❑ **Single stub impedance matching**
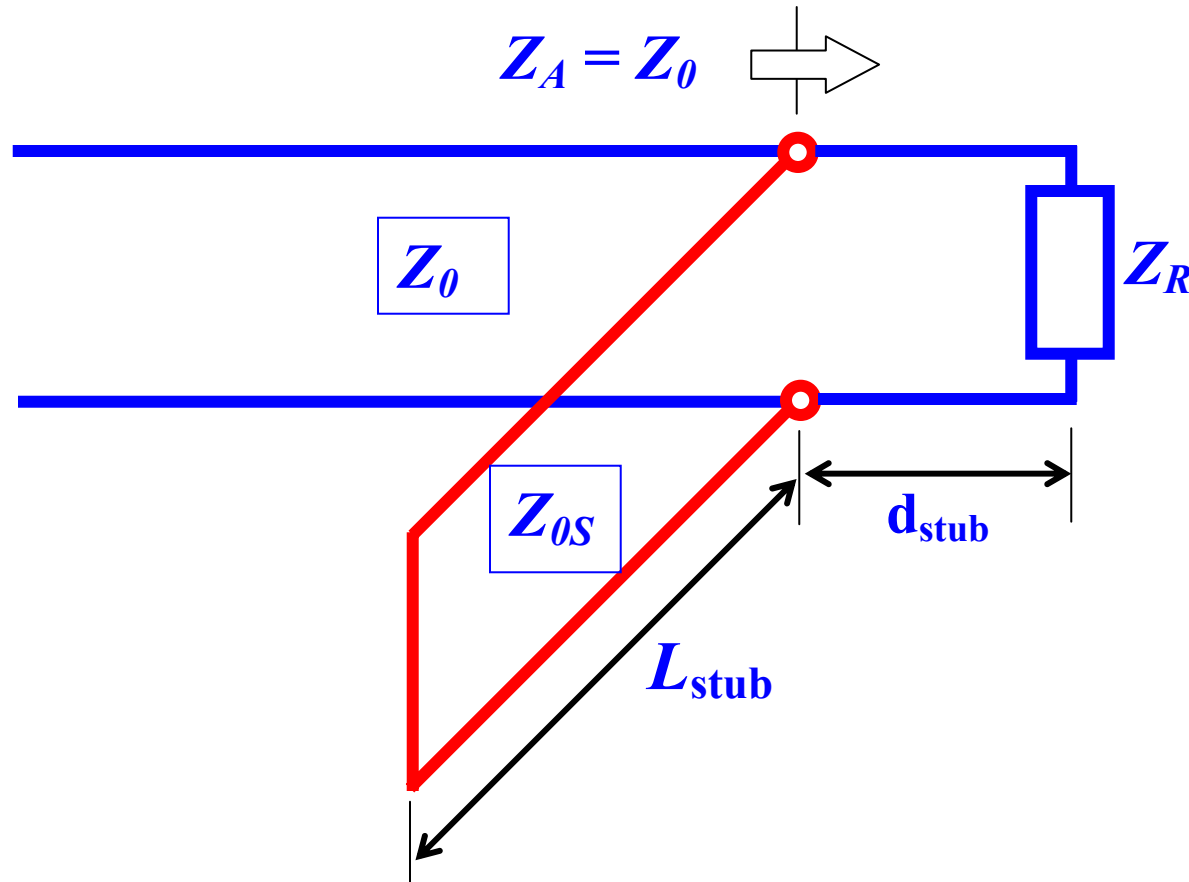
Impedance matching can be achieved by inserting another transmission line (stub) as shown in the diagram below

$$Z_A = Z_0$$

Z_0

Z_R

Z_{0S}

d_{stub}

L_{stub}

There are two design parameters for <mark>single stub matching</mark>:

❑   **The location of the stub with reference to the load d_{stub}**

❑   **The length of the stub line L_{stub}**

Any load impedance can be matched to the line by using single stub technique. <mark>The drawback of this approach is that if the load is changed, the location of insertion may have to be moved</mark>.

The transmission line realizing the stub is normally terminated by a short or by an open circuit. In many cases it is also convenient to select the same characteristic impedance used for the main line, although this is not necessary. The choice of open or shorted stub may depend in practice on a number of factors. A short circuited stub is less prone to leakage of electromagnetic radiation and is somewhat easier to realize. On the other hand, an open circuited stub may be more practical for certain types of transmission lines, for example microstrips where one would have to drill the insulating substrate to short circuit the two conductors of the line.

Since the circuit is based on insertion of a parallel stub, it is more convenient to work with admittances, rather than impedances.
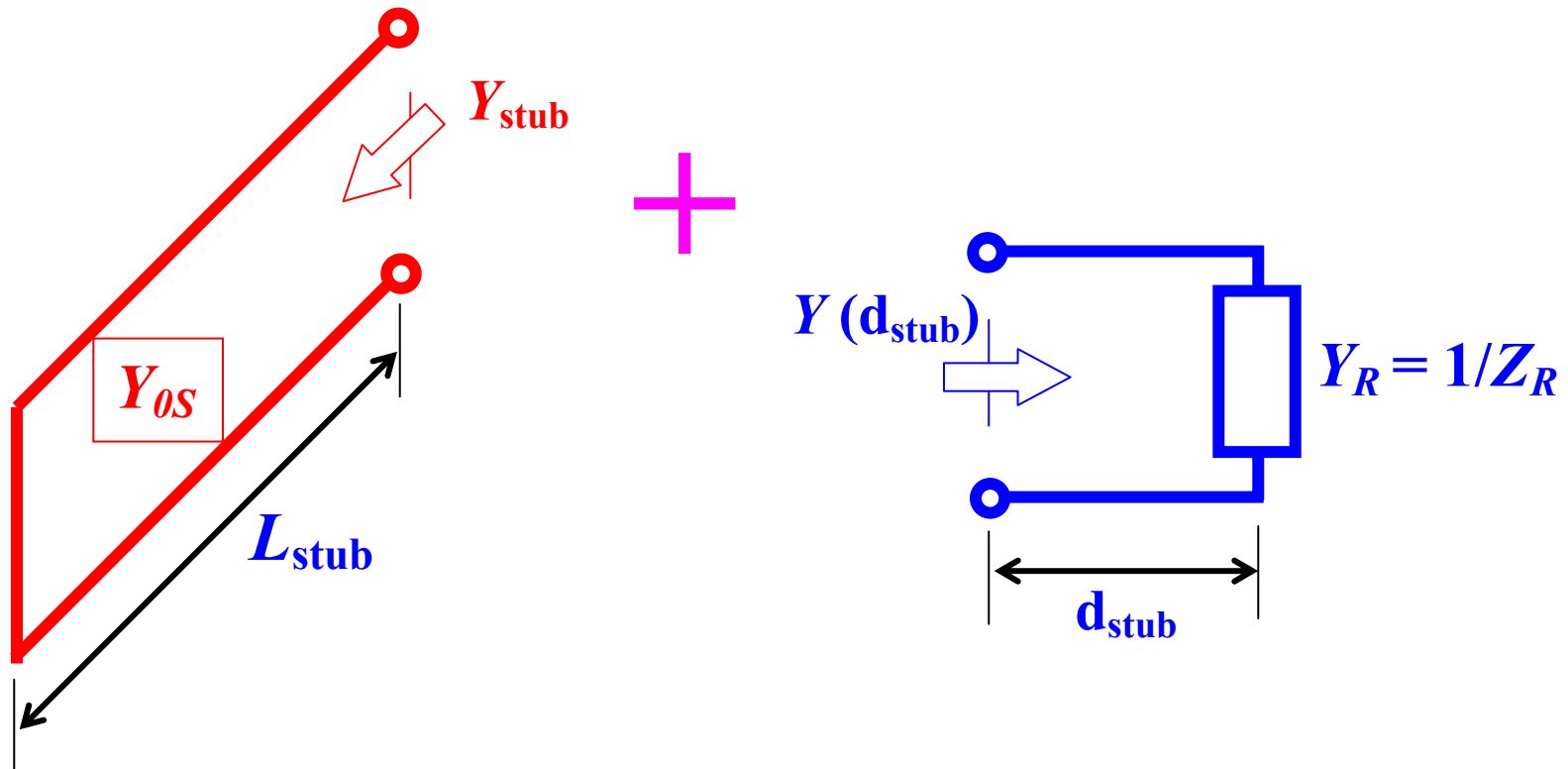
$$Y_A = Y_0 \Rightarrow$$

