CSE622/622A IQC: Homework 2

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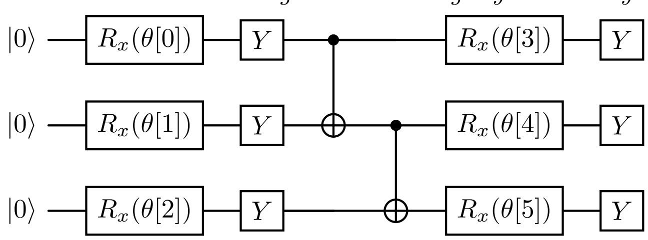
Use bra-ket notation. You may need to do linear-algebraic calculations; either do them by hand (submit calculations with the homework) or use numpy (submit python or ipynb/Jupyter notebook file as attachment on Classroom).

Q1 [7+4=11 points] Ansatz are parameterized circuits that consists of repetition (*aka.* layers) of blocks of circuits like the one below. Ansatz form the building blocks of quantum optimization and machine learning algorithms.

(a) What is the output statevector of the following ansatz if we use $\theta = [\pi/2, \pi/2, \pi/2, \pi/3, \pi/3, \pi/3]$. *Warning: This is a messy exercise.*

(b) Suppose I measure the first and the third qubit using the X and Z observables. What would be the expected outcome? *Warning: This is a slightly less messy if you do it correctly.*



Q2 [3 points]

1. Let $U_1 = |0\rangle\langle 0| \otimes U$ is unitary, where U is unitary. Is U_1 unitary? Prove or disprove.
2. Prove that $U_2 = |0\rangle\langle 0| \otimes U + |1\rangle\langle 1| \otimes V$ is unitary, where U and V are unitary.
3. Prove that $|0\rangle\langle 0| \otimes U + |1\rangle\langle 1| \otimes V = (|0\rangle\langle 0| \otimes U + |1\rangle\langle 1| \otimes I)\,(|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes V)$, where U and V are unitary.

Q3 [1+3+3+2=9 points] (a) Write down the amplitude encoding for this vector $V = [2, -3, -1, 2]$. Denote the state by $|\psi\rangle$. Normalize it first.

(b) Draw a quantum circuit (on paper, or attach a digital image) to load $|\psi\rangle$ from $|00\rangle$ (upto a global phase). Your circuit can use arbitrary single-qubit gates and arbitrary 2-qubit control gates. The single-qubit gates must come from the set $\{X, Y, Z, H, S, R_X, R_Y, R_Z\}$, and the control-gates must be a controlled-version of one of the single-qubit gates from that set. If you are using parameterized rotation gates, then you have to specify their parameters.

(c) Write a program using Qiskit to load V (you can ignore any global phase difference). You can use Q2(c) to decompose a complex controlled-gate into simpler forms. You can use the `gate.control()` method in Qiskit to create controlled-version of any gate that operates as Q2.(b) but none of the other methods to create a controlled-gate is acceptable. Use any of the methods listed on `https://quantum.cloud.ibm.com/docs/en/api/qiskit/circuit_library` to implement single-qubit gates.

(d) Obtain the output state of your quantum circuit on $|00\rangle$ using the `Statevector` class [1]. Multiply it by the normalization factor from (a) and print it.

[1] `https://quantum.cloud.ibm.com/docs/en/api/qiskit/qiskit.quantum_info.Statevector`. `sv = Statevector(circuit)` returns the statevector from the circuit on the all-zero state.

Submit the PDF of the notebook or the python file for (c) and (d).

Q4 [1+1=2 points] We discussed the SWAP operator in class whose action on any two-qubit tensor-product state is the following: $|\psi\rangle \otimes |\phi\rangle \rightarrow |\phi\rangle \otimes |\psi\rangle$.

(a) Write down an expression for the 3-qubit c-SWAP (controlled-SWAP) operator as a sum of outer-product of standard basis states and their adjoints. This operator is also known as the Fredkin operator.

(b) The GHZ state is the following 3-qubit state: $\frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|111\rangle$. Observe that the Fredkin operator is also a Hermitian operator, and hence, an observable. What is the expected value of the measurement of the GHZ state using the Fredkin observable. *Hint: We discussed an elegant approach in an earlier lecture to compute the expected outcome of an observable on any state.*