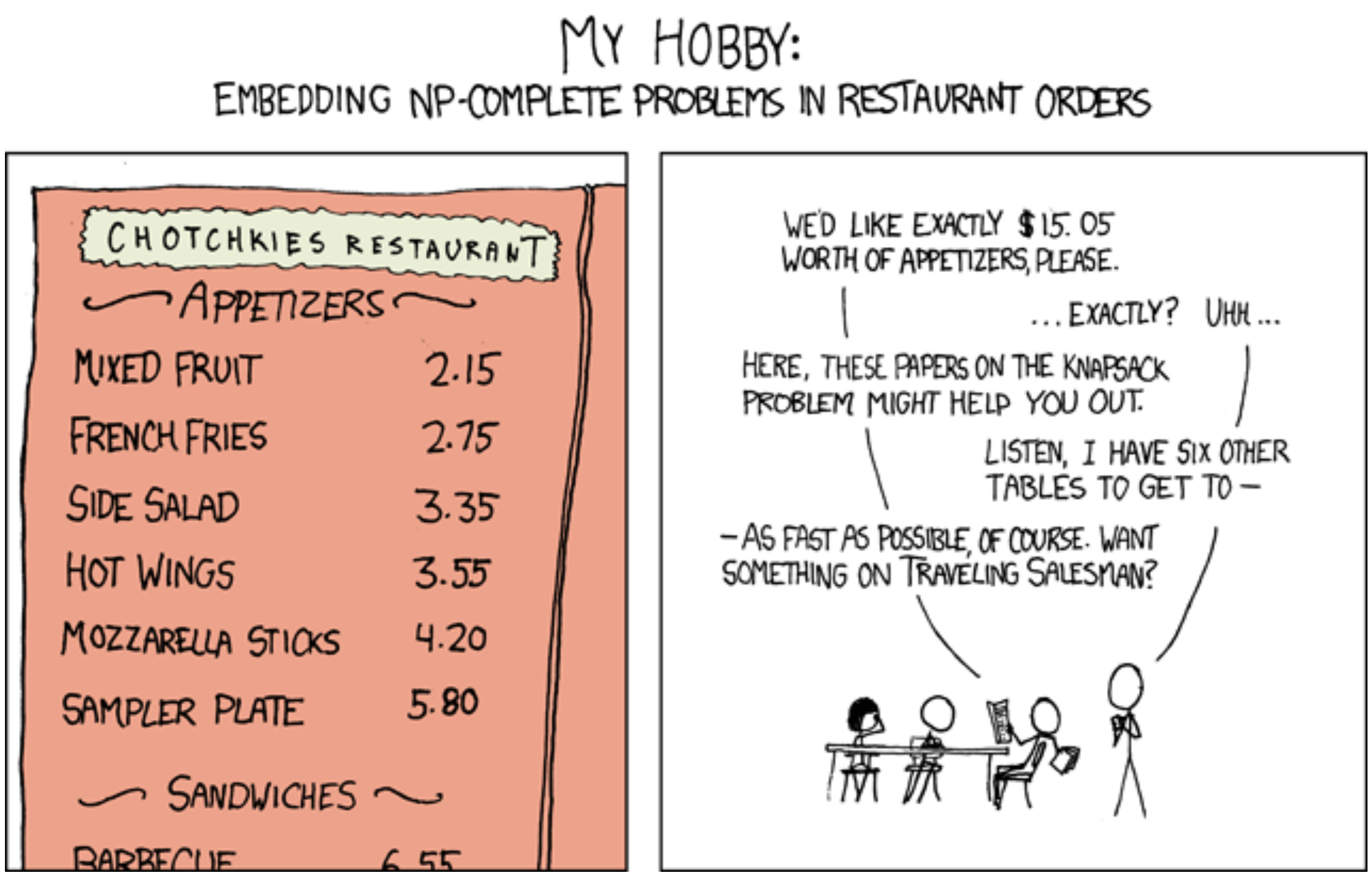
CSE322 Theory of Comput. (L26)

NP(k) = NTIME(n^k)

NP = P(0) U P(1) U P(2) U ...