$$Y_0 = 1/Z_0$$

$$Y_R = 1/Z_R$$

$$Y_{0S}$$

$$d_{stub}$$

$$L_{stub}$$
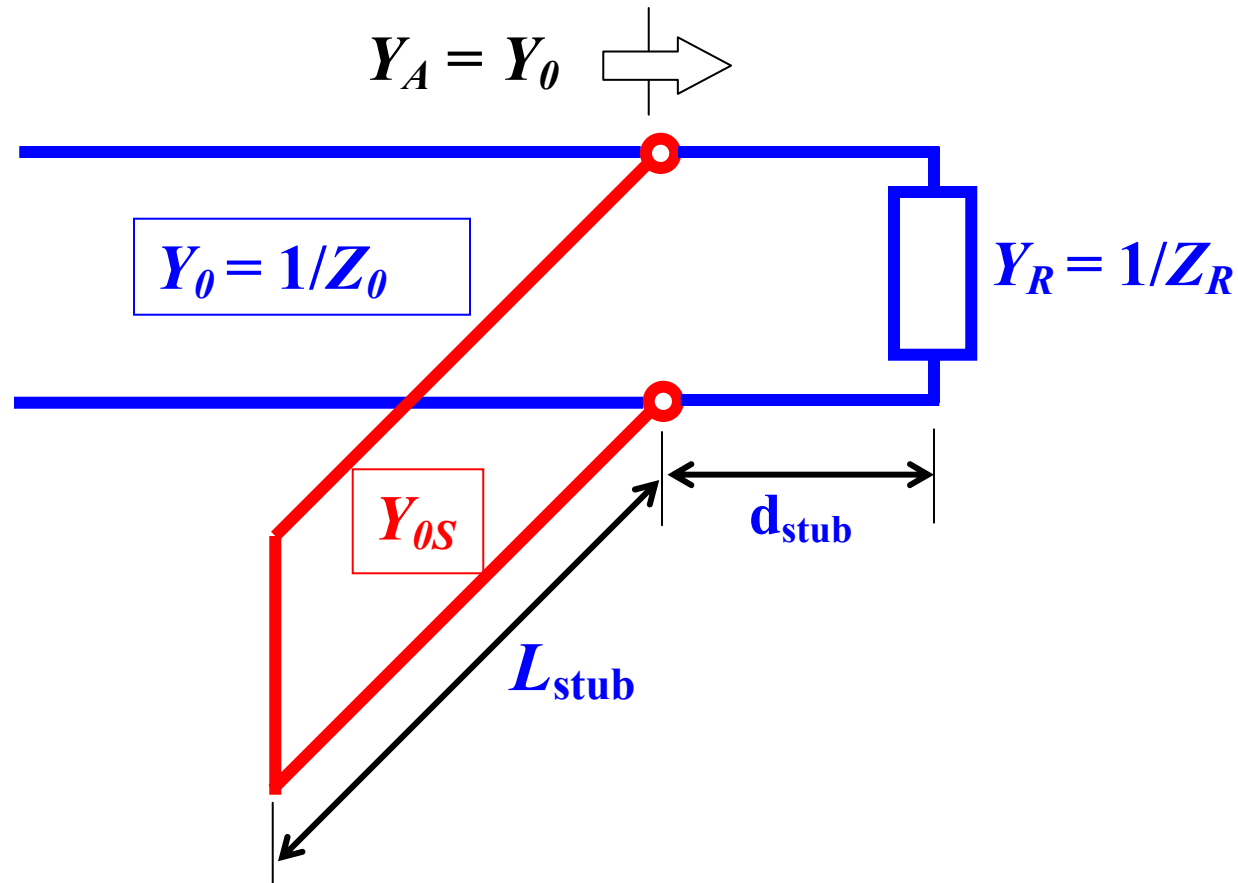
For proper impedance match:

$$Y_A = Y_{\text{stub}} + Y(\mathbf{d}_{\text{stub}}) = Y_0 = \frac{1}{Z_0}$$

| Input admittance of the stub line | Line admittance at location $\mathbf{d}_{\text{stub}}$ before the stub is applied |

Y_{stub}

Y_{0S}

L_{stub}

+

$Y(\mathbf{d}_{\text{stub}})$

$Y_R = 1/Z_R$

$\mathbf{d}_{\text{stub}}$

In order to complete the design, we have to find an appropriate **location** for the stub. Note that the **input admittance** of a **stub** is always **imaginary** (inductance if negative, or capacitance if positive)

$$Y_{stub} = jB_{stub}$$

A stub should be placed at a location where the line admittance has real part equal to Y_0

