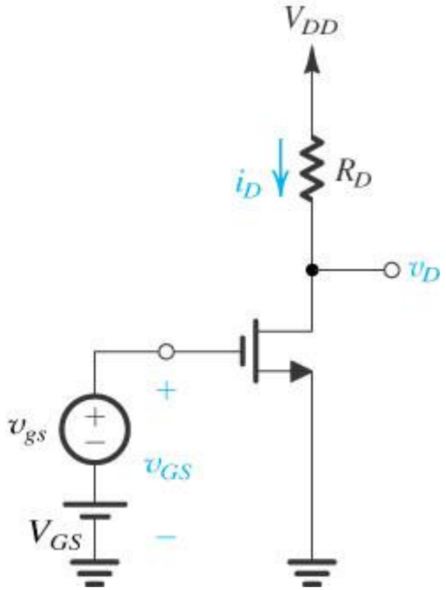
ECE315 / ECE515

Lecture – 4 **Date: 11.08.2016**

- MOSFET Small Signal Operation, Models, Analysis

MOSFET – Small Signal Operation

- Consider this circuit, which has both a **DC** and an AC **small signal** source. As a result, each voltage and current in the circuit has **both** a DC and small-signal AC.



- If the MOSFET is in **saturation**, then the **total** drain current is: $i_D = K\left[V_{GS} + v_{gs} - V_T\right]^2$

$$\Rightarrow i_D = K\left[V_{GS} - V_T\right]^2 + 2K\left[V_{GS} - V_T\right]v_{gs} + Kv_{gs}^2$$

$\mathbf{I_D}$ $\quad$ $\mathbf{i_d}$

Very Small If: $v_{gs} \ll 2\left[V_{GS} - V_T\right]$

We call this equation the **small-signal** condition.

Now, $i_d = 2K\left[V_{GS} - V_T\right]v_{gs} \quad \Rightarrow \dfrac{i_d}{v_{gs}} = 2K\left[V_{GS} - V_T\right]$ $\quad \mathbf{g_m}$

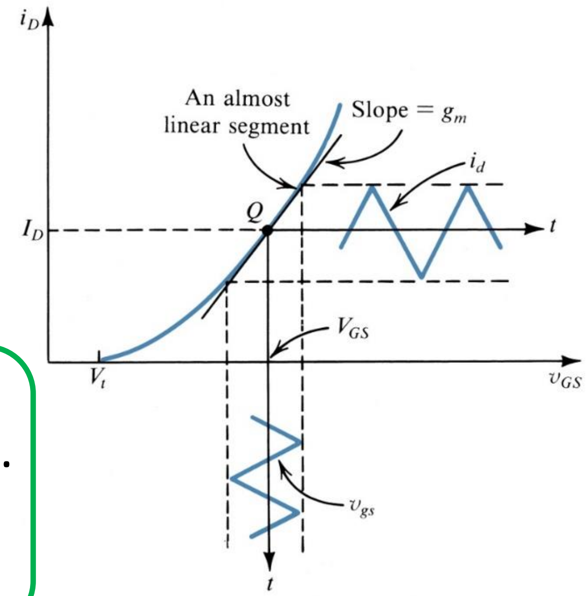
Alternatively, $\Rightarrow g_m = 2KV_{OV}$

- The small-signal parameter $\mathbf{g_m}$ can also be **derived** from small-signal analysis of the drain current:

$$i_d = \left.\frac{di_D}{dV_{GS}}\right|_{v_{GS}=V_{GS}} \left(v_{gs}\right) = 2K\left[v_{GS} - V_T\right]\Big|_{v_{GS}=V_{GS}} \left(v_{gs}\right)$$

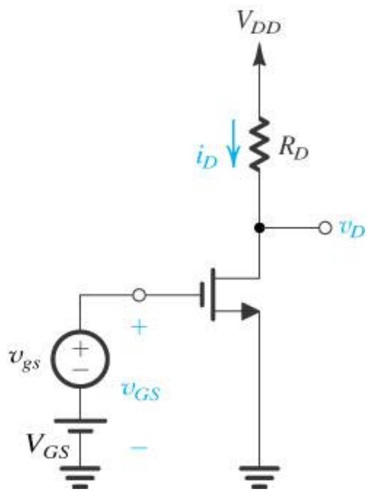
MOSFET – Small Signal Operation (contd.)

$$i_d = 2K\left[V_{GS} - V_T\right]\left(v_{gs}\right) \Rightarrow i_d = g_m\left(v_{gs}\right)$$



Physical meaning of the g_m → formal definition

The MOSFET transconductance relates a small **change** in v_{GS} to a small **change** in drain current i_D. This change is completely dependent on the **DC bias** point of the MOSFET, V_{GS} and I_D.



- The total instantaneous drain voltage v_D is given by:

$$v_{DS} = v_D = V_{DD} - i_D R_D$$

- Under small signal condition it changes to:

$$v_D = V_{DD} - R_D\left(I_D + i_d\right)$$

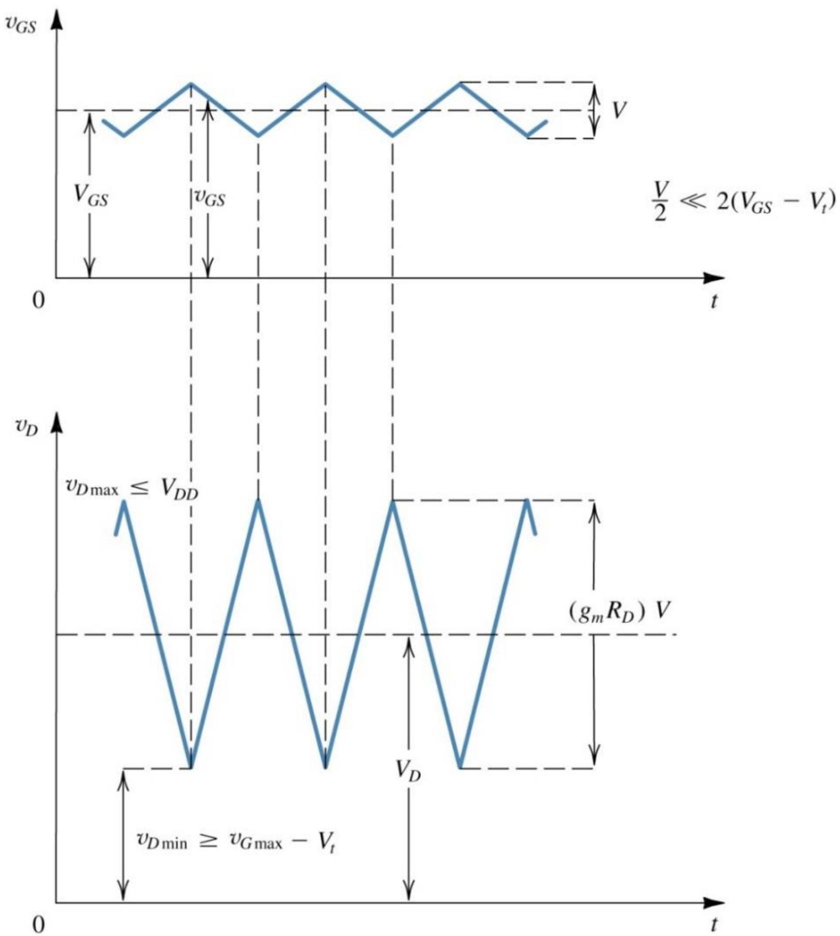
$$\Rightarrow V_D + v_d = V_{DD} - I_D R_D - i_d R_D$$

