Lecture – 4

- The MOSFET as a Switch and Amplifier
- MOSFET Small Signal Operation, Models, Analysis
Example – 3

- Consider the **PMOS** circuit below, find the value of unknown of resistor $R$.

\[
\begin{align*}
R_2 &= 1K \\
R &= 15.0 \text{ V} \\
R_1 &= 1K \\
R_3 &= 1K \\
V_{GS} &= -4.0 \text{ V} \\
V_T &= -2.0 \text{ V} \\
K &= 0.75 \text{ mA/V}^2
\end{align*}
\]
The MOSFET as a Switch and an Amplifier

• Consider this simple MOSFET circuit:

\[ V_i \]  
\[ K = 0.75 \text{ mA/}V^2 \]  
\[ V_o \]

\[ R_D = 1K \]

\[ 5.0 \text{ V} \]

\[ \]

Q: Oh, goody—you’re going to waste my time with another of these pointless academic problems. Why can’t you discuss a circuit that actually does something?

A: Actually, this circuit is a fundamental electronic device! To see what this circuit does, we need to determine its transfer function \( V_o = f(V_i) \).
Q: **Transfer function**! How can we determine the transfer function of a MOSFET circuit!? 

A: **Same** as with junction diodes—we determine the output \( V_o \) for each device mode, and then determine **when** (i.e., for what values of \( V_i \)) the device is in that mode!

- First, note that **regardless** of the MOSFET mode:
  \[
  V_{GS} = V_i - 0.0 = V_i \\
  V_{DS} = V_o - 0.0 = V_o
  \]

- From KVL, we can likewise conclude that:
  \[
  V_{DS} = V_o = 5.0 - I_D R_D
  \]
The MOSFET as a Switch and an Amplifier (contd.)

• Now let’s ASSUME that the MOSFET is in cutoff, thus ENFORCING $I_D = 0$.

$$V_{DS} = V_o = 5.0 - I_D R_D$$

\[ V_o = 5.0 - 0 \times (1 \times 10^3) \]

\[ \therefore V_o = 5.0 \text{ V} \]

• Now, we know that MOSFET is in cutoff when: $V_{GS} = V_i < V_T = 1.0 \text{ V}$

• Thus, we conclude that:

$$V_o = 5.0 \text{ V} \quad \text{when} \quad V_i < 1.0 \text{ V}$$
The MOSFET as a Switch and an Amplifier (contd.)

• Now, let’s ASSUME that the MOSFET is in saturation, thus ENFORCE:

\[ I_D = K(V_{GS} - V_T)^2 \]

• And thus the output voltage is:

\[ V_0 = 5.0 - I_D R_D = 5.0 - 0.75 \times 10^{-3} \times (V_i - 1.0)^2 \times 1 \times 10^3 \]

\[ \therefore V_0 = 5.0 - 0.75 \times (V_i - 1.0)^2 \]

• We know that MOSFET is in saturation when:

\[ V_{GS} = V_i > V_T = 1.0 \, V \quad \text{and} \quad V_{DS} = V_0 > V_{GS} - V_T = V_i - 1.0 \]

• The second inequality means:

\[ V_o > V_i - 1.0 \quad \therefore 5.0 - 0.75 \times (V_i - 1.0)^2 > V_i - 1.0 \]
The MOSFET as a Switch and an Amplifier (contd.)

\[ 0 > 0.75 \times (V_i - 1.0)^2 + (V_i - 1.0) - 5.0 \]

- Solving this quadratic, we find that the **only** consistent solution is:
  \[ V_i - 1.0 < 2.0 \quad \Rightarrow \quad V_i < 3.0 \]

- Thus we conclude that the MOSFET in saturation:
  \[ V_o = 5.0 - 0.75 \times (V_i - 1.0)^2 \quad \text{when} \quad 1.0 < V_i < 3.0 \text{ V} \]

- Finally, let’s **ASSUME** that the MOSFET is in **triode** mode, thus we **ENFORCE**:
  \[ I_D = K[2(V_{GS} - V_T)V_{DS} - (V_{DS})^2] \]

