Lecture – 14 Date: 03.10.2016

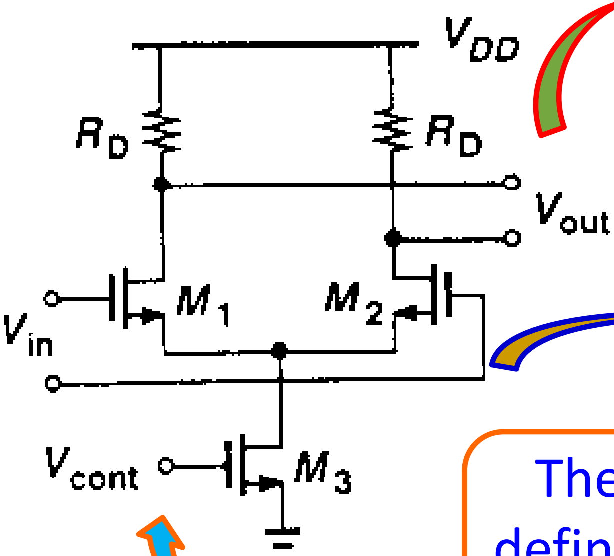
- Gilbert Cell
- Introduction to Frequency Response
- 3-dB Frequency
- Bode's Rules
- Association of Poles with Nodes
- Miller's Theorem

Gilbert Cell

Two important aspects of differential pair

- **small signal gain is function of tail current**
- **the whole tail current can be steered to one of two paths by some means**

- **These features can be utilized to design very interesting and useful circuit known as Gilbert Cell [a differential pair whose gain can be varied by control voltage]**



$$A_v = \frac{V_{out}}{V_{in}}$$

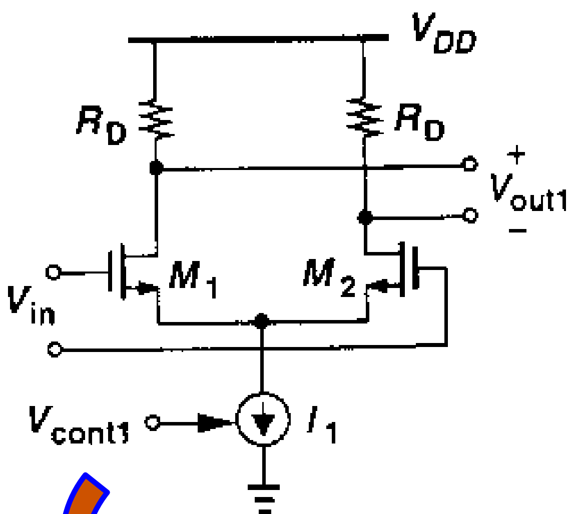
Varies from ZERO (if ID_3=0) to a limit set by the voltage headroom and device dimensions

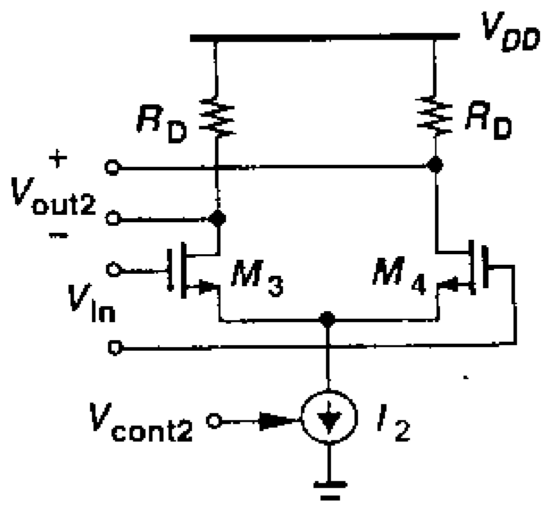
The control voltage defines the tail current and hence the gain

Simple Variable Gain Amplifier (VGA)

Gilbert Cell (contd.)

- Suppose we require an amplifier whose gain can be continuously varied from a NEGATIVE value to a POSITIVE value.
- Definitely, two VGA with opposite gains.





