Lecture – 11

- MOS Differential Pair
- Quantitative Analysis – differential input
- Small Signal Analysis

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MOS Differential Pair

Variation of input CM level regulates the bias currents of $M_1$ and $M_2$ → Undesired!!

Solution??

Current source is ideal: constant current, infinite output impedance

To overcome the issues emanating from non-ideal CM level

$M_1$ and $M_2$ are perfectly matched (at least in theory!)

ensures $M_1$ and $M_2$ in saturation

$R_{D1} = R_{D2} = R_D$
MOS Differential Pair
Qualitative Analysis – differential input

- Let us check the effect of $V_{in1} - V_{in2}$ variation from $-\infty$ to $\infty$

- $V_{in1}$ is much more $-$ve than $V_{in2}$ then:
  - $M_1$ if OFF and $M_2$ is ON
  - $I_{D2} = I_{SS}$
  - $V_{out1} = V_{DD}$ and $V_{out2} = V_{DD} - I_{SS}R_D$

- $V_{in1}$ is brought closer to $V_{in2}$ then:
  - $M_1$ gradually turns ON and $M_2$ is ON
    - Draws a fraction of $I_{SS}$ and lowers $V_{out1}$
    - $I_{D2}$ decreases and $V_{out2}$ rises

- $V_{in1} = V_{in2}$
  - $V_{out1} = V_{out2} = V_{DD} - I_{SS}R_D/2$
MOS Differential Pair

Qualitative Analysis – differential input

- Let us check the effect of $V_{in1} - V_{in2}$ variation from $-\infty$ to $\infty$

- $V_{in1}$ becomes more +ve than $V_{in2}$ then:
  - $M_1$ if ON and $M_2$ is ON
  - $M_1$ carries greater $I_{SS}$ than $M_2$

- For sufficiently large $V_{in1} - V_{in2}$:
  - All of the $I_{SS}$ goes through $M_1 \rightarrow M_2$ is OFF
  - $V_{out1} = V_{DD} - I_{SS}R_D$ and $V_{out2} = V_{DD}$
MOS Differential Pair

Qualitative Analysis – differential input

- Plotting $V_{\text{out1}} - V_{\text{out2}}$ versus $V_{\text{in1}} - V_{\text{in2}}$

The maximum and minimum levels at the output are well defined and is independent of input CM level ($V_{\text{in,cm}}$)

Minimum Slope $\leftrightarrow$ Minimum Gain

Maximum Slope $\leftrightarrow$ Maximum Gain

The circuit becomes more nonlinear as the input voltage swing increases (i.e., $V_{\text{in1}} - V_{\text{in2}}$ increases) $\leftrightarrow$ at $V_{\text{in1}} = V_{\text{in2}}$, the circuit is said to be in equilibrium
MOS Differential Pair

Qualitative Analysis – common mode input

• Now let us consider the common mode behavior of the circuit

As mentioned, the tail current source is used to suppress the effect of input CM level variation \( (V_{in,cm}) \)

Does this enable us to set any arbitrary level of input CM \( (V_{in,cm}) \)

To understand this:
• Set \( V_{in1} = V_{in2} = V_{in,CM} \)
• Then vary \( V_{in,CM} \) from 0 to \( V_{DD} \)
• Also implement \( I_{SS} \) with an NFET

• Lower bound of \( V_{in,cm} \): \( V_p \) should be sufficiently high in order for \( M_3 \) to act as a current source.
• Upper bound of \( V_{in, cm} \): \( M_1 \) and \( M_2 \) need to remain in saturation.
MOS Differential Pair

Qualitative Analysis – common mode input

• What happens when $V_{\text{in,CM}} = 0$?
  • $M_1$ and $M_2$ will be OFF and $M_3$ can be in triode for high enough $V_b$
  • $I_{D1} = I_{D2} = 0 \iff$ circuit is incapable of amplification

