Lecture – 15 Date: 17.10.2016

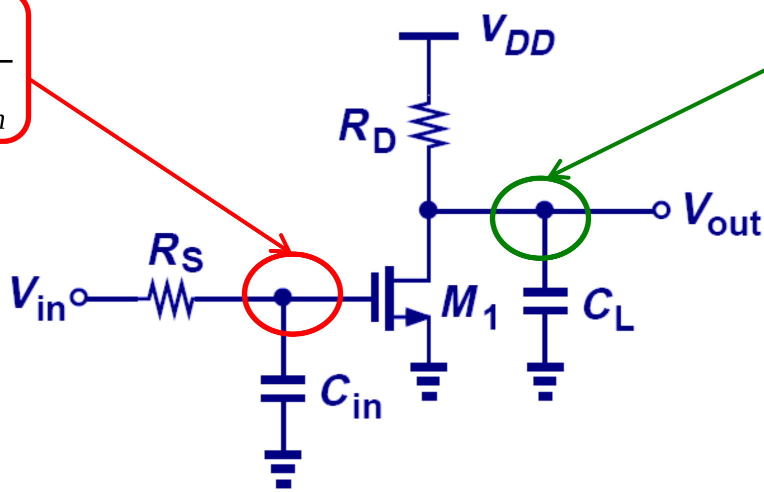
- General Frequency Response
- High Frequency MOSFET Model and Transit Frequency
- Determination of 3-dB Frequency
- CS Stage - Analysis using Miller’s Approximation, OCTC Method, Exact Technique
- CD and CG Stages

Frequency Response

Association of poles with nodes: poles of a circuit transfer function is key in the frequency response → it aids in determining the speed of various parts of the circuit

$$\left|\omega_{p1}\right| = \left|\omega_{in}\right| = \frac{1}{R_S C_{in}}$$

$$\left|\omega_{p2}\right| = \left|\omega_{out}\right| = \frac{1}{R_D C_L}$$



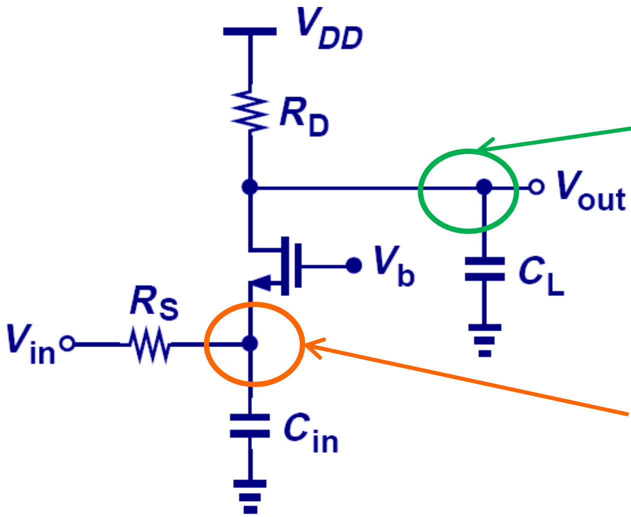
Low Frequency Small Signal Gain

$$\Rightarrow \left|\frac{V_{out}}{V_{in}}(j\omega)\right| = A_v \frac{1}{\sqrt{\left(1+\omega^2/\omega_{p1}^2\right)}} \cdot \frac{1}{\sqrt{\left(1+\omega^2/\omega_{p2}^2\right)}}$$

$$\therefore \left|\frac{V_{out}}{V_{in}}\right| = \frac{g_{m1} R_D}{\sqrt{\left(1+\omega^2/\omega_{p1}^2\right)\left(1+\omega^2/\omega_{p2}^2\right)}}$$

Frequency Response (contd.)

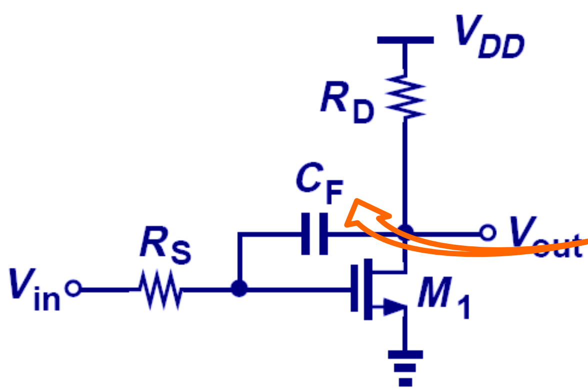
Example:



$$|\omega_{p2}| = |\omega_{out}| = \frac{1}{R_D C_L}$$

$$|\omega_{p1}| = |\omega_{in}| = \frac{1}{(R_S \| 1/g_{m1})C_{in}}$$

Example: circuit with floating capacitor

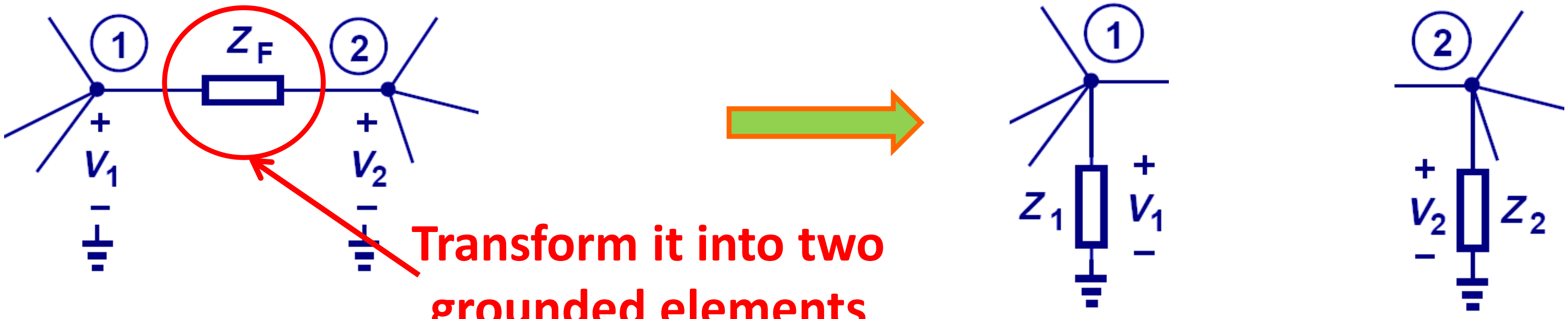


Now the capacitor C_F isn't connected between just one node and ground. What do we do?

Miller's Theorem Provides the Solution

Miller's Theorem

- It coverts floating impedance element into two grounded elements



- The current drawn by Z_F from node 1 must be equal to that drawn by Z_1

$$\frac{V_1 - V_2}{Z_F} = \frac{V_1}{Z_1}$$

- Current injected to node 2 must be equal to that injected to node 2 in both situations:

$$\frac{V_1 - V_2}{Z_F} = -\frac{V_2}{Z_2}$$

$$Z_1 = Z_F \frac{V_1}{V_1 - V_2} = \frac{Z_F}{1 - A_v}$$

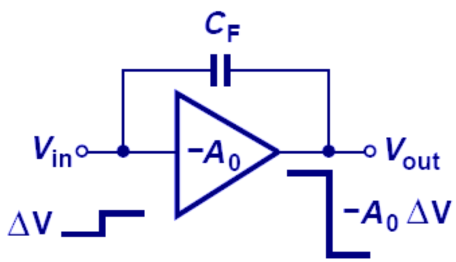
$$Z_2 = Z_F \frac{V_1}{V_1 - V_2} = \frac{Z_F}{1 - \frac{1}{A_v}}$$

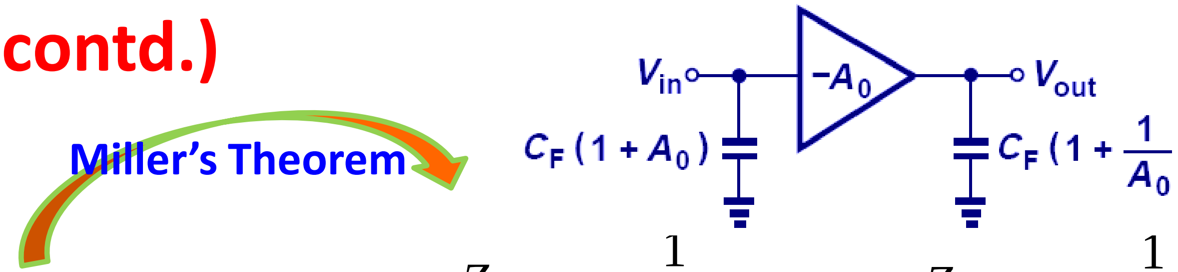
A_v = low frequency small-signal gain

If A_v is the gain from node 1 to 2, then a floating impedance Z_F can be converted to two grounded impedances Z_1 and Z_2.

Miller's Theorem (contd.)

- If Z_F is capacitive and amplifier is inverting then:

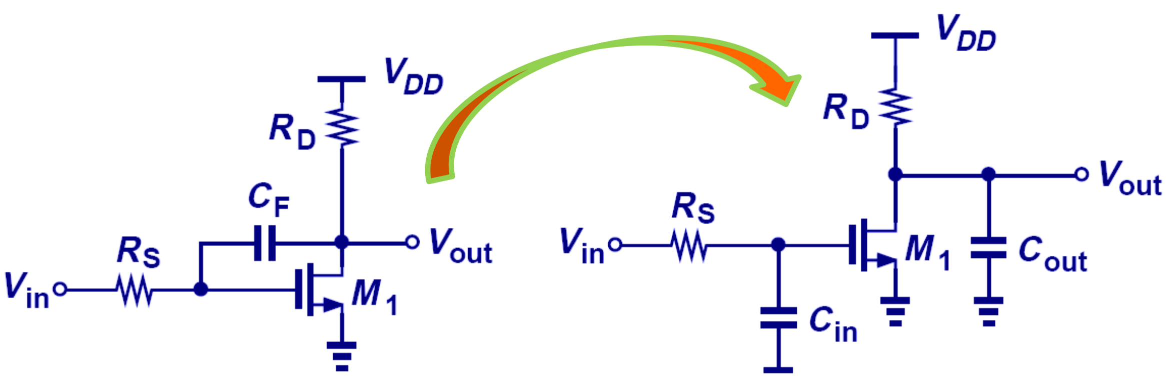




$$Z_1 = \frac{1}{(1+A_0)C_F s} \qquad Z_2 = \frac{1}{\left(1+\frac{1}{A_0}\right)C_F s}$$

With Miller's theorem, we can separate the floating capacitor. However, the input capacitor is larger than the original floating capacitor. We call this Miller multiplication.

Example: determine poles of the following circuit



$$C_{in} = (1+A_0)C_F = (1+g_m R_D)C_F$$

$$C_{out} = \left(1+\frac{1}{A_0}\right)C_F = \left(1+\frac{1}{g_m R_D}\right)C_F$$

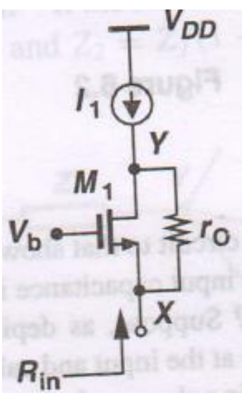
Miller's Theorem (contd.)

