Lecture – 9 Date: 02.09.2014

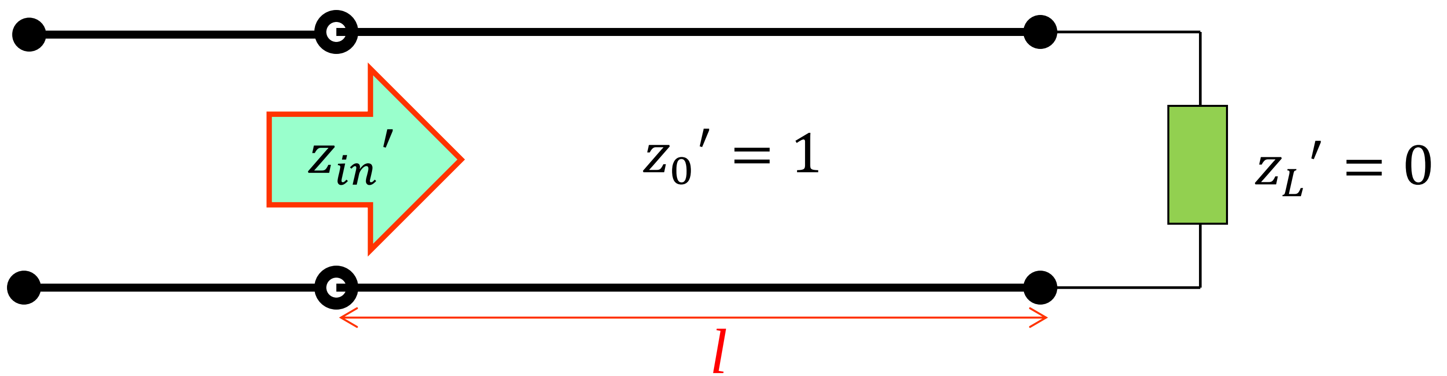
- Examples
- Admittance Transformation
- Examples
- Admittance Smith Chart
- High Frequency Network Analysis (intro)

Example – 1

- determine the input impedance of a transmission line that is terminated in a **short circuit**, and whose length is:

$$a)\ l = {}^{\lambda}/_{8} = 0.125\lambda \quad \Rightarrow \quad 2\beta l = 90°$$

$$\text{b)}\ l = {}^{3\lambda}/_{8} = 0.375\lambda \quad \Rightarrow \quad 2\beta l = 270°$$



- **Solution:**

a) Rotate **clockwise** $90°$ from $\Gamma = -1.0 = e^{j180°}$ and find z_{in}'. $z_{in}' = j$

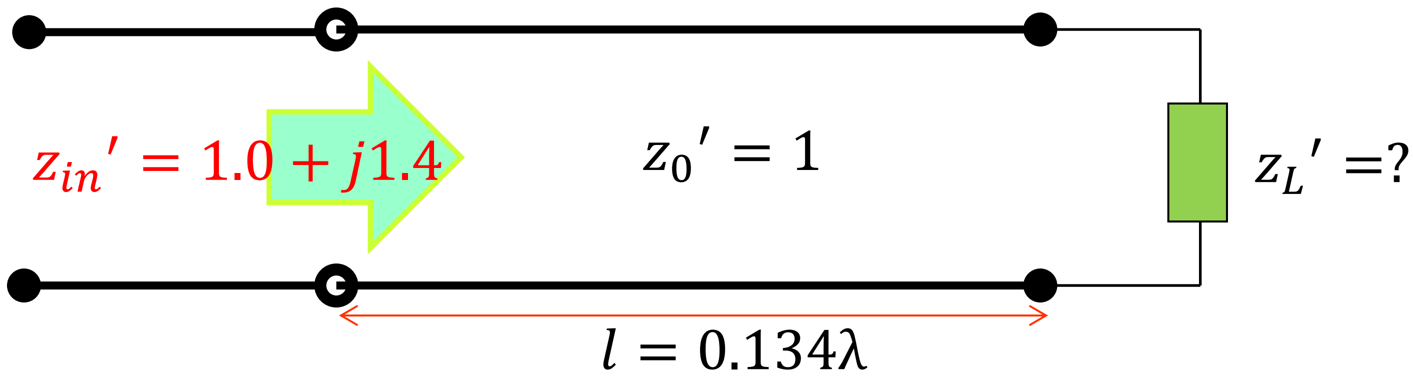
b) Rotate **clockwise** $270°$ from $\Gamma = -1.0 = e^{j180°}$ and find z_{in}'. $z_{in}' = -j$

Example – 2

- we know that the **input** impedance of a transmission line length $l = 0.134\lambda$ is:

$$z_{in}' = 1.0 + j1.4$$

→ determine the impedance of the **load** that is terminating this line.



- **Solution:**

Locate z_{in}' on the Smith Chart, and then rotate **counter clockwise** (yes, I said **counter**-clockwise) $2\beta l = 96.5°$. Essentially, you are removing the phase shift associated with the transmission line. When you stop, lift your pen and find z_L'!

Example – 3

- A load **terminating** at transmission line has a normalized impedance $z_L' = 2.0 + j2.0$. What should the **length** l of transmission line be in order for its input impedance to be:
  a) Purely **real** (i.e., $X_{in} = 0$)
  b) Have a real (resistive) part equal to **one** (i.e., $r_{in} = 1.0$)
- **Solution:**

a) Find $z_L' = 2.0 + j2.0$ on your Smith Chart, and then rotate **clockwise** until you "bump into" the contour $x = 0$ (recall this contour lies on the $\Gamma_r -$ **axis!**).

- When you reach the $x = 0$ contour—**stop!** Lift your pen and note that the impedance value of this location is **purely real** (after all, $x = 0!$).
- Now, measure the **rotation angle** that was required to move clockwise from $z_L' = 2.0 + j2.0$ to an impedance on the $x = 0$ contour—this **angle** is equal to $2\beta l$!

You can now **solve** for l, or alternatively use the **electrical length scale** surrounding the Smith Chart.

Example – 3 (contd.)

One more important point—there are **two** possible solutions!

$z_{in}' = 4.2 + j0$ → $2\beta l = 30°$ → $l = 0.042\lambda$

$z_{in}' = 0.24 + j0$ → $2\beta l = 210°$ → $l = 0.292\lambda$

b) Find $z_L' = 2.0 + j2.0$ on your Smith Chart, and then rotate **clockwise** until you "bump into" the **circle** $r = 1$ (recall this circle intersects the **center** point of the Smith Chart!).

- When you reach the $r = 1$ circle—**stop**! Lift your pencil and note that the impedance value of this location has a real value equal to **one** (after all, $r = 1$!).
- Now, measure the **rotation angle** that was required to move clockwise from $z_L' = 2.0 + j2.0$ to an impedance on the $r = 1$ circle—this **angle** is equal to $2\beta l$!

Example – 3 (contd.)

You can now **solve** for l, or alternatively use the **electrical length scale** surrounding the Smith Chart.

Again, we find that there are **two** solutions!

$z_{in}' = 1.0 - j1.6$ → $2\beta l = 82°$ → $l = 0.114\lambda$

$z_{in}' = 1.0 + j1.6$ → $2\beta l = 339°$ → $l = 0.471\lambda$

Q: Hey! For part b), the solutions resulted in $z_{in}' = 1.0 - j1.6$ and $z_{in}' = 1.0 + j1.6$ --the **imaginary** parts are equal but **opposite!** Is this just a coincidence?

A: Hardly! Remember, the two impedance solutions must result in the **same magnitude** for Γ --for this example we find $\Gamma\,(z) = 0.625$.

Example – 3 (contd.)

- Thus, for impedances where $r = 1$ (i.e., $z' = 1 + jx$):

$$\Gamma = \frac{z'-1}{z'+1} = \frac{(1+jx)-1}{(1+jx)+1} = \frac{jx}{2+jx}$$

- and therefore:

$$|\Gamma|^2 = \frac{|jx|^2}{|2+jx|^2} = \frac{x^2}{4+x^2}$$

$$x^2 = \frac{4|\Gamma|^2}{1-|\Gamma|^2}$$

there are **two** equal by opposite solutions!

$$x = \pm\frac{2|\Gamma|}{\sqrt{1-|\Gamma|^2}}$$

Which for **this** example gives us our solutions $x = \pm 1.6$.