• Now suppose $V_{\text{in,CM}}$ becomes more +ve
  • $M_1$ and $M_2$ will turn ON if $V_{\text{in,CM}}$ exceeds $V_T$
  • $I_{D1}$ and $I_{D2}$ will continue to rise with the increase in $V_{\text{in,CM}}$
  • $V_P$ will track $V_{\text{in,CM}}$ as $M_1$ and $M_2$ work like a source follower
  • For high enough $V_{\text{in,CM}}$, $M_3$ will be in saturation as well

• If $V_{\text{in,CM}}$ rises further
  • $M_1$ and $M_2$ will remain in saturation if:
    \[ V_{\text{in,CM}} - V_T \leq V_{\text{out1}} \]
    \[ V_{\text{in,CM}} \leq V_{DD} - \frac{I_{SS}}{2} R_D + V_T \]
MOS Differential Pair

Qualitative Analysis – common mode input

- For $M_1$ and $M_2$ to remain in saturation:

$$V_{GS_{1,2}} - V_T \leq V_{DS_{1,2}} \quad \Rightarrow V_{in,CM} - V_T \leq V_{DD} - \frac{I_{SS}}{2} R_D \quad \Rightarrow V_{in,CM} \leq V_T + V_{DD} - \frac{I_{SS}}{2} R_D$$

$$\therefore (V_{in,CM})_{\text{max}} = V_T + V_{DD} - \frac{I_{SS}}{2} R_D$$

- The lowest value of $V_{in,CM}$ is determined by the need to keep the constant current source operational:

$$V_{in,CM} - V_{GS_{1,2}} \geq V_{GS_3} - V_T \quad \Rightarrow V_{in,CM} \geq V_{GS_{1,2}} + (V_{GS_3} - V_T)$$

$$V_{GS_{1,2}} + (V_{GS_3} - V_T) \leq V_{in,CM} \leq \min \left[ V_{DD} - \frac{I_{SS}}{2} R_D + V_T, V_{DD} \right]$$
MOS Differential Pair

Qualitative Analysis – common mode input

• Thus, $V_{in,CM}$ is bounded as:

$$V_{GS1,2} + (V_{GS3} - V_T) \leq V_{in,CM} \leq \min \left[ V_{DD} - \frac{I_{SS}}{2} R_D + V_T, V_{DD} \right]$$

• Summary:
MOS Differential Pair

Qualitative Analysis – common mode input

- How large can the output voltage swings of a differential pair be?

\[ V_{\text{out}, \text{max}} = V_{DD} \]

\[ V_{\text{out}, \text{min}} = V_{\text{in, CM}} - V_T \]

The higher the input CM level, the smaller the allowable output swings.
MOS Differential Pair

Quantitative Analysis – differential input

For +ve \( V_{in1} \rightarrow V_{GS1} \) is greater than \( V_{GS2} \rightarrow \)
\( I_{D1} \) will be greater than \( I_{D2} \)

\[
V_{out2}(=V_{DD} - I_{D2}R_D) > V_{out1}(=V_{DD} - I_{D1}R_D)
\]

For +ve \( V_{in2} \rightarrow V_{GS2} \) is greater than \( V_{GS1} \rightarrow \)
\( I_{D2} \) will be greater than \( I_{D1} \)

\[
V_{out1}(=V_{DD} - I_{D1}R_D) > V_{out2}(=V_{DD} - I_{D2}R_D)
\]

It is thus apparent that the differential pair respond to differential-mode signals → by providing differential output signal between the two drains
Differential Pair – Large Signal Analysis

Quantitative Analysis – differential input

- The idea is to define $I_{D1}$ and $I_{D2}$ in terms of input differential signal $V_{in1} - V_{in2}$
- The circuit doesn’t include connection details considering that these drain current equations do not depend on the external circuitries

- Assumptions: $M_1$ and $M_2$ are always in saturation; differential pair is perfectly matched; channel length modulation is not present
Differential Pair – Large Signal Analysis