Signal component of drain voltage (v_d)

$$v_d = -i_d R_D = -g_m v_{gs} R_D \Rightarrow A_v = \frac{v_d}{v_{gs}} = -g_m R_D$$

MOSFET – Small Signal Operation (contd.)

Thus, if $g_m R_D \gg 1$, we have small-signal **voltage gain**.



$$A_v = \frac{v_d}{v_{gs}} = -g_m R_D$$

It indicates that v_d is 180° out of phase with respect to v_{gs}

The input has been assumed very small as compared to overdrive voltage [$v_{gs} << 2(V_{GS}-V_T)$]

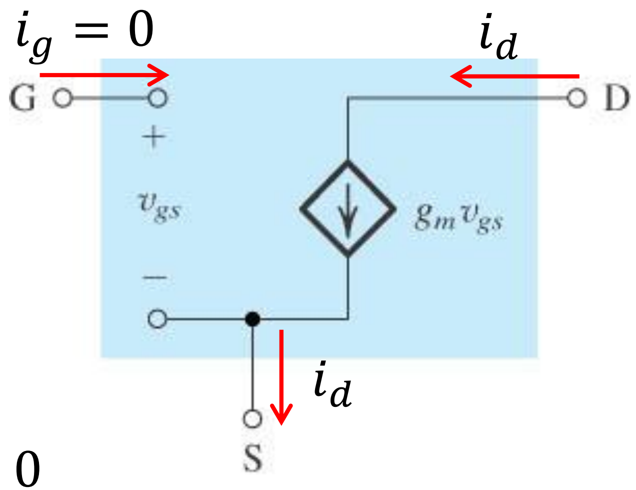
<u>For saturation:</u>

$$v_{D\min} \ge v_{G\max} - V_T$$

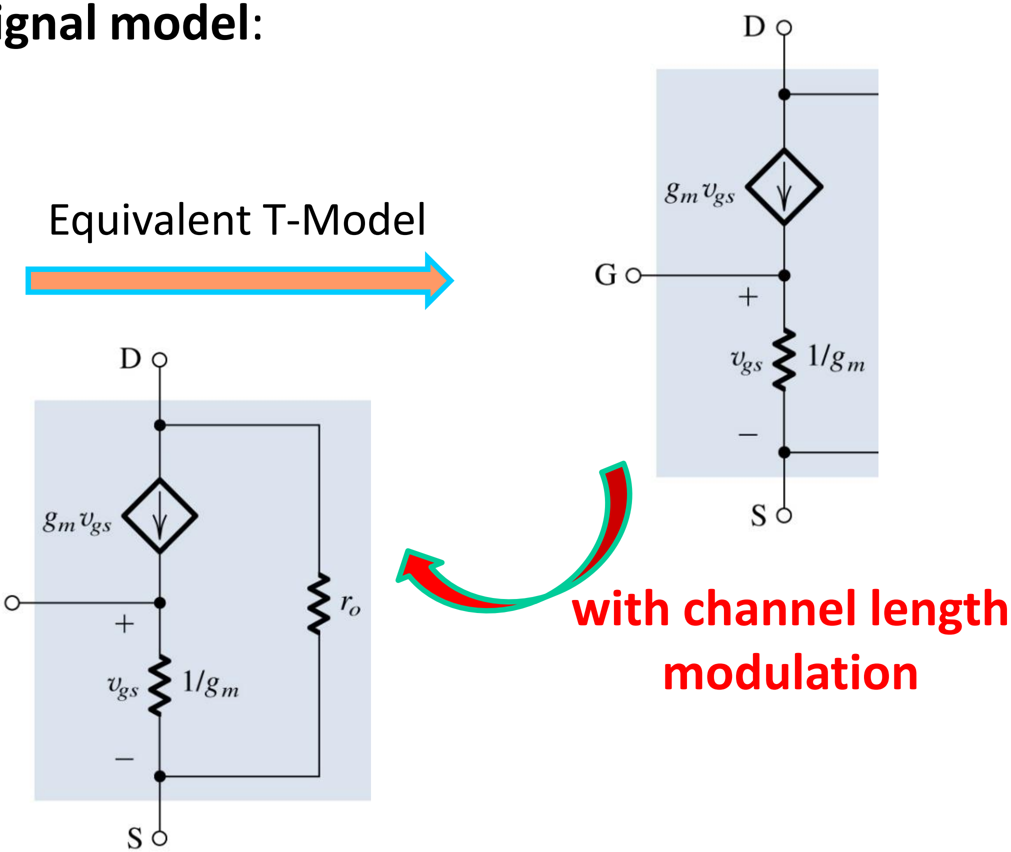
$$v_{D\max} \le V_{DD}$$

MOSFET – Small Signal Operation (contd.)

- To determine the small-signal performance of a given MOSFET amplifier circuit, we can replace it with its **small-signal model**:



$$i_g = 0$$
$$i_d = g_m v_{gs} + \frac{v_{ds}}{r_o}$$
$$i_s = i_d$$



- Recall that due to **channel-length modulation**, the MOSFET drain current is **slightly** dependent on v_{DS}, and thus is more accurately described as:

$$i_D = K\left(v_{GS} - V_T\right)^2\left(1 + \lambda v_{DS}\right)$$

MOSFET – Small Signal Model (contd.)