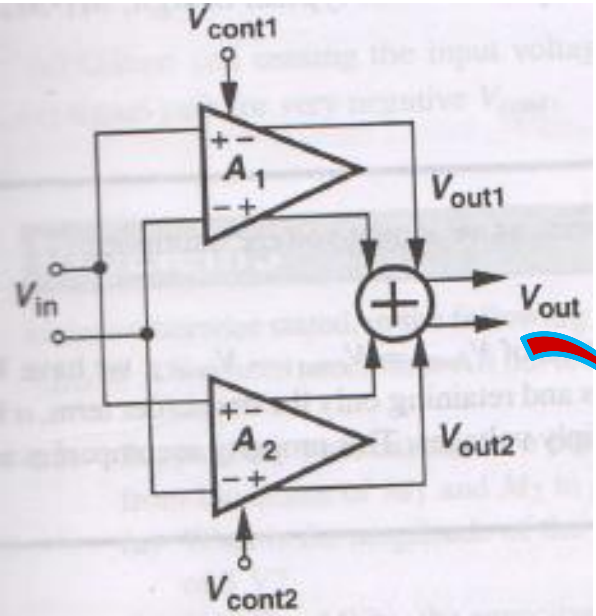
This pair of VGA should be designed in such a way that they give opposite gains

$V_{out1}/V_{in} = -g_m R_D$ and $V_{out2}/V_{in} = +g_m R_D$ ↔ require variation of I_1 and I_2 in opposite directions

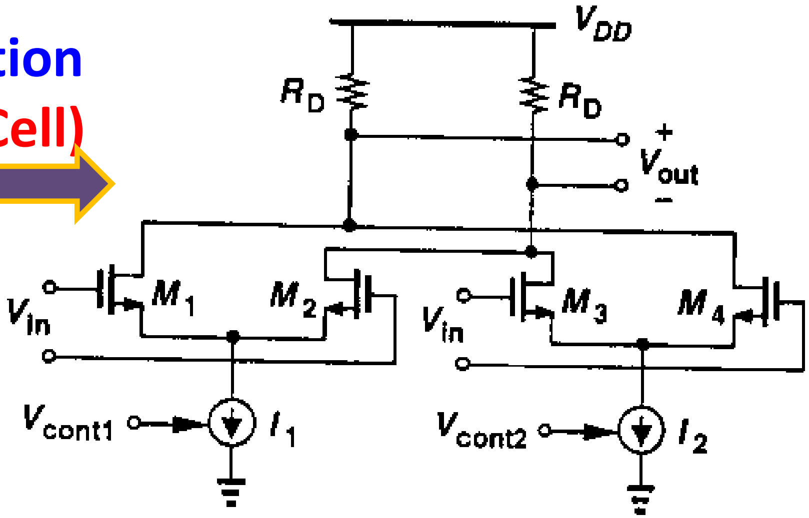
How to combine V_{out1} and V_{out2} in a single output?

Gilbert Cell (contd.)

How to combine V_{out1} and V_{out2} in a single output?



Actual Implementation (i.e, Gilbert Cell)



$$V_{out} = V_{out1} + V_{out2} = A_1 V_{in} + A_2 V_{in}$$

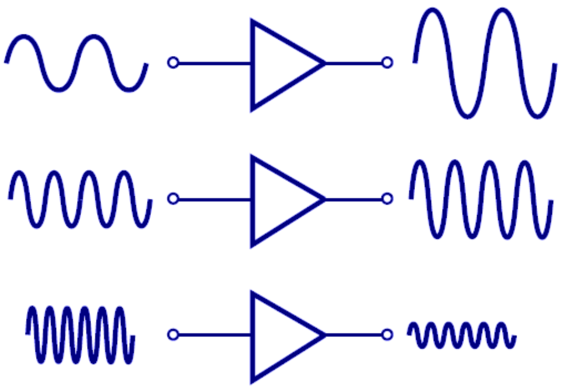
A_1 and A_2 controlled by V_{cont1} and V_{cont2} respectively

Here, instead of adding V_{out1} and V_{out2}, the drain terminals are shorted to sum the currents and subsequently generate the output voltage

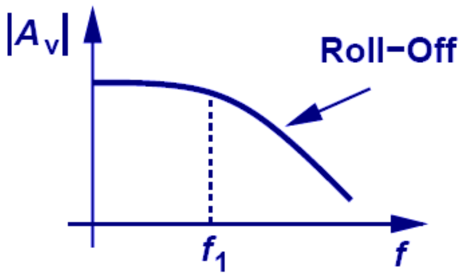
Intro to Frequency Response

What is meant by frequency response of an amplifier?