$$Y(d_{stub}) = Y_0 + jB(d_{stub})$$

For matching, we need to have

$$B_{stub} = -B(d_{stub})$$

Depending on the length of the transmission line, there may be a number of possible locations where a stub can be inserted for impedance matching. It is very convenient to analyze the possible solutions on a Smith chart.
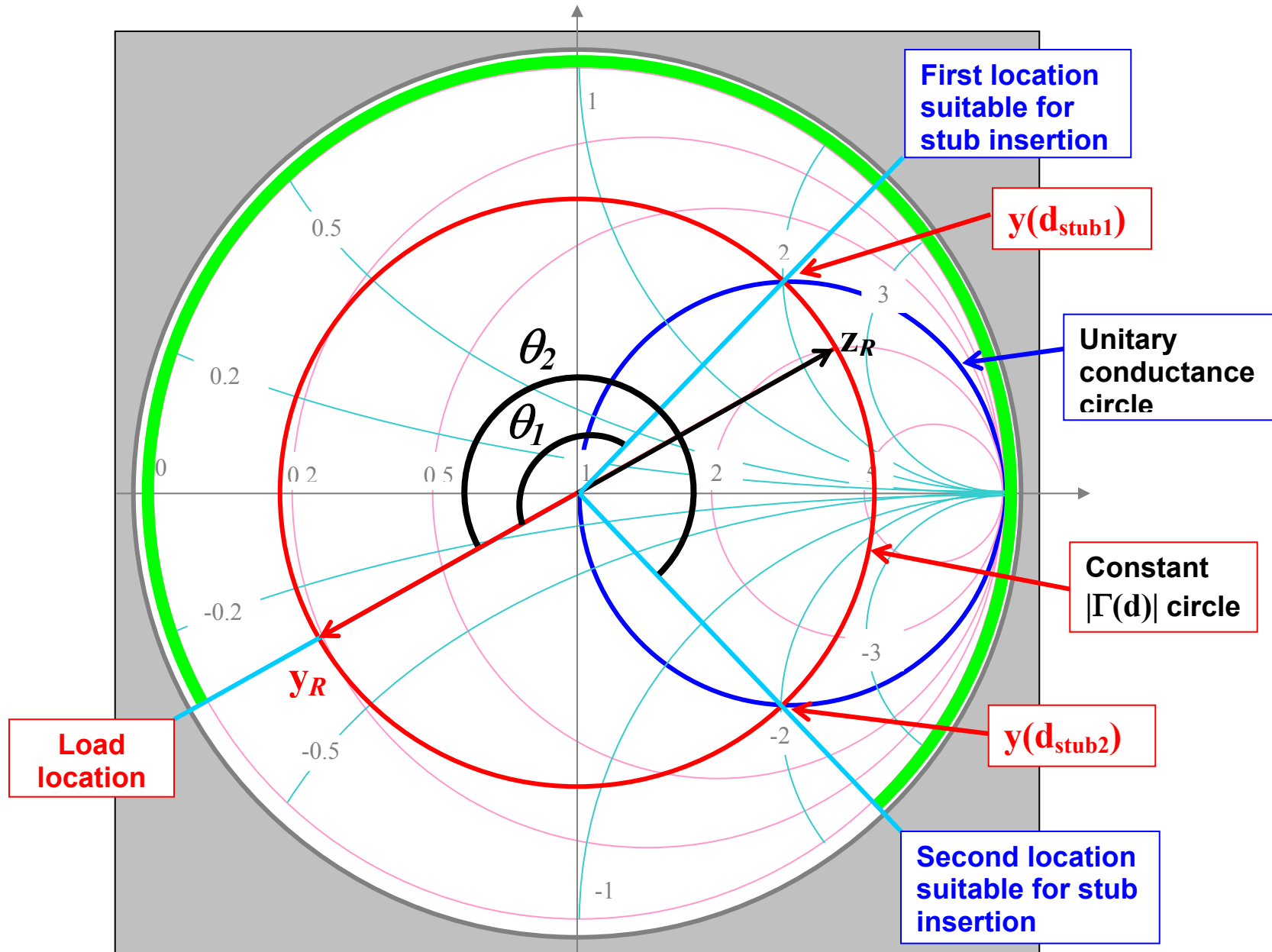
The **red arrow** on the example indicates the **load admittance**. This provides on the "admittance chart" the physical reference for the load location on the transmission line. Notice that in this case the load admittance falls outside the unitary conductance circle. If one moves **from load to generator** on the line, the corresponding chart location moves from the reference point, in **clockwise** motion, according to an angle θ (indicated by the light green arc)

$$\theta = 2\beta d = \frac{4\pi}{\lambda} d$$

The value of the **admittance** rides on the **red circle** which corresponds to constant magnitude of the line reflection coefficient, $|\Gamma(d)| = |\Gamma_R|$, imposed by the load.

Every circle of constant $|\Gamma(d)|$ intersects the circle **Re { y } = 1** (**unitary normalized conductance**), in correspondence of two points. Within the **first revolution**, the two intersections provide the locations **closest to the load** for possible stub insertion.

The **first solution** corresponds to an **admittance** value with **positive** imaginary part, in the **upper portion** of the chart

Line Admittance - Actual: $\qquad Y\left(d_{stub1}\right) = Y_0 + j\,B\left(d_{stub1}\right)$

$\qquad\qquad\qquad$ **Normalized:** $\qquad y\left(d_{stub1}\right) = 1 + j\,b\left(d_{stub1}\right)$

Stub Location: $\quad d_{stub1} = \dfrac{\theta_1}{4\pi}\,\lambda$

Stub Admittance - Actual: $\qquad -j\,B\left(d_{stub1}\right)$

$\qquad\qquad\qquad$ **Normalized:** $\qquad -j\,b\left(d_{stub1}\right)$

Stub Length: $\quad L_{stub} = \dfrac{\lambda}{2\pi}\tan^{-1}\left(\dfrac{1}{Z_{0s}B\left(d_{stub1}\right)}\right)$ **(short)**

$\qquad\qquad\qquad L_{stub} = \dfrac{\lambda}{2\pi}\tan^{-1}\left(Z_{0s}B\left(d_{stub1}\right)\right)$ **(open)**

The second solution corresponds to an admittance value with negative imaginary part, in the lower portion of the chart

Line Admittance - Actual :
$$Y\left(d_{stub_2}\right) = Y_0 - j\, B\left(d_{stub_2}\right)$$

Normalized :
$$y\left(d_{stub_2}\right) = 1 - j\, b\left(d_{stub_2}\right)$$

Stub Location :
$$d_{stub_2} = \frac{\theta_2}{4\pi}\lambda$$

Stub Admittance - Actual :
$$j\, B\left(d_{stub_2}\right)$$

Normalized :
$$j\, b\left(d_{stub_2}\right)$$

Stub Length :
$$L_{stub} = \frac{\lambda}{2\pi}\tan^{-1}\left(-\frac{1}{Z_{0s}B\left(d_{stub_2}\right)}\right) \quad \textbf{(short)}$$

$$L_{stub} = \frac{\lambda}{2\pi}\tan^{-1}\left(-Z_{0s}B\left(d_{stub_2}\right)\right) \quad \textbf{(open)}$$

If the normalized **load** admittance falls **inside the unitary conductance circle** (see next figure), the **first** possible stub location corresponds to a line admittance with **negative** imaginary part. The **second** possible location has line admittance with **positive** imaginary part. **In this case, the formulae given above for first and second solution exchange place.**