- In order to determine the relationship between the small-signal voltage v_{gs} and small-signal current i_d we can apply a **small signal analysis** of this equation:

$$i_d = \left.\frac{di_D}{dv_{GS}}\right|_{v_{GS}=V_{GS}} (v_{gs}) \Rightarrow i_d = 2K(V_{GS} - V_T)v_{gs} \Rightarrow \therefore i_d = g_m v_{gs}$$

Note that we evaluated the derivative at the DC **bias** point V_{GS}. The result, as we expected, was the **transconductance** g_m

- We can likewise determine the relationship between small-signal voltage v_{ds} and the small-signal current i_d:

$$i_d = \left.\frac{di_D}{dv_{DS}}\right|_{v_{GS}=V_{GS}} (v_{ds}) \Rightarrow i_d = \lambda K(V_{GS} - V_T)^2 v_{ds} \Rightarrow \therefore i_d = \frac{v_{ds}}{r_o}$$

- where we **recall** that r_o is the MOSFET **output resistance**:

$$r_o = \frac{1}{\lambda K(V_{GS} - V_T)^2} \Rightarrow r_o = \frac{1}{\lambda I_D}$$

- The small signal drain current i_d of a MOSFET (biased at a DC operating point V_{GS} *and* I_D) is therefore:

$$i_d = g_m v_{gs} + \frac{v_{ds}}{r_o}$$

MOSFET – Small Signal Analysis Steps

- Complete **each** of these steps if you **choose** to correctly complete a MOSFET Amplifier **small-signal** analysis.

Step 1: Complete a **D.C. Analysis**

Turn **off** all **small-signal** sources, and then complete a circuit analysis with the remaining **D.C. sources** only.

- Complete this DC analysis exactly, precisely, the same way you performed the DC analysis in **last lecture**. That is, you assume (the **saturation** mode), enforce, analyze, and **check (do not forget to check!)**.
- Note that you enforce and check exactly, precisely the same equalities and inequalities as discussed in **last lecture**.
- Remember, if we "turn off" a **voltage** source (e.g., $v_i(t) = 0$), it becomes a **short** circuit.
- However, if we "turn off" a **current** source (e.g., $i_i(t) = 0$), it becomes an **open** circuit!

- Small-signal amplifiers frequently employ large **capacitors**. Remember, the impedance of a capacitor at **DC** is infinity—a DC **open** circuit.

Step 1: Complete a **D.C. Analysis (contd.)**

- The goal of this DC analysis is to determine:
  1) The DC voltage V_{GS} for **each** MOSFET.
  2) The DC voltage V_{DS} for **each** MOSFET (you need this value for the CHECK).

You do not **necessarily** need to determine any other DC currents or voltages within the amplifier circuit!

Once you have found these values, you can **CHECK** your saturation assumption, and then move on to **step 2**.

Step 2: Calculate the **small-signal circuit parameters** for each MOSFET.

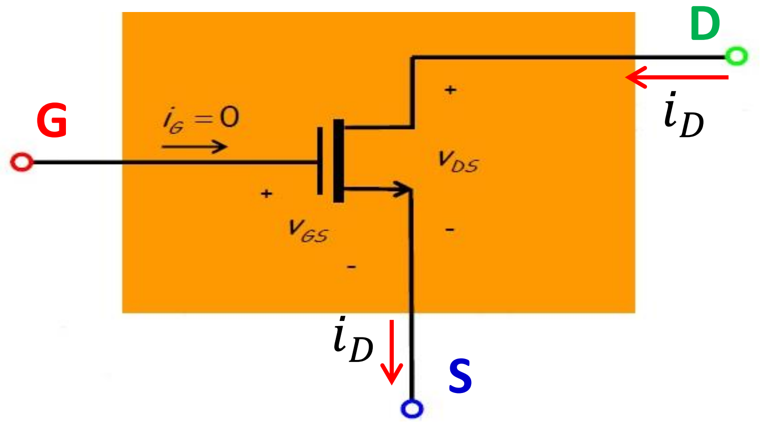
$$g_m = 2K(V_{GS} - V_T) \qquad r_o = \frac{1}{\lambda K (V_{GS} - V_T)^2}$$

Step 3: Carefully replace all MOSFETs with their **small-signal circuit model.**

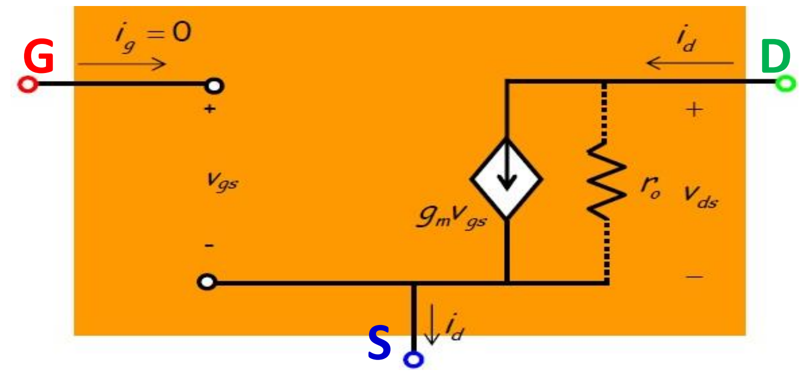
This step often gives students fits!

However, it is actually a **very simple** and straight-forward step. It does require four important things from the student— **patience**, **precision, persistence** and **professionalism**!

- First, note that a **MOSFET** is: a device with **three** terminals, called the gate, drain, and source. Its behavior is described in terms of current i_D and voltages v_{GS}, v_{DS}.



- Now, **contrast** the MOSFET with its small-signal circuit model. A MOSFET small-signal circuit model is: a device with **three** terminals, called the gate, drain, and source. Its behavior is described in terms of current i_d and voltages v_{gs}, v_{ds}.



Exactly the **same**—what a coincidence!

Therefore, replacing a MOSFET with its small-signal circuit model is very simple—you simply change the stuff **within** the orange box!

- The parts of the circuit **external** to the orange box do not change! i.e.,

1) **every** device attached to the MOSFET **terminals** (i.e, gate, drain, source) is attached in **precisely** the same way to the terminals of the **circuit model**.
2) **every** external voltage or current (e.g., v_i, v_o, i_R) is defined in **precisely** the same way both before and after the MOSFET is replaced with its circuit model is (e.g., if the output voltage is the drain voltage in the MOSFET circuit, then the output voltage is **still** the drain voltage in the small-signal circuit!).

Step 4: Set all **D.C. sources** to zero.

- A zero voltage DC source is a **short**.
- A zero current DC source is an **open**.
- Replace the **large** capacitors with a (AC) **short**.

The schematic now in front of you is called the **small-signal circuit**. Note that it is **missing** two things—**DC sources** and MOSFET **transistors**!