Quantitative Analysis – differential input

\[ V_P = V_{in1} - V_{GS1} = V_{in2} - V_{GS2} \]

\[ \therefore V_{in1} - V_{in2} = V_{GS1} - V_{GS2} \]

We also know:

\[ (V_{GS} - V_T)^2 = \frac{2I_D}{\mu_n C_{ox} \frac{W}{L}} \]

\[ \Rightarrow V_{GS} = \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}} + V_T \]

Therefore:

\[ V_{in1} - V_{in2} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} \frac{W}{L}}} \]

\[ (V_{in1} - V_{in2})^2 = \frac{2}{\mu_n C_{ox} \frac{W}{L}} \left( I_{D1} + I_{D2} - 2\sqrt{I_{D1}I_{D2}} \right) \]

Squaring
**Differential Pair – Large Signal Analysis**

**Quantitative Analysis – differential input**

\[
(V_{in1} - V_{in2})^2 = \frac{2}{\mu_n C_{ox} \frac{W}{L}} (I_{D1} + I_{D2} - 2\sqrt{I_{D1}I_{D2}})
\]

\[
\therefore (V_{in1} - V_{in2})^2 = \frac{2}{\mu_n C_{ox} \frac{W}{L}} (I_{SS} - 2\sqrt{I_{D1}I_{D2}})
\]

\[
\frac{1}{4} \left( \mu_n C_{ox} \frac{W}{L} \right)^2 (V_{in1} - V_{in2})^4 + I_{SS}^2 - I_{SS} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2})^2 = 4I_{D1}I_{D2}
\]

\[
\frac{1}{4} \left( \mu_n C_{ox} \frac{W}{L} \right)^2 (V_{in1} - V_{in2})^4 + I_{SS}^2 - I_{SS} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2})^2 = (I_{D1} + I_{D2})^2 - (I_{D1} - I_{D2})^2
\]

**Squaring**

\[
\frac{1}{2} \left( \mu_n C_{ox} \frac{W}{L} \right) (V_{in1} - V_{in2})^2 - I_{SS} = -2\sqrt{I_{D1}I_{D2}}
\]
Differential Pair – Large Signal Analysis

Quantitative Analysis – differential input

\[
(I_{D1} - I_{D2})^2 = -\frac{1}{4}(\mu_n C_{ox} \frac{W}{L})^2 (V_{in1} - V_{in2})^4 + I_{SS} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2})^2
\]

\[
I_{D1} - I_{D2} = \frac{1}{2}(\mu_n C_{ox} \frac{W}{L})(V_{in1} - V_{in2}) \sqrt{\frac{4I_{SS}}{\mu_n C_{ox}} \frac{W}{L} - (V_{in1} - V_{in2})^2}
\]

Observations

- \(I_{D1} - I_{D2}\) falls to zero for \(V_{in1} = V_{in2}\) and \(|I_{D1} - I_{D2}|\) increases with increase in \(|V_{in1} - V_{in2}|\)
- Therefore, \(I_{D1} - I_{D2}\) is an odd function of \(V_{in1} - V_{in2}\)
- Its important to notice that \(I_{D1}\) and \(I_{D2}\) are even functions of their respective gate-source voltage
Differential Pair – Large Signal Analysis

Quantitative Analysis – differential input

- Equivalent $G_m$ of $M_1$ and $M_2 \rightarrow$ its effectively the slope of the characteristics

**Lets denote:**

\[ I_{D1} - I_{D2} = \Delta I_D \]
\[ V_{in1} - V_{in2} = \Delta V_{in} \]

\[ \Delta I_D = \frac{1}{2} \left( \mu_n C_{ox} \frac{W}{L} \right) \Delta V_{in} \left[ \sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} - \Delta V_{in} \right] \]
\[ \frac{\partial \Delta I_D}{\partial \Delta V_{in}} = \frac{1}{2} \left( \mu_n C_{ox} \frac{W}{L} \right) \frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - \Delta V_{in} \]