- The idea is to apply varying frequency signal to the amplifier and then observe the behavior → the gain may present three different scenario → falling, rising or constant with frequency



As frequency of operation increases, the gain of amplifier decreases

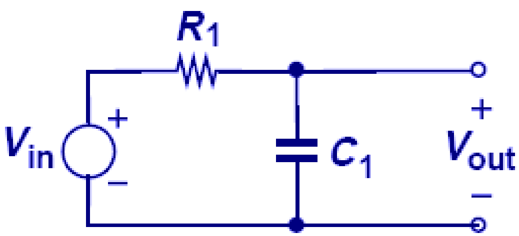


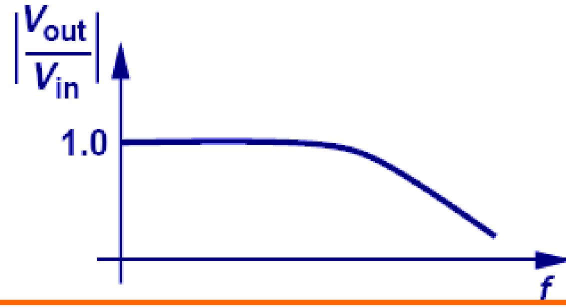
In this example, the gain rolls-off with the increase in frequency → the frequency 'f_1' is termed as the useful bandwidth of the amplifier

Intro to Frequency Response (contd.)

- **Gain Roll-Off**

Simple Low-Pass Filter





In this simple example, as frequency increases the impedance of C_1 decreases and therefore the voltage divider consisting of C_1 and R_1 attenuates V_{in} to a greater extent at the output

Why is it important?

What happens if a CDMA signal has to pass through a GSM standard transceiver?

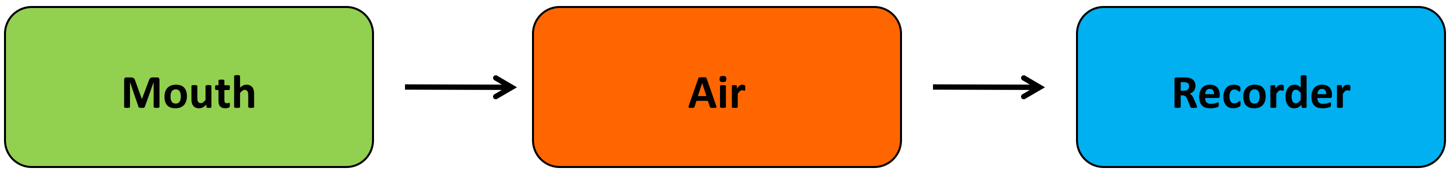
What happens if your DVB-H enabled hand phone possess circuit components capable of transmitting only CW signals?

What happens if the bandwidth of video system in your computer is insufficient to process video signals?

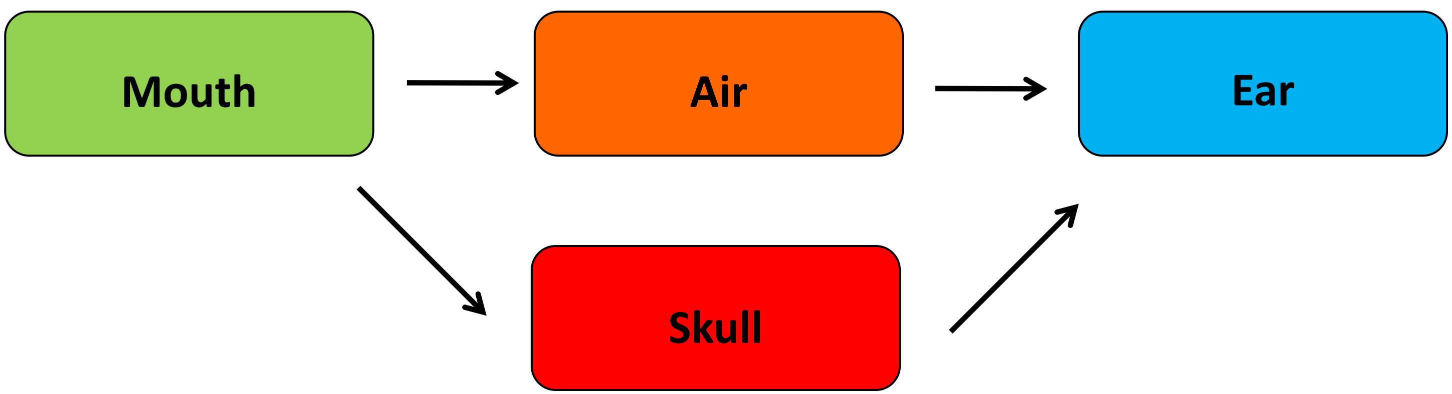
Intro to Frequency Response (contd.)