- And thus the output voltage is:
  \[ V_o = 5.0 - I_D R_D = 5.0 - 0.75 \times [2(V_i - 1.0)V_o - (V_o)^2] \]
Rearranging the equation, we get the quadratic form:

\[ 0.75(V_o)^2 - (1.5V_i - 0.5)V_o + 5.0 = 0 \]

The solutions of which are:

\[ V_o = \frac{(1.5V_i - 0.5) \pm \sqrt{(1.5V_i - 0.5)^2 - 15.0}}{1.5} \]

Note because of the ±, there are two possible solutions. However, to be in triode region, the MOSFET must not be in pinchoff, i.e.:

\[ V_{DS} = V_o < V_{GS} - V_T = V_i - 1.0 \]

This condition is satisfied with the smaller of the two solutions (i.e., the solution with the minus sign!):
The MOSFET as a Switch and an Amplifier (contd.)

\[
V_o = \frac{(1.5V_i - 0.5) - \sqrt{(1.5V_i - 0.5)^2 - 15.0}}{1.5}
\]

This expression provides us with the output voltage if the MOSFET is in triode mode. The question remaining is thus when (i.e., for what values of \( V_i \)) is the MOSFET in triode mode?

We could do a lot more math to find this answer, but this answer is actually quite obvious!

- Recall that we have already determined that:
  a) The MOSFET is in cutoff when \( V_i < 1.0V \).
  b) The MOSFET is in saturation when \( 1.0V < V_i < 3.0V \).
The MOSFET as a Switch and an Amplifier (contd.)

- Since there are only three modes of a MOSFET device, and since the transfer function must—well—be a function, we can conclude (correctly) that the MOSFET will be in triode region when \( V_i \) is the value of the only region that is left: \( V_i > 3.0 \) \( V \).

- Thus we can conclude that:

\[
V_o = \frac{(1.5V_i - 0.5) - \sqrt{(1.5V_i - 0.5)^2 - 15.0}}{1.5}
\]

when \( V_i > 3.0 \) \( V \)
The MOSFET as a Switch and an Amplifier (contd.)

- We now have determined the complete, continuous **transfer function** of this circuit!

\[
V_o = \begin{cases} 
5.0 V & \text{when } V_i < 1.0 V \\
5.0 - 0.75 \times (V_i - 1.0)^2 & \text{when } 1.0 < V_i < 3.0 V \\
\frac{(1.5V_i - 0.5) - \sqrt{(1.5V_i - 0.5)^2 - 15.0}}{1.5} & \text{when } V_i > 3.0 V 
\end{cases}
\]
The MOSFET as a Switch and an Amplifier (contd.)

- NMOS in Cutoff
- NMOS in Saturation
- NMOS in Triode
The MOSFET as a Switch and an Amplifier (contd.)

Q: I thought you said this circuit did something. It appears to be just as pointless as all the others!

A: To see how this circuit is useful, consider what happens when the input voltage $V_i$ is 0 V and 5 V.
The MOSFET as a Switch and an Amplifier (contd.)

- From the transfer function, we find that if $V_i = 0V$, the output voltage will be $V_o = 5V$. Likewise, if the input voltage is $V_i = 5V$, the output voltage will be small.

- Let’s summarize these results in a table:

<table>
<thead>
<tr>
<th>$V_i$</th>
<th>$V_o$</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>5.0</td>
<td>Cutoff</td>
</tr>
<tr>
<td>5.0</td>
<td>Small ($\sim0.5$)</td>
<td>Triode</td>
</tr>
</tbody>
</table>
Why, this device is not useless at all! It is clearly a:

Switch

• This circuit provides a simple example of one of the primary applications of MOSFET devices—digital circuit design. We can use MOSFETs to make digital devices such as logic gates (AND, OR, NOR, etc.), flip-flops, and digital memory.

• We typically find that, just like this circuit, when a MOSFET digital circuit is in either of its two binary states (i.e., “0” or “1”), the MOSFETs in the circuit will either be in cutoff \( I_D = 0 \) or in triode \( V_{DS} \) small) modes.

