Test-2 Solution

(1) We note that attaching transmission li
of char. impedance Z_0 to each port of our
ckt forms a continuous transmission
line of char. impedance Z_0.

Thus, the voltage along the TL has the form:

$$V(z) = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$$

let us define: $z_{1p} = -l$, $z_{2p} = 0$

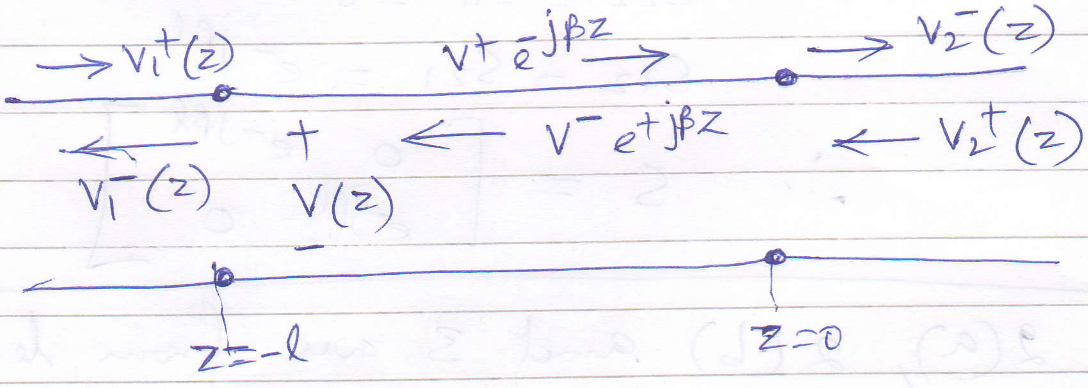
we can then express:

$$V_1^+(z) = V^+ e^{-j\beta z} \quad (z \le -l)$$

$$V_1^-(z) = V^- e^{+j\beta z} \quad (z \le -l)$$

$$V_2^+(z) = V^- e^{+j\beta z} \quad (z \ge 0)$$

$$V_2^-(z) = V^+ e^{-j\beta z} \quad (z \ge 0)$$



let us say that the TL is terminated into
a matched load at port-2. We know that

in such a case the -z wave must be zero ($V^- = 0$), and so: $\boxed{V(z) = V^+ e^{-j\beta z}}$

Therefore:

$$V_1^+(z) = V^+ e^{-j\beta z} \qquad (z \leq -l)$$
$$V_1^-(z) = 0 \qquad (z \leq -l)$$
$$V_2^+(z) = 0 \qquad (z \geq 0)$$
$$V_2^-(z) = V^+ e^{\cancel{+}-j\beta z} \qquad (z \geq 0)$$

Now, because Port-2 is terminated into match:

$$S_{11} = \frac{V_1^-(z = z_{1p})}{V_1^+(z = z_{1p})} = \frac{0}{V^+ e^{+j\beta l}} = 0$$

$$S_{21} = \frac{V_2^-(z = z_{2p})}{V_1^+(z = z_{1p})} = \frac{V^+ e^{-j\beta(0)}}{V^+ e^{-j\beta(-l)}} = e^{-j\beta l}$$

From symmetry of the structure:

$$S_{22} = S_{11} = 0$$
$$S_{12} = S_{21} = e^{-j\beta l}$$

$$\therefore \quad S = \begin{bmatrix} 0 & e^{-j\beta l} \\ e^{-j\beta l} & 0 \end{bmatrix}$$

2(a), 2(b) and 3 are from lecture slides