- When you record your voice and listen to it, it sounds little different from the way you directly hear. Why?

Path traveled by the human voice to the voice recorder



Path traveled by the human voice to the human ear

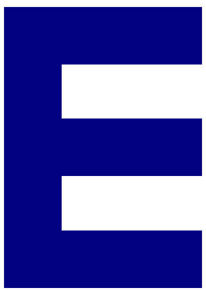


Since the paths are different, the results will also be different

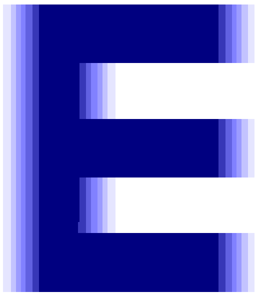
Actually, skull passes some frequencies more easily than air

Intro to Frequency Response (contd.)

- Example: The graphics card delivering the video signal to the display must provide at least 5MHz bandwidth. What happens if the bandwidth is not sufficient?



High Bandwidth



Low Bandwidth

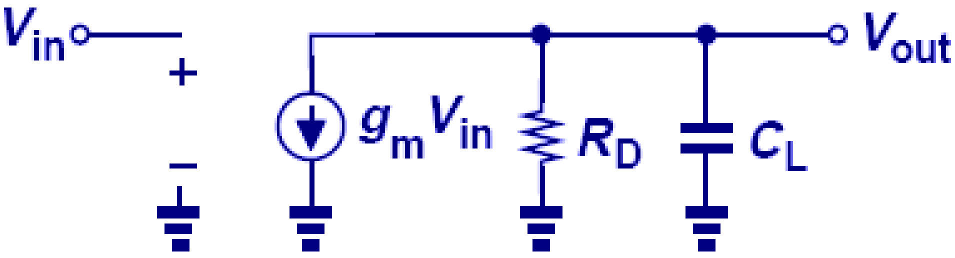
Insufficient bandwidth leads to soft edges
→ yields fuzzy picture

This is due to the fact that the circuit driving the display is not fast enough to abruptly change the contrast from e.g., complete dark to complete white from one pixel to the next

Frequency Response (contd.)

What causes the roll-off in frequency response?

Impedance, current, voltage?



$$V_{out}(s) = -g_m V_{in}(s)\left(R_D \parallel \frac{1}{X_{C_L}(s)}\right)$$

At low frequencies

The signal current produced by FET flows through R_D → reason is the almost open condition provided by C_L

At high frequencies

The signal current produced by FET flows through the parallel combination of R_D and the impedance contributed by C_L

Reduces with increase in frequency

Results in gain reduction with increase in frequency

The capacitive load, C_L, is the culprit for gain roll-off since at high frequency, it will “steal” away some signal current and shunt it to ground

Frequency Response (contd.)