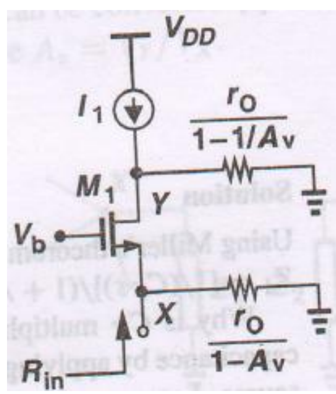
$$\therefore \omega_{in} = \frac{1}{R_S C_{in}} = \frac{1}{R_S\left(1+g_m R_D\right)C_F}$$

$$\therefore \omega_{out} = \frac{1}{R_D C_{out}} = \frac{1}{R_D\left(1+\frac{1}{g_m R_D}\right)C_F}$$

Miller's Theorem requires that the floating impedance and voltage gain be computed at the same frequency. However, apparently we always use low-frequency gain even at high frequencies. It is done for simplifying the analysis, otherwise the use of Miller Theorem will be no simpler. Therefore it is often called Miller's Approximation

Q: Calculate the input resistance of the following:





Here,

$$A_v = 1 + (g_m + g_{mb})r_o$$

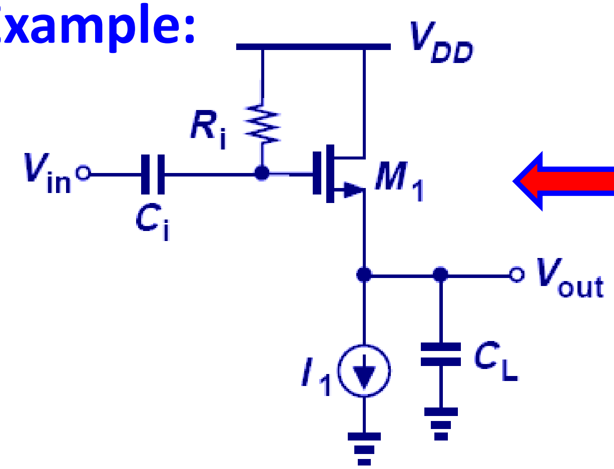
$$R_{in} = \frac{r_o}{1-A_v} \,||\, \frac{1}{g_m + g_{mb}}$$

$$R_{in} = \frac{r_o}{1-[1+(g_m+g_{mb})r_o]} \,||\, \frac{1}{g_m+g_{mb}}$$

$$\therefore R_{in} \approx \infty$$

General Frequency Response

Example:



In high quality audio amplifier: R_i establishes a gate bias voltage equal to V_{DD} for M_1, and I_1 defines the drain bias current. Assume $\lambda=0$, $g_m=1/(200\Omega)$, and $R_i=100k\Omega$. Determine the minimum required value of C_i and the maximum tolerable value of C_L

- The input network consisting of R_i and C_i attenuates the signal at low frequencies. The roll-off frequency for audio signal is given as:

$$2\pi * (20Hz) = \frac{1}{R_i C_i} = \frac{1}{100*10^3 * C_i}$$

$$\therefore C_i = 79.6nF$$ **Min. Value**

- The load capacitance creates a pole at the output node, lowering the gain at the high frequencies. Let us suppose pole frequency at 20kHz (upper end of audio):

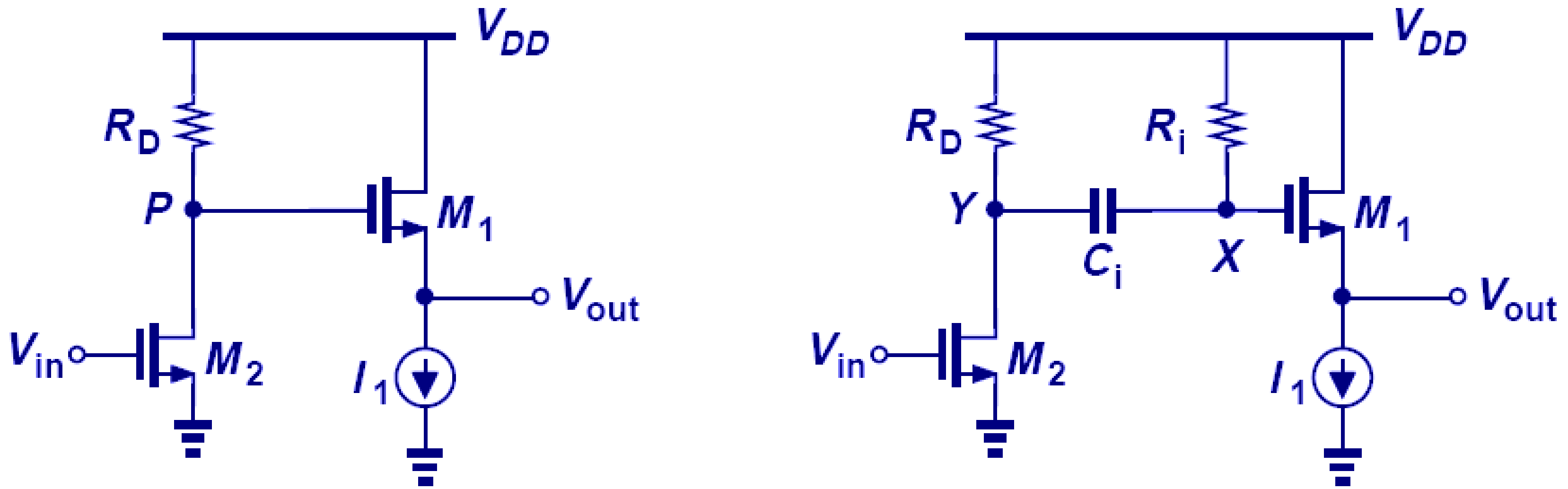
$$\omega_{p,out} = \frac{g_m}{C_L} = 2\pi * 20 * 10^3$$

$$\therefore C_L = 39.8nF$$ **Max. Value**

General Frequency Response (contd.)

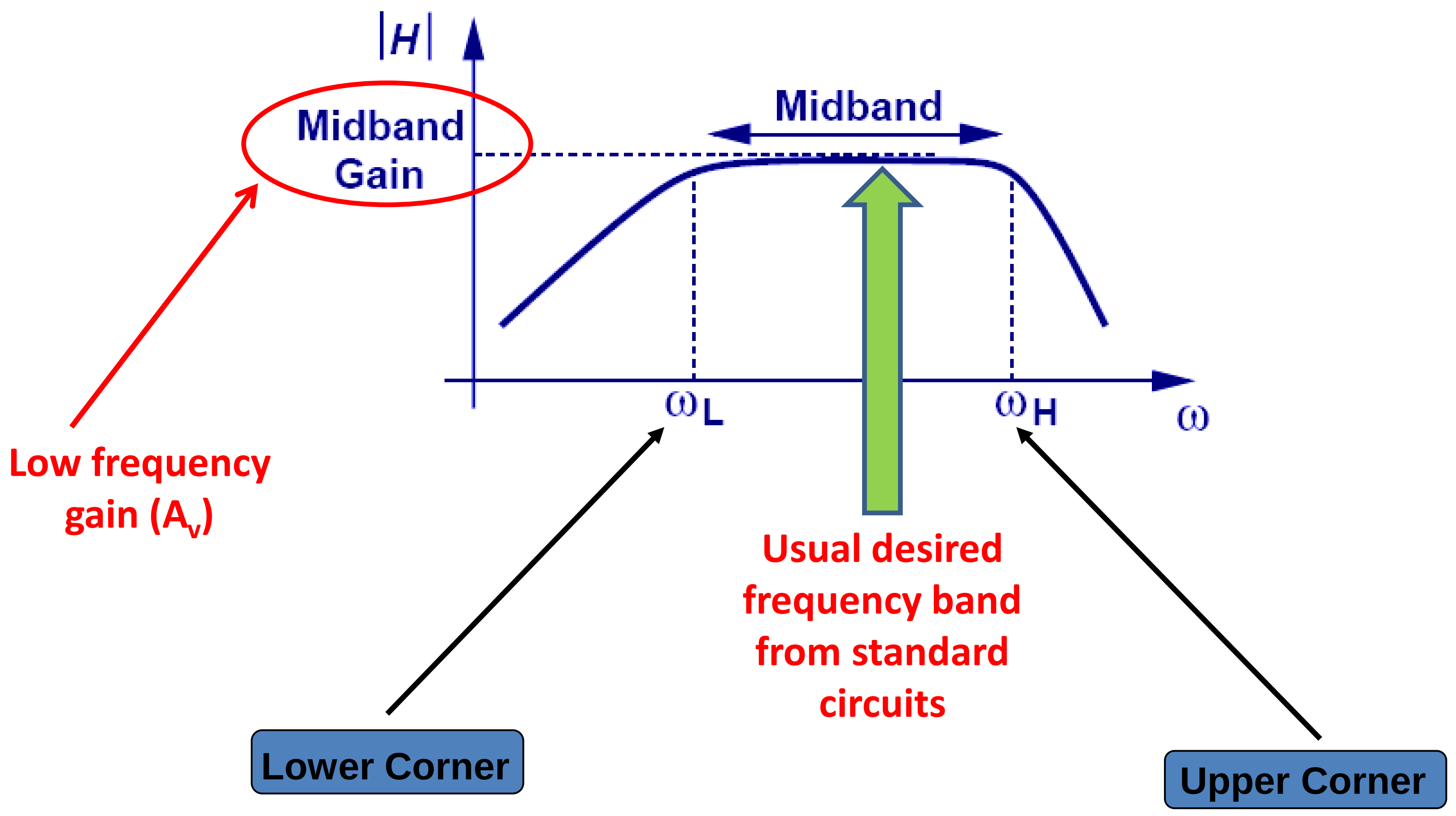
Why do we need capacitor C_i at the input in the previous example?

The absence of C_i could be blessing as it will not affect the performance at low frequencies → we would be saved from computing C_i as well

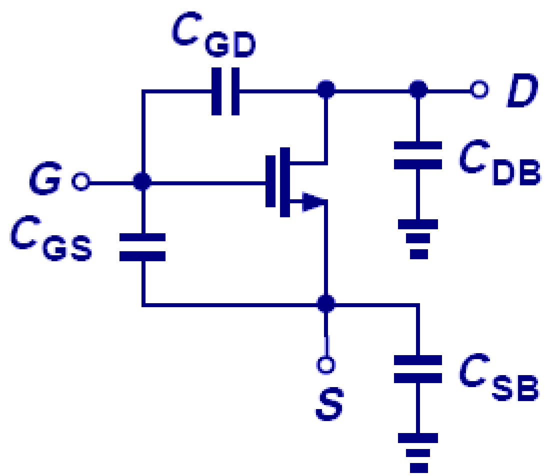


- Capacitive coupling, also known as AC coupling, passes AC signals from Y to X while blocking DC contents.
- This technique allows independent bias conditions between stages. Direct coupling does not.

