CSE622/622A: IQC W26 Quiz

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Duration: 45 Minutes Total marks: 40

You should not use vectors or matrices in the final answer that you submit for grading. You can use them for rough calculations.

1. **(5 marks)** Let $|\psi\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{i}{2}|1\rangle$. Describe what happens when $|\psi\rangle$ is measured in the $\{|+i\rangle, |-i\rangle\}$ basis.

2. **(3+5=8 marks)** This question is about the Bell state $|\beta_{11}\rangle$, also written as $|\Psi^-\rangle$.
   (a) Construct a quantum circuit using only Hadamard and CNOT gates that transforms $|11\rangle$ into $|\beta_{11}\rangle$. Show the state of the qubits after each gate.
   (b) Describe what happens when the first qubit of $|\beta_{11}\rangle$ is measured in the Hadamard basis.

3. **(3+5=8 marks)** We can write any unitary operator as

$$U = e^{i\alpha} R_{\vec{n}}(\theta),$$

   where $R_{\vec{n}}(\theta)$ is a rotation about the unit vector $\vec{n}$ in the Bloch sphere by angle θ, and $e^{i\alpha}$ is a global phase.
   (a) What are α, $\vec{n}$, and θ for $S = |0\rangle\langle 0| + \iota |1\rangle\langle 1|$, without ignoring global phase? $\vec{n}$ should be a Bloch vector (give the 3D coordinates), and the other two are scalar angles from $[0, 2\pi]$.
   (b) Verify that $e^{i\alpha} R_{\vec{n}}(\theta)$ is the same operator as S.

4. **(8 marks)** Draw a quantum circuit to prepare the state $e^{i\pi/4}(\cos\pi/8\,|0\rangle + e^{i\pi/6}\sin\pi/8\,|1\rangle)$ from $|0\rangle$. Trace the states of the qubit after each gate.

5. **(3+3+5=11 marks)** This question is about implementing one type of controlled-$R_Y(\theta)$ operator using CNOT and single-qubit gates. Let θ be some known angle.
   (a) Prove that $XY = -YX$.
   (b) Prove that $XR_Y(-\theta)X = R_Y(\theta)$.
   (c) Use the answers to the above questions design the following 2-qubit conditional operator.

$$|0\rangle\langle 0| \otimes R_Y(-\theta) + |1\rangle\langle 1| \otimes R_Y(\theta)$$

   You should use only CNOT and single-qubit gates for implementation. Explain why your construction is correct.

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