$\leq_p^m$: polytime manyone reduction

$|x|^{K_1}$

def AlgoA(x):

1. y = Red(x)

2. do what AlgoB(y) does

$|y|^{K_2}$

1. If A <= B and B <= C, then A <= C

Red n^{K_1} AlgoB n^{K_2}

2. If A <= B and B is in P, then A is in P
3. If A <= B and A is not in P, then B is not in P

Proof by contradiction.

Total: $|x|^{K_1} + |y|^{K_2}$

$\leq |x|^{K_1} + |x|^{K_1 K_2}$ ($\because |y| \leq |x|^{K_1}$)

$= 2|x|^{K_1 K_2}$

Tutorial: question on $\leq_p^m$.

$\leqslant_m$: poly-time many-one redn.

$\exists$NDTM decider M for H running in poly time

NP-complete

L is NP-hard if for EVERY H in NP, H <=mp L.

L is NP-complete if (a) L is in NP, and (b) L is NP-hard.

Lemma: Suppose H is NP-complete. If H <=mp L, then L is NP-hard.

$H \in NP$
H is NPH $\Rightarrow$ $\boxed{\forall H' \in NP,}$ $H' \leq^p_m H \leq^p_m L \Rightarrow \boxed{H' \leq^p_m L}$

How to prove that L is NP-complete?

1. Prove L is in NP

2. Prove L is NP-hard by reducing FROM some NP-complete H.

First NP-complete problem: SAT

NP-completeness

$B \in NP$
$\forall H \in NP, H \le^p_m B \Rightarrow H \in P$

Q : If B is NP-complete and B is in P then ... $P = NP$

Q : If B is NP-complete and B is not in P then ... $P \neq NP$
(then no NP-complete L is in P)

Q : If B is NP-complete, C is in NP and B <= C then ...

C is NP-hard

C is NP-complete

Q : If B is NP-complete, then B-compl is ...

$\forall H \in NP, H \le B$

$\overline{B} \in coNP$

$\forall H \in coNP, \overline{H} \in NP, \overline{H} \le B \Rightarrow H \le \overline{B}$