General Frequency Response (contd.)



High Frequency MOSFET Model



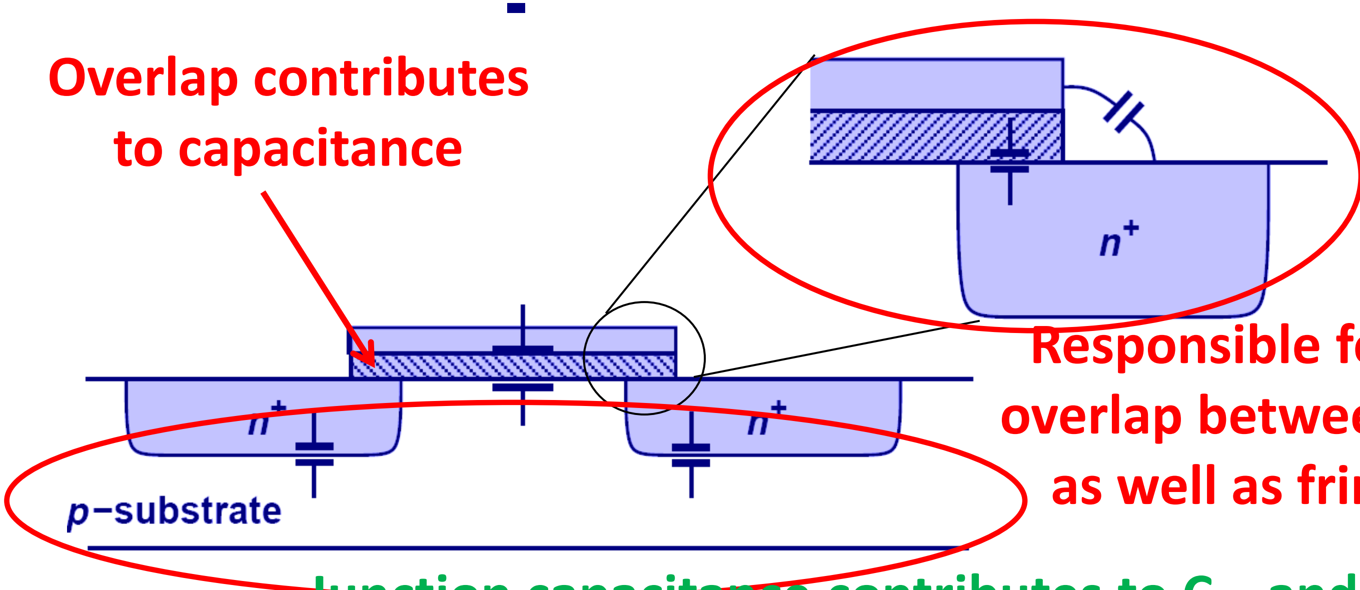
C_{GD}: gate-drain capacitance

C_{GS}: gate-source capacitance

C_{SB}: source-bulk capacitance

C_{DB}: drain-bulk capacitance

Overlap contributes to capacitance

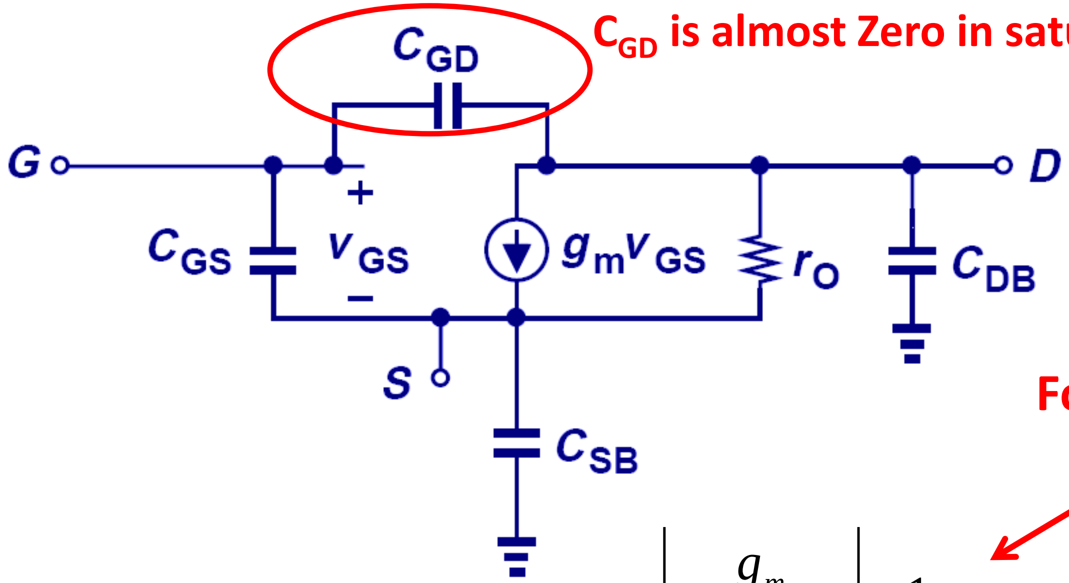


Responsible for C_{GD} and C_{GS}: overlap between gate and S/D as well as fringe field lines

Junction capacitance contributes to C_{SB} and C_{DB}

Cut-off Frequency or Transit Frequency

- So many capacitances in the MOSFET reduces the performance of amplifiers → cut-off or transit frequency, f_T, regulates the speed of MOSFET
- It is the frequency at which the small-signal current gain falls to unity



C_{GD} is almost Zero in saturation

Input Current $= j\omega C_{GS} V_{GS}$

Output Current $= g_m V_{GS}$

For f_T:

$$\left| \frac{g_m V_{GS}}{\omega_T C_{GS} V_{GS}} \right| = 1$$

$$\Rightarrow \left| \frac{g_m}{2\pi f_T C_{GS}} \right| = 1$$

$$\therefore f_T = \left| \frac{g_m}{2\pi C_{GS}} \right|$$

The source-bulk and drain-bulk capacitance doesn't affect the speed of transistor

Determination of 3-dB frequency (f_H)

- As a designer it is important to understand the implications of various capacitive effects (present in the circuit) on the overall performance of the circuit
- In order to understand such implications there are three different techniques to determine f_H (a key parameter in high frequency performance estimation)
- Miller's Approximation Technique: It is useful for certain cases when the input resistance is relatively large and output capacitance (C_L) is relatively small → in such a case the high-frequency response is dominated by the pole formed at the input node
- OCTC Method: Its useful for circuits when its not easy to determine the poles and zeros by hand analysis → is an approximate method
- Exact Analysis: Involves full analysis of the circuit to find the transfer function

Determination of 3-dB frequency (f_H) – contd.

Physical Significance of Poles and Zeros in a Transfer Function:

- Think of Poles and Zeros as INFINITY's and ZEROs.
- At Zeros: the system produces ZERO output
- At Poles: the system produces INFINITE output
- Obviously, you cannot produce infinite voltage with any electronics

→ So, it means that, the output will be unbounded (in theory) and saturated at the highest possible value (in practice)

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Now, let's talk about a specific case: The TRANSFER FUNCTION can be the IMPEDANCE of a filter, it will be zero (short circuit) at zeros, and INFINITY (open circuit) at poles

Miller Approximation Technique

- **High Frequency Gain function of an amplifier can be given as:**

$$A(s) = A_M . F_H(s)$$

Mid-band gain → small-signal gain

Transfer function of amp

- **$F_H(s)$ can be represented in terms of poles and zeros as:**

$$F_H(s) = \frac{(1+s/\omega_{z1})(1+s/\omega_{z2})......(1+s/\omega_{zn})}{(1+s/\omega_{p1})(1+s/\omega_{p2})......(1+s/\omega_{pn})}$$

- **If a dominant pole (ω_{p1}) exists then:**

$$F_H(s) \cong \frac{1}{(1+s/\omega_{p1})}$$

Assuming that zeros are usually either at infinity or possess very high value

Miller Approximation Technique (contd.)

- Thus presence of a dominant pole provides 3-dB roll-off frequency as: $\omega_H \cong \omega_{p1}$
- Condition for the existence of dominant pole: the lowest-frequency pole is at least two octave away from the nearest pole or zero.
- If a dominant pole doesn't exist then: $F_H(s) = \dfrac{(1+s/\omega_{z1})(1+s/\omega_{z2})}{(1+s/\omega_{p1})(1+s/\omega_{p2})}$ **For 2-pole and 2-zero network**

$$\Rightarrow F_H(j\omega) = \frac{(1+j\omega/\omega_{z1})(1+j\omega/\omega_{z2})}{(1+j\omega/\omega_{p1})(1+j\omega/\omega_{p2})} \qquad \Rightarrow |F_H(j\omega)|^2 = \frac{(1+\omega^2/\omega_{z1}^2)(1+\omega^2/\omega_{z2}^2)}{(1+\omega^2/\omega_{p1}^2)(1+\omega^2/\omega_{p2}^2)}$$

- For $\omega=\omega_H \rightarrow |F_H|^2 = 1/2$ and therefore: $\Rightarrow \dfrac{1}{2} = \dfrac{(1+\omega_H^2/\omega_{z1}^2)(1+\omega_H^2/\omega_{z2}^2)}{(1+\omega_H^2/\omega_{p1}^2)(1+\omega_H^2/\omega_{p2}^2)}$
- ω_H is smaller than all other poles and zeros and as a consequence terms with ω_H^4 could be neglected. Therefore simplification gives:

$$\omega_H \cong \frac{1}{\sqrt{\left(\dfrac{1}{\omega_{p1}^2}+\dfrac{1}{\omega_{p2}^2}\right)-2\left(\dfrac{1}{\omega_{z1}^2}+\dfrac{1}{\omega_{z2}^2}\right)}}$$

Open Circuit Time Constant (OCTC) Method

- Its not always straightforward to apply Miller technique and determine the poles and zeros
- In such cases OCTC method prove handy
- Alternate form of $F_H(s)$ for n-zero and n-pole network is:

$$F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \ldots\ldots + a_n s_n}{1 + b_1 s + b_2 s^2 + \ldots\ldots + b_n s_n}$$

Where, **a** and **b** are related to zeros and poles respectively. For example, $\mathbf{b_1}$ is given by:

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \ldots\ldots + \frac{1}{\omega_{pn}}$$

Ref: Paul E. Gray and Campbell L. Searle, Electronic Principles: Physics, Models, and Circuits (1969), John Wiley & Sons Inc., New York

- b_1 can be determined by considering various capacitances in the network one at a time while reducing all other capacitors to zero i.e, replacing them with open circuits
- Determine C_iR_i for each capacitors and then compute:

$$b_1 = \sum_{i=1}^{n} C_i R_i$$

Open Circuit Time Constant (OCTC) Method (contd.)