Admittance Transformation

- RF/Microwave network, similar to any electrical network, has impedance elements in series and parallel
- Impedance Smith chart is well suited while working with series configurations while admittance Smith chart is more useful for parallel configurations
- The impedance Smith chart can easily be used as an admittance calculator

$$z_{in}(z)=\frac{1+\Gamma(z)}{1-\Gamma(z)} \quad \Leftarrow \quad y_{in}(z)=\frac{Y_{in}(z)}{Y_0}=\frac{1/Z_{in}(z)}{1/Z_0}=\frac{1}{Z_{in}(z)/Z_0}=\frac{1}{z_{in}(z)}$$

- Hence, $y_{in}(z)=\frac{1-\Gamma(z)}{1+\Gamma(z)} \Rightarrow y_{in}(z)=\frac{1+e^{-j\pi}\Gamma(z)}{1-e^{-j\pi}\Gamma(z)}$

It means, to obtain normalized admittance → take the normalized impedance and multiply associated reflection coefficient by -1 = $e^{-j\pi}$ → it is equivalent to a 180^{o} rotation of the reflection coefficient in complex Γ-plane

Example – 4

- Convert the following normalized input impedance z_{in}' into normalized input admittance y_{in}' using the Smith chart:

$$z_{in}' = 1 + j1 = \sqrt{2}e^{j(\pi/4)}$$

First approach: The normalized admittance can be found by direct inversion as:

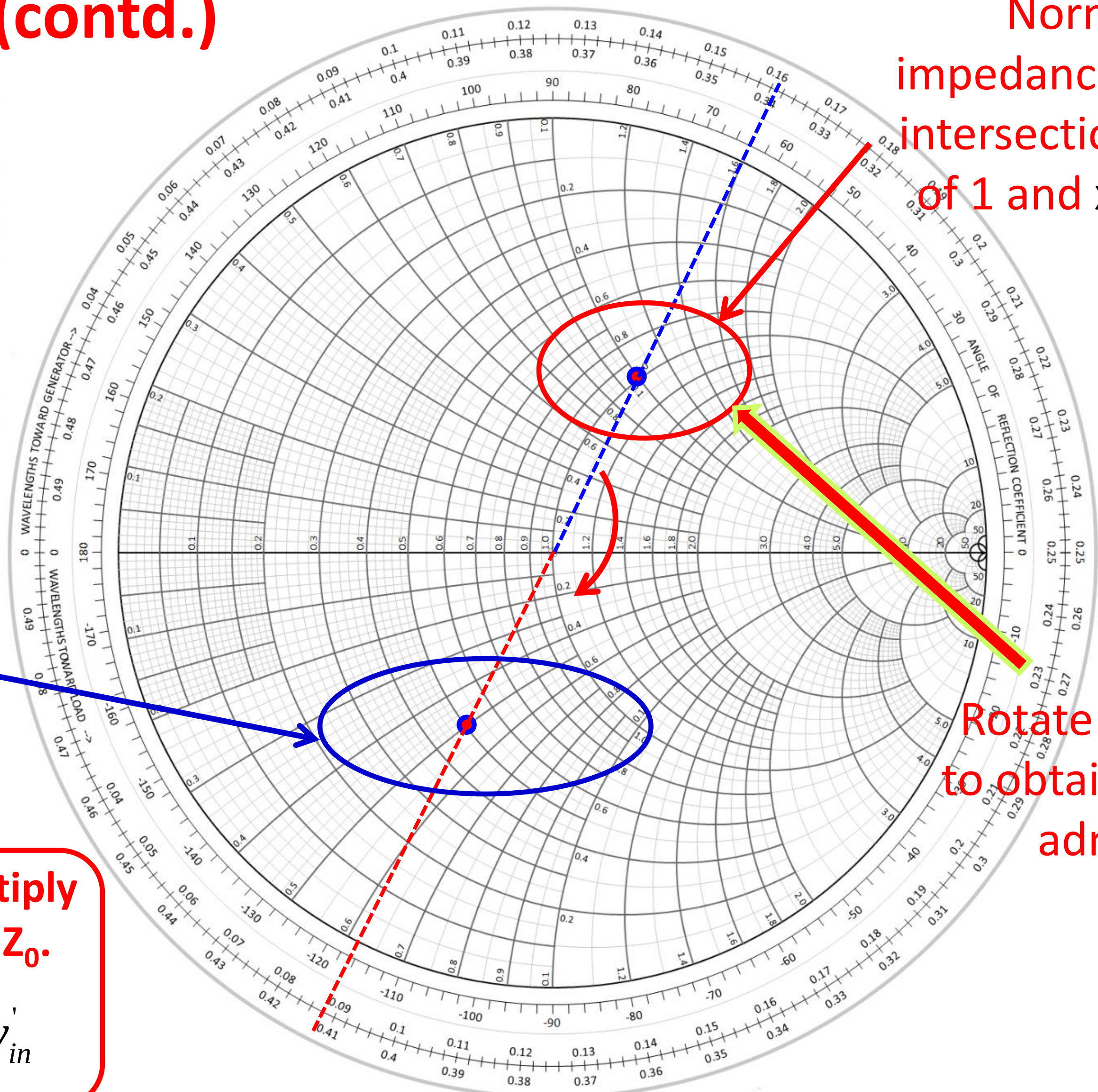
$$y_{in}' = \frac{1}{z_{in}'} = \frac{1}{1+j1} = \frac{1}{\sqrt{2}}e^{-j(\pi/4)} = \frac{1}{2} - j\frac{1}{2}$$

Alternative approach:

- Mark the normalized impedance on Smith chart
- Identify phase angle and magnitude of the associated reflection coefficient
- Rotate the reflection coefficient by 180^{0}
- Identify the x-circle and r-circle intersection of the rotated reflection coefficient

Example – 4 (contd.)

Quick investigation show that the normalized impedance (y_{in}') is the intersection of r-circle of 1/2 and x-circle of -1/2



Normalized impedance (z_{in}') is the intersection of r-circle of 1 and x-circle of 1

Rotate this by 180° to obtain normalized admittance

To denormalize, multiply with the inverse of Z_0.

$$Y_{in} = y_{in}' \frac{1}{Z_0} = Y_0 y_{in}'$$

Example – 5

Given: $z_{in}^{'} = 1 + j2$

➢ Find the normalized admittance $\lambda/8$ away from the load

Steps:

1. Mark the normalized impedance on Smith Chart
2. Clockwise rotate it by 180^{o}
3. Identify the normalized impedance and the phase angle of the associated reflection coefficient
4. Clockwise rotate the reflection coefficient (associated with the normalized admittance) by $2\beta l$ (here $l = \lambda/8$)
5. The new location gives the required normalized admittance

Example – 5 (contd.)

