Bode magnitude plot



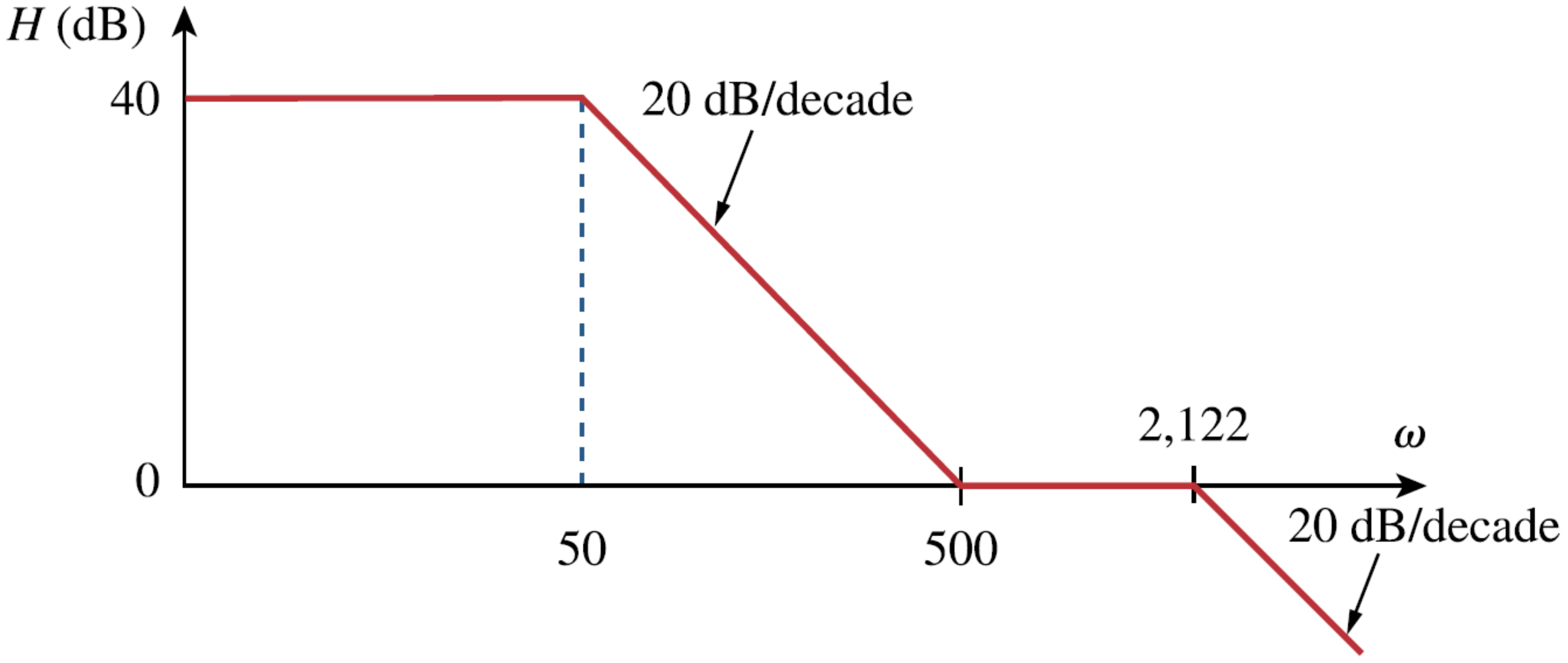
Example – 2

- Find the transfer function **H**(ω) with this Bode magnitude plot



Example – 3

The following magnitude plot represents the transfer function of a preamplifier. Find $H(s)$.