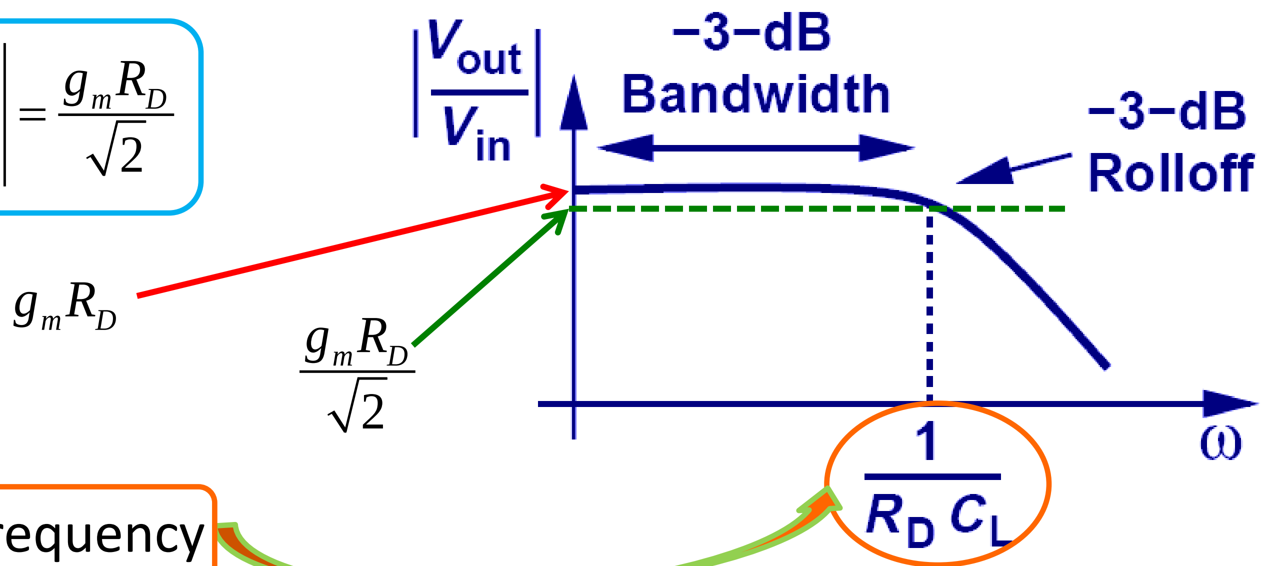
$$\Rightarrow \left|\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\right| = g_m\left(R_D \,||\, \frac{1}{X_{C_L}(j\omega)}\right) = \frac{g_m R_D}{\sqrt{R_D^2 X_{C_L}^2 \omega^2 + 1}}$$

At low frequency, the capacitor is effectively open and the gain is flat ($g_m R_D$). As frequency increases, the capacitor tends to a short and the gain starts to decrease.

- A special frequency is $\omega = 1/(R_D C_L)$, where the gain drops by 3dB.

$$\Rightarrow \left|\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\right| = \frac{g_m R_D}{\sqrt{2}}$$

3dB – Gain



3-dB frequency

Frequency Response (contd.)

Bode's Rules: The task of obtaining |H(jω)| from H(s) is often tedious. In such cases, it is easier to approximate the response by looking at H(s) → Bode's rules help in approximation

- As ω passes each pole frequency, the slope of |H(jω)| decreases by 20dB/dec. (A slope of 20dB/dec simply means a ten fold decrease in H for a ten fold increase in frequency)
- As ω passes each zero frequency, the slope of |H(jω)| increases by 20dB/dec.

$$H(s) = A_v \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$

- When we hit a zero, ω_{zj}, the Bode magnitude rises with a slope of +20dB/dec.
- When we hit a pole, ω_{pj}, the Bode magnitude falls with a slope of -20dB/dec

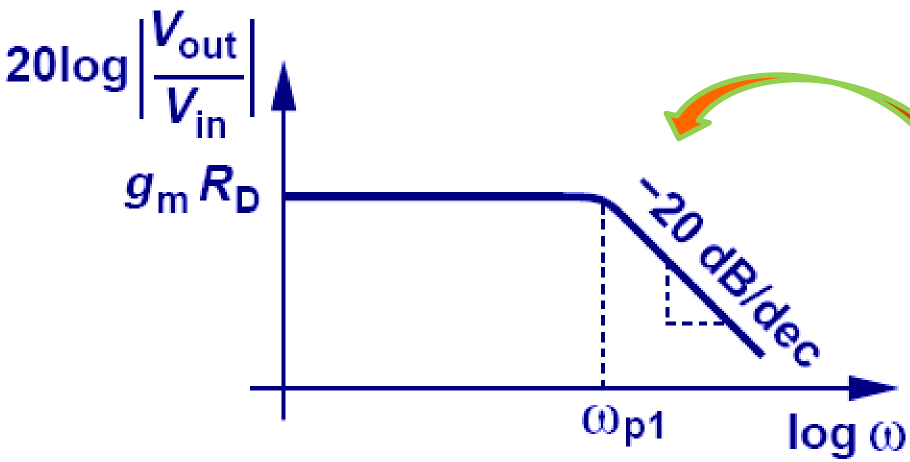
Bode's Rules (contd.)