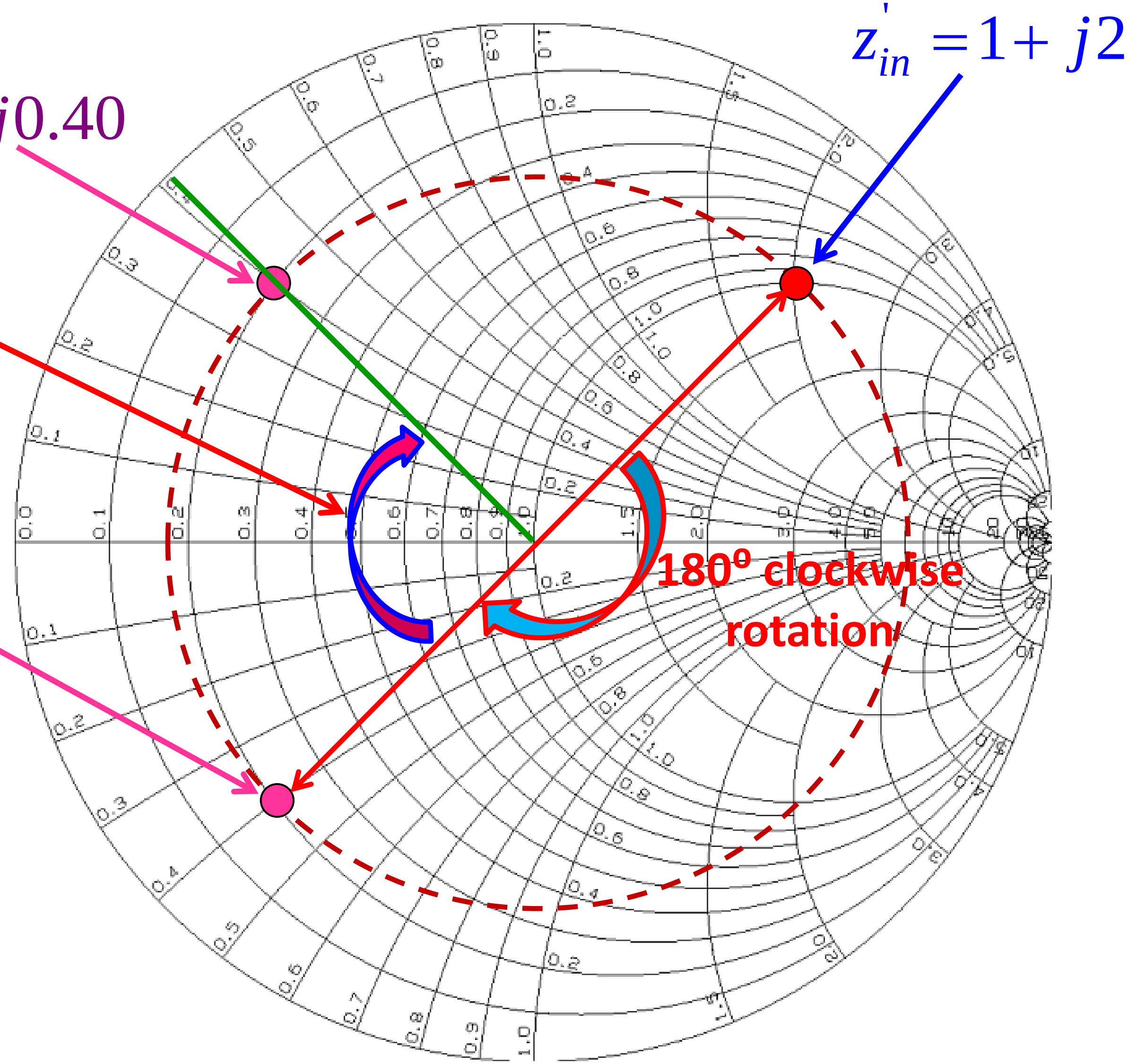
$y_{in}^{'} = 0.20 + j0.40$

Clockwise rotation by

$$2\beta l = 4\pi\left(\frac{l}{\lambda}\right)$$

$y_{in}^{'} = 0.2 - j0.4$

$$l = \lambda/8 \rightarrow 2\beta l = 90^{o}$$



Admittance Smith chart

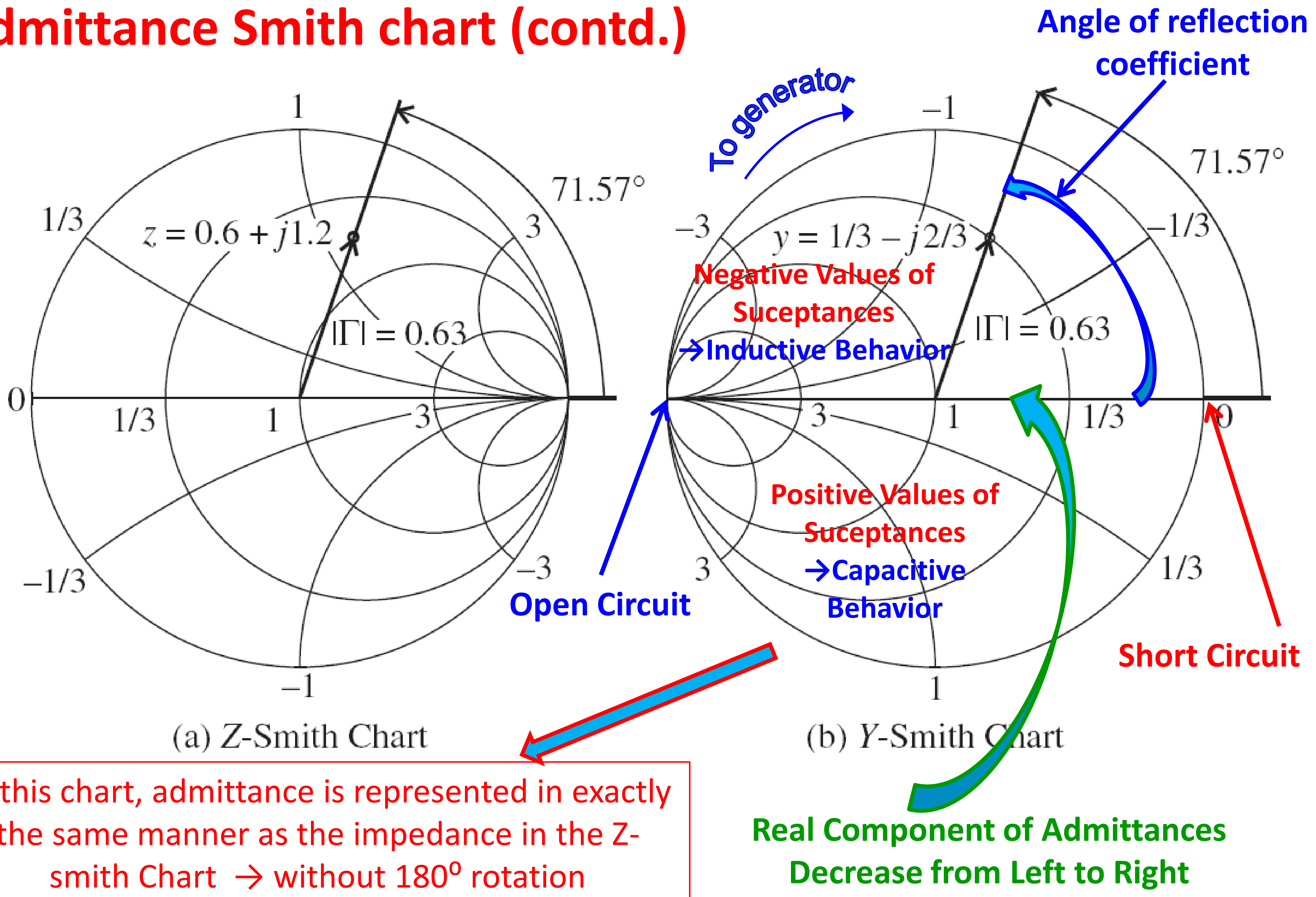
- Alternative approach to solve parallel network elements is through 180^o rotated Smith chart
- This rotated Smith chart is called **admittance Smith chart** **or** **Y-Smith chart**
- The corresponding normalized resistances become normalized conductances & normalized reactances become normalized suceptances

$$r = \frac{R}{Z_0} \Rightarrow g = \frac{G}{Y_0} = Z_0 G$$

$$x = \frac{X}{Z_0} \Rightarrow b = \frac{b}{Y_0} = Z_0 B$$

- **The Y-Smith chart preserves:**
  - The direction in which the angle of the reflection coefficient is measured
  - The direction of rotation (either toward or away from the generator)