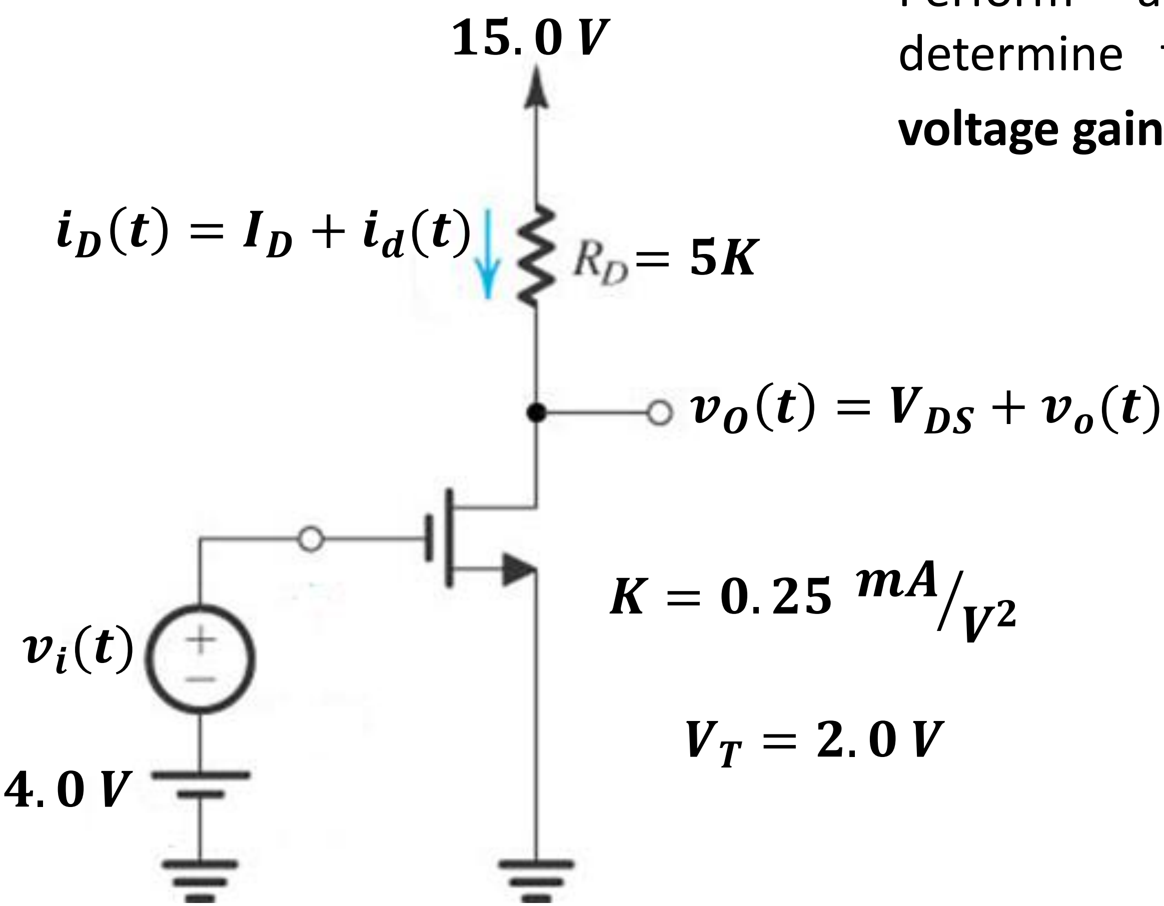
Note that steps **three** and **four** are **reversible**. You could turn off the DC sources **first**, and then replace all MOSFETs with their small-signal models—the resulting small-signal circuit will be the **same**!

Step 5: Analyze **small-signal circuit**.

- For small-signal **amplifiers**, we typically attempt to find the small-signal output voltage v_o in terms of the small-signal input voltage v_i. From this result, we can find the **voltage gain** of the amplifier.
- Do **not** attempt to insert any MOSFET knowledge into your small-signal circuit analysis—there are **no** MOSFETs in a small-signal circuit!!!!!
- Remember, the MOSFET circuit model contains **all** of our MOSFET small-signal knowledge, we **do** not—indeed **must** not—add any more information to the analysis.

You must **trust** completely the MOSFET small-signal circuit model. It **will** give you the correct answer!

Example – 1



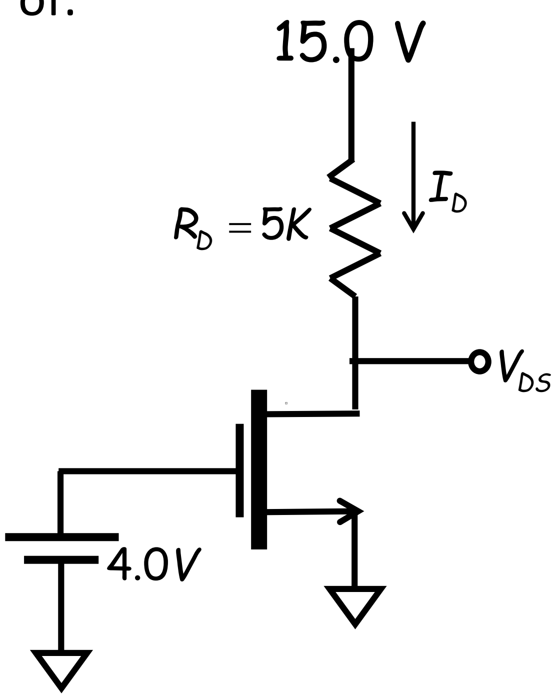
$K = 0.25\ {}^{mA}/_{V^2}$

$V_T = 2.0\ V$

Perform a small-signal analysis to determine the small-signal open-circuit **voltage gain** $A_v = {}^{v_o(t)}/_{v_i(t)}$

Step 1: DC Analysis

- Turning **off** the small signal source leaves a DC circuit of:



- We **ASSUME** saturation, so that we **ENFORCE**:

$$I_D = K\left(V_{GS} - V_t\right)^2$$

- It is evident that: $V_{GS} = 4.0\text{ V}$

- Therefore the DC drain current is:

$$\begin{aligned} I_D &= K\left(V_{GS} - V_t\right)^2 \\ &= 0.25(4-2)^2 \\ &= 1.0\text{ mA} \end{aligned}$$

- Thus, the DC voltage V_{DS} can be determined from *KVL* as:

$$\begin{aligned} V_{DS} &= 15.0 - I_D R_D \\ &= 15.0 - 1(5) \\ &= 10.0\text{ V} \end{aligned}$$

- We **CHECK** our results and find:

$V_{GS} = 4.0 > V_t = 2.0$ ✓

$V_{DS} = 10.0 > V_{GS} - V_t = 2.0$ ✓

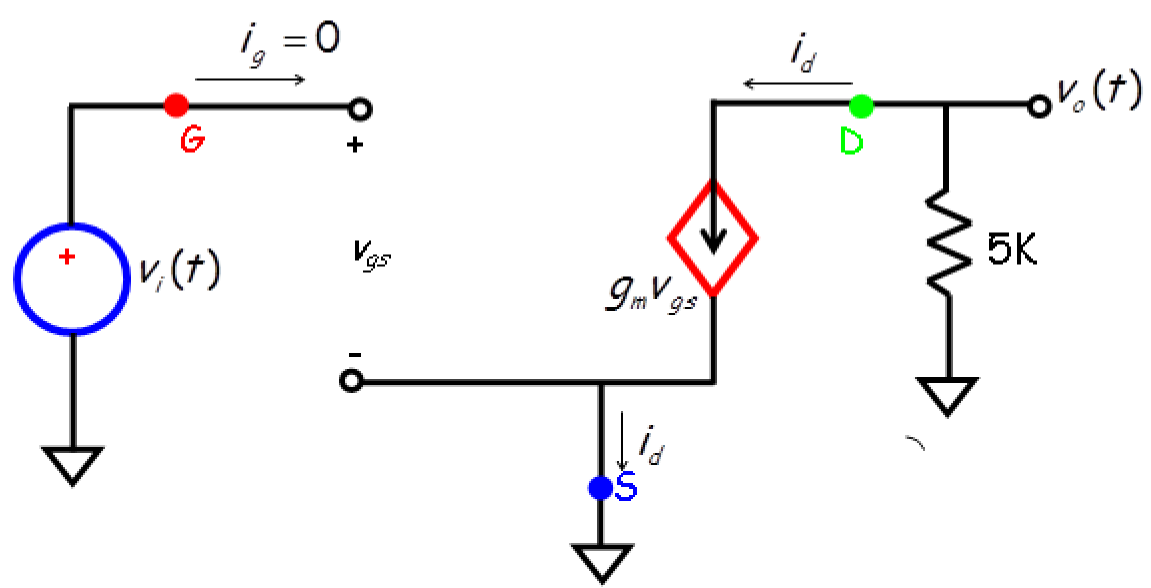
Step 2: Determine the small-signal parameters

$$g_m = 2K\left(V_{GS} - V_t\right) = 2(0.25)(4.0 - 2.0) = 1 \ \mathrm{mA}/\mathrm{V}$$

- Note that **no** value of λ was given, so we will assume $\lambda = 0$, and thus **output resistance** $r_o = \infty$.

Steps 3 and 4: Determine the small-signal circuit

We now turn off the **two** DC voltage source, and replace the MOSFET with its **small signal model**. The result is our **small-signal circuit**.



Step 5: Analyze the small-signal circuit

- The analysis of this small-signal circuit is fairly **straightforward**. First, we note from KVL that:

$$v_{gs} = v_i$$

- and that:

$$\begin{aligned} i_d &= g_m v_{gs} \\ &= 1.0 v_{gs} \\ &= v_{gs} \end{aligned}$$

- and that from Ohm's Law:

$$v_o = -5 i_d$$

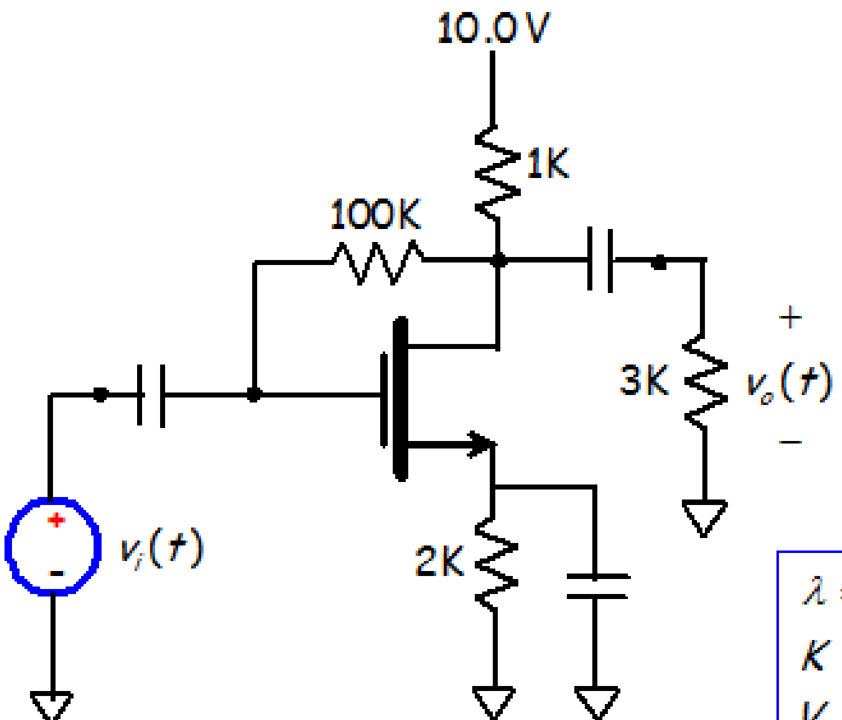
- **Combining** these equations, we find that: $v_o = -5 v_i$

- And thus the **small-signal** open-circuit voltage gain of this amplifier is:

$$A_{vo} = \frac{v_o(t)}{v_i(t)} = -5.0$$

Example – 2

- Perform a small-signal analysis to determine the small-signal open-circuit **voltage gain** $A_v = {}^{v_o(t)}/_{v_i(t)}$



Here the C's are large

$\lambda = 0.005\ V^{-1}$

$K = 0.4\ mA/V^2$

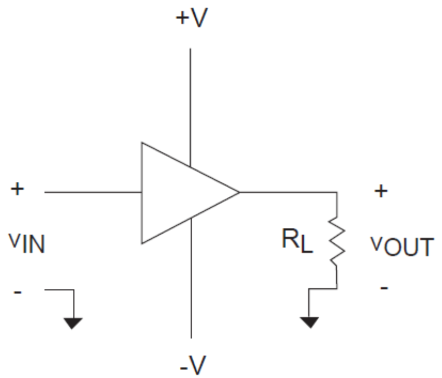
$V_t = 2.0\ V$

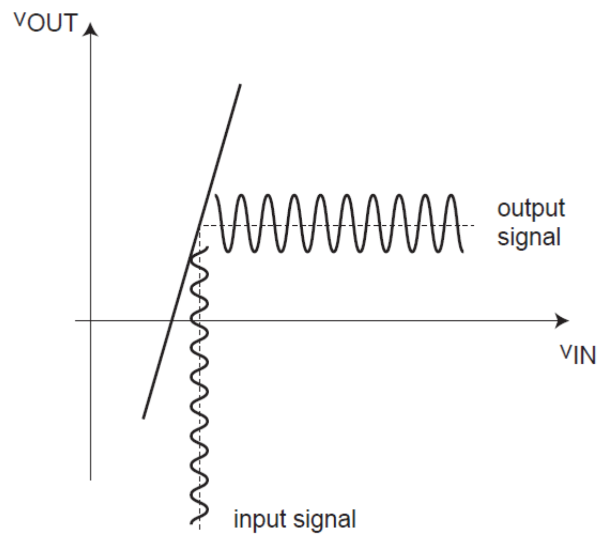
Amplifier Fundamentals

Key questions

- What are the key figures of merit of an amplifier?
- How can one make a voltage amplifier with a single MOSFET and a resistor?
- How can this amplifier be improved?