$$\Rightarrow \left|\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\right| = \frac{g_m R_D}{\sqrt{R_D^2 X_{C_L}^2 \omega^2 + 1}}$$

At this frequency the slope changes from 0 to –20 dB/dec

- In this a pole frequency is:

$$|\omega_{p1}| = \frac{1}{R_D C_L}$$



Bode's rule ignores the 3dB roll-off effect at the pole frequency

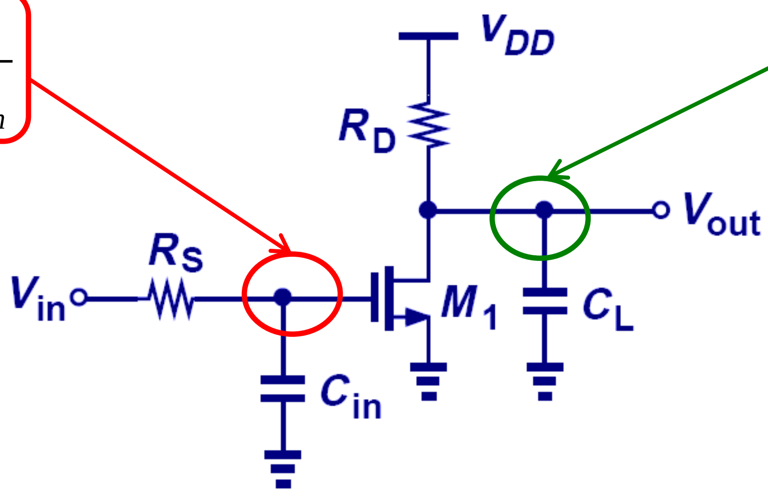
The circuit only has one pole (no zero) at $1/(R_D C_L)$, so the slope drops from 0 to -20dB/dec as we pass ω_{p1}

Frequency Response (contd.)

Association of poles with nodes: poles of a circuit transfer function is key in the frequency response → it aids in determining the speed of various parts of the circuit

$$|\omega_{p1}| = |\omega_{in}| = \frac{1}{R_S C_{in}}$$

$$|\omega_{p2}| = |\omega_{out}| = \frac{1}{R_D C_L}$$



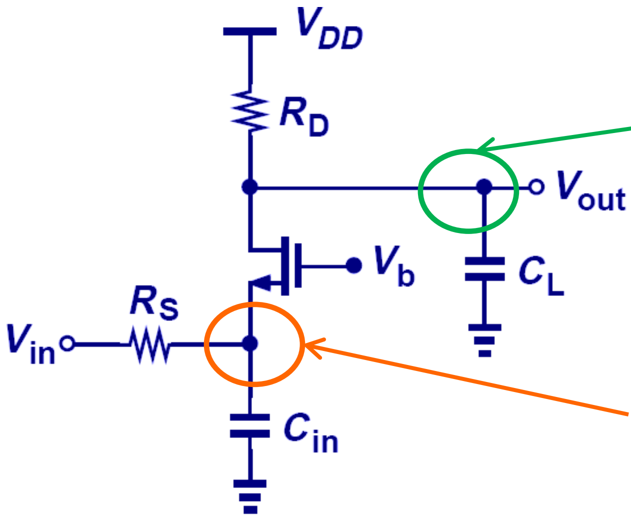
Low Frequency Small Signal Gain

$$\Rightarrow \left|\frac{V_{out}}{V_{in}}(j\omega)\right| = A_v \frac{1}{\sqrt{\left(1+\omega^2/\omega_{p1}^2\right)}} \cdot \frac{1}{\sqrt{\left(1+\omega^2/\omega_{p2}^2\right)}}$$

$$\therefore \left|\frac{V_{out}}{V_{in}}\right| = \frac{g_{m1}R_D}{\sqrt{\left(1+\omega^2/\omega_{p1}^2\right)\left(1+\omega^2/\omega_{p2}^2\right)}}$$

Frequency Response (contd.)