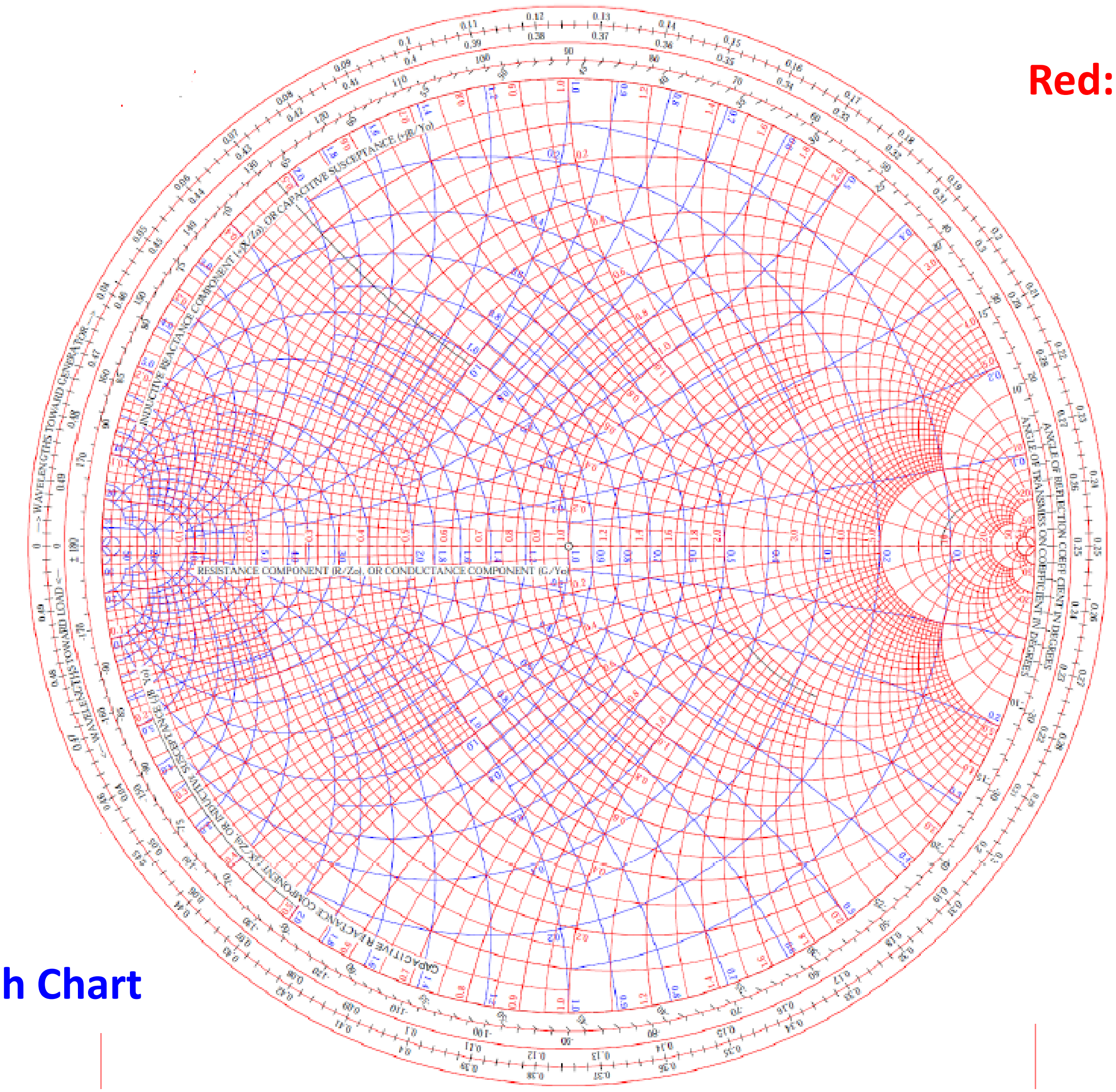
Admittance Smith chart (contd.)



In this chart, admittance is represented in exactly the same manner as the impedance in the Z-smith Chart → without 180^{o} rotation

Combined Z- and Y- Smith Charts

Red: Z – Smith Chart

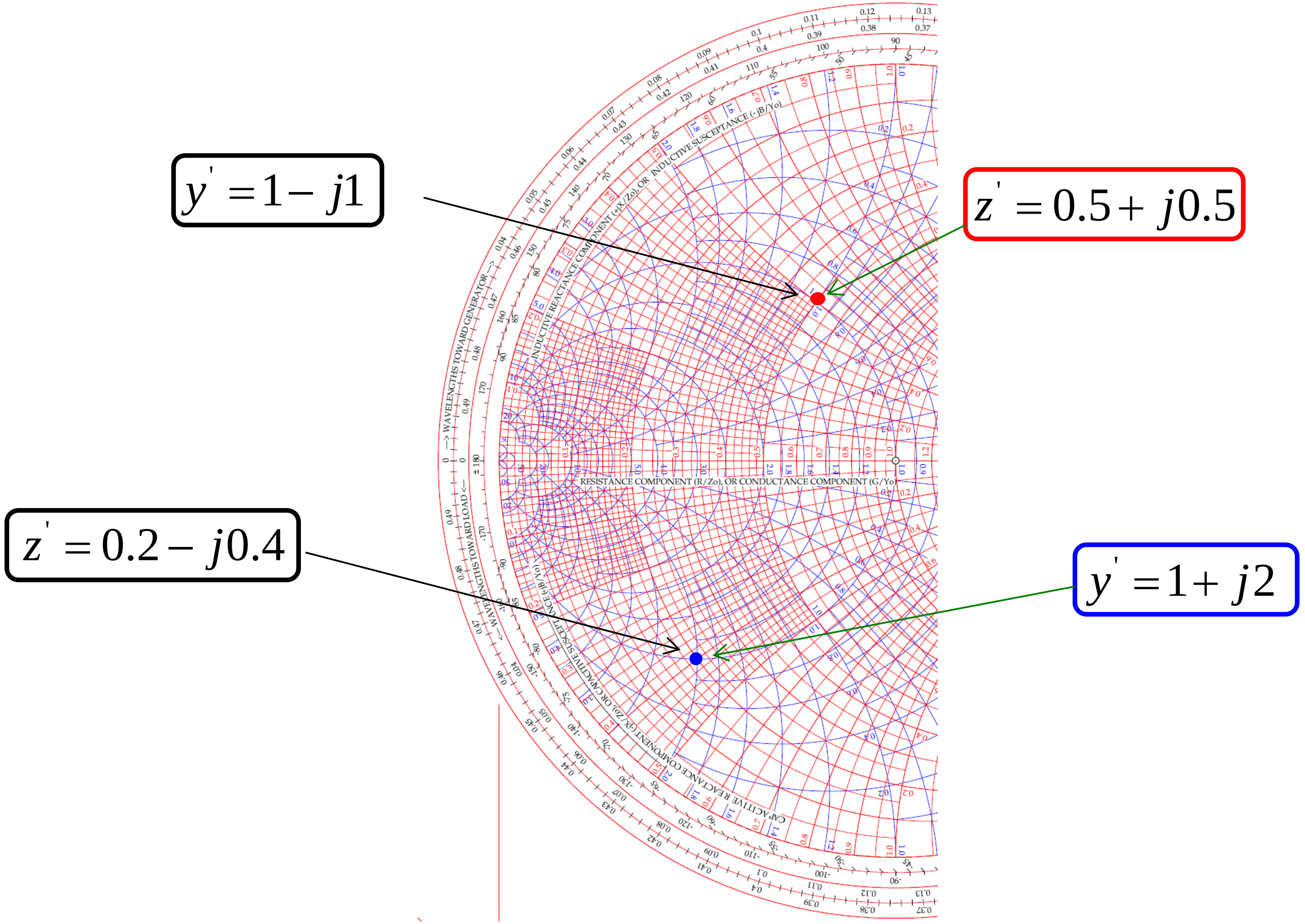


Blue: Y – Smith Chart

Example – 6

- Identify (a) the normalized impedance $z' = 0.5 + j0.5$, and (b) the normalized admittance value $y' = 1 + j2$ in the combined ZY-Smith Chart and find the corresponding values of normalized admittance and impedance

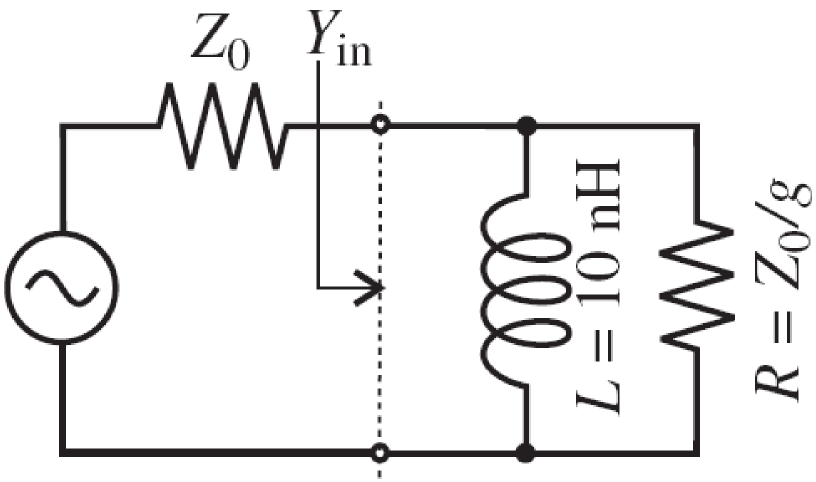
Example – 6 (contd.)