- If one of the poles is dominant (say P1) then: $b_1 \cong \frac{1}{\omega_{p1}}$ $\Rightarrow \omega_H \simeq \frac{1}{b_1} = \frac{1}{\sum_i C_i R_i}$

Advantage of OCTC method:

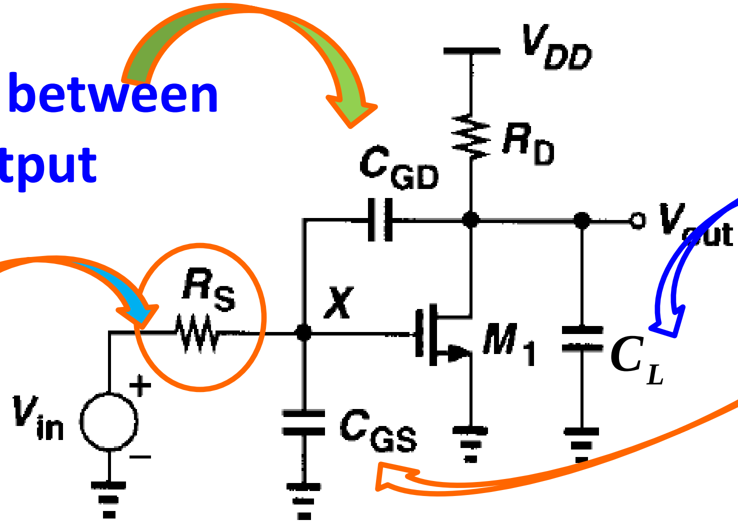
- It tells the circuit designer which of the various capacitances is significant in determining the network (amplifier) frequency response
- The relative contribution of the various capacitances to the effective time constant b_1 is immediately obvious
- For example, if in any amplifier the contribution of $C_{GD}R_{GD}$ in the overall time constant is maximum → then C_{GD} is dominant capacitor in determining f_H → to increase f_H, either use MOSFET with smaller C_{GD} or for a given MOSFET reduce R_{GD} by either reducing the load impedance or by employing smaller source impedance → furthermore, if source impedance is also fixed then the only way to increase f_H (and hence the bandwidth) is by reducing the load impedance
- Reduction in load impedance → leads to reduction in A_M

Common Source Amplifier

Floating Capacitor between Input and Output

Driven by finite source resistance

Grounded Capacitor



A_V	I (µA)	L(µm)	W(µm)	g_m (uS)	C_{DB} (fF)	C_{GD}(fF)	C_{GS}(fF)
10	10	2	5.78	3.613	5.19	1.84	98.16
15	10	2	32.5	5.33	27.5	10.4	517.8
20	10	2	668.2	6.66	319.6	239.8	6,041.1

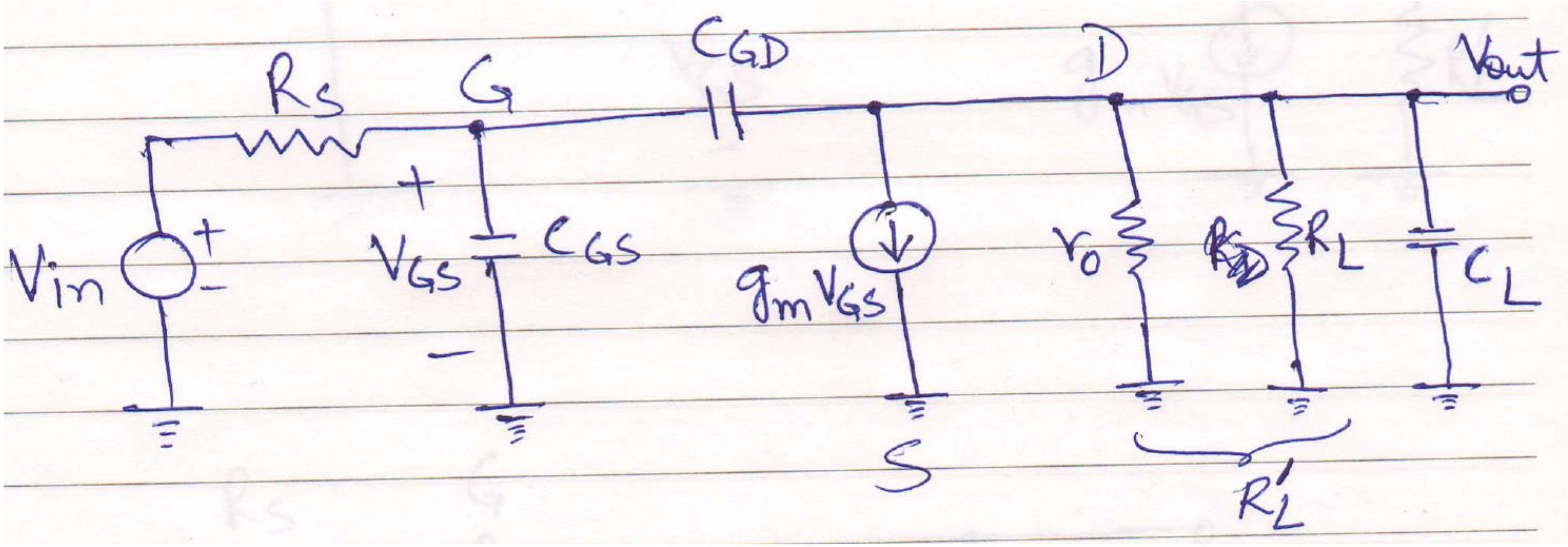
Increasing Gain

Constant Current

Increasing C_{GS} ↔ Reduced Speed

Difficult to achieve high gain and high speed at the same time!

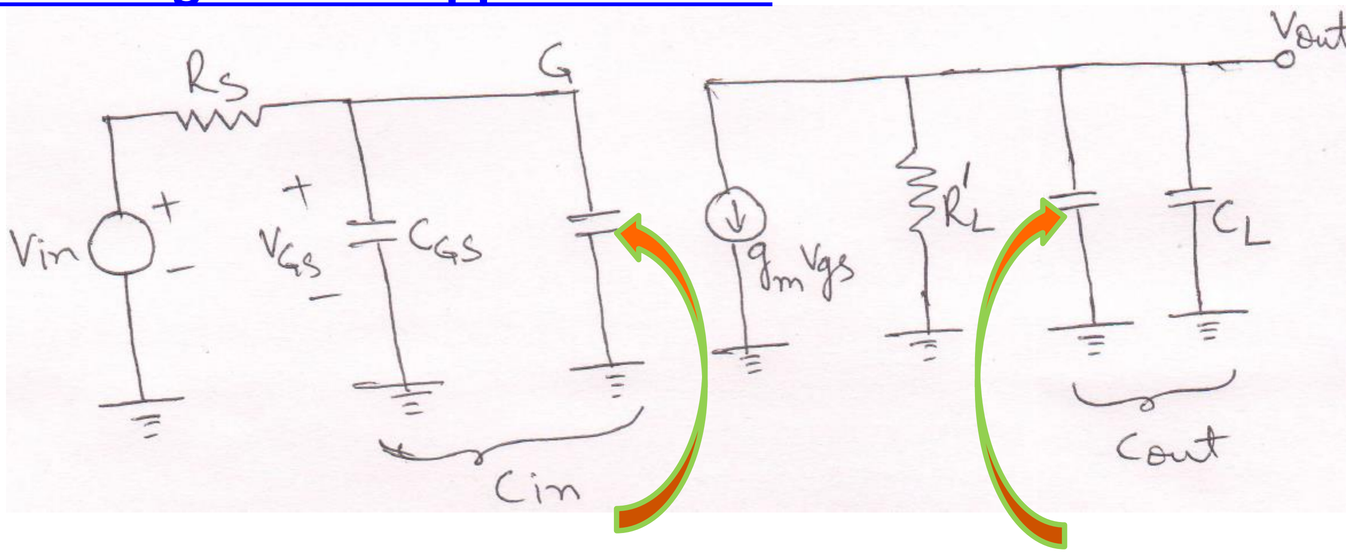
Common Source Amplifier (contd.)



- R_S: also includes the resistance due to the biasing network
- R_L: includes R_D → usually R_L is of the order of r_o
- C_L: represents the total capacitance between the drain and the ground → includes C_{DB} and input capacitance of succeeding amplifier stage → C_L in an IC is substantial

Common Source Amplifier (contd.)

Analysis using Miller's Approximation



$$C_A = (1 - A_v)C_{GD} = \left(1 + g_m R_L^{'}\right)C_{GD} \qquad C_B = \left(1 - A_v^{-1}\right)C_{GD} \approx C_{GD}$$

Therefore the poles are:

$$\omega_{in} = \frac{1}{R_S C_{in}} = \frac{1}{R_S\left(C_{GS} + C_A\right)} = \frac{1}{R_S\left(C_{GS} + \left(1 + g_m R_L^{'}\right)C_{GD}\right)}$$

$$\omega_{out} = \frac{1}{R_L^{'} C_{out}} = \frac{1}{R_L^{'}\left(C_L + C_B\right)} = \frac{1}{R_L^{'}\left(C_L + C_{GD}\right)}$$

Common Source Amplifier (contd.)

Then the transfer function is given by:

$$H(s) = \frac{V_{out}}{V_{in}}(s) = \frac{-g_m R_D}{\left(1+\frac{s}{\omega_{in}}\right)\left(1+\frac{s}{\omega_{out}}\right)}$$

When R_S is large and C_L is small

ω_{in} dominates, and the transfer function becomes:

$$H(s) = \frac{-g_m R_D}{\left(1+\frac{s}{\omega_{in}}\right)}$$

$$H(s) = \frac{A_M}{1+\frac{s}{\omega_H}}$$

Dominant Pole

3-dB Frequency: $f_H = \frac{1}{2\pi C_{in} R_S}$

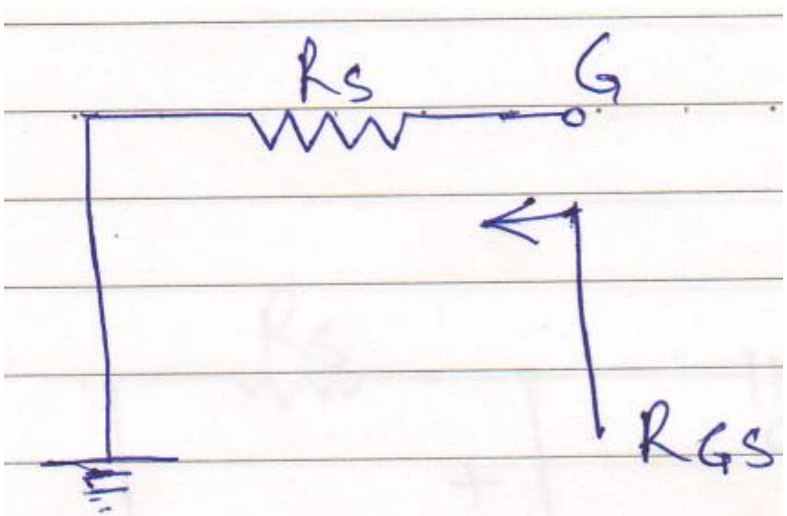
Where, $C_{in} = C_{GS} + C_{GD}(1+g_m R_L^{'})$

The main error in this expression is that the presence of zero has not been considered

Common Source Amplifier (contd.)

