# Lecture – 5

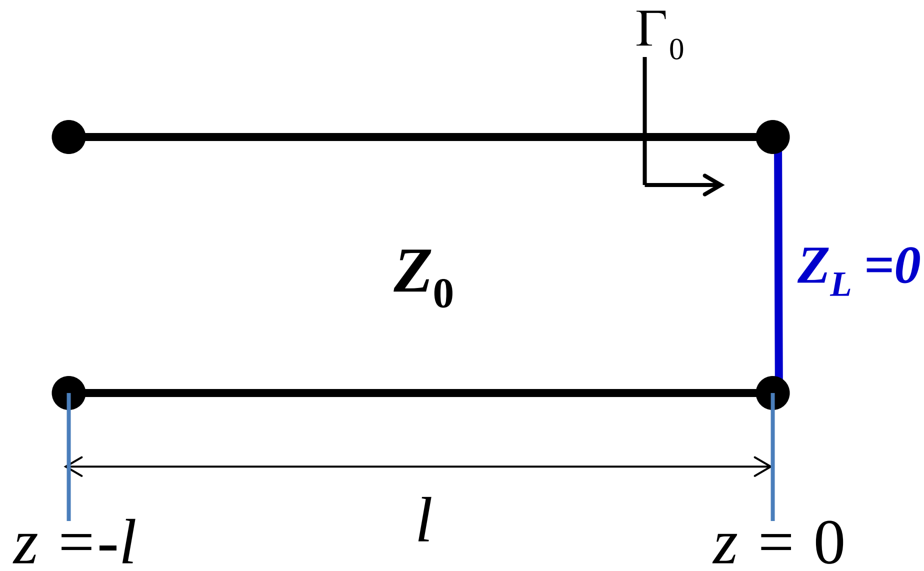
**Date: 19.08.2014**

- Review – Lecture 4
- Reflection Coefficient Transformation
- Power Considerations on a TL
- Return Loss, Insertion Loss, SWR etc.
- Sourced and Loaded TL
- Lossy TL

# Review – Lecture 4

- **Short-Circuited Line**

$$\Gamma_0 = \frac{0 - Z_0}{0 + Z_0} = -1$$




- The current and voltage along the TL is:

$$V(z) = V_0^+\left[e^{-j\beta z} - e^{+j\beta z}\right] = -j2V_0^+\sin(\beta z)$$

$$I(z) = \frac{2V_0^+}{Z_0}\cos(\beta z)$$

- The line impedance is:

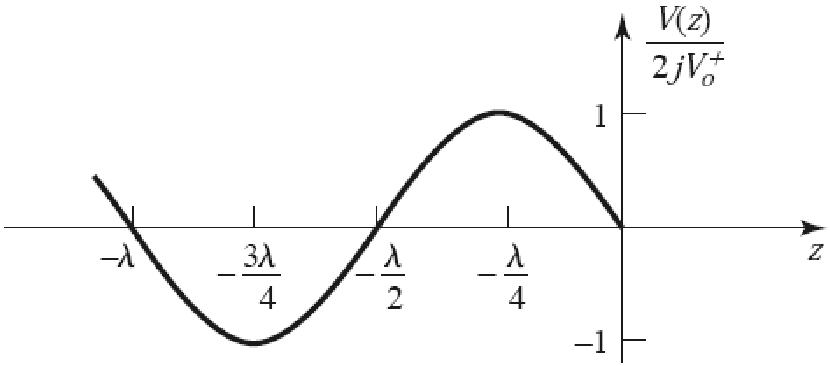
$$Z(z) = -jZ_0\tan(\beta z)$$

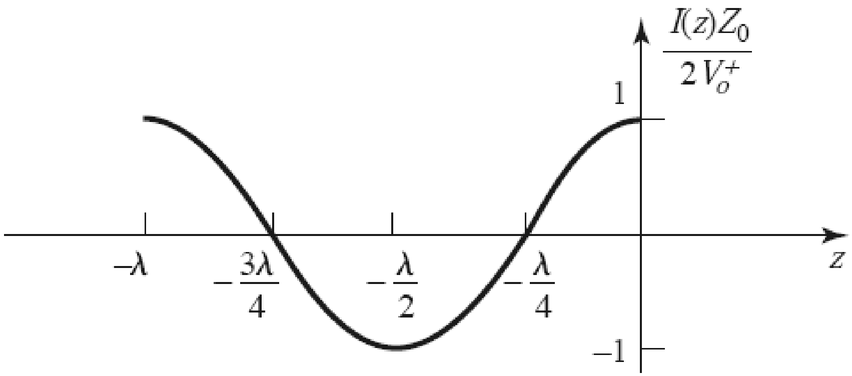
**Alternatively**

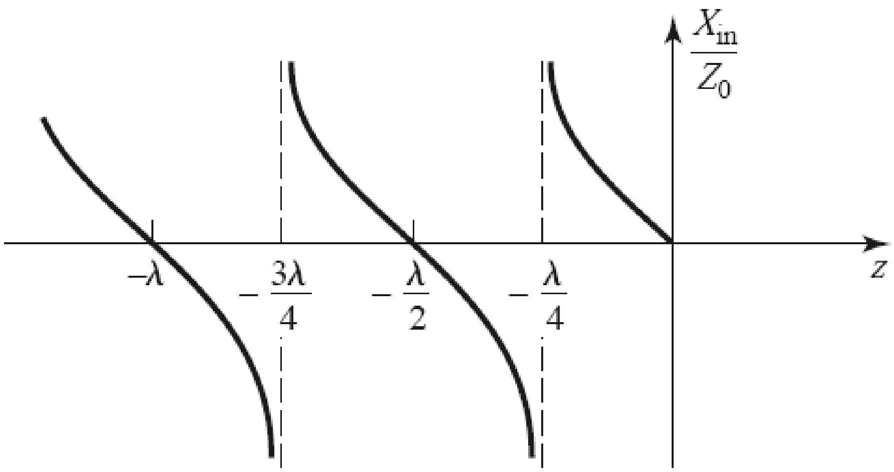
$$Z(z) = -jZ_0\tan\left(\frac{2\pi z}{\lambda}\right)$$

# Review – Lecture 4

- **Short-Circuited Line**