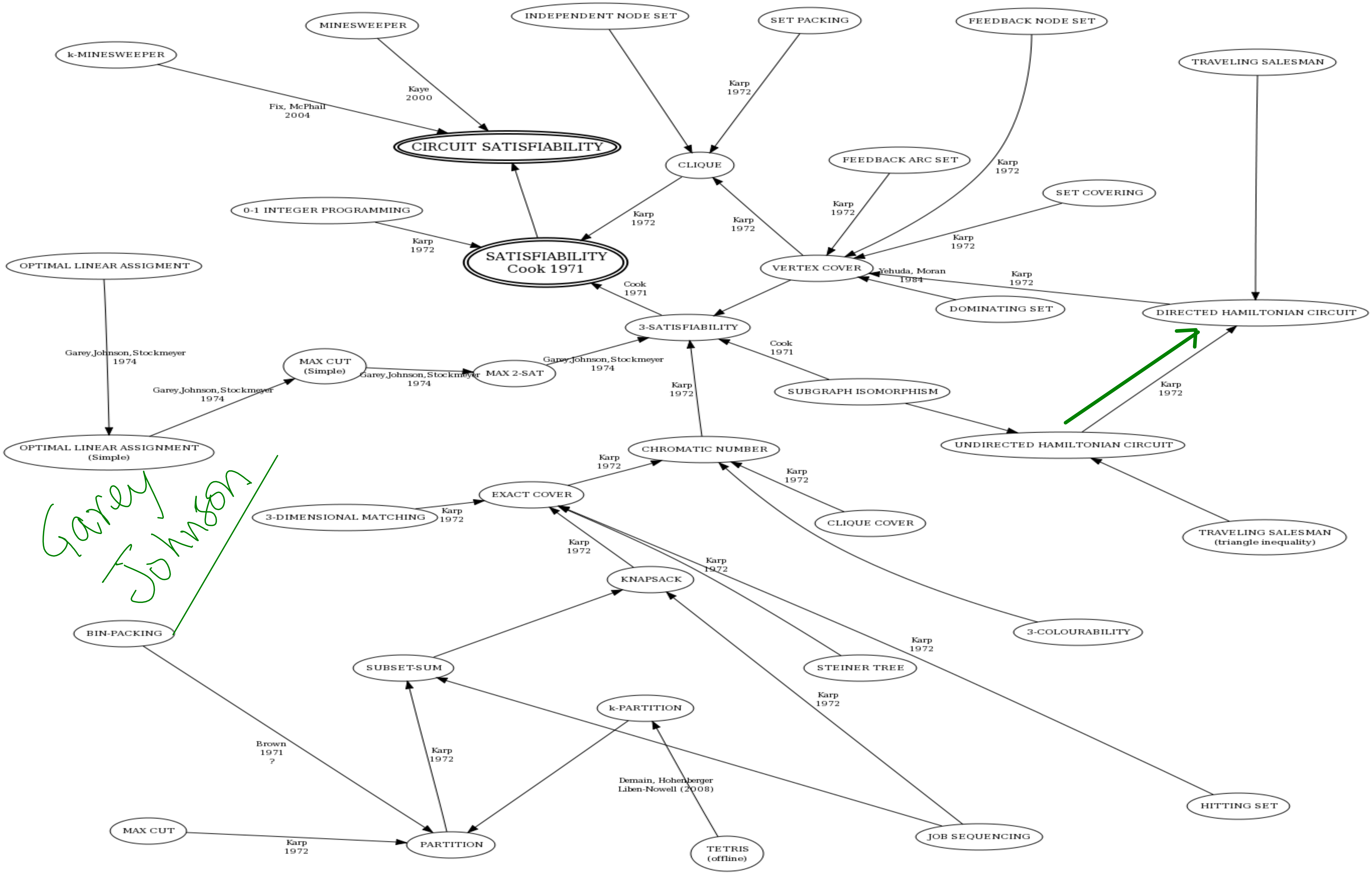
$\therefore \overline{B}$ is coNP-hard

To show L is NP-complete ...

(i) Show L is in NP ...

(ii) Choose any NP-complete language MYFAVNP

(iii) Show that MYFAVNP <= L by a polytime many-one red.



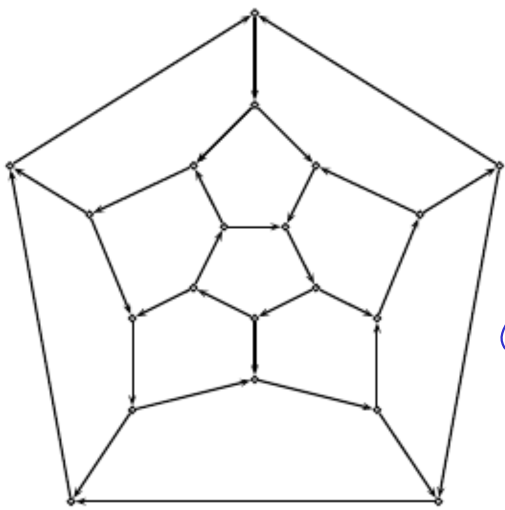
Prove that anyone of these is in P and get A+

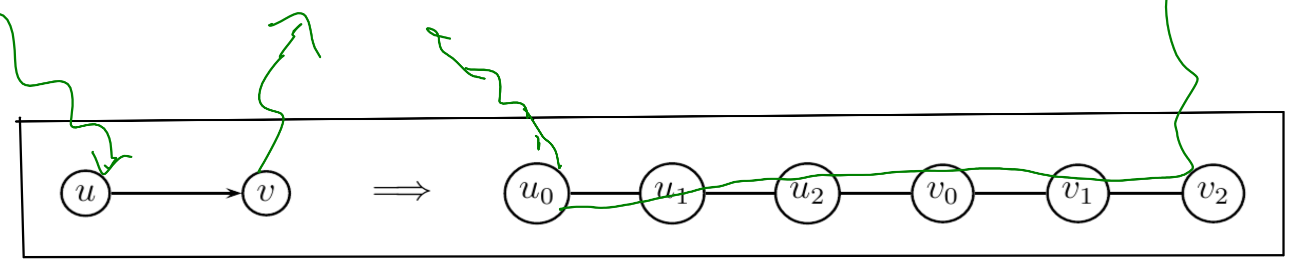
(reductions between specific NP-complete problems (Ch 7.5) are not in syllabus)

Undirected HC is NP-complete.