Analysis using OCTC Method

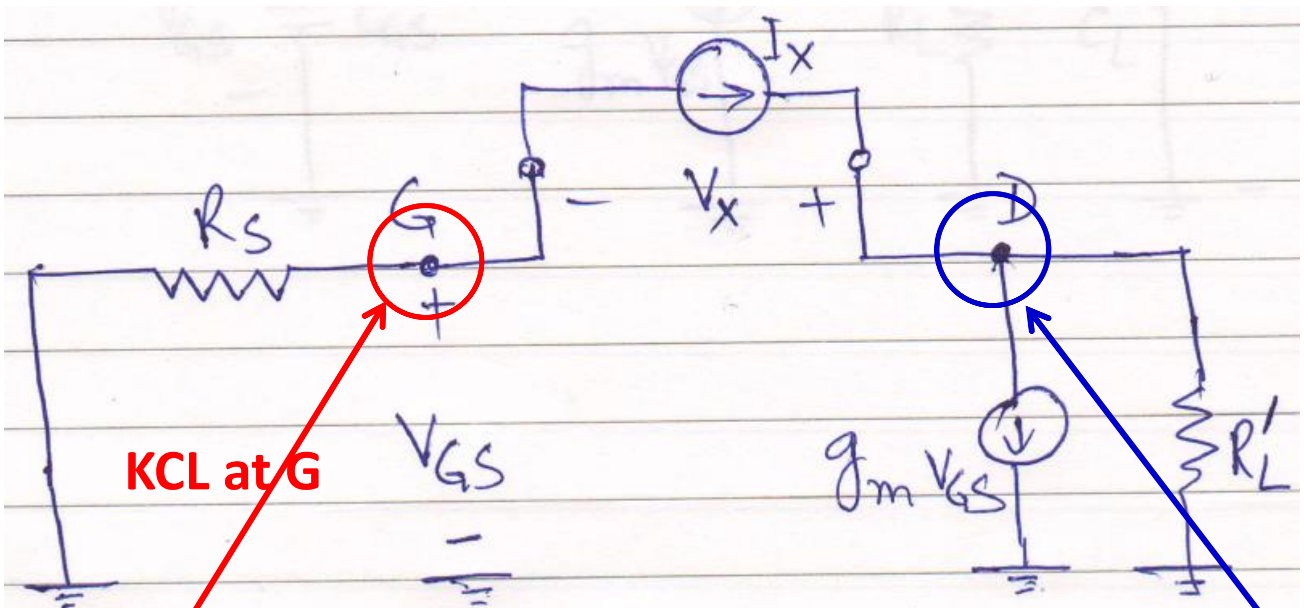
- Considering only C_{GS} → open other capacitances and short the voltage sources and open the current sources
- For R_{GS} we get:



$$R_{GS} = R_S$$

Common Source Amplifier (contd.)

- Considering only C_{GD} → open C_{GS} and C_L



KCL at G

KCL at D

Then, $R_{GD} = \dfrac{V_X}{I_X}$

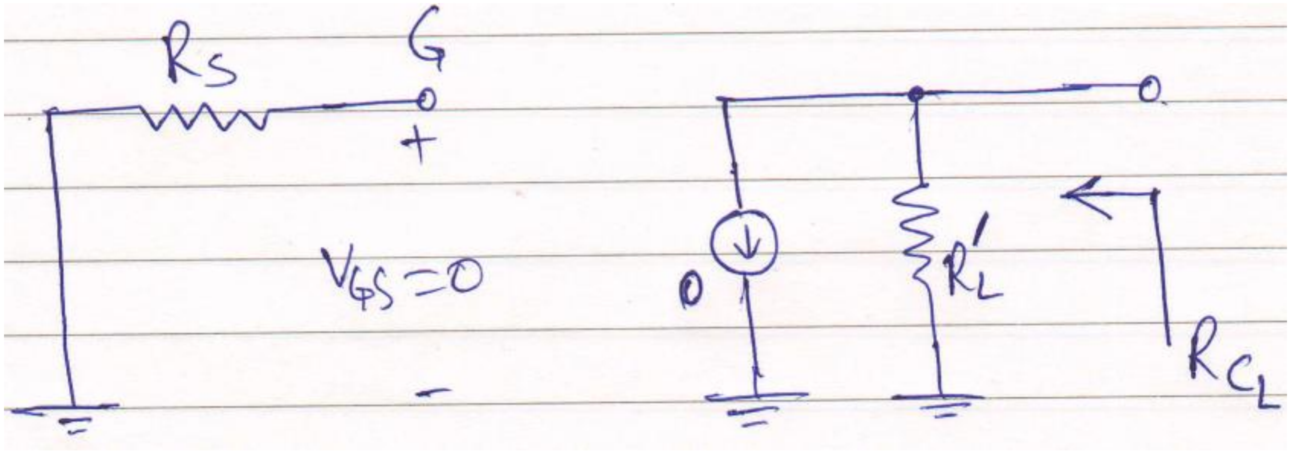
$$I_X = g_m V_{GS} + \frac{V_{GS} + V_X}{R_L^{'}}$$

$$\frac{V_{GS}}{R_S} + I_X = 0$$

$$\Rightarrow R_{GD} = \frac{V_X}{I_X} = R_S + (1 + g_m R_S) R_L^{'}$$

Common Source Amplifier (contd.)

- Considering only C_L → open C_{GS} and C_{GD}



$$R_{C_L} = R_L^{'}$$

Thus, the effective time constant: $\tau_H = C_{GS}R_{GS} + C_{GD}R_{GD} + C_L R_{C_L}$

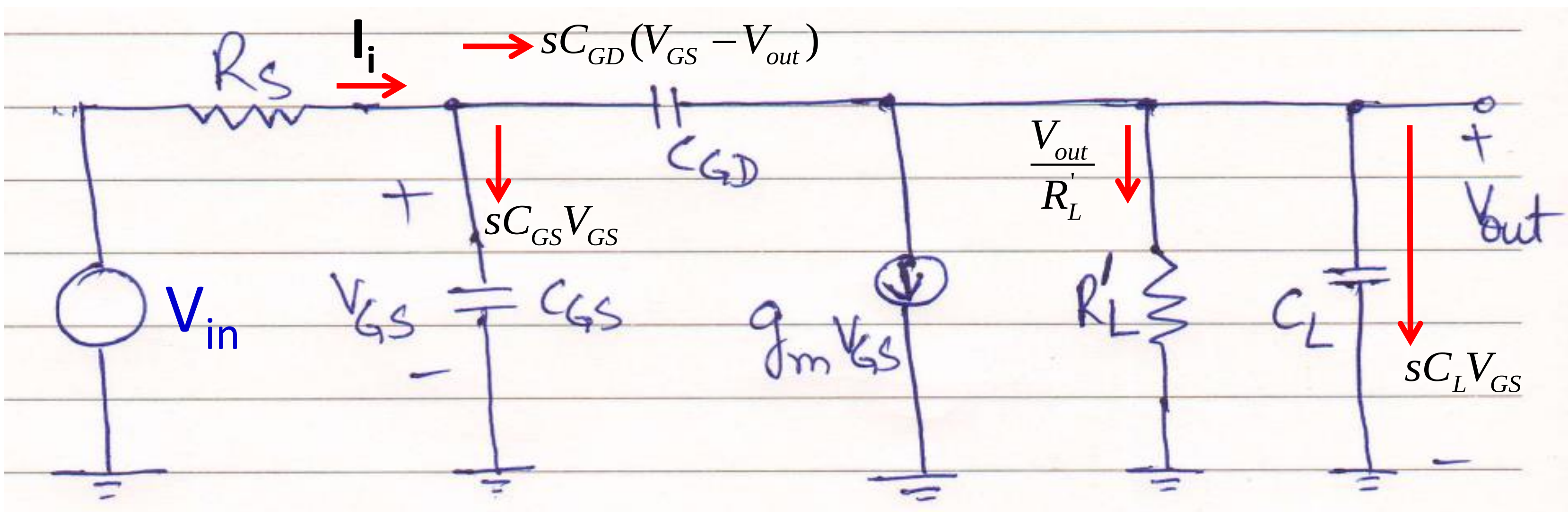
Therefore the 3-dB roll-off frequency is: $f_H = \dfrac{1}{2\pi\tau_H}$

Provides a better estimate than Miller's approximation

Common Source Amplifier (contd.)

Exact Analysis

- Miller's Approx and OCTC Technique provides insight about the impact of various capacitances on the high frequency response of amplifier
- However, for simple circuits its imperative to carry out exact analysis



KCL at the drain: $sC_{GD}\left(V_{GS}-V_{out}\right)=g_mV_{GS}+\frac{V_{out}}{R_L'}+sC_LV_{out}$

$$V_{GS}=\frac{-V_{out}}{g_mR_L'}\frac{1+s\left(C_L+C_{GD}\right)R_L'}{1-\left(sC_{GD}/g_m\right)}$$

Common Source Amplifier (contd.)

KVL at the gate: $V_{in} = I_i R_S + V_{GS}$

KCL at the gate: $I_i = sC_{GS} V_{GS} + sC_{GD}\left(V_{GS} - V_{out}\right)$

$$V_{in} = V_{GS}\left[1 + s(C_{GS} + C_{GD})R_S\right] - sC_{GD} R_S V_{out}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{-\left(g_m R_L^{'}\right)\left[1 - s\left(C_{GD} / g_m\right)\right]}{1 + sA + s^2 B}$$

Where,

$$A = \left[C_{GS} + C_{GD}\left(1 + g_m R_L^{'}\right)\right]R_S + \left(C_L + C_{GD}\right)R_L^{'}$$

$$B = \left[\left(C_L + C_{GD}\right)C_{GS} + C_L C_{GD}\right]R_S R_L^{'}$$

Observations

- **There exists one zero → not known through the approximate analysis**
- **2^{nd} order denominator [D(s)] → presence of two poles**
- **There are three capacitances → why only two poles and one zero**

Common Source Amplifier (contd.)

