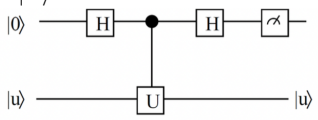
Due:
Date: March 17, 2023

HW3 (25 points)

Problem [6 points]. Consider a quantum circuit C_1 for computing QFT over basis states $|0\rangle \dots |12\rangle$ and C_2 for computing QFT over basis states $|0\rangle \dots |18\rangle$ (observe that 13 and 19 are prime to each other). Show how to obtain a circuit for computing QFT over a $13 * 19 = 247$ dimensional space H_{247} using C_1 and C_2. You may use additional ancillæ as required, usual single qubits gates including (conditional) R_k gates of the form $|x\rangle \rightarrow e^{2\pi i x/k} |x\rangle$ (and controlled-R_k gates too). Draw the circuit and explain the idea. Show all relevant analysis.
Hint: Represent a standard basis in H_{247}, say $|x\rangle$, as $|x_1\rangle |x_2\rangle$ such that $x = 13 * x_1 + x_2$.

Problem [2+2+2+1+3=10 points]. (a) Derive the output state of the circuit given below where $|u\rangle$ is a state such that $U |u\rangle = e^{2\pi i\alpha} |u\rangle$.



(b) Write down the density matrix for the 1st qubit.

(c) Suppose we measure using the Z observable. What is the expected value $\langle Z \rangle$ of the measurement; recall that Z observable is equivalent to a random variable with values -1 and 1 (the eigenvalues of Z)? You should be able to relate $\langle Z \rangle$ to α.

(d) Explain how to obtain an estimate of α? (e) $|1\rangle$ is an eigenvector of the T-gate with eigenvalue $2\pi i\alpha$ where $\alpha = 0.001$ (in binary). Implement the above idea to estimate α upto 3 bits from multiple measurements of Z using a simulator. Plot the value of the estimate against the number of measurements, where the latter is varied as 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.

Problem [2+2+2+2+1=9 points]. Finish the tasks specified in Steps A,B,C,D,E of Part 1.2 of

https://qiskit.org/textbook/ch-labs/Lab04_IterativePhaseEstimation.html.

You will be estimating the eigenvalue corresponding to the eigenvector $|1\rangle$ of the T gate using the iterative phase estimation method. You shall also use the *reset* operation of QISKIT. You will submit the codes for the steps and the plot produced in Step E.
