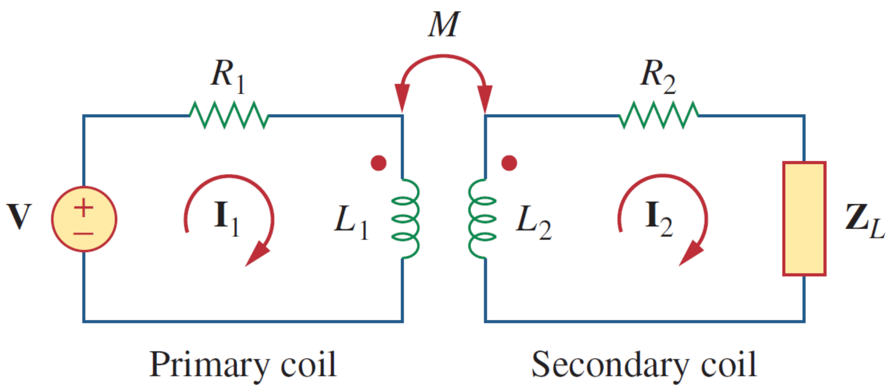
Lecture – 13 **Date: 26.09.2016**

- Transformer

Transformer

A transformer is generally a four-terminal device comprising two (or more) magnetically coupled coils.



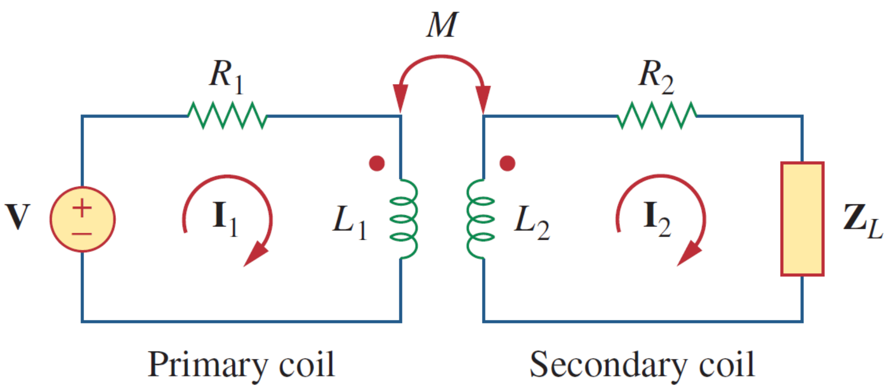
- The resistances R_1 and R_2 account for the losses (power dissipation) in the coils.

- The transformer is *linear* if the coils are wound on a magnetically linear material—a material for which the magnetic permeability is constant. Such materials include air, plastic, Bakelite, and wood.
- In fact, most materials are magnetically linear.

A linear transformer may also be regarded as one whose flux is proportional to the currents in its windings.

Linear transformers are sometimes called *air-core transformers*, although not all of them are necessarily air-core.

Transformer (contd.)



$$\mathbf{V} = (R_1 + j\omega L_1)\mathbf{I}_1 - j\omega M \mathbf{I}_2$$

$$0 = -j\omega M \mathbf{I}_1 + (R_2 + j\omega L_2 + \mathbf{Z}_L)\mathbf{I}_2$$

- the input impedance Z_{in} as seen from the source:

$$\mathbf{Z}_{in} = \frac{\mathbf{V}}{\mathbf{I}_1} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$$

primary impedance

due to the coupling between the primary and secondary windings

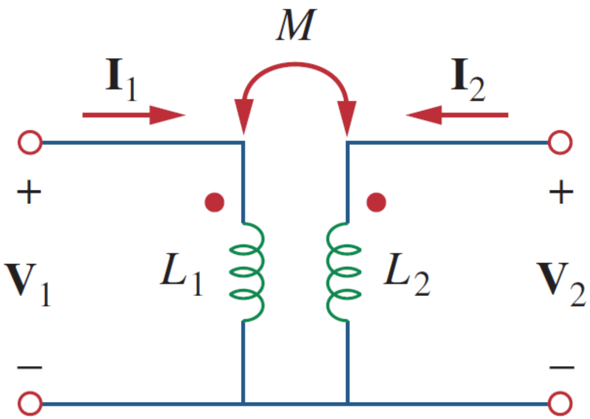
It is as though this impedance is reflected to the primary and is called *reflected impedance* Z_R

$$\mathbf{Z}_R = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$$

Not affected by the location of the dots on the transformer

Transformer (contd.)

- it is sometimes convenient to replace a magnetically coupled circuit by an equivalent circuit with no magnetic coupling.