If one moves further away from the load, other suitable locations for stub insertion are found by moving toward the generator, at distances multiple of half a wavelength from the original solutions. These locations correspond to the same points on the Smith chart.
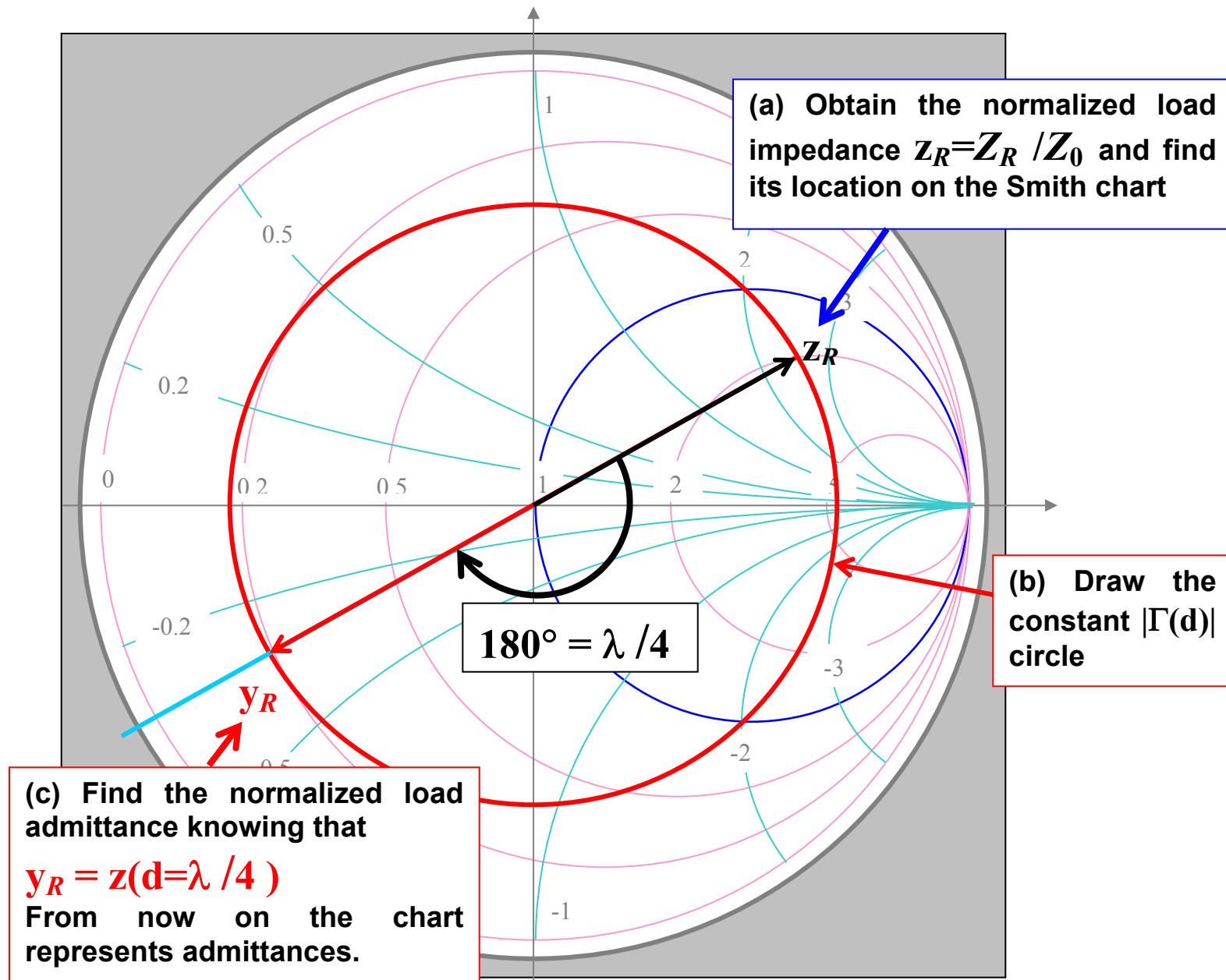
First set of locations $= \mathbf{d_{stub_1}} + n\dfrac{\lambda}{2}$

Second set of locations $= \mathbf{d_{stub_2}} + n\dfrac{\lambda}{2}$
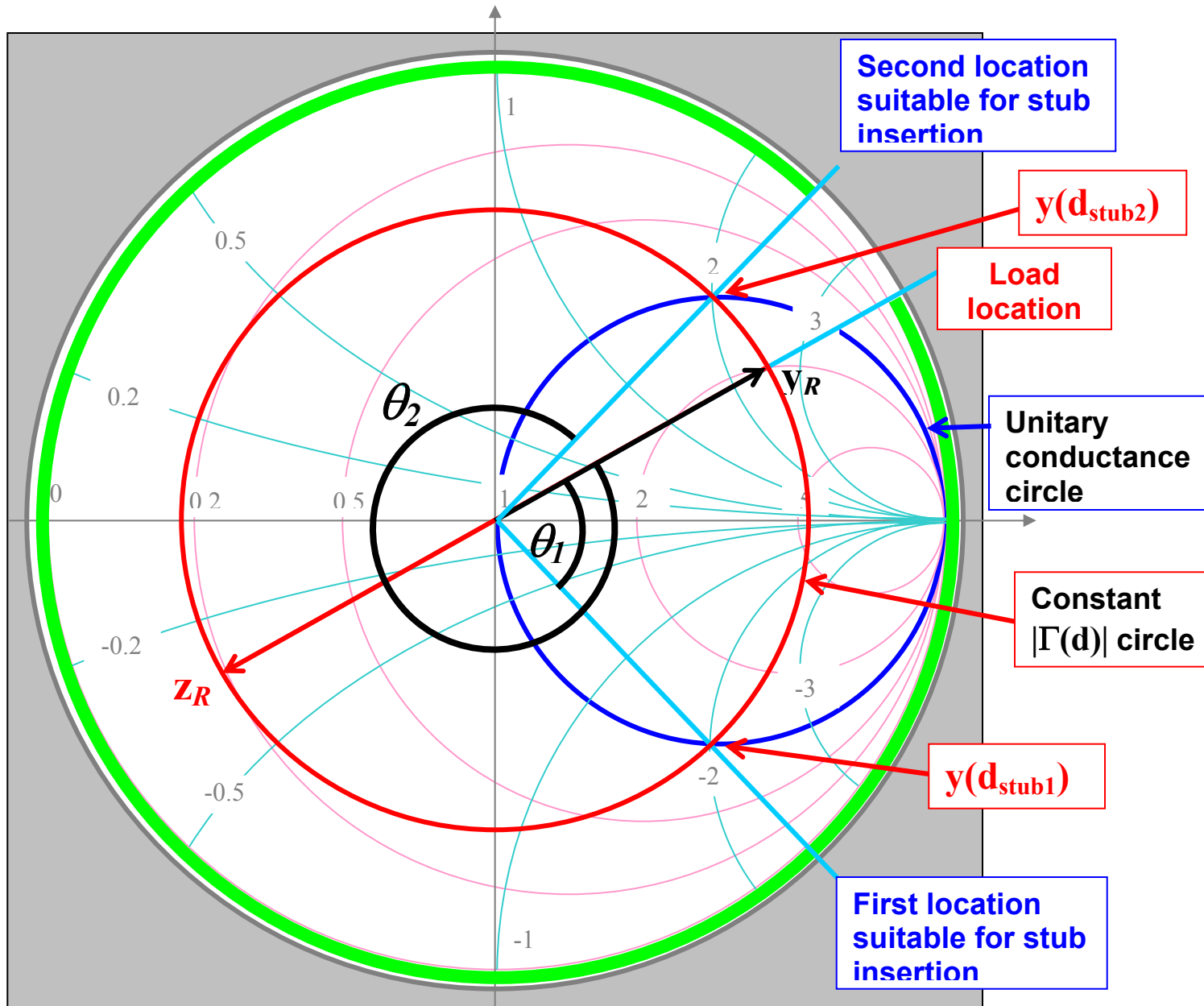
Single stub matching problems can be solved on the Smith chart graphically, using a compass and a ruler. This is a step-by-step summary of the procedure:

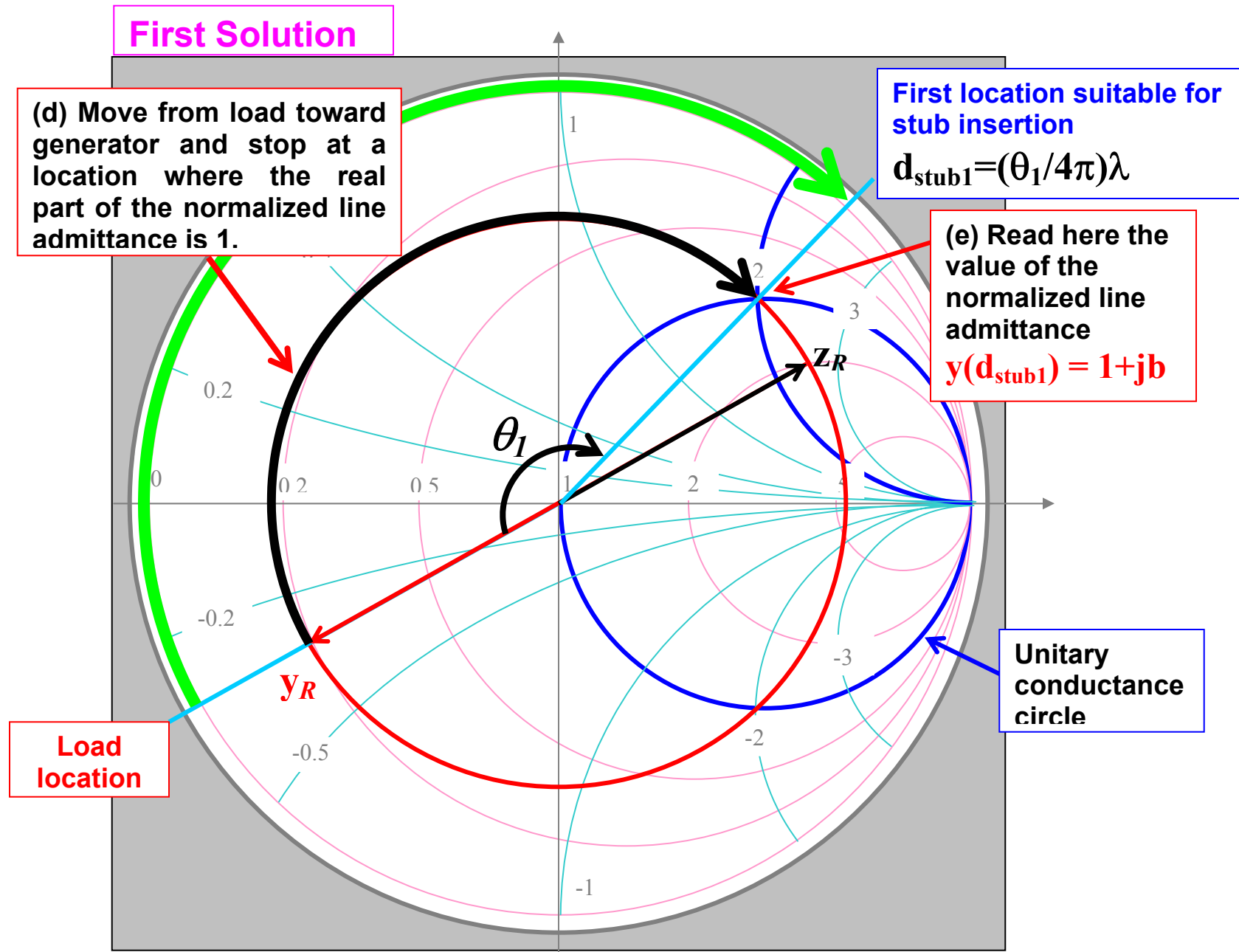(a) Find the normalized load impedance and determine the corresponding location on the chart.

(b) Draw the circle of constant magnitude of the reflection coefficient $|\Gamma|$ for the given load.

(c) Determine the normalized load admittance on the chart. This is obtained by rotating 180° on the constant $|\Gamma|$ circle, from the load impedance point. From now on, all values read on the chart are normalized admittances.

(a) **Obtain the normalized load impedance $z_R = Z_R / Z_0$ and find its location on the Smith chart**

(b) **Draw the constant $|\Gamma(d)|$ circle**

$180° = \lambda / 4$

(c) **Find the normalized load admittance knowing that**

$y_R = z(d = \lambda / 4)$

From now on the chart represents admittances.

z_R

y_R

(d) Move from load admittance toward generator by riding on the constant $|\Gamma|$ circle, until the intersections with the unitary normalized conductance circle are found. These intersections correspond to possible locations for stub insertion. Commercial Smith charts provide graduations to determine the angles of rotation as well as the distances from the load in units of wavelength.

(e) Read the line normalized admittance in correspondence of the stub insertion locations determined in (d). These values will always be of the form

$$y(d_{stub}) = 1 + jb \qquad \text{top half of chart}$$
$$y(d_{stub}) = 1 - jb \qquad \text{bottom half of chart}$$

First Solution

(d) Move from load toward generator and stop at a location where the real part of the normalized line admittance is 1.

First location suitable for stub insertion

$$d_{stub1} = (\theta_1/4\pi)\lambda$$

(e) Read here the value of the normalized line admittance

$$y(d_{stub1}) = 1 + jb$$

θ_1

z_R

y_R

Load location

Unitary conductance circle

Second Solution

(d) Move from load toward generator and stop at a location where the real part of the normalized line admittance is 1.