Parallel and Series Connections of RLC Elements

Parallel Connection of R and L

- Let us consider the following circuit



Normalized admittance y_{in}' will be in upper part of Y-Smith Chart

$$y_{in}^{'}(\omega) = g - j\frac{Z_0}{\omega L}$$

- We can compute the normalized admittance as:

$$g = \frac{Z_0}{R} \qquad b_L = \frac{Z_0}{\omega L}$$

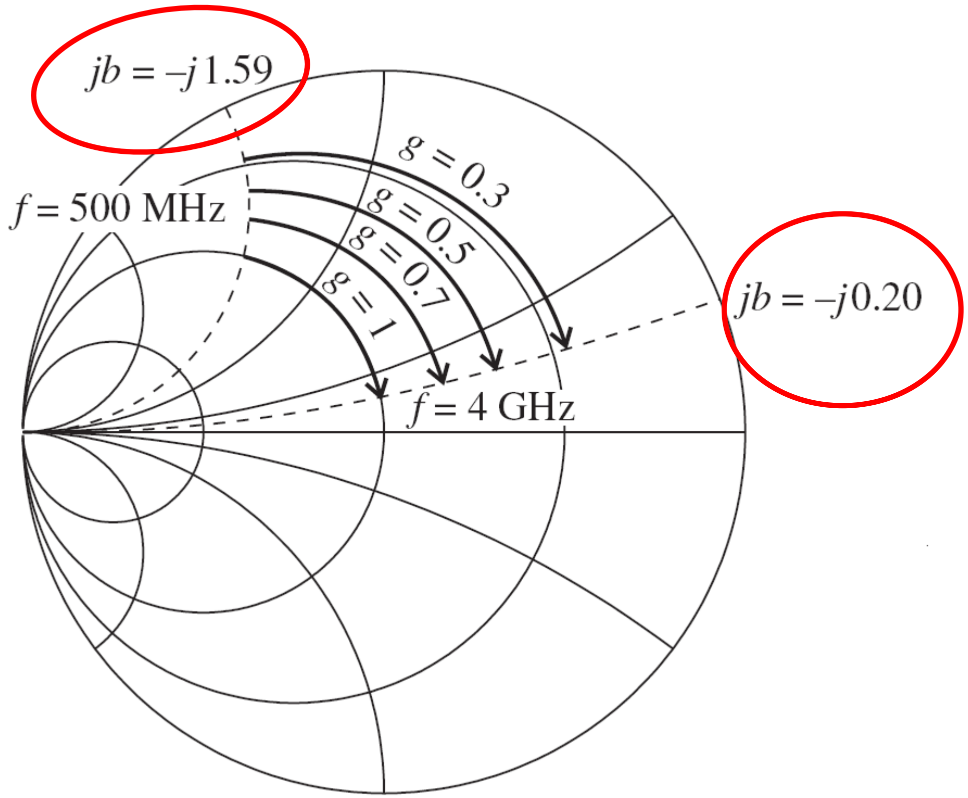
For a constant conductance (g) circle and variable frequency → admittance will be a curve along the conductance circle

Parallel and Series Connections of RLC Elements (contd.)

Parallel Connection of R and L

- Frequency dependent admittance behavior → for conductance values g = 0.3, 0.5, 0.7, and 1 for 500 MHz to 4 GHz range → for fixed inductance of 10 nH and Z_0 = 50Ω.

Susceptance at 500 MHz

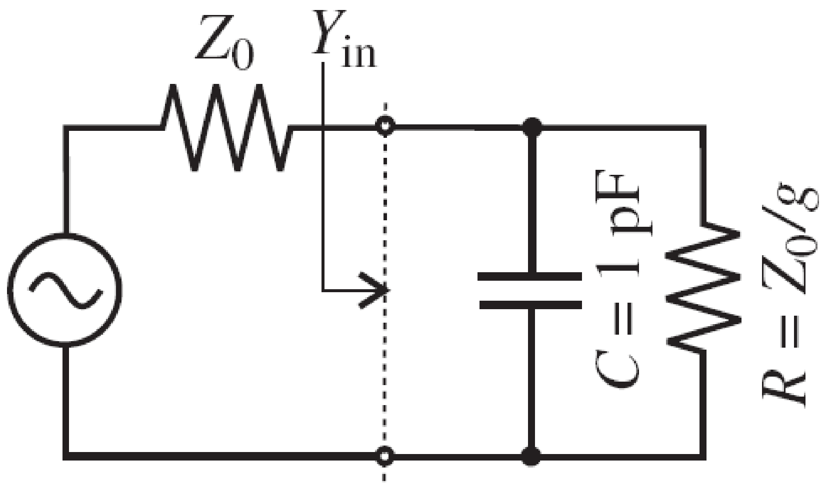


Susceptance at 4 GHz

Parallel and Series Connections of RLC Elements (contd.)

Parallel Connection of R and C

- Let us consider the following circuit



- We can compute the normalized admittance as:

$$g = \frac{Z_0}{R} \qquad b_C = Z_0 \omega C$$

Normalized admittance y_{in}' will be in lower part of Y-Smith Chart

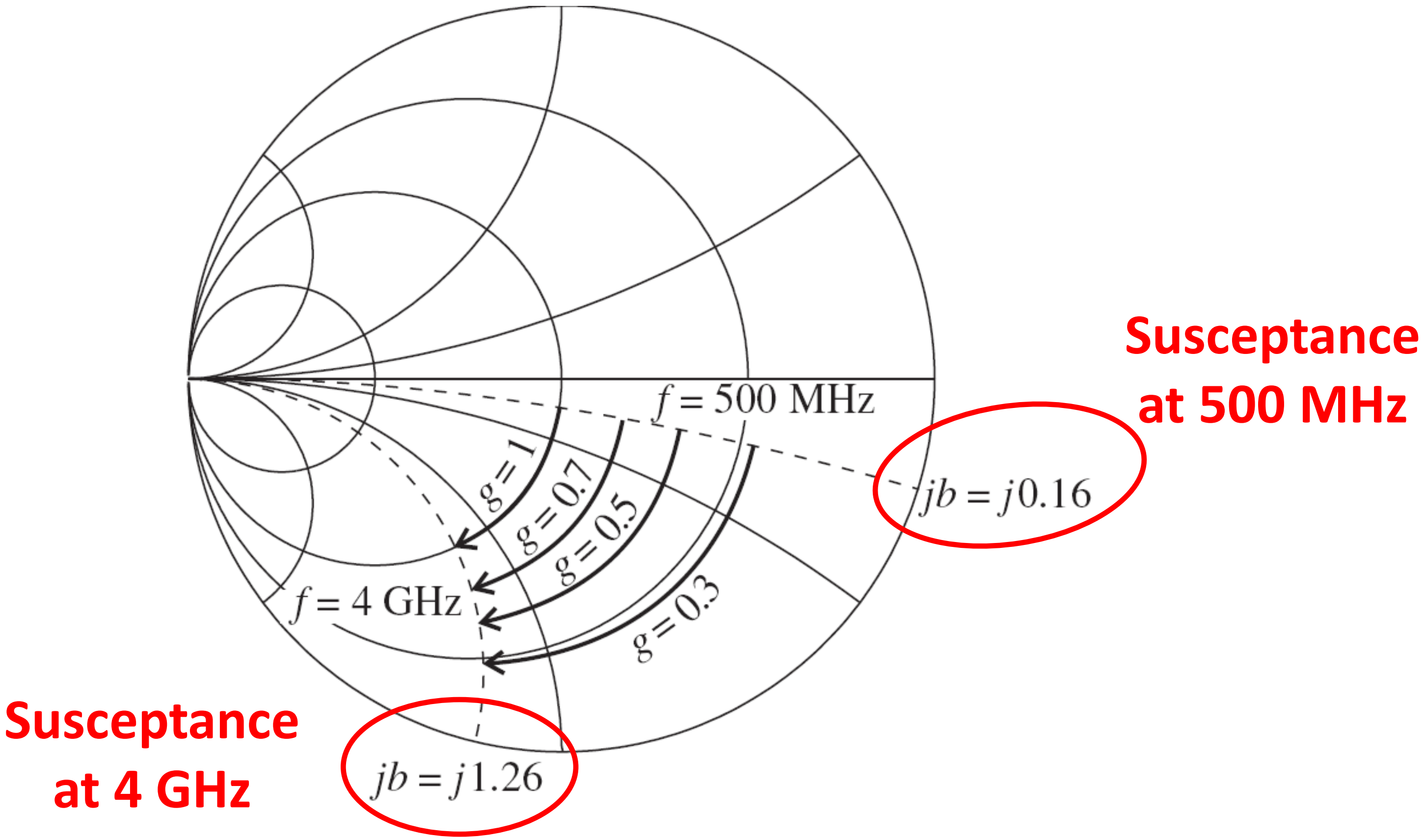
$$y_{in}'(\omega) = g + jZ_0\omega C$$

For a constant conductance (g) circle and variable frequency → admittance will be a curve along the conductance circle

Parallel and Series Connections of RLC Elements (contd.)