Poles Determination

- As s → 0, the transfer function approaches: $\Rightarrow \frac{V_{out}}{V_{in}} = -\left(g_m R_L^{'}\right)$ DC Gain
- Let ω_{p1} and ω_{p2} be the two poles then: $$D(s) = \left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right) = 1 + s\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1}\omega_{p2}}$$
- If ω_{p1} is dominant then: $$D(s) \cong 1 + \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}}$$
- Now, equating the coefficients: $$\omega_{p1} = \frac{1}{\left[C_{GS} + C_{GD}\left(1 + g_m R_L^{'}\right)\right]R_S + \left(C_L + C_{GD}\right)R_L^{'}}$$

$$\omega_{p1}\omega_{p2} = \frac{1}{\left[\left(C_L + C_{GD}\right)C_{GS} + C_L C_{GD}\right]R_S R_L^{'}}$$

$$\Rightarrow \omega_{p2} = \frac{\left[C_{GS} + C_{GD}\left(1 + g_m R_L^{'}\right)\right]R_S + \left(C_L + C_{GD}\right)R_L^{'}}{\left[\left(C_L + C_{GD}\right)C_{GS} + C_L C_{GD}\right]R_S R_L^{'}}$$

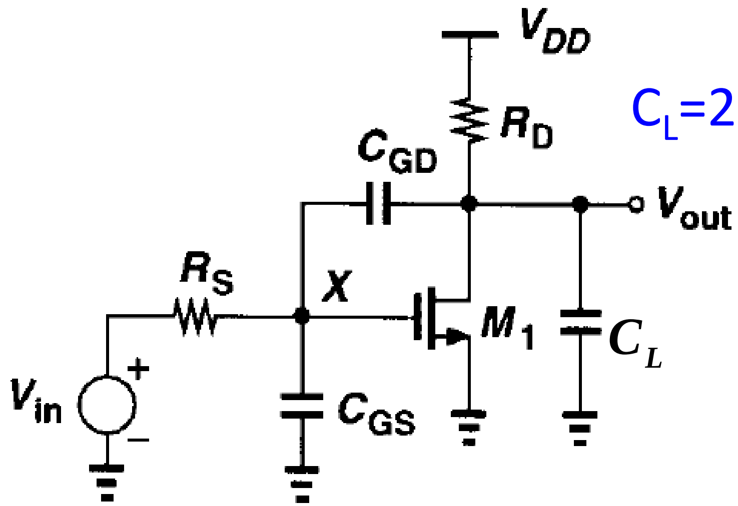
Very similar to the pole determined using OCTC method with the only addition being $R_L^{'}(C_{GD} + C_L)$

Common Source Amplifier (contd.)

Example:

R_S=50 Ohms
L=2.0 um
A_V=15

f_{in}=4.65 GHz
f_{out}=69.9 MHz

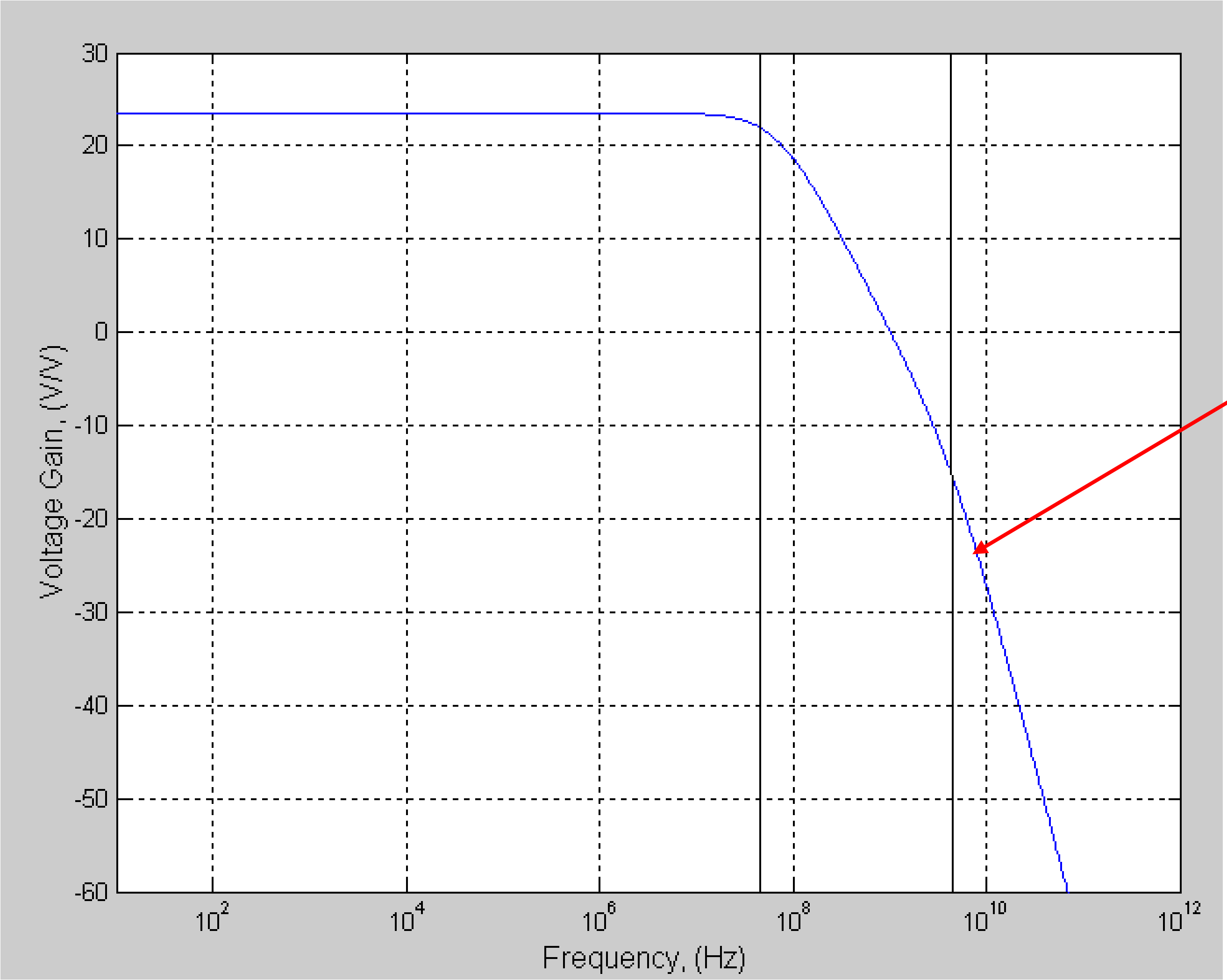


C_L=27.51 fF, R_L'=60 KOhm

$$\omega_{in} = \frac{1}{R_S\left(C_{GS} + \left(1 + g_m R_L^{'}\right)C_{GD}\right)}$$

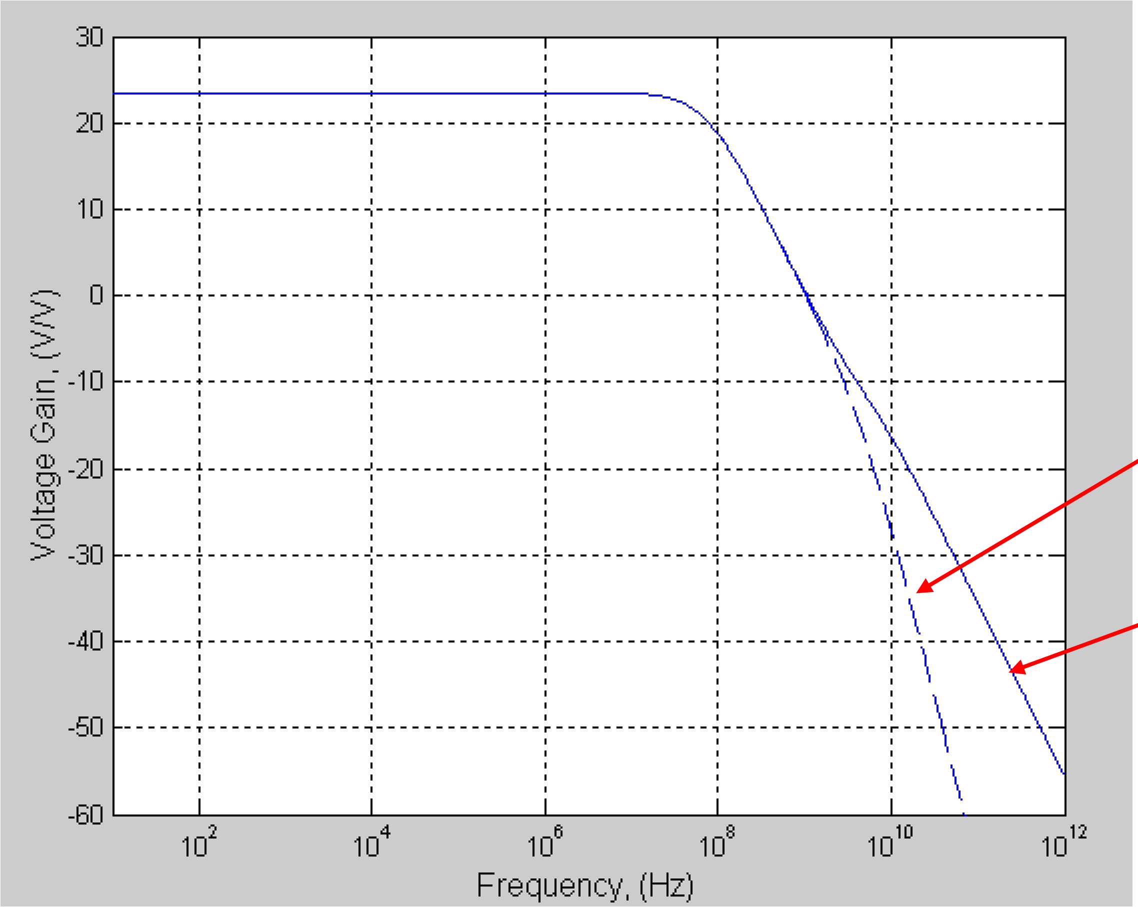
$$\omega_{out} = \frac{1}{R_L^{'}\left(C_L + C_{GD}\right)}$$

Transfer Function



Miller Approximation

Transfer Function



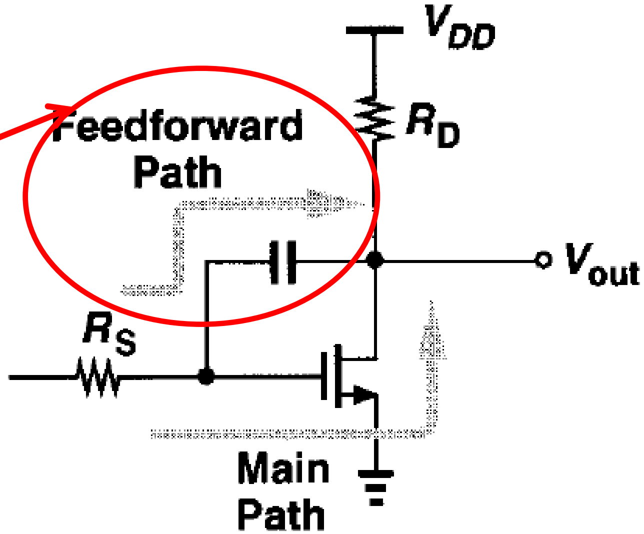
Miller Approximation

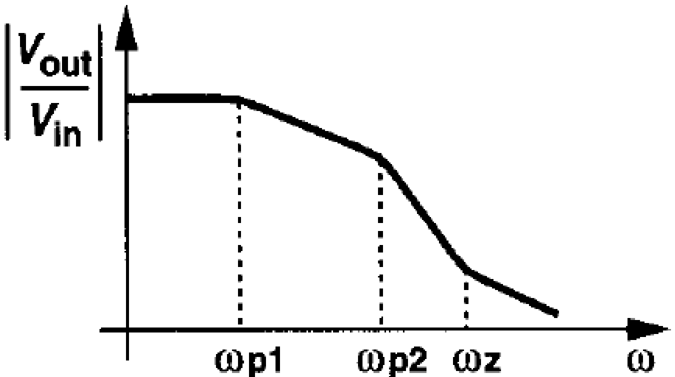
Exact Analysis

Common Source Amplifier (contd.)