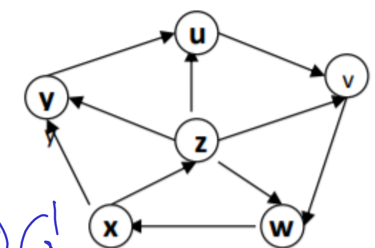
- UHC ∈ NP. ✓
- UHC is NP-hard.

(using the fact that Directed HC is NP-complete)





G has a HamCycle iff G' has a HamCycle.



G = (V, E)

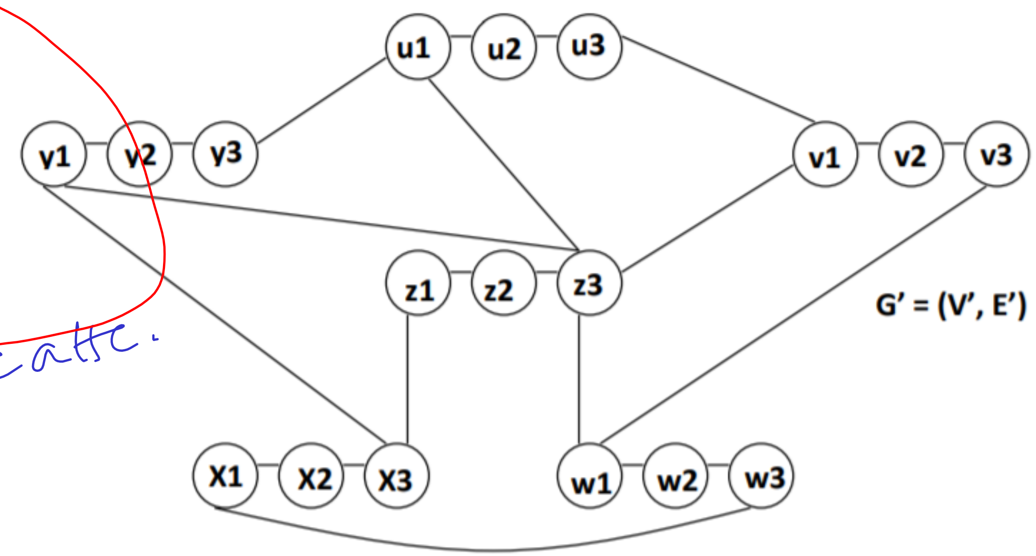
def Red(G directed): // output undirected G'

G' = remove directions

return G'

⇒ If G has a HC, then G' has a HC

⇒ Even if G doesn't have a HC, G' may have a HC.



G' = (V', E')

P vs NP ???

How to show P != NP ?

How to show P = NP ?

$P \underset{?}{\subseteq} NP \underset{?}{\subseteq} EXP$

It is known that P is a proper subset of EXP

7.37 Suppose P = NP.

3COL ∈ P

decider D for 3COL

Show how to compute size of the largest clique in polynomial time.