Parallel Connection of R and C

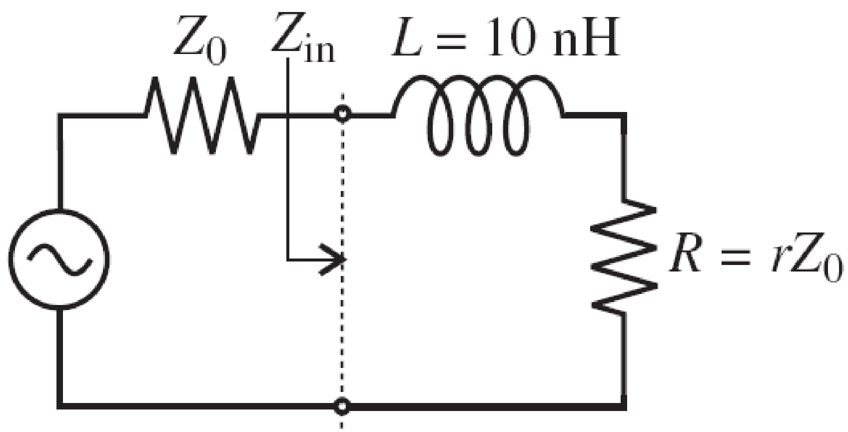
- Frequency dependent admittance behavior → for conductance values g = 0.3, 0.5, 0.7, and 1 for 500 MHz to 4 GHz range → for fixed capacitance of 1 pF and Z_0 = 50Ω.



Parallel and Series Connections of RLC Elements (contd.)

Series Connection of R and L

- Let us consider the following circuit



Normalized impedance z_{in}' will be in upper part of Z-Smith Chart

$$z_{in}^{'}(\omega) = r + j\frac{\omega L}{Z_0}$$

- We can compute the normalized impedance as:

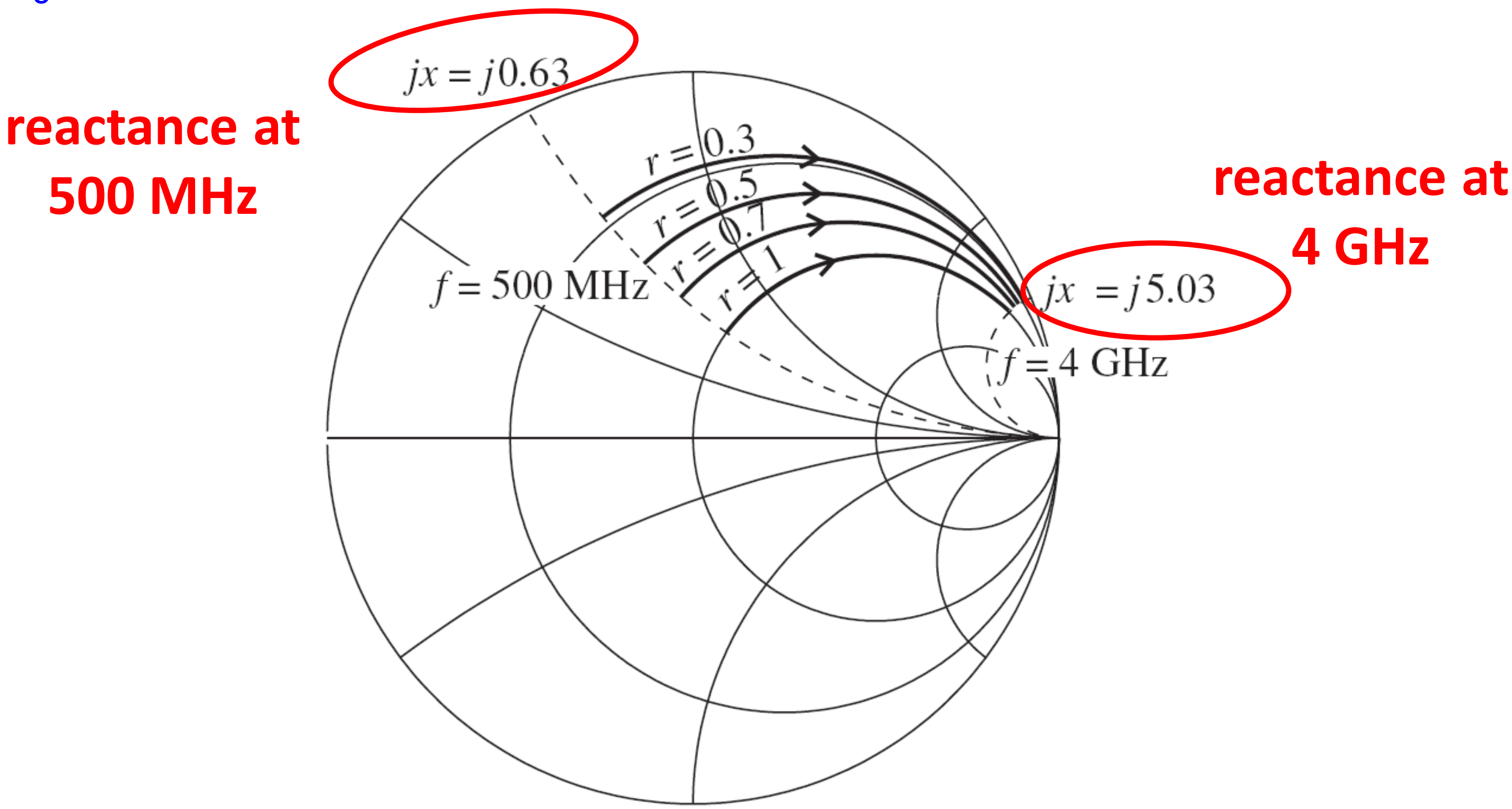
$$r = \frac{R}{Z_0} \qquad x_L = \frac{\omega L}{Z_0}$$

For a constant resistance (r) circle and variable frequency → impedance will be a curve along the resistance circle

Parallel and Series Connections of RLC Elements (contd.)

Series Connection of R and L

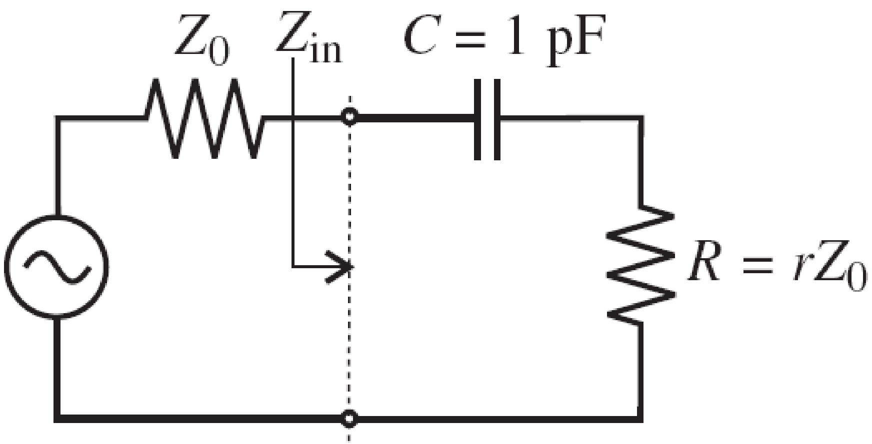
- Frequency dependent impedance behavior → for resistance values r = 0.3, 0.5, 0.7, and 1 for 500 MHz to 4 GHz range → for fixed inductance of 10 nH and Z_0 = 50Ω.



Parallel and Series Connections of RLC Elements (contd.)