Goal of amplifiers: signal amplification.





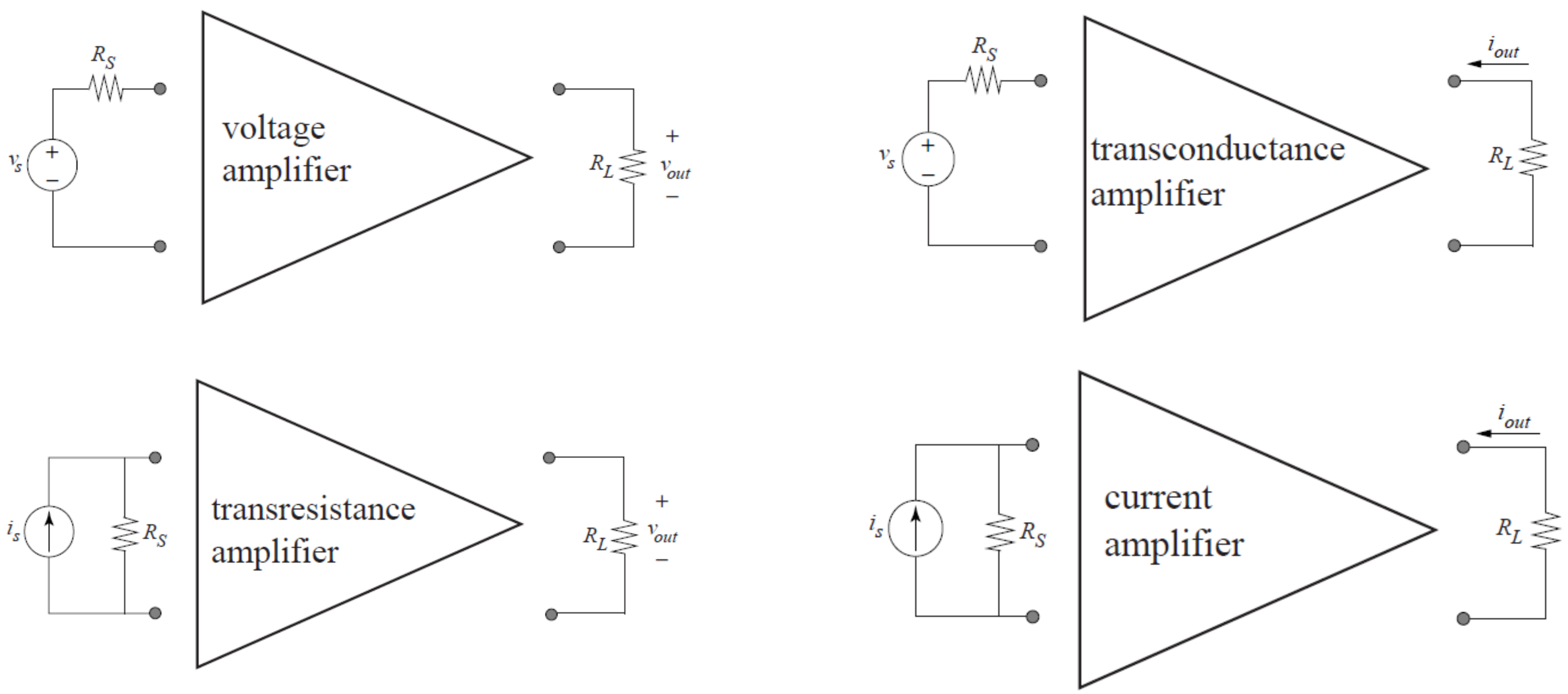
Features of amplifier:

- Output signal is faithful replica of input signal but amplified in magnitude.
- Active device is at the heart of amplifier.
- Need linear transfer characteristics for distortion not to be introduced.

Amplifier Fundamentals (contd.)

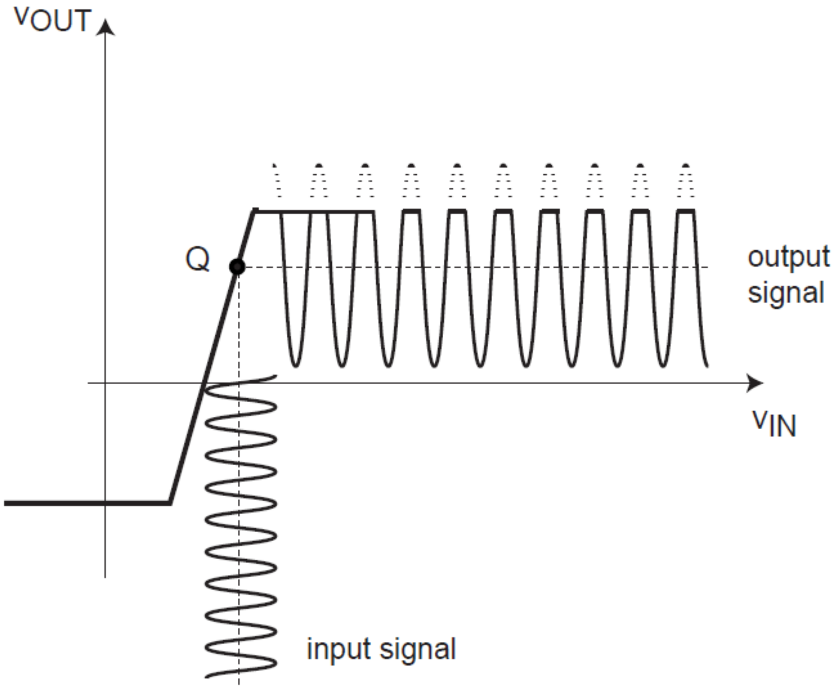
Signal could be represented by current or voltage

⇒ four distinct configurations:



Amplifier Fundamentals (contd.)

- More realistic transfer characteristics:



- Transfer characteristics linear over limited range of voltages: amplifier saturation.
- Amplifier saturation limits signal swing.
- Signal swing also depends on choice of bias point, Q (also called quiescent point or operating point).