→ Cutoff and Triode are the MOSFET modes associated with digital circuits and applications!
Q: So, just what good is the MOSFET Saturation Mode??

A: Actually, we will find the MOSFET saturation mode to be extremely useful!

Sir, it appears to me that the Saturation region is just a useless MOSFET mode between cutoff and triode!
The MOSFET as a Switch and an Amplifier (contd.)

• To see why, let us take the derivative of the above circuit’s transfer function (i.e., \( \frac{dV_o}{dV_i} \)).

• We note that in cutoff and triode:
  \[ |\frac{dV_o}{dV_i}| \approx 0 \]

• While in the saturation mode:
  \[ |\frac{dV_o}{dV_i}| \gg 1 \]
Q: Oh goody. The slope of the transfer function is large when the MOSFET is in saturation. Am I supposed to be impressed by that?! How are these results even remotely important!?

A: Since in cutoff and triode $|\frac{dV_o}{dV_i}| \approx 0$, a small change in input voltage $V_i$ will result in almost no change in output voltage $V_o$.

Contrast this with the saturation region, where $|\frac{dV_o}{dV_i}| \gg 1$. This means that a small change in input voltage $V_i$ results in a large change in the output voltage $V_o$!
The MOSFET as a Switch and an Amplifier (contd.)

• To see how this is important, consider the case where the input signal has both a DC and a small-signal (AC) component:

\[ v_i(t) = V_i + v_i(t) \]

• As a result, the output voltage likewise has both a DC and small signal component:

\[ v_o(t) = V_o + v_o(t) \]

Now, let’s consider only the DC components. We can select the DC input \( V_i \) such that the MOSFET is placed in saturation. The value \( V_i \), along with the resulting DC output \( V_o \), sets a DC bias point for this circuit.

By selecting the right value of \( V_i \) we could set this DC bias point to where the transfer function slope is the greatest.
The MOSFET as a Switch and an Amplifier (contd.)

Now, say we add a small-signal $v_i$ to this input DC voltage (i.e., $v_I(t) = V_i + v_i(t)$).

This small signal simply represents a small change in the input voltage from its average (i.e., DC) value. The result is of course as small change in the output voltage— the small-signal output voltage $v_o(t)$!
Now for the **interesting** part (I bet you were wondering when I would get around to it)! The small change in the output voltage will have a much **larger** magnitude than the small change in the input!

*For example*, if the input voltage changes by 1 mV (i.e., $v_i = 1\text{mV}$), the output might change by, say, 5 mV (i.e., $v_o = 5\text{mV}$).

**Q:** Goodness! By how much would the **output** change in our example circuit? How can we **determine** the small signal **output** $v_o$??
The MOSFET as a Switch and an Amplifier (contd.)

- Determining how much the output voltage of our circuit will change when we change the input voltage by a small amount is very straightforward—we simply take the derivative of the output voltage \( v_O \) with respect to input voltage \( v_I \).

- By taking the derivative of \( v_O \) with respect to \( v_I \) (when the MOSFET is in saturation, we find:

\[
\frac{dv_O}{dv_I} = \frac{d\left(5.0 - 0.75(v_I - 1.0)^2\right)}{dv_I} = -1.50(v_I - 1.0)
\]

when \( 1.0 < v_I < 3.0 \) V

The expression describes the slope of our circuit’s transfer function (for \( 1.0 < v_I < 3.0 \) V). Note the slope with the largest magnitude occurs when \( v_I = 3.0 \) V, providing a slope of -3.0 mV/mV.
The MOSFET as a Switch and an Amplifier (contd.)

- Thus, if we DC bias this circuit with $V_i = 3.0$ V (resulting in $V_o = 2.0$ V), we find that the small signal output will be **3 times** the small signal input!