Unitary conductance circle

z_R

θ_2

y_R

Load location

(e) Read here the value of the normalized line admittance
$$y(d_{stub2}) = 1 - jb$$

Second location suitable for stub insertion
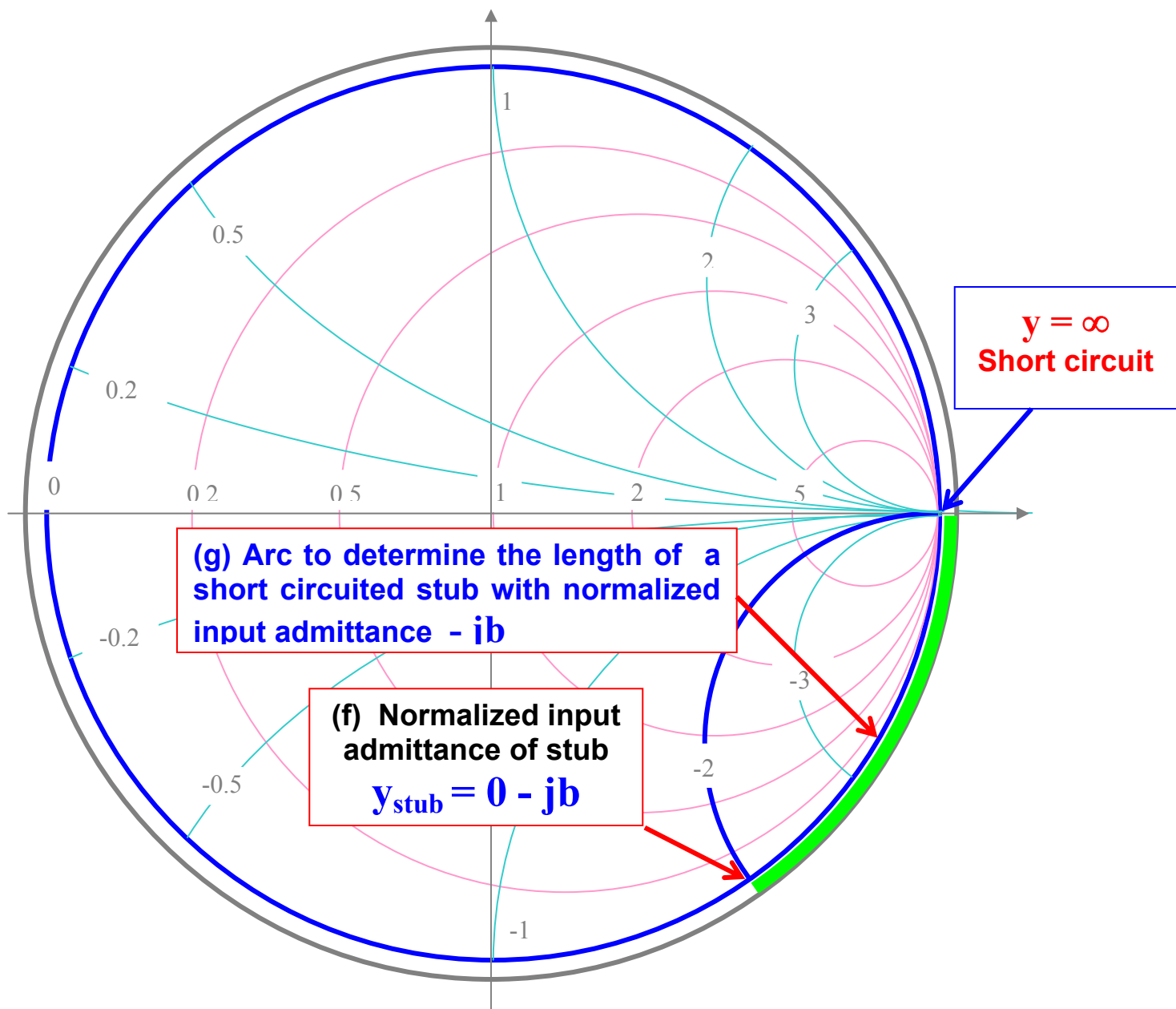$$d_{stub2} = (\theta_2/4\pi)\lambda$$
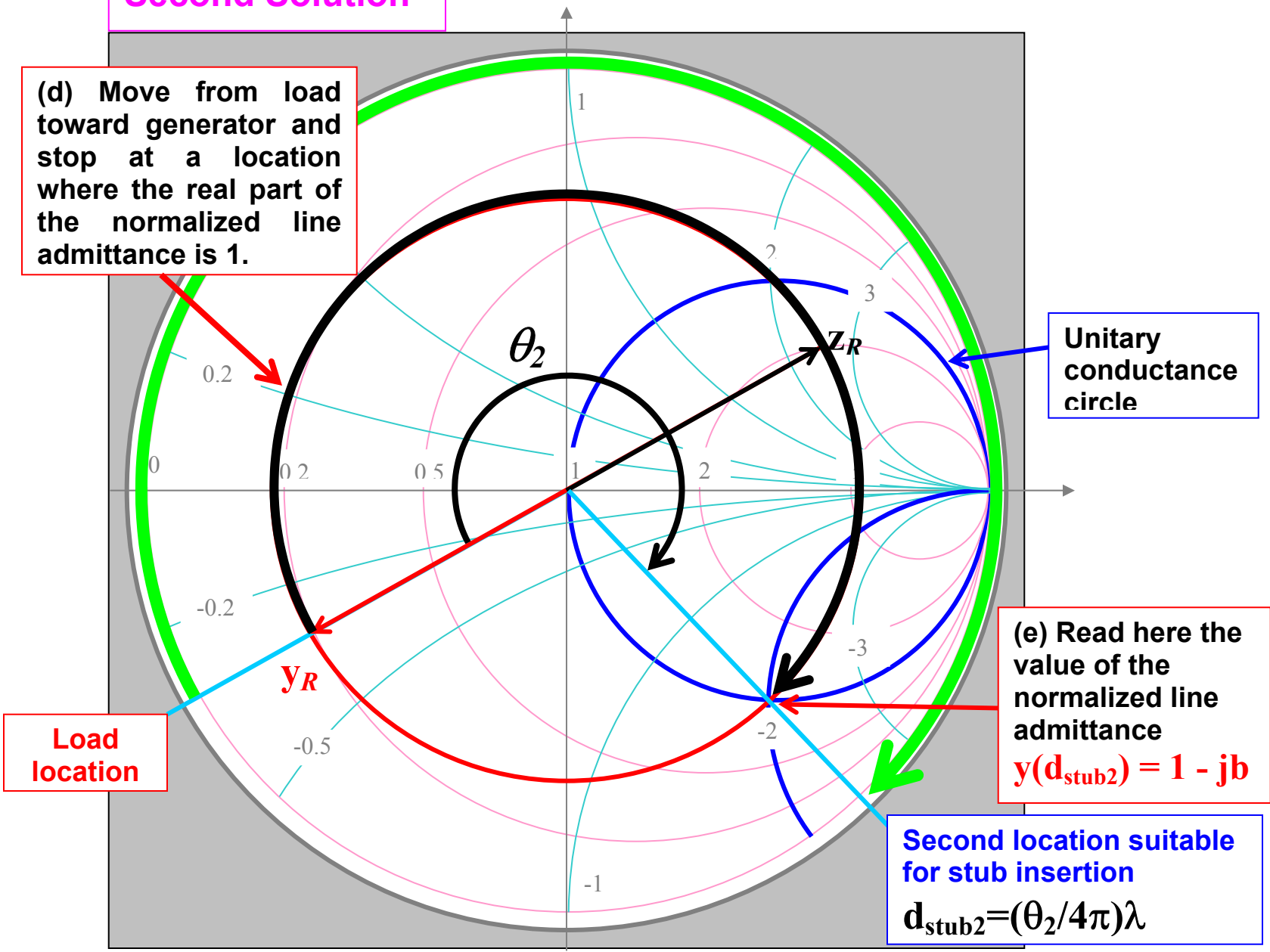