- There exists one zero given by: $\omega_{z1} = \frac{g_m}{C_{GD}}$
- This zero results from the direct coupling of the input and output through C_{GD} at high frequencies
- the capacitor provides a feed-forward path

Only at high frequencies

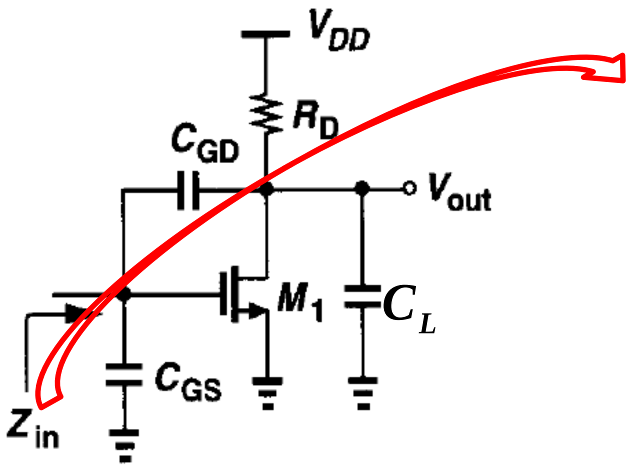




It results in a slope in the frequency response that is less negative than -20dB/dec

Common Source Amplifier (contd.)

- In high speed applications, the <u>input impedance</u> of the common source stage is extremely important

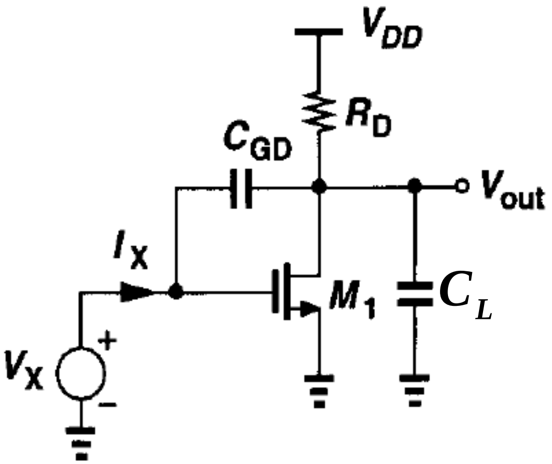


<u>At high enough frequency, we can derive</u>

$$Z_{in} = \frac{1}{\left[C_{GS} + \left(1 + g_m R_D\right) C_{GD}\right] s}$$

But at extremely high frequencies where Miller's approximation doesn't give appropriate performance, it's a must to take into account the contribution of output node

Common Source Amplifier (contd.)



For simplification, C_{GS} has been ignored

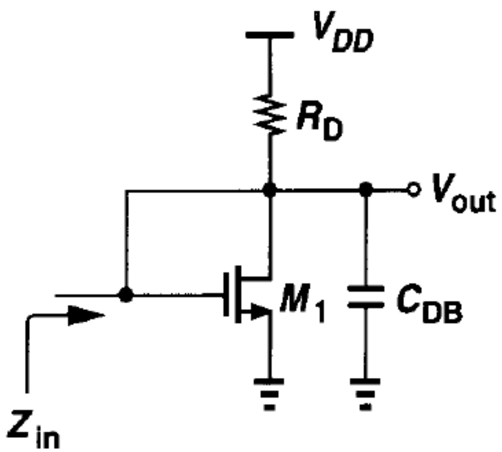
Using small signal model:

$$\frac{V_X}{I_X} = \frac{1 + R_D\left(C_{GD} + C_{DB}\right)s}{C_{GD}s\left[\left(1 + g_m R_D + R_D C_L s\right)\right]}$$

Therefore:

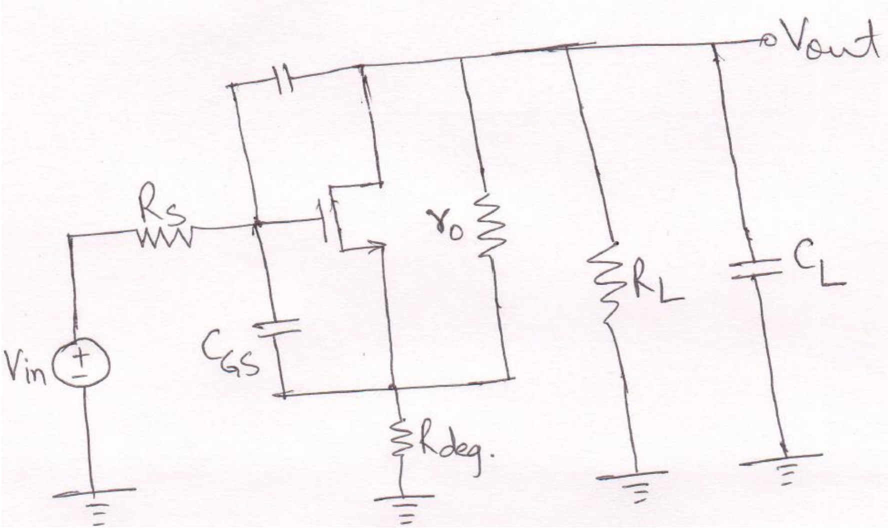
$$Z_{in} = X_{C_{GS}} \parallel \frac{V_X}{I_X}$$

At extremely high frequency



This is the case when C_{GD} is very high → provides a low impedance path between G and D

CS Amplifier with Source Degeneration



We know,

$$R_{out} = r_o\left[1+\left(g_m+g_{mb}\right)R_{\deg}\right]$$

$$G_m = \frac{g_m}{1+(g_m+g_{mb})R_{\deg}}$$

- **To determine the effective time constant, use OCTC by considering one capacitor at a time.**
- **Consider C_{GD} first:** $R_{GD} = R_S\left(1+G_mR_L^{'}\right)+R_L^{'}$ **Where,** $R_L^{'} = R_L \parallel R_{out}$
- **Then Consider C_L:** $R_{C_L} = R_L \parallel R_{out} = R_L^{'}$
- **Finally Consider C_{GS}:** $R_{GS} = \dfrac{R_S+R_{\deg}}{1+(g_m+g_{mb})R_{\deg}\left(\dfrac{r_o}{r_o+R_L}\right)}$

CS Amplifier with Source Degeneration (contd.)

- **Now, the effective time constant:** $$\tau_H = C_{GS}R_{GS} + C_{GD}R_{GD} + C_L R_{C_L}$$

- **For relatively large R_S the contribution of $C_{GD}R_{GD}$ in open circuit time constants (τ_H) will be largest.**

$$\Rightarrow \tau_H = C_{GD}R_{GD} \qquad \therefore f_H \cong \frac{1}{2\pi C_{GD}R_{GD}}$$

Comment

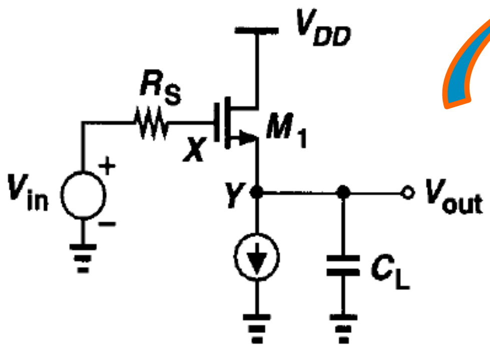
- **If R_{deg} is increased → the mid-band gain A_M will decrease → this causes reduction in R_{GD} → as a result f_H increases**

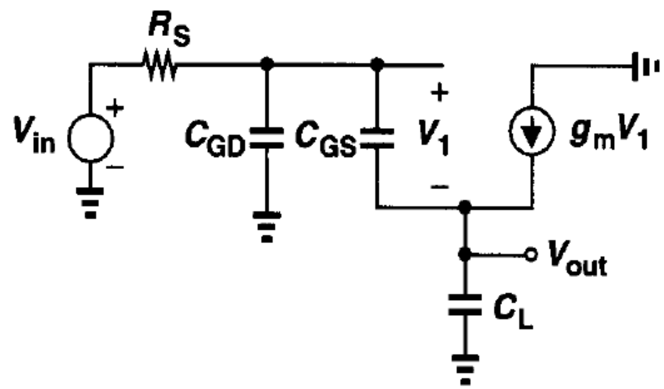
- **As $G_m R_L'$ >>1 and $G_m R_S$>>1 the term R_{GD} can be approximated as:** $$R_{GD} \cong G_m R_L' R_S = |A_M| R_S$$

$$\Rightarrow f_H = \frac{1}{2\pi C_{GD}|A_M|R_S}$$

Gain bandwidth product ($f_H.|A_M|$) remains constant for fixed R_S → however other capacitances make it variable

Common Drain





Strong interaction between XY, making it difficult to associate each pole with each node

$$H(s) = \frac{g_m + C_{GS}s}{R_S\left(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L\right)s^2 + \left(g_m R_S C_{GD} + C_L + C_{GS}\right)s + g_m}$$

Denominator

$$D = \left(\frac{s}{\omega_{p1}} + 1\right)\left(\frac{s}{\omega_{p2}} + 1\right) = \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + 1$$

Dominant Pole

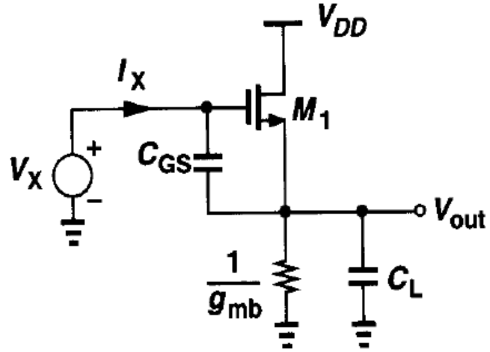
$$\omega_{p1} \approx \frac{g_m}{g_m R_S C_{GD} + C_L + C_{GS}} = \frac{1}{R_S C_{GD} + \dfrac{C_L + C_{GS}}{g_m}}$$

If, R_S = 0: then

$$\omega_{p1} = \frac{g_m}{C_L + C_{GS}}$$

Common Drain Amplifier (contd.)

Input Impedance



$$V_X = \frac{I_X}{C_{GS}s} + \left(I_X + \frac{g_m I_X}{C_{GS}s}\right)\left(\frac{1}{g_{mb}} \parallel \frac{1}{C_L s}\right)$$

$$Z_{in} = \frac{V_X}{I_X} = \frac{1}{C_{GS}s} + \left(1 + \frac{g_m}{C_{GS}s}\right)\frac{1}{g_{mb} + C_L s}$$

At relatively low frequencies:

$$Z_{in} \approx \frac{V_X}{I_X} = \frac{1}{C_{GS}s}\left(1 + \frac{g_m}{g_{mb}}\right) + \frac{1}{g_{mb}}$$

$$g_{mb} \gg |C_L s|$$

Equivalent Input Capacitance :

$$C_{in\_eq} = \frac{g_{mb} C_{GS}}{g_m + g_{mb}}$$

Can be obtained using Miller's Theorem

At high frequencies:

$$Z_{in} \approx \frac{1}{C_{GS}s} + \frac{1}{C_L s} + \frac{g_m}{C_{GS} C_L s^2}$$

$$g_{mb} \ll |C_L s|$$

For a given, s=jω

$$Z_{in} = \frac{1}{j\omega C_{GS}} + \frac{1}{j\omega C_L} - \frac{g_m}{\omega^2 C_{GS} C_L}$$

Negative Resistance

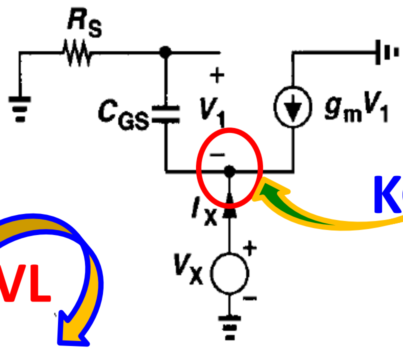
must be some positive feedback at the input node

could be useful in oscillator design

Common Drain Amplifier (contd.)

