HW5 (25 points)

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Problem 1. *(Points :6)* Consider a quantum circuit C_1 for computing QFT_{13} over basis states $|0\rangle \ldots |12\rangle$ and C_2 for computing QFT_{19} over basis states $|0\rangle \ldots |18\rangle$ (observe that 13 and 19 are prime to each other).

Design a quantum circuit to implement QFT_{247} over a $13 * 19 = 247$ dimensional space using C_1 and C_2. You may use additional ancillæ as required, usual single qubits gates including (conditional) R_k gates of the form $|x\rangle \to e^{2\pi i x/k} |x\rangle$ (and controlled-R_k gates too). Draw the circuit and explain the idea. Show all relevant analysis.
Hint: Represent a standard basis state of the 247-dimensional space, say $|x\rangle$, as $|x_1\rangle |x_2\rangle$ such that $x = 13 * x_1 + x_2$.

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Problem 2. *(Points :1+2+3+2=8)* Consider the function $f(a) = 7a \pmod{10}$.

1. What is the period r of f?

2. Show how Shor's algorithm finds the period of f, using a Fourier transform over 128 elements, i.e., 7 qubits. (a) First, draw the quantum circuit for quantum part of the algorithm. (b) Then, write down all intermediate superpositions of the algorithm for this case (don't just copy the general expressions from the notes, but instantiate them with actual numbers as much as possible, incl. with the value of the period found in (1)). (c) Finally, explain the classical post-processing by choosing some basis state as the observed output. You may assume you're lucky, meaning the first run of the algorithm already gives a measurement outcome that, after classical post-processing, is a co-prime multiple of $\frac{1}{r}$.

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Problem 5. *(Points :1+1+1+2+2+4=11)* Solve all parts of Ch-4 Q(4) of `https://homepages.cwi.nl/~rdewolf/qcnotes.pdf`.

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