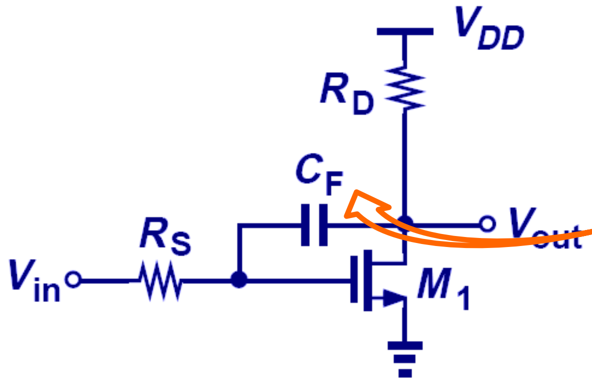
Example:



$$|\omega_{p2}| = |\omega_{out}| = \frac{1}{R_D C_L}$$

$$|\omega_{p1}| = |\omega_{in}| = \frac{1}{(R_S \| 1/g_{m1}) C_{in}}$$

Example: circuit with floating capacitor

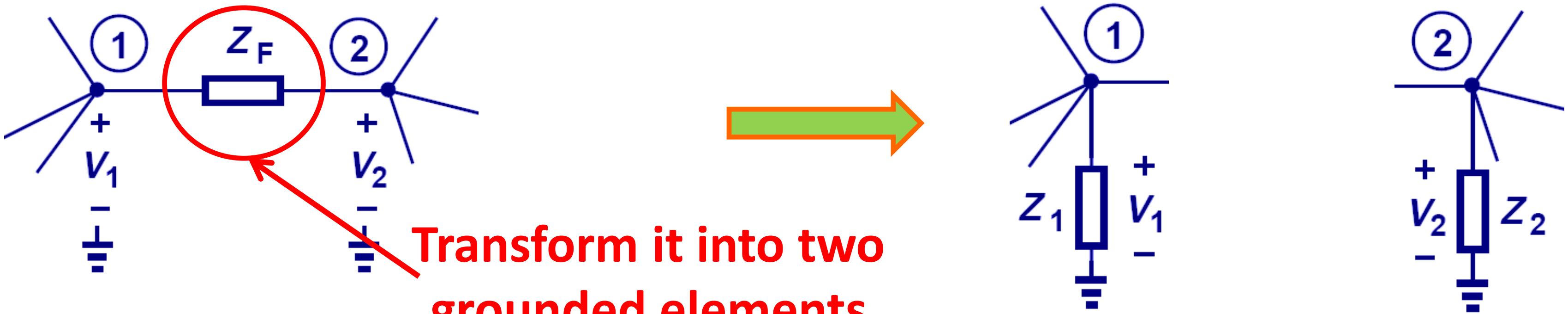


Now the capacitor C_F isn't connected between just one node and ground. What do we do?

Miller's Theorem Provides the Solution

Miller's Theorem

- It coverts floating impedance element into two grounded elements



- The current drawn by Z_F from node 1 must be equal to that drawn by Z_1

$$\frac{V_1 - V_2}{Z_F} = \frac{V_1}{Z_1}$$

- Current injected to node 2 must be equal to that injected to node 2 in both situations:

$$\frac{V_1 - V_2}{Z_F} = -\frac{V_2}{Z_2}$$

$$Z_1 = Z_F \frac{V_1}{V_1 - V_2} = \frac{Z_F}{1 - A_v}$$

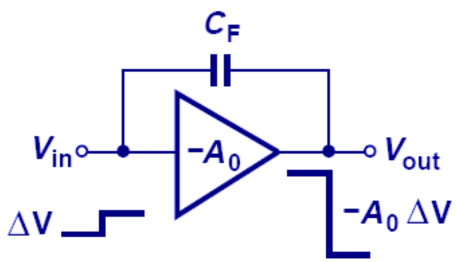
$$Z_2 = Z_F \frac{V_1}{V_1 - V_2} = \frac{Z_F}{1 - \frac{1}{A_v}}$$

A_v = low frequency small-signal gain

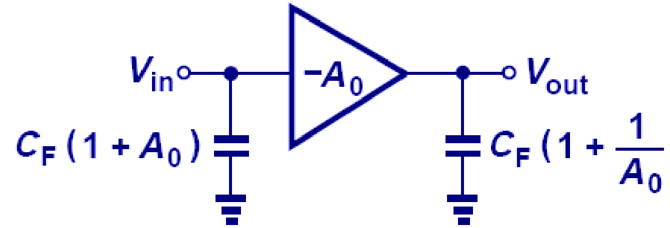
If A_v is the gain from node 1 to 2, then a floating impedance Z_F can be converted to two grounded impedances Z_1 and Z_2.