(f) **Select the input normalized admittance of the stubs, by taking the opposite of the corresponding imaginary part of the line admittance**
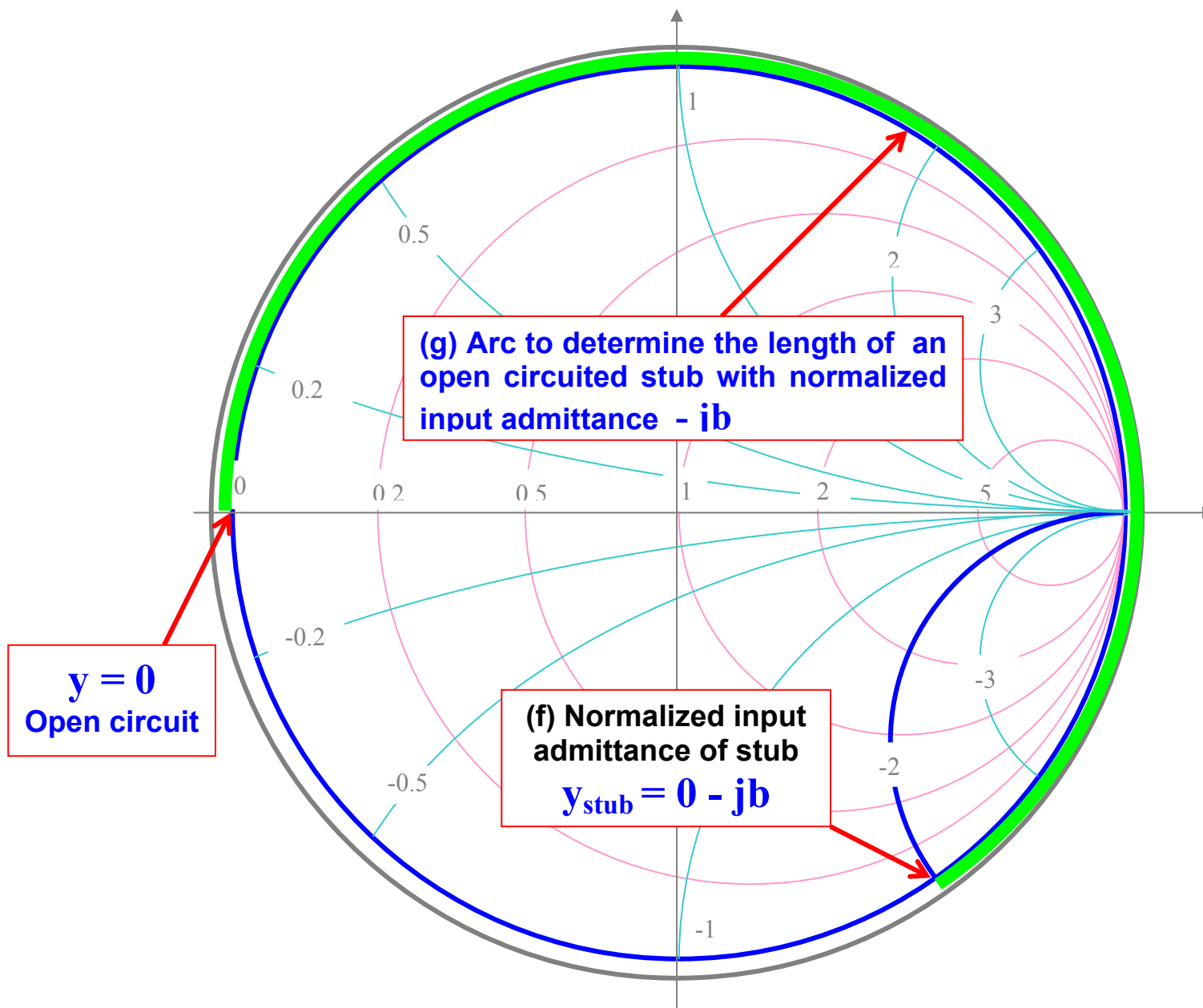
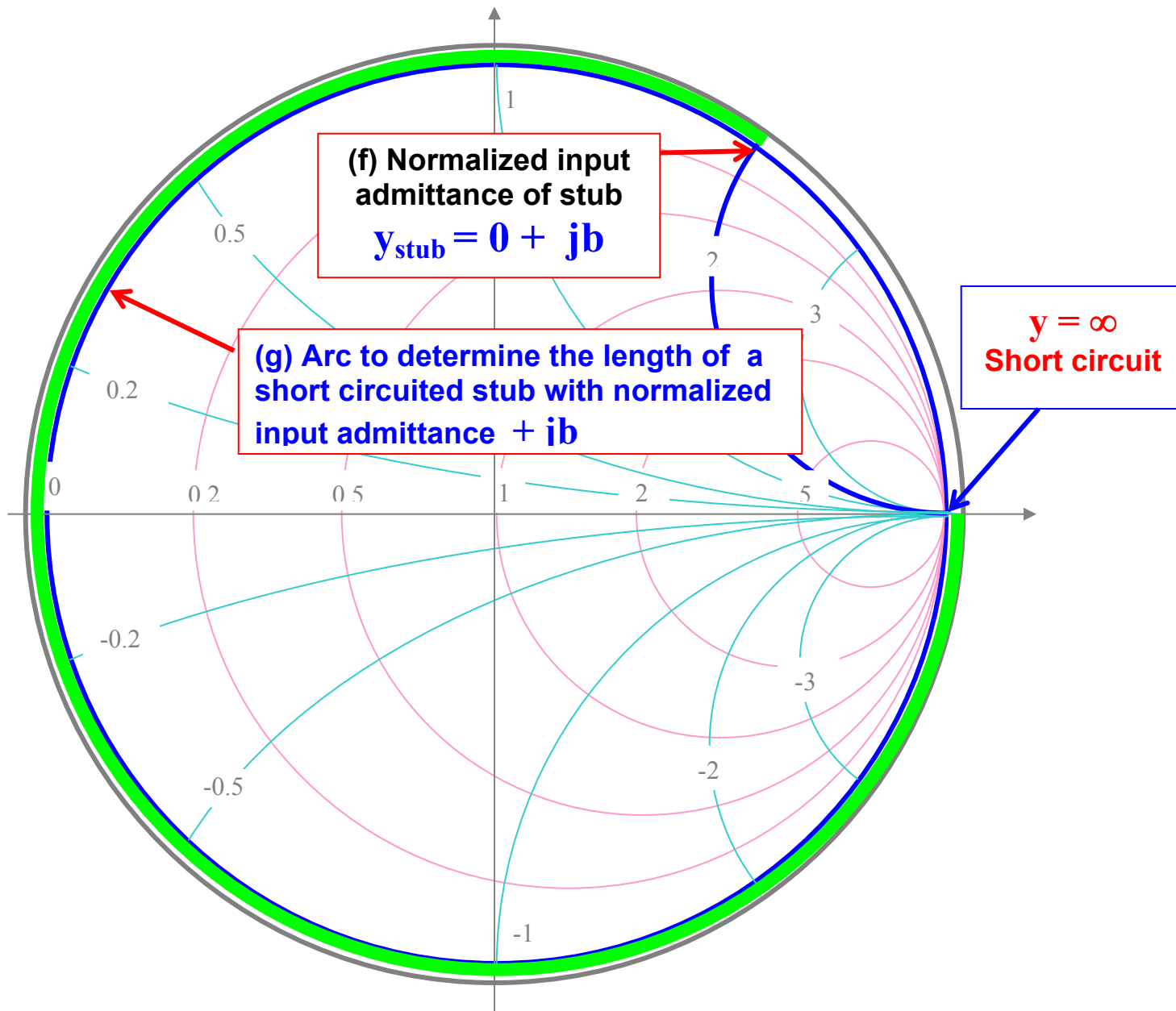$$\text{line: } y(d_{stub}) = 1 + jb \qquad \rightarrow \qquad \text{stub: } y_{stub} = -jb$$