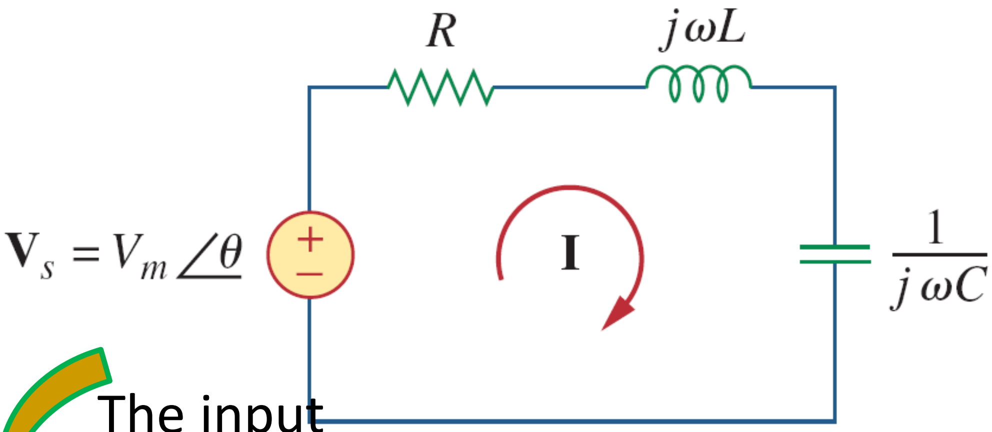
Series Resonance

- Resonance occurs in any system that has a complex conjugate pair of poles; it is the cause of oscillations of stored energy from one form to another.
- It allows frequency discrimination in communications networks.

Resonance is a condition in an *RLC* circuit in which the capacitive and inductive reactances are equal in magnitude, thereby resulting in a purely resistive impedance.

Resonant circuits (series or parallel) are useful for constructing highly frequency selective filters. They are used in many applications such as selecting the desired stations in radio and TV receivers.

Series Resonance (contd.)



The input impedance:

$$\mathbf{Z} = \mathbf{H}(\omega) = \frac{\mathbf{V}_s}{\mathbf{I}} = R + j\omega L + \frac{1}{j\omega C}$$

$$\mathbf{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

For Resonance

$$\text{Im}(\mathbf{Z}) = \omega L - \frac{1}{\omega C} = 0$$

The value of ω at which the imaginary impedance vanishes is called the *resonant frequency* ω_0.

$$\omega_0 L = \frac{1}{\omega_0 C} \quad \Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s} \quad \Rightarrow \quad f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

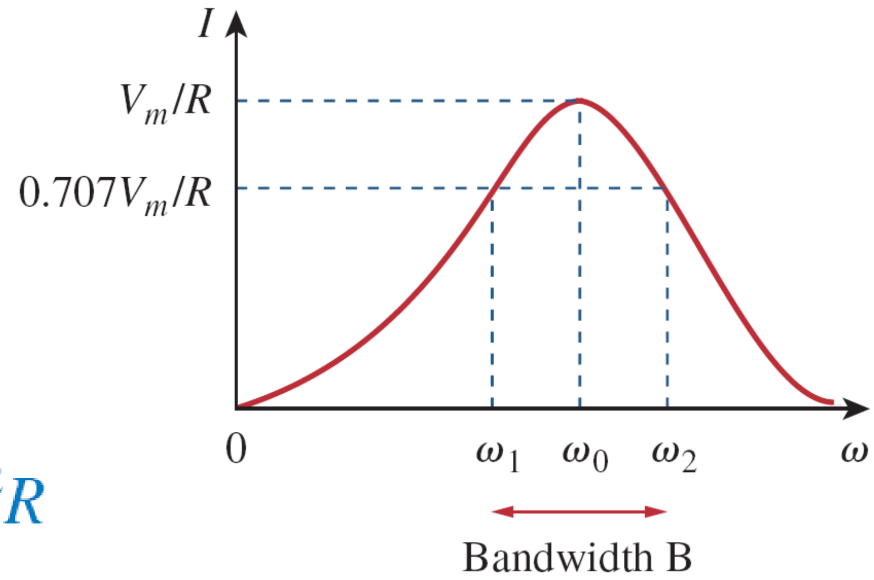
Series Resonance (contd.)