Miller's Theorem (contd.)

- If Z_F is capacitive and amplifier is inverting then:



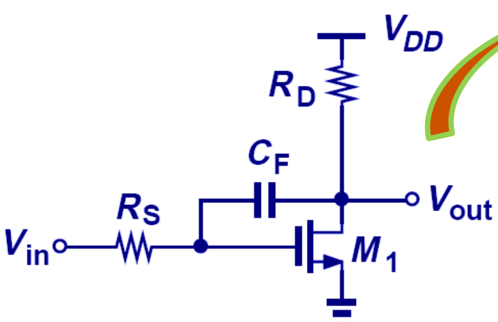
Miller's Theorem

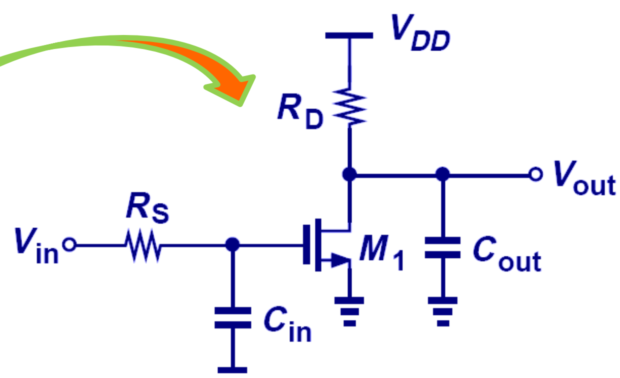


$$Z_1 = \frac{1}{(1 + A_0)C_F s} \qquad Z_2 = \frac{1}{\left(1 + \frac{1}{A_0}\right)C_F s}$$

With Miller's theorem, we can separate the floating capacitor. However, the input capacitor is larger than the original floating capacitor. We call this Miller multiplication.

Example: determine poles of the following circuit





$$C_{in} = (1 + A_0)C_F = (1 + g_m R_D)C_F$$

$$C_{out} = \left(1 + \frac{1}{A_0}\right)C_F = \left(1 + \frac{1}{g_m R_D}\right)C_F$$

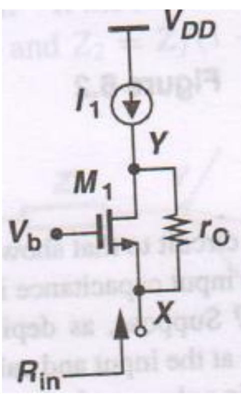
Miller's Theorem (contd.)

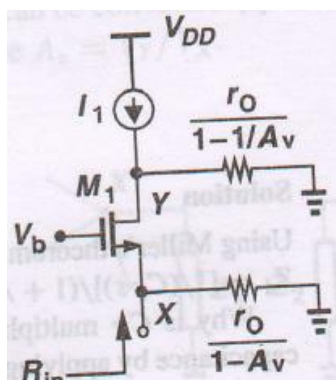
$$\therefore \omega_{in} = \frac{1}{R_S C_{in}} = \frac{1}{R_S\left(1+g_m R_D\right)C_F}$$

$$\therefore \omega_{out} = \frac{1}{R_D C_{out}} = \frac{1}{R_D\left(1+\dfrac{1}{g_m R_D}\right)C_F}$$

Miller's Theorem requires that the floating impedance and voltage gain be computed at the same frequency. However, apparently we always use low-frequency gain even at high frequencies. It is done for simplifying the analysis, otherwise the use of Miller Theorem will be no simpler. Therefore it is often called Miller's Approximation

Q: Calculate the input resistance of the following:





Here,

$$A_v = 1+(g_m+g_{mb})r_o$$

$$R_{in} = \frac{r_o}{1-A_v} \,\|\, \frac{1}{g_m+g_{mb}}$$

$$R_{in} = \frac{r_o}{1-[1+(g_m+g_{mb})r_o]} \,\|\, \frac{1}{g_m+g_{mb}}$$

$$\therefore R_{in} \approx \infty$$