- **For example**, say that the input to our circuit is:
  $$v_I(t) = 3.0 + 0.01 \cos(\omega t) \ \text{V} \quad \text{Here:} \quad V_i = 3.0 \ \text{V} \quad v_i = 0.01 \ \text{cos}(\omega t)$$

- We would find that the **output voltage** would approximately be:
  $$v_O(t) = 2.0 - 0.03 \cos(\omega t) \ \text{V} \quad \text{Here:} \quad V_o = 2.0 \ \text{V} \quad v_o = -0.03 \ \text{cos}(\omega t)$$

  In other words, the magnitude of the small-signal output has a magnitude **three times larger** than the input magnitude.

  We say then that our signal provides **small-signal gain**—our circuit is also a **small-signal amplifier**!
I see. A **small** voltage change results in a **big** voltage change—it’s **voltage gain**!

The **MOSFET saturation** mode turns out to be—**excellent**.

- Even the simple circuit of this example is sufficient to demonstrate the **two primary applications** of MOSFET transistors—**digital** circuits and signal **amplification**.
- Whereas the important MOSFET regions for **digital** devices are **triode** and **cutoff**, MOSFETs in amplifier circuits are typically biased into the **saturation** mode!
MOSFET – Small Signal Operation

• Consider this circuit, which has both a **DC** and an AC **small signal** source. As a result, each voltage and current in the circuit has **both** a DC and small-signal component.

• If the MOSFET is in **saturation**, then the **total** drain current is:

\[
 i_D = K \left[ V_{GS} + v_{gs} - V_T \right]^2 \\
\Rightarrow i_D = K \left[ V_{GS} - V_T \right]^2 + 2K \left[ V_{GS} - V_T \right] v_{gs} + K v_{gs}^2
\]

**Very Small If:** \( v_{gs} \ll 2 [V_{GS} - V_T] \)

We call this equation the **small-signal** condition.
MOSFET – Small Signal Operation (contd.)

Now,

\[ i_d = 2K(V_{GS} - V_T) v_{gs} \]

\[ \Rightarrow \frac{i_d}{v_{gs}} = 2K(V_{GS} - V_T) \]

Alternatively,

\[ \Rightarrow g_m = 2KV_{OV} \]

- The small-signal parameter \( g_m \) can likewise be derived from a small-signal analysis of the drain current:

\[ i_d = \frac{di_D}{dV_{GS}} \bigg|_{v_{GS}=V_{GS}} \]

\[ \left( v_{gs} \right) = 2K \left[ v_{GS} - V_T \right] \bigg|_{v_{GS}=V_{GS}} \left( v_{gs} \right) \]

\[ i_d = 2K \left[ V_{GS} - V_T \right] \left( v_{gs} \right) \]

\[ i_d = g_m \left( v_{gs} \right) \]

Physical meaning of the \( g_m \)

→ formal definition
The MOSFET transconductance relates a small change in $v_{GS}$ to a small change in drain current $i_D$. This change is completely dependent on the DC bias point of the MOSFET, $V_{GS}$ and $I_D$. 
MOSFET – Small Signal Operation (contd.)

- The total instantaneous drain voltage $v_D$ is given by:
  \[ v_{DS} = v_D = V_{DD} - i_D R_D \]
- Under small signal condition it changes to:
  \[ v_D = V_{DD} - R_D (I_D + i_d) \]
  \[
  \Rightarrow V_D + v_d = V_{DD} - I_D R_D - i_d R_D
  \]
  Signal component of drain voltage ($v_d$)

\[
  v_d = -i_d R_D = -g_m v_{gs} R_D
  \]
  \[
  \Rightarrow A_v = \frac{v_d}{v_{gs}} = -g_m R_D
  \]

Thus, if $g_m R_D \gg 1$, we have small-signal **voltage gain**.
It indicates that $v_d$ is $180^\circ$ out of phase with respect to $v_{gs}$.

The input has been assumed very small as compared to overdrive voltage [$v_{gs} << 2(V_{GS} - V_T)$]

For saturation:  

$V_{D_{min}} \geq V_{G_{max}} - V_T$  

$V_{D_{max}} \leq V_{DD}$