At resonance:

- The impedance is purely resistive, i.e., the *LC* series combination acts like a short circuit, and the entire voltage is across *R*.
- Voltage and Current are in phase and therefore the power factor is unity.
- The magnitude of the transfer function is minimum.
- The inductor voltage and capacitor voltage can be much more than the source voltage.

- The frequency response of the circuit's current magnitude:

$$I = |\mathbf{I}| = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$



- The average power dissipated by the *RLC* circuit:

$$P(\omega) = \frac{1}{2} I^2 R$$

Series Resonance (contd.)

- The highest power dissipation happens at the resonance, when current peak of $I = \frac{V_m}{R}$ exists.

$$P(\omega_0) = \frac{1}{2}\frac{V_m^2}{R}$$

- Lets assume that half power is dissipated at frequencies of ω_1 and ω_2:

$$P(\omega_1) = P(\omega_2) = \frac{(V_m/\sqrt{2})^2}{2R} = \frac{V_m^2}{4R}$$

- The half-power frequencies can be obtained by setting Z equal to $\sqrt{2}$R.

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R$$

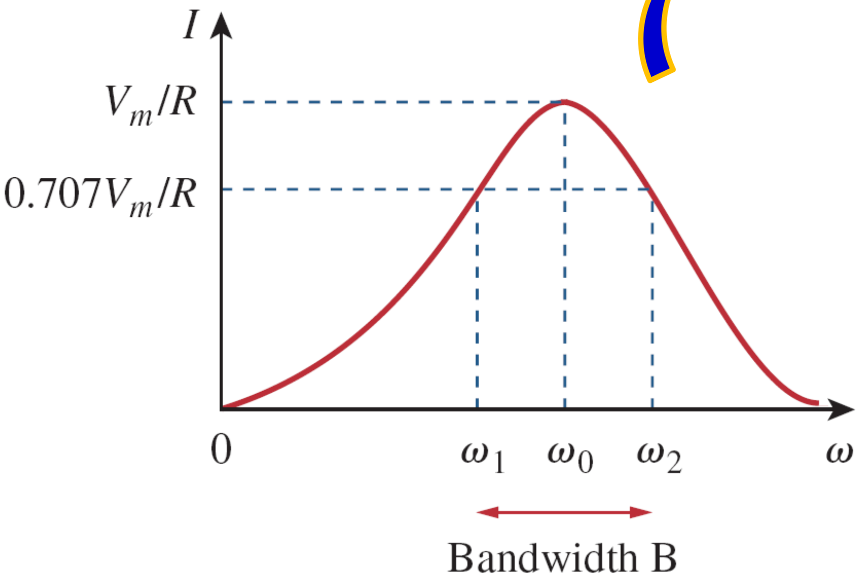
$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

In general ω_1 and ω_2 are not symmetrical around the resonant frequency, because the frequency response is not generally symmetrical.

Series Resonance (contd.)



The height of the curve is determined by *R*, the width of the curve depends on the *bandwidth B* defined as:

$$B = \omega_2 - \omega_1$$

B is essentially the half-power bandwidth, because it is the width of the frequency band between the half-power frequencies.

A metric known as *quality factor Q* relates the maximum or peak energy stored to the energy dissipated in the circuit per cycle of oscillation.

$$Q = 2\pi \frac{\text{Peak energy stored in the circuit}}{\text{Energy dissipated by the circuit in one period at resonance}}$$

Series Resonance (contd.)

- *Q* is also regarded as a measure of the energy storage property of a circuit in relation to its energy dissipation property.

$$Q = 2\pi \frac{\frac{1}{2}LI^2}{\frac{1}{2}I^2R(1/f_0)} = \frac{2\pi f_0 L}{R}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