For $\Delta V_{in} = 0$:

\[ G_m = \frac{\partial \Delta I_D}{\partial \Delta V_{in}} = \sqrt{\mu_n C_{ox} \frac{W}{L}} I_{SS} \]

**Furthermore:**

\[ V_{out1} - V_{out2} = R_D \Delta I = R_D G_m \Delta V_{in} \]

\[ \therefore |A_v| = \frac{V_{out1} - V_{out2}}{\Delta V_{in}} = \sqrt{\mu_n C_{ox} \frac{W}{L}} I_{SS} R_D \]
Differential Pair – Large Signal Analysis

Quantitative Analysis – differential input

\[ G_m = \frac{1}{2} \left( \mu_n C_{ox} \frac{W}{L} \right) \left( \frac{4I_{ss} W}{\mu_n C_{ox} \frac{W}{L}} - 2\Delta V_{in}^2 \right) \]

\[ \Delta V_{in} = \sqrt{\frac{2I_{ss} W}{\mu_n C_{ox} \frac{W}{L}}} \]

G_m falls to zero for \( \Delta V_{in1} \)

\[ \Delta V_{in1} \]

\( \Delta V_{in1} \) represents the maximum differential signal a differential pair can handle.

Beyond \( |\Delta V_{in1}| \), only one transistor is ON and therefore draws all of the \( I_{ss} \)

\[ I_{D1} \]

\[ I_{D2} \]

\( \Delta V_{in} \)
Differential Pair – Large Signal Analysis

Quantitative Analysis – differential input

Linearity of a differential pair can be improved by decreasing W/L and/or increasing $I_{SS}$. 

$I_{SS}$ Constant

Reduce $\Delta V_{in1} \rightarrow$ by increasing W/L

W/L Constant

Increase $\Delta V_{in1} \rightarrow$ by increasing $I_{SS}$
MOS Differential Pair – small signal analysis

Quantitative Analysis – differential input

• From large signal analysis we achieved:

\[
|A_v| = \frac{V_{out1} - V_{out2}}{\Delta V_{in}} = g_m R_D
\]

\[
\therefore |A_v| = \frac{V_{out1} - V_{out2}}{\Delta V_{in}} = \mu_n C_{ox} \frac{W}{L} \frac{I_{SS}}{R_D}
\]

At equilibrium, this is \(g_m\)

• We apply small signals to \(V_{in1}\) and \(V_{in2}\) and assume \(M_1\) and \(M_2\) are already operating in saturation.

• How to arrive at this result using small signal analysis?
  • Two techniques
    • Superposition method
    • Half-circuit concept
MOS Differential Pair – small signal analysis

- **Method-I:** Superposition technique – the idea is to see the effect of $V_{in1}$ and $V_{in2}$ on the output and then combine to get the differential small signal voltage gain

  - First set, $V_{in2} = 0$
  - Then let us calculate $V_X/V_{in1}$

**Simplified Circuit**

- Input impedance of $M_2$ provides degeneration resistance to CS-stage of $M_1$

**Equations**

\[
\frac{V_X}{V_{in1}} = \frac{-R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}
\]

\[
\frac{V_X}{V_{in2}} = \frac{-R_{D1}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}
\]
MOS Differential Pair – small signal analysis

- **Superposition technique**
  - Now calculate $\frac{V_Y}{V_{in1}}$
  - Replace $M_1$ by its Thevenin Equivalent Circuit

This is open for small signal analysis

- Combine the expressions to calculate small signal voltage only due to $V_{in1}$

\[
\begin{align*}
\frac{V_Y}{V_{in1}} &= \frac{R_D}{1 + \frac{1}{g_{m2}} + \frac{1}{g_{m1}}} \\
\left(\frac{V_X - V_Y}{V_{in1}}\right)_{due\ to\ V_{in1}} &= -\frac{2R_D}{1 + \frac{1}{g_{m1}} + \frac{1}{g_{m2}}}
\end{align*}
\]
**MOS Differential Pair – small signal analysis**