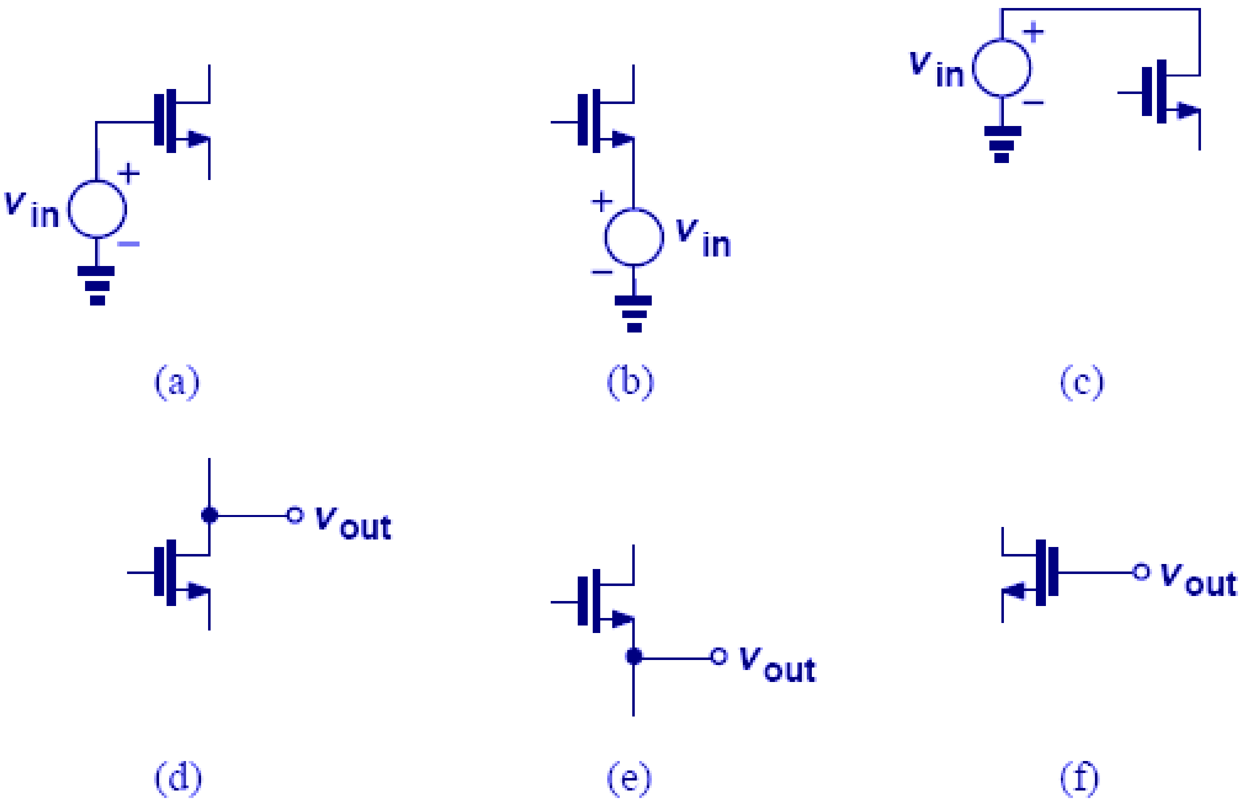
Other features desired in amplifiers:

- Low power consumption.
- Wide frequency response.
- Robust to process and temperature variations.
- Inexpensive: must minimize use of unusual components, must be small.

CMOS Amplifier Configurations

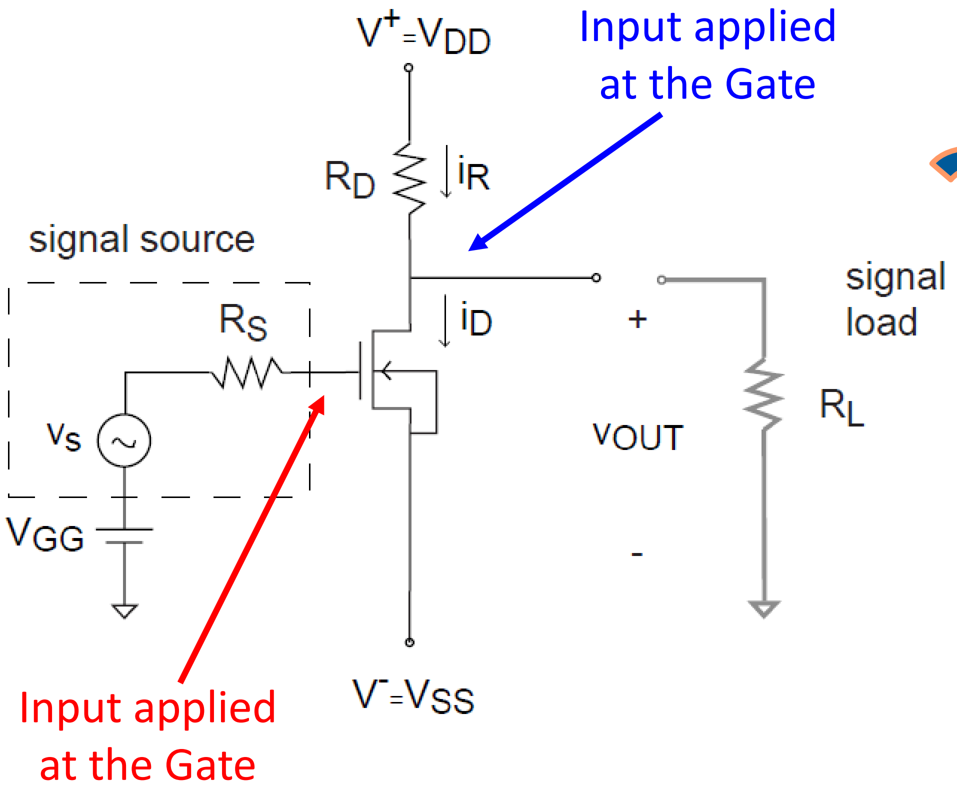
Of all the possible I/O connections to a MOS transistor, only (a,d), (a,e) and (b,d) are functional.