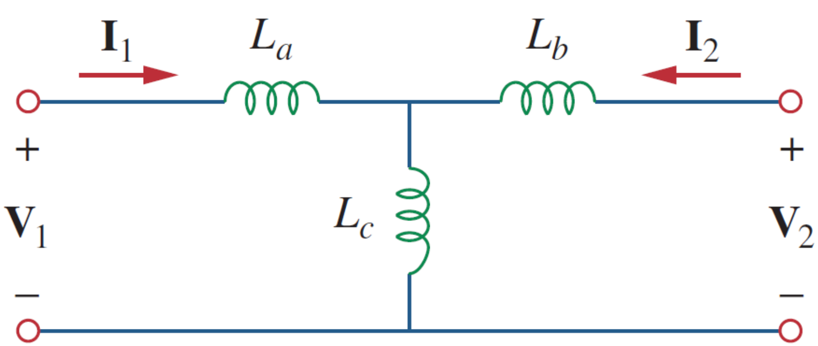


$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

Matrix Inversion

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{L_2}{j\omega(L_1 L_2 - M^2)} & \dfrac{-M}{j\omega(L_1 L_2 - M^2)} \\ \dfrac{-M}{j\omega(L_1 L_2 - M^2)} & \dfrac{L_1}{j\omega(L_1 L_2 - M^2)} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

For a T-Network



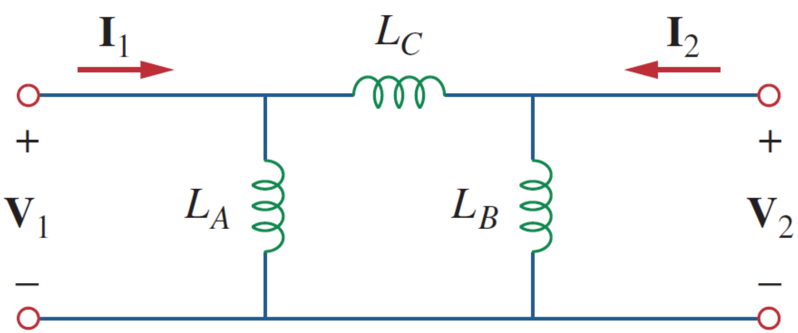
$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} j\omega(L_a + L_c) & j\omega L_c \\ j\omega L_c & j\omega(L_b + L_c) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

Transformer (contd.)

- For the T-model to be equivalent to the linear transformer:

$$L_a = L_1 - M, \qquad L_b = L_2 - M, \qquad L_c = M$$

For a π-Network



$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{j\omega L_A} + \dfrac{1}{j\omega L_C} & -\dfrac{1}{j\omega L_C} \\ -\dfrac{1}{j\omega L_C} & \dfrac{1}{j\omega L_B} + \dfrac{1}{j\omega L_C} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

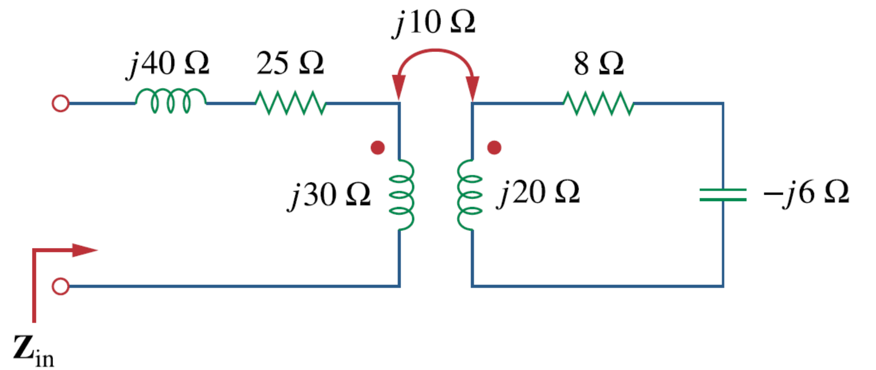
- For the π-model to be equivalent to the linear transformer:

$$L_A = \frac{L_1 L_2 - M^2}{L_2 - M}, \qquad L_B = \frac{L_1 L_2 - M^2}{L_1 - M} \qquad L_C = \frac{L_1 L_2 - M^2}{M}$$

In the T- and π- Models, the inductors are not magnetically coupled

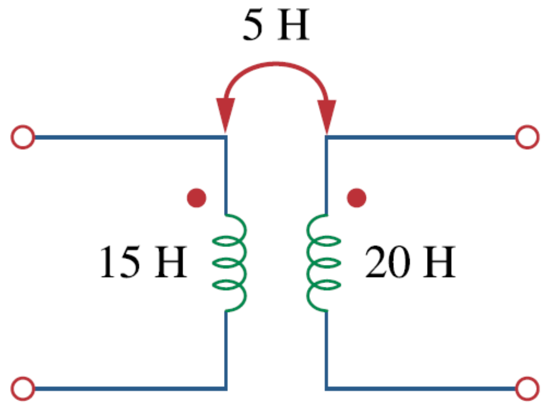
Example – 1

(a) Find the input impedance of the circuit using the concept of reflected impedance.
(b) Obtain the input impedance by replacing the linear transformer by its T equivalent.



Example – 2

Find:
(a) the T-equivalent circuit,
(b) the π-equivalent circuit.



Example – 3

Two linear transformers are cascaded as shown below. Show:

$$\mathbf{Z}_{\text{in}} = \frac{\omega^2 R(L_a^2 + L_a L_b - M_a^2 + j\omega^3(L_a^2 L_b + L_a L_b^2 - L_a M_b^2 - L_b M_a^2)}{\omega^2(L_a L_b + L_b^2 - M_b^2) - j\omega R(L_a + L_b)}$$