For matched transistors:

\[
(V_X - V_Y)_{\text{due to } V_{in1}} = -g_m R_D V_{in1}
\]

- Similarly:

\[
(V_X - V_Y)_{\text{due to } V_{in2}} = g_m R_D V_{in2}
\]

- Superposition gives:

\[
A_v = \frac{(V_X - V_Y)_{\text{total}}}{V_{in2} - V_{in1}} = -g_m R_D
\]

- **Half Circuit Approach**
  - If a fully symmetric differential pair senses differential inputs (i.e., the two inputs change by equal and opposite amounts from the equilibrium condition), then the concept of half circuit can be applied.

Change this by \(\Delta V_T\)

Node is said to be ac-grounded

Potential at node P will remain unchanged

- The magnitude of differential gain is \(g_m R_D\) regardless of how the inputs are applied.
- The gain will be halved if single ended output is considered.

Potential at node P will remain unchanged

Change this by \(-\Delta V_T\)
MOS Differential Pair – small signal analysis

- Half Circuit Approach

Ac grounding of node P leads to

We can write:

\[
\frac{V_X}{V_{in1}} = -g_m R_D
\]

\[
\frac{V_Y}{-V_{in1}} = g_m R_D
\]

Therefore the differential output can be expressed as:

\[
V_X - V_Y = 2V_{in1}(-g_m R_D)
\]

Thus the small signal voltage given is:

\[
A_v = \frac{V_X - V_Y}{2V_{in1}} = -g_m R_D
\]
MOS Differential Pair – small signal analysis

- How does the gain of a differential amplifier compare with a CS stage?
  - For a given total bias current $I_{SS}$, the value of equivalent $g_m$ of a differential pair is $\frac{1}{\sqrt{2}}$ times that of $g_m$ of a single transistor biased at the $I_{SS}$ with the same dimensions. Thus the total gain is proportionally less.
  - Equivalently, for given device dimensions and load impedance, a differential pair achieves the same gain as a CS stage at the cost of twice the bias current.
- What is the advantage of differential stage then?
  - Definitely the noise suppression capability. Right?
MOS Differential Pair – small signal analysis

- How is gain affected if channel length modulation is considered?

  - Effect of $r_0$ on the gain

    → the circuit is still symmetric → the voltage at node P will be zero

    - No current through $R_{SS}$

    - $R_{SS}$ plays no role in differential gain

Finite output resistance of current source
MOS Differential Pair – small signal analysis

- The virtual ground on the source allows division of two identical CS amplifiers: → differential half circuits

\[ V_X = -g_m V_{in1} \left( R_D \parallel r_o \right) \]
\[ V_Y = g_m V_{in1} \left( R_D \parallel r_o \right) \]

\[ \Rightarrow V_X - V_Y = -g_m \left( 2V_{in1} \right) \left( R_D \parallel r_o \right) \]

\[ \therefore A_v = \frac{V_X - V_Y}{2V_{in1}} = -g_m \left( R_D \parallel r_o \right) \]
Small signal analysis – asymmetric inputs

Transform the inputs as

Simplify

Differential Inputs

Simplified Circuit

Common Mode Inputs
Small signal analysis – asymmetric inputs

If the circuit is fully symmetric and $I_{SS}$ is ideal current source, then $M_1$ and $M_2$ draws half of $I_{SS}$ and is independent of $V_{in,CM}$. The $V_X$ and $V_Y$ experience no change as $V_{in,CM}$ varies. In essence, the circuit simply amplifies the difference between $V_{in1}$ and $V_{in2}$ while eliminating the effect of $V_{in,CM}$.