The body effect and C_{SB} yields an impedance in parallel to the output → lets ignore this for now

Output Impedance



KCL

$$-I_X = V_1 C_{GS} s + g_m V_1$$

KVL

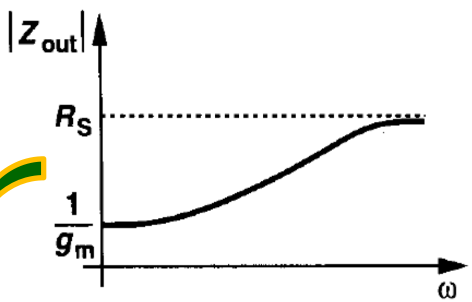
$$V_1 C_{GS} s R_S + V_1 = V_X$$

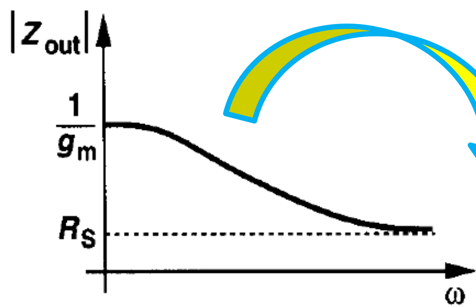
$$\therefore Z_{out} = \frac{Z_X}{I_X} = \frac{R_S C_{GS} s + 1}{g_m + C_{GS} s}$$

At low frequencies: $Z_{out} = \frac{1}{g_m}$ **Expected**

At very high frequencies: $Z_{out} = R_S$

Due to the fact that C_{GS} shorts the G and S



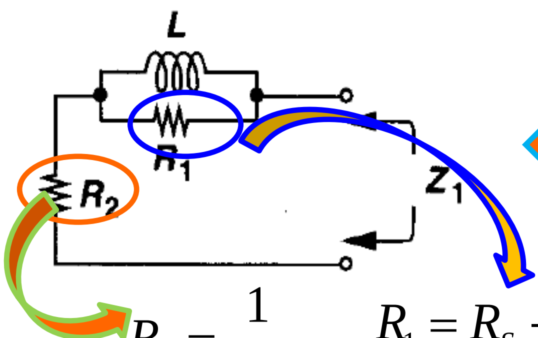


This is required considering that CD works as buffer and its purpose is to lower the output impedance i.e., $(1/g_m) < R_S$

The output impedance increases with frequency ↔ clear indication of the presence of inductive element

Common Drain Amplifier (contd.)

Output Network – first order model



$$R_2 = \frac{1}{g_m} \qquad R_1 = R_S - \frac{1}{g_m}$$

This Ensures

$$Z_1 = \frac{1}{g_m}$$ **At low frequencies**

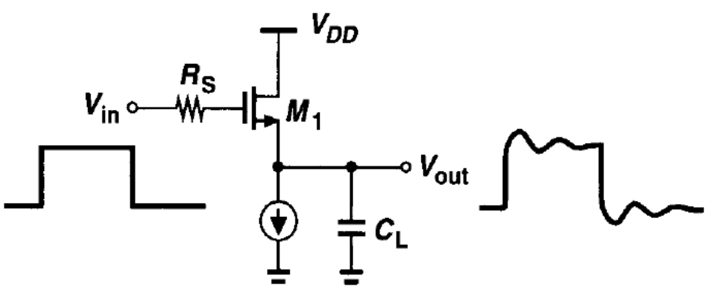
$$Z_1 = R_S$$ **At very high frequencies**

- **For the determination of L, equate the Z_1 expression to that of Z_{out}**

$$L = \frac{C_{GS}}{g_m}\left(R_S - \frac{1}{g_m}\right)$$

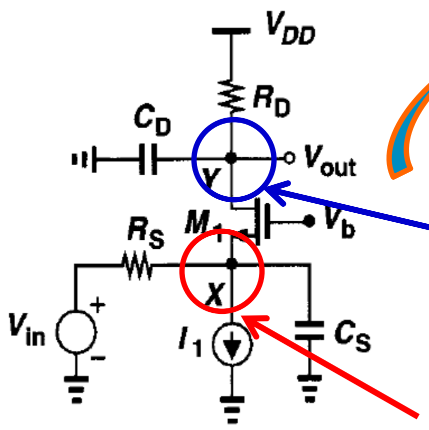
Its apparent that if a CD stage is driven by a R_S, then it exhibits inductive behavior at the output

It has detrimental effect on the output characteristics



For example, the ringing problem in step response

Common Gate Amplifier



Very important: no Miller multiplication of capacitances

⇓

Primary factor for providing wide-band performance

$$\omega_{out} = [C_D R_D]^{-1} \quad \leftarrow \quad C_D = C_{GD} \,||\, C_{DB}$$

$$\omega_{in} = \left[C_S \left(R_S \,||\, \frac{1}{g_m + g_{mb}} \right) \right]^{-1} \quad \leftarrow \quad C_S = C_{GS} \,||\, C_{SB}$$

Therefore the transfer function:

$$\frac{V_{out}}{V_{in}} = \frac{(g_m + g_{mb})R_D}{1+(g_m + g_{mb})R_S} \frac{1}{\left(1+\dfrac{C_S}{g_m + g_{mb} + R_S^{-1}} s\right)(1+R_D C_D s)}$$

A_v

- If we consider channel length modulation → not easy to associate a pole to the input node → direct analysis is needed to get the transfer function
- The low input impedance may load the preceding stage

Common Gate Amplifier (contd.)

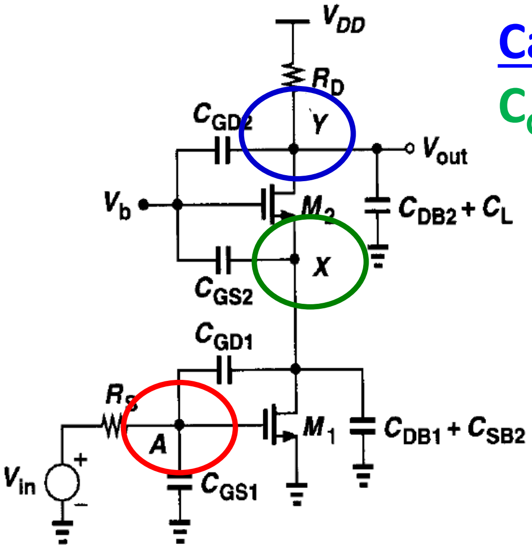
- The voltage drop across RD is typically maximized to obtain a reasonable gain, therefore the dc level of input signal must be low
- CG stage with relatively large capacitance at the input → possesses low output impedance → good for cascode configuration

Cascode Stage

Why do we need cascode stage?

- High input impedance – good in a sense that it doesn't disturb the previous stage and doesn't get affected by the previous stage
- **High gain – important any way!**
- Relatively higher output impedance – doesn't disturb the succeeding stages
- **How about freq response?**
  - Provides a broader bandwidth of operation →due to CG stage (no Miller approximation of intrinsic capacitor) → it was the initial motivating factor for cascode stage!!!

Cascode Stage (contd.)



Capacitance at node A:

C_{GS1} in parallel with $(1-A_M)C_{GD1}$.

Where A_M is given by: $-\dfrac{g_{m1}}{g_{m2}+g_{mb2}}$

With assumption that R_D is small and the channel length modulation is negligible

For equal dimensions of M_1 and M_2: A_M approximately equals 1 → therefore C_{GD} gets multiplied by a factor of roughly 2 both at node A as well as at node X → much smaller multiplication factor as compared to a single CS stage

Therefore the pole associated with node A is:

$$\omega_{p,A} = \frac{1}{R_S\left[C_{GS1} + \left(1+\dfrac{g_{m1}}{g_{m2}+g_{mb2}}\right)C_{GD1}\right]}$$

Capacitance at node X: $2C_{GD1} + C_{DB1} + C_{SB2} + C_{GS2}$

Therefore the pole associated with node X is:

$$\omega_{p,X} = \frac{g_{m2}+g_{mb2}}{2C_{GD1}+C_{DB1}+C_{SB2}+C_{GS2}}$$

Cascode Stage (contd.)

Capacitance at node Y: $C_{DB2} + C_L + C_{GD2}$

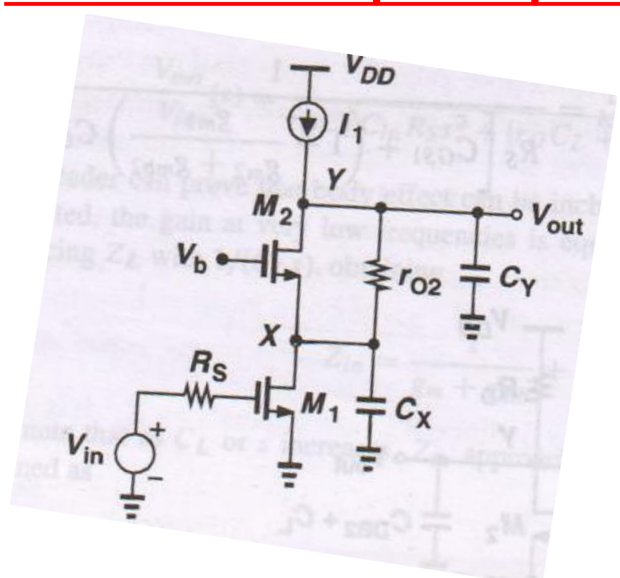
Therefore the pole associated with node Y is:

$$\omega_{p,Y} = \frac{1}{R_D\left(C_{DB2} + C_L + C_{GD2}\right)}$$

- Do you have any control on the choice of poles?
- Yes → through selection of appropriate devices
- Usually $\omega_{p,X}$ is chosen very high → to obtain better stability

Instead of R_D → if a constant current source is used → what happens?
→ do the designer have control on the choice of poles?

How about output impedance?



If we ignore C_{GD1} and the capacitance at the node Y then:

$$Z_{out} = \left(1 + g_{m2}r_{o2}\right)Z_X + r_{o2}$$

$$= r_{o1} \,||\, (C_X s)^{-1}$$

Z_{out} contains a pole at $(r_{o1}C_X)^{-1}$

Z_{out} falls above this frequency