Series Connection of R and C

- Let us consider the following circuit



Normalized impedance z_{in}' will be in lower part of Z-Smith Chart

$$z_{in}'(\omega) = r - j\frac{1}{\omega C Z_0}$$

- We can compute the normalized impedance as:

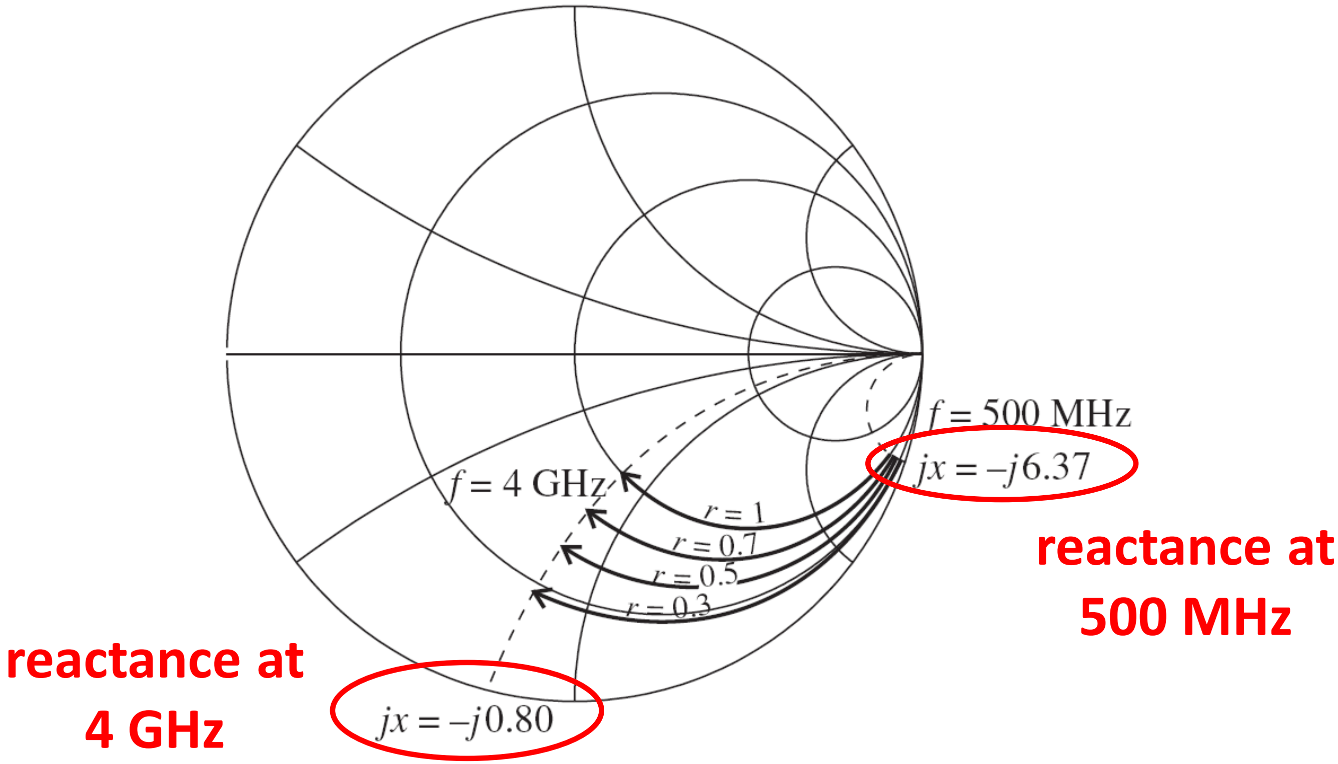
$$r = \frac{R}{Z_0} \qquad x_C = -\frac{1}{\omega C Z_0}$$

For a constant resistance (r) circle and variable frequency → impedance will be a curve along the resistance circle

Parallel and Series Connections of RLC Elements (contd.)

Series Connection of R and C

- Frequency dependent impedance behavior → for resistance values r = 0.3, 0.5, 0.7, and 1 for 500 MHz to 4 GHz range → for fixed capacitance of 1 pF and Z_0 = 50Ω.



High Frequency Networks

- **Requirement of Matrix Formulation**

Current/Voltage or Incident/Reflected Traveling Wave ⟺ directional coupler (more than one port) ⟺ Current/Voltage or Incident/Reflected Traveling Wave

Can we characterize this using an impedance or admittance!

These are called networks

NO!!

What is the way?

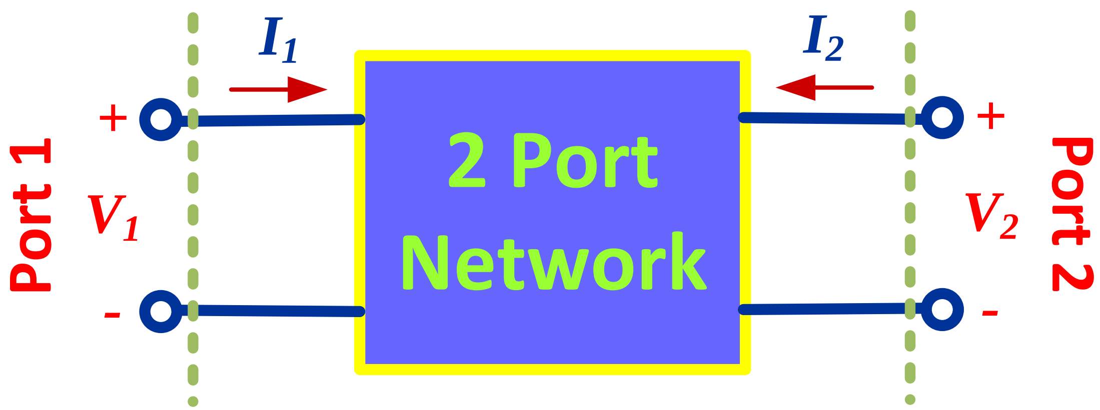
Impedance or Admittance Matrix. Right?

In principle, N by N impedance matrix completely characterizes a linear N-port device. Effectively, the impedance matrix defines a multi-port device the way a Z_L describes a single port device (e.g., a load)

Linear networks can be completely characterized by parameters measured at the network ports without knowing the content of the networks.

Multiport Networks

- Networks can have any number of ports – however, analysis of a 2-port, 3-port or 4-port network is sufficient to explain the theory and the associated concepts



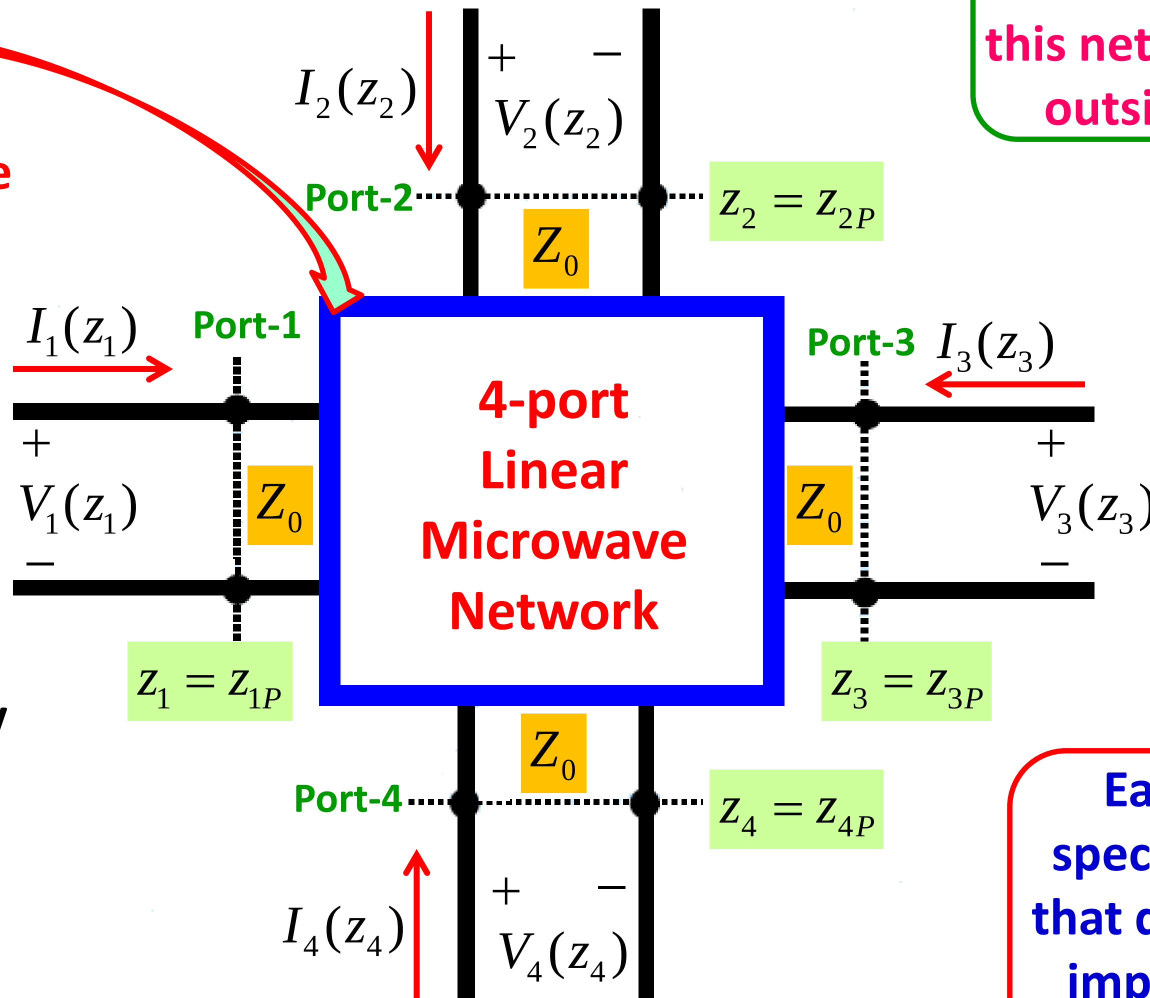
- The ports can be characterized with many parameters (Z, Y, S, ABDC). Each has a specific advantage.
- For 2-port Network, each parameter set is related to 4 variables:
  - 2 independent variables for excitation
  - 2 dependent variables for response