$$\text{line: } y(d_{stub}) = 1 - jb \qquad \rightarrow \qquad \text{stub: } y_{stub} = +jb$$

(g) **Use the chart to determine the length of the stub. The imaginary normalized admittance values are found on the circle of zero conductance on the chart. On a commercial Smith chart one can use a printed scale to read the stub length in terms of wavelength. We assume here that the stub line has characteristic impedance Z_0 as the main line. If the stub has characteristic impedance $Z_{0S} \neq Z_0$ the values on the Smith chart must be renormalized as**

$$\pm jb' = \pm jb \frac{Y_0}{Y_{0s}} = \pm jb \frac{Z_{0s}}{Z_0}$$

y = ∞
Short circuit

(g) Arc to determine the length of a short circuited stub with normalized input admittance - jb

(f) Normalized input admittance of stub
$y_{stub} = 0 - jb$

(g) Arc to determine the length of an open circuited stub with normalized input admittance $- jb$

y = 0
Open circuit

(f) Normalized input admittance of stub

$$y_{stub} = 0 - jb$$

(f) Normalized input admittance of stub

$$y_{stub} = 0 + jb$$

(g) Arc to determine the length of a short circuited stub with normalized input admittance $+jb$

$y = \infty$
Short circuit

(f) Normalized input admittance of stub

$$y_{stub} = 0 + jb$$

(g) Arc to determine the length of an open circuited stub with normalized input admittance $+jb$
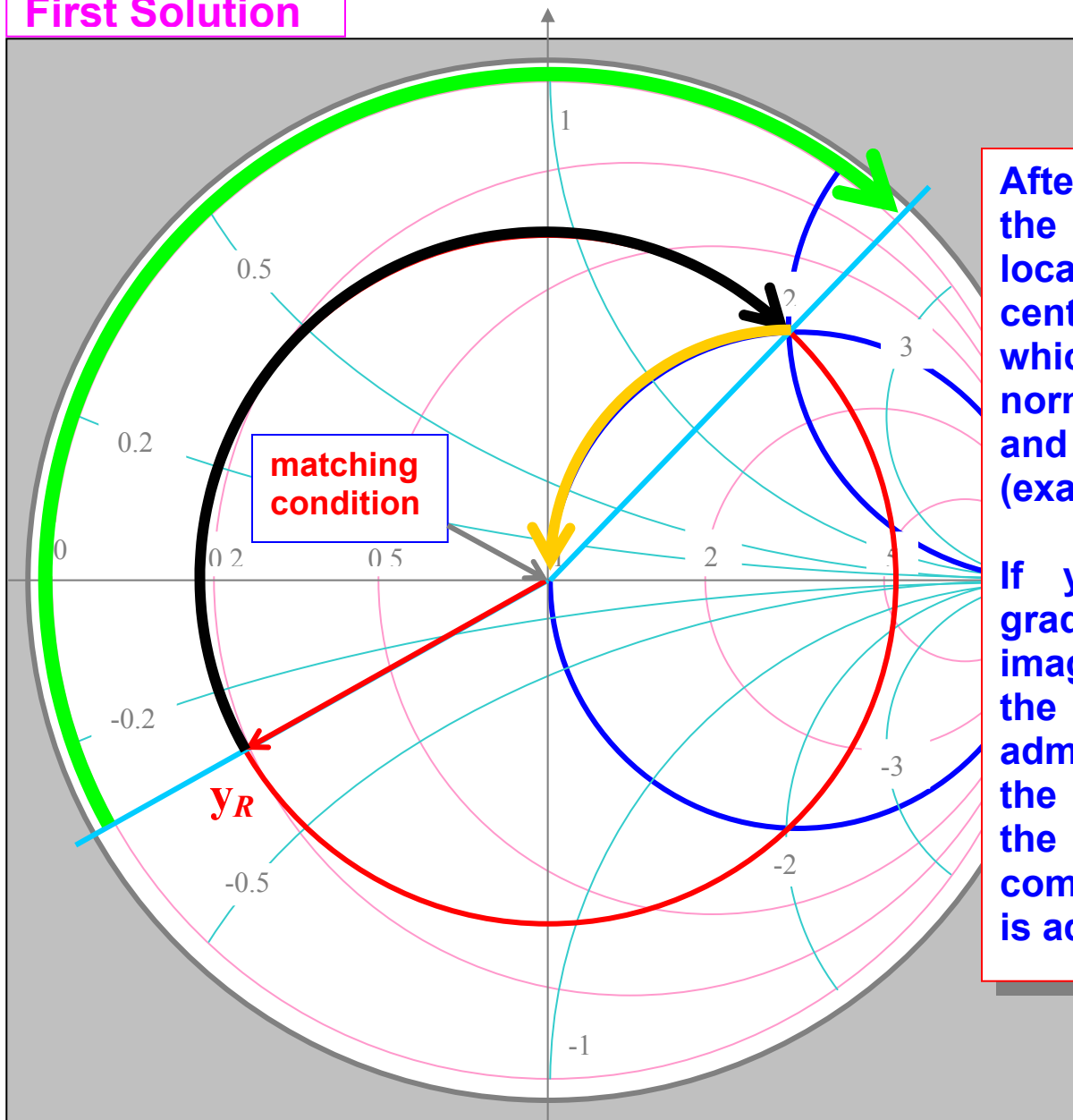
$$y = 0$$
Open circuit

First Solution

After the stub is inserted, the admittance at the stub location is moved to the center of the Smith chart, which corresponds to normalized admittance 1 and reflection coefficient 0 (exact matching condition).

If you imagine to add gradually the negative imaginary admittance of the inserted stub, the total admittance would follow the yellow arrow, reaching the match point when the complete stub admittance is added.

First Solution

If the stub does not have the proper normalized input admittance, the matching condition is not reached

Effect of a stub with negative susceptance of insufficient magnitude

Effect of a stub with positive susceptance

y_R

Effect of a stub with negative susceptance of excessive magnitude