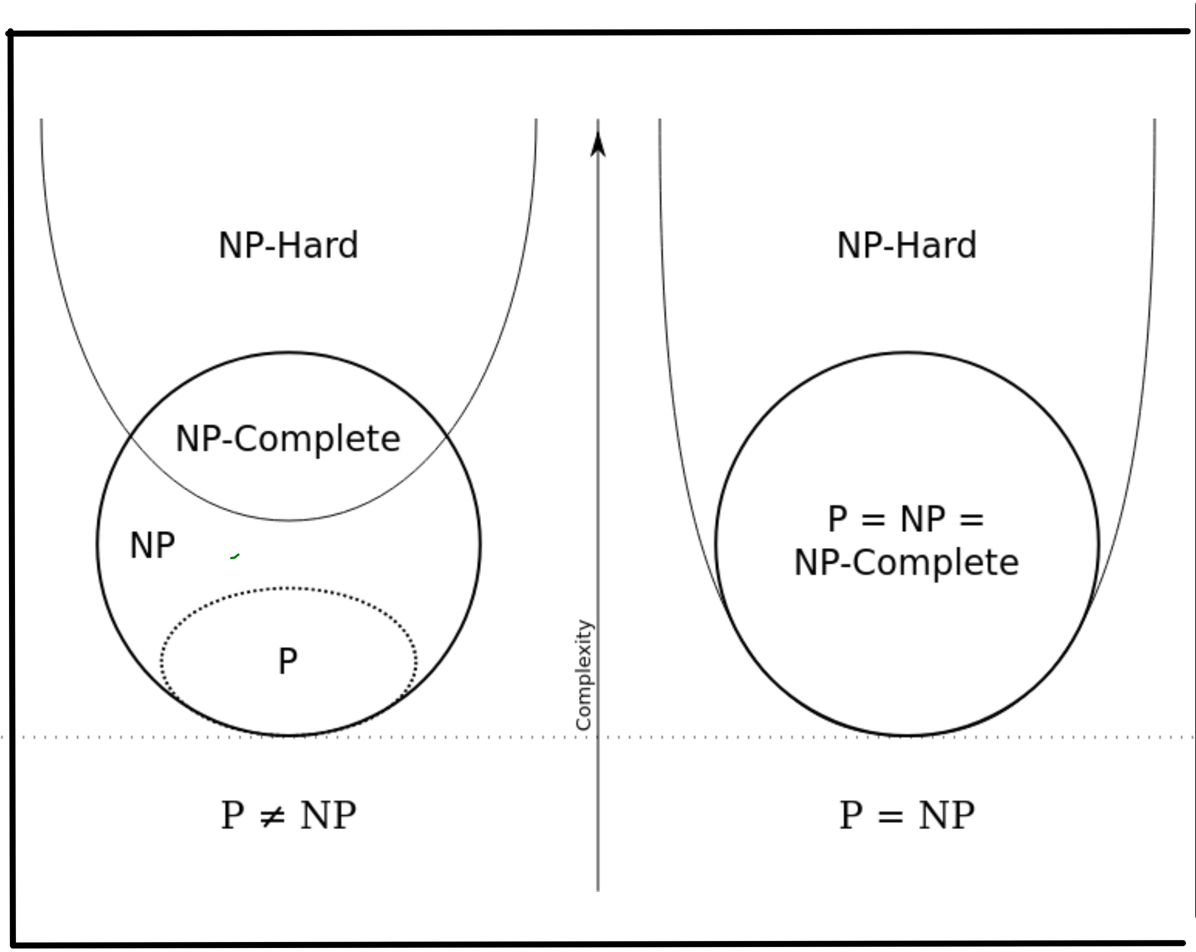
Twbr

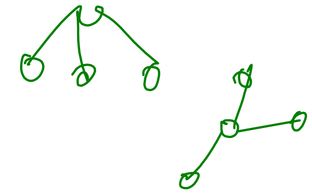
7.6 Show that P is closed under union, concatenation

Show that NP is closed under Kleene star.

Can you solve them ? (and get A+)

NP-intermediate





Not in syllabus

1. Given t, can the numbers $1 \ldots n$ be put in t boxes s.t. no box has any triple s.t. $x+y=z$.
2. Given two graphs, are they isomorphic?
3. ~~Given a number, is it prime?~~
4. Given integers n, $t1$ and $t2$, does n have a factor between $t1,t2$?
5. Turnpike problem (reconstruct a point set from the pairwise distances)
6. Given two binary trees T1 and T2, and an integer t, can T1 be changed to T2 using at most t rotations?

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