The Impedance Matrix

- Let us consider the following 4-port network:

Four identical TLs used to connect this network to the outside world

This could be a simple linear device or a large/complex linear microwave system



Either way, the network can be fully described by its impedance matrix

Each TL has specific location that defines input impedances to the network

The arbitrary locations are known as ports of the network

The Impedance Matrix (contd.)

- In principle, the current and voltages at the port-n of networks are given as:

$$V_n(z_n = z_{nP}) \qquad\qquad I_n(z_n = z_{nP})$$

- However, the simplified formulations are:

$$V_n = V_n(z_n = z_{nP}) \qquad\qquad I_n = I_n(z_n = z_{nP})$$

- If we want to say that there exists a non-zero current at port-1 and zero current at all other ports then we can write as:

$$I_1 \neq 0 \qquad\qquad I_2 = I_3 = I_4 = 0$$

- In order to define the elements of impedance matrix, there will be need to measure/determine the associated voltages and currents at the respective ports. Suppose, if we measure/determine current at port-1 and then voltage at port-2 then we can define:

$$Z_{21} = \frac{V_2}{I_1} \Rightarrow \textbf{Trans-impedance}$$

The Impedance Matrix (contd.)

- Similarly, the trans-impedance parameters Z_{31} and Z_{41} are:

$$Z_{31} = \frac{V_3}{I_1} \qquad Z_{41} = \frac{V_4}{I_1}$$

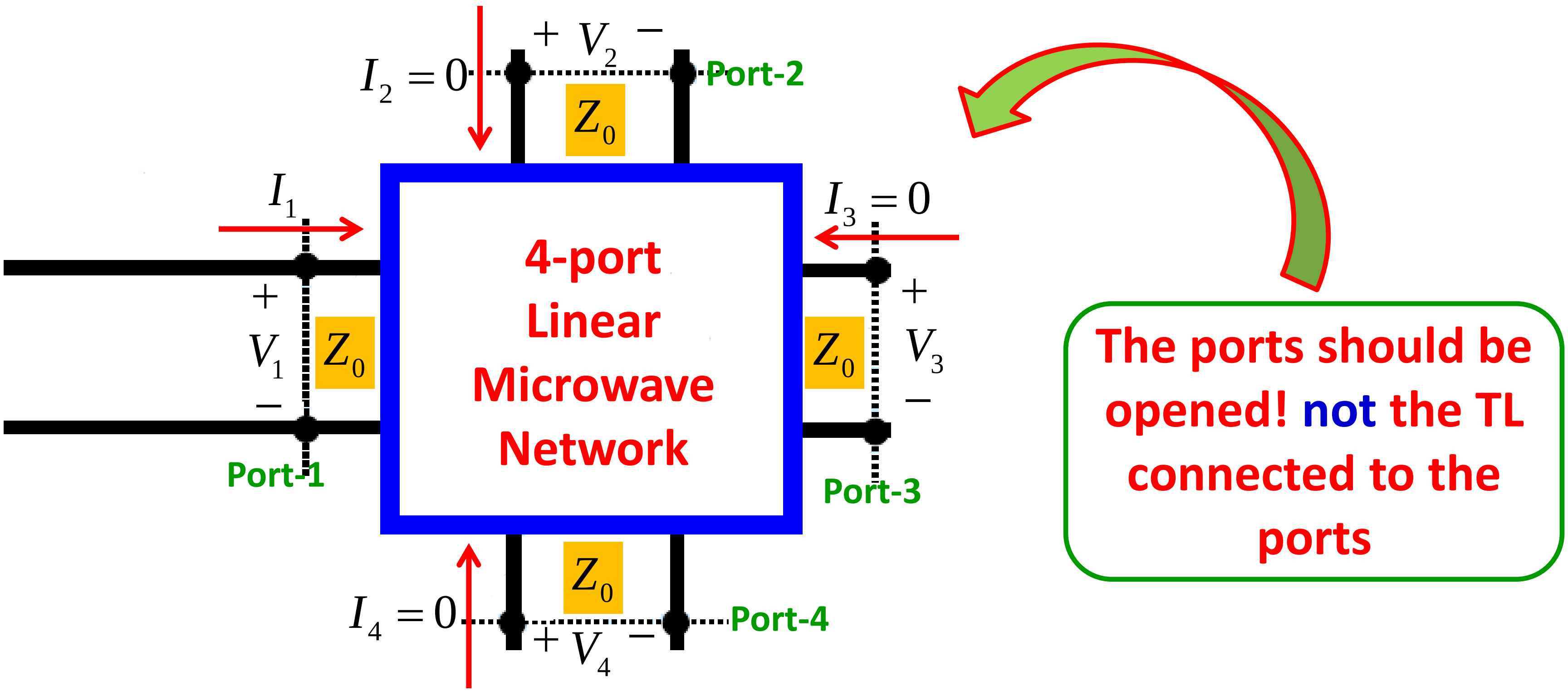
- We can also define other trans-impedance parameters such as Z_{34} as the ratio between the complex values I_4 (the current into port-4) and V_3 (the voltage at port-3), given that the currents at all other ports (1, 2, and 3) are zero.
- Therefore, the more generic form of trans-impedance is:

$$Z_{mn} = \frac{V_m}{I_n}$$ **(given that $I_k = 0$ for all $k \neq n$)**

How do we ensure that all but **one port** current is zero?

The Impedance Matrix (contd.)

- Open the ports where the current needs to be zero



- We can then define the respective trans-impedances as:

$$Z_{mn} = \frac{V_m}{I_n}$$

(given that all ports k≠n are open)

The Impedance Matrix (contd.)

- Once we have defined the trans-impedance terms by opening various ports, it is time to formulate the impedance matrix
- Since the network is **linear**, the voltage at any port due to all the port currents is simply the coherent **sum** of the voltage at that port due to **each** of the currents
- For example, the voltage at **port-3** is:

$$V_3 = Z_{34} I_4 + Z_{33} I_3 + Z_{32} I_2 + Z_{31} I_1$$

- Therefore we can generalize the voltage for **N-port** network as:

$$V_m = \sum_{n=1}^{N} Z_{mn} I_n$$

$$\Rightarrow \mathbf{V} = \mathbf{ZI}$$

- Where **I** and **V** are vectors given as:

$$\mathbf{V} = \left[\mathrm{V}_1, \mathrm{V}_2, \mathrm{V}_3, ..., \mathrm{V}_\mathrm{N} \right]^T \qquad \mathbf{I} = \left[\mathrm{I}_1, \mathrm{I}_2, \mathrm{I}_3, ..., \mathrm{I}_\mathrm{N} \right]^T$$

The Impedance Matrix (contd.)

- The term **Z** is matrix given by:

$$\mathbf{Z} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & & & \vdots \\ \vdots & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mn} \end{bmatrix}$$

Impedance Matrix

- The values of elements in the impedance matrix are frequency dependents and often it is advisable to describe impedance matrix as:

$$\mathbf{Z}(\omega) = \begin{bmatrix} Z_{11}(\omega) & Z_{12}(\omega) & \cdots & Z_{1n}(\omega) \\ Z_{21}(\omega) & & & \vdots \\ \vdots & & & \\ Z_{m1}(\omega) & Z_{m2}(\omega) & \cdots & Z_{mn}(\omega) \end{bmatrix}$$