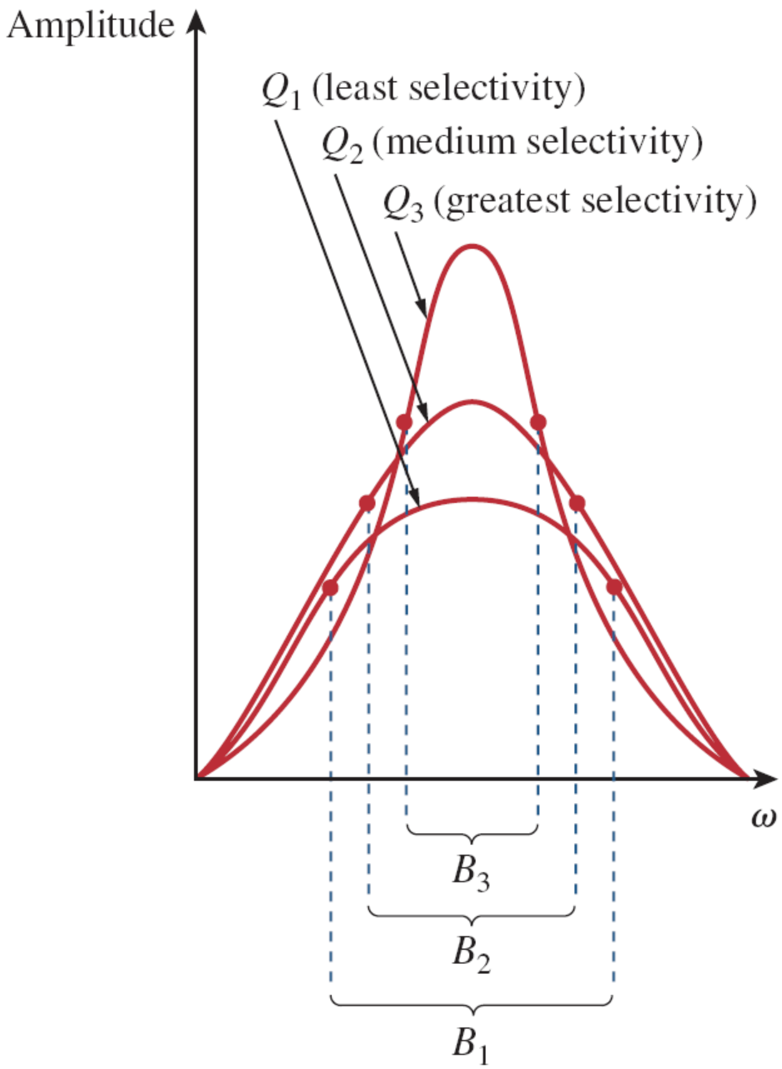
Further Simplification:

$$B = \frac{R}{L} = \frac{\omega_0}{Q}$$

$$B = \omega_0^2 CR$$

The quality factor of a resonant circuit is the ratio of its resonant frequency to its bandwidth.

Series Resonance (contd.)



- the higher the value of *Q*, the more selective the circuit is but the smaller the bandwidth.
- The *selectivity* of an *RLC* circuit is the ability of the circuit to respond to a certain frequency and discriminate against all other frequencies.
- If the band of frequencies to be selected or rejected is narrow, the quality factor of the resonant circuit must be high.
- If the band of frequencies is wide, the quality factor must be low.

Series Resonance (contd.)

- A resonant circuit is designed to operate at or near its resonant frequency.
- It is said to be a *high-Q circuit* when its quality factor is equal to or greater than 10 ($Q \geq 10$).
- For high-*Q* circuits the half-power frequencies are, for all practical purposes, symmetrical around the resonant frequency and can be approximated as:

$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2}$$

a resonant circuit is characterized by five related parameters: the two half-power frequencies ω_1 and ω_2, the resonant frequency ω_0, the bandwidth *B*, and the quality factor *Q*.

Example – 4

A series *RLC* network has *R* = 2 kΩ, *L* = 40 mH, and *C* = 1 μF. Calculate the impedance at resonance and at one-fourth, one-half, twice, and four times the resonant frequency.