Possible I/O Connections to a MOS transistor



- **I/O connections (a,d): Common Source (CS)**
- **I/O connections (a,e): Common Drain (CD)**
- **I/O connections (b,d): Common Gate (CG)**

Common Source (CS) Amplifier

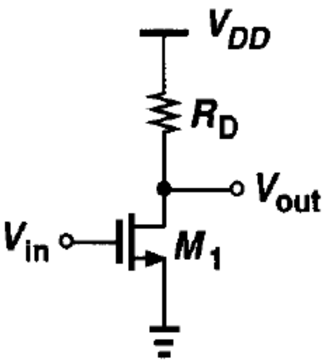


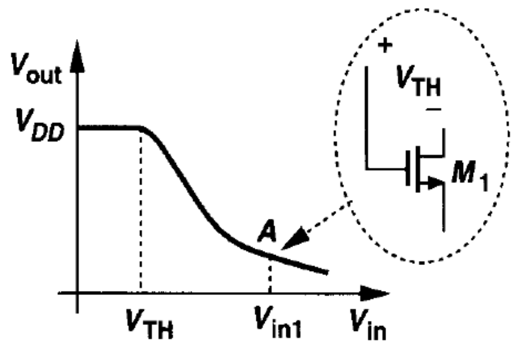
If the input is applied to the gate and the output is sensed at the drain, the circuit is called a "common-source" (CS) stage.

Common Source (CS) Amplifier

Large Signal Analysis

Initially, M_1 is off and $V_{out} = V_{DD}$.





- **Increase V_{in}, M_1 begins to turn on once V_{in} reaches V_T** → draws current from R_D and lowers V_{out}. **For adequate level of V_{DD},** M_1 turns on in saturation and we have:

$$V_{out} = V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_T)^2$$

- For the transistor's operation in the saturation region ($V_{out} > V_{in} - V_T$), i.e, in the region left of point A.

→ **the small signal gain is:**

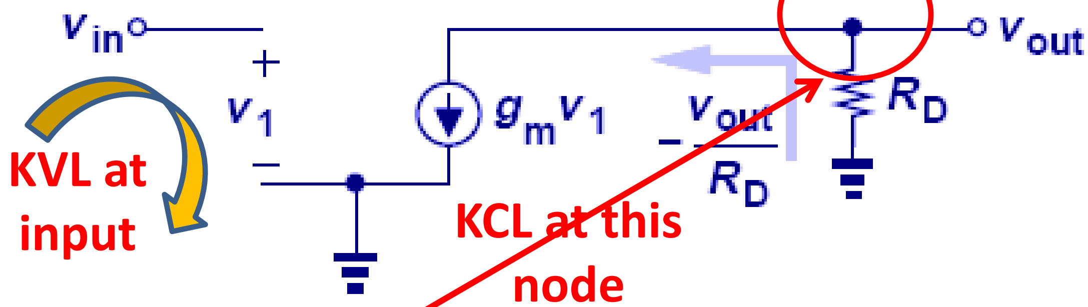
$$A_v = \frac{\partial V_{out}}{\partial V_{in}} = -R_D \underbrace{\mu_n C_{ox}\frac{W}{L}(V_{in} - V_T)}_{g_m}$$

- Can also be seen from a simple observation → M_1 converts input voltage change ΔV_{in} to a drain current change $g_m \Delta V_{in}$ → an output voltage change $-g_m R_D \Delta V_{in}$.

$$\boxed{\Rightarrow A_v = \frac{\Delta V_{out}}{\Delta V_{in}} = -\frac{g_m R_D \Delta V_{in}}{\Delta V_{in}} = -g_m R_D}$$

Common Source (CS) Amplifier (contd.)

- Alternatively, let us look through the small signal representation of the CS stage:

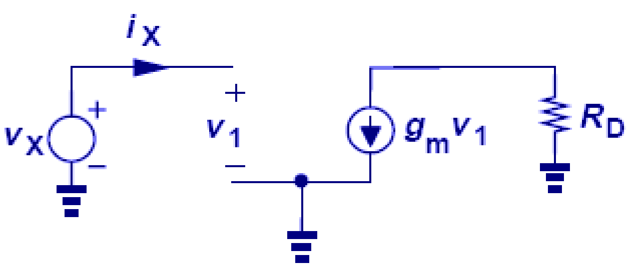


$$v_1 = v_{in}$$

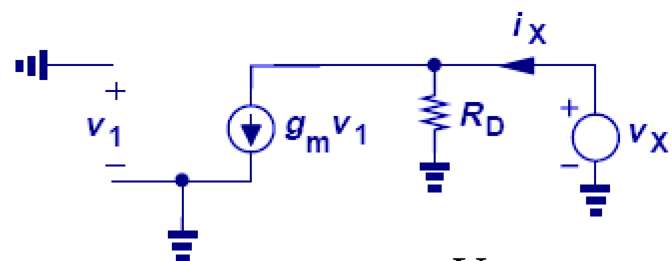
$$g_m v_1 = -\frac{v_{out}}{R_D} \Rightarrow g_m v_{in} = -\frac{v_{out}}{R_D}$$

$$\boxed{\therefore A_v = \frac{v_{out}}{v_{in}} = -g_m R_D}$$

Input and Output Impedances:



$$R_{in} = \frac{V_x}{I_x} = \infty$$



$$R_{out} = \frac{V_x}{I_x} = R_D$$

Common Source (CS) Amplifier (contd.)

Observations and Discussions

$$A_v = -g_m R_D$$

- g_m changes substantially if the input signal is large → if the gain changes significantly with the signal swing then the circuit operates in large signal mode.
- The dependence of the gain (A_v) upon the signal level leads to nonlinearity → undesirable condition.
- To minimize the nonlinearity, the gain (A_v) should be a weak function of g_m → design and layout of amplifier circuit critical.

  - Alternative expression for gain: $$\boxed{\Rightarrow A_v = -\sqrt{2\mu_n C_{ox}\frac{W}{L}}\frac{V_{RD}}{\sqrt{I_D}}}$$ V_{RD}: voltage drop across R_D

- A_v can be ↑ by making W/L ↑ or V_{RD} ↑ or making I_D ↓ by keeping other parameters are fixed.
- However, large W/L leads to greater device capacitance, and a higher V_{RD} limits the maximum voltage swings.
- If V_{RD} remains constant and I_D is reduced, then R_D must increase → results in greater time constant at the output node.

Common Source (CS) Amplifier (contd.)

- In addition, for large R_D the effect of channel length modulation comes into play and therefore the output voltage becomes:

$$V_{out} = V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_T)^2(1+\lambda V_{out})$$

$$A_v = \frac{\partial V_{out}}{\partial V_{in}} = -R_D\mu_n C_{ox}\frac{W}{L}(V_{in} - V_T)(1+\lambda V_{out}) - R_D\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_T)^2\lambda\frac{\partial V_{out}}{\partial V_{in}}$$

$$\Rightarrow A_v = -R_D g_m - R_D I_D \lambda A_v$$

Where, $I_D \approx -R_D\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_T)^2$

$$\therefore A_v = -\frac{g_m R_D}{1+R_D\lambda I_D}$$

$$\Rightarrow A_v = -g_m\frac{r_o R_D}{r_o + R_D}$$

Where, $r_o = \frac{1}{\lambda I_D}$

Eventual Lower Gain