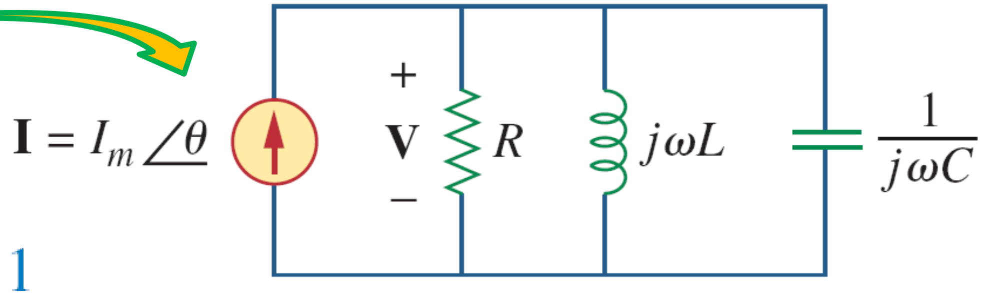
Example – 5

A coil with resistance 3Ω and inductance 100 mH is connected in series with a capacitor of 50 pF, a resistor of 6Ω and a signal generator that gives 110 V rms at all frequencies. Calculate ω_0, *Q*, and *B* at resonance of the resultant series *RLC* circuit.

Example – 6

Design a series *RLC* circuit with *B* = 20 rad/s and $\omega_0 = 1{,}000$ rad/s. Find the circuit's *Q*. Let *R* = 10Ω.

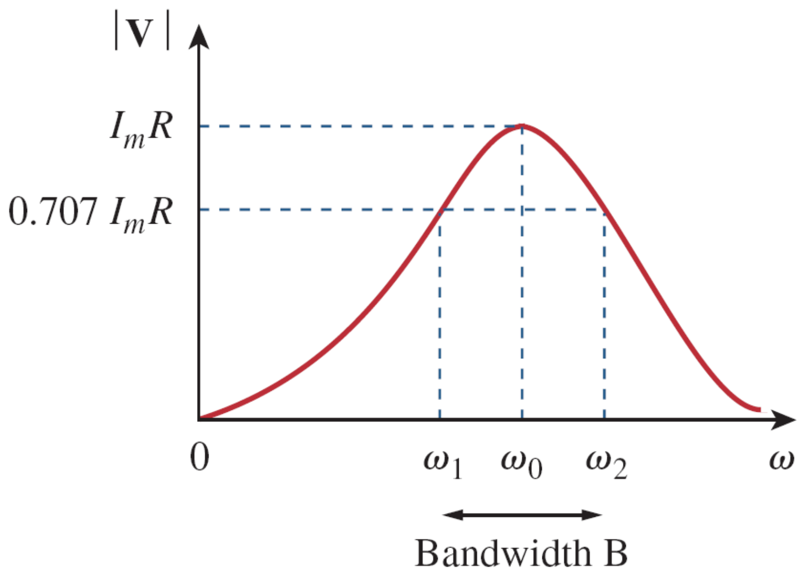
Parallel Resonance



$$\mathbf{Y} = H(\omega) = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

$$\mathbf{Y} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

At resonance: $\omega C - \frac{1}{\omega L} = 0$ $\Rightarrow$ $\omega_0 = \frac{1}{\sqrt{LC}}$ rad/s



- The voltage |V| as a function of frequency.
- At resonance, the parallel *LC* combination acts like an open circuit, so that the entire current flows through *R*.

Parallel Resonance (contd.)

- For parallel resonance, we can derive:

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$B = \omega_2 - \omega_1 = \frac{1}{RC}$$

$$Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}$$

- Half-power frequencies in terms of the quality factor:

$$\omega_1 = \omega_0\sqrt{1 + \left(\frac{1}{2Q}\right)^2} - \frac{\omega_0}{2Q},$$

$$\omega_2 = \omega_0\sqrt{1 + \left(\frac{1}{2Q}\right)^2} + \frac{\omega_0}{2Q}$$

- For high-Q circuits: $\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2}$

Example – 7

Find:

(a) the resonant frequency ω_0

(b) $Z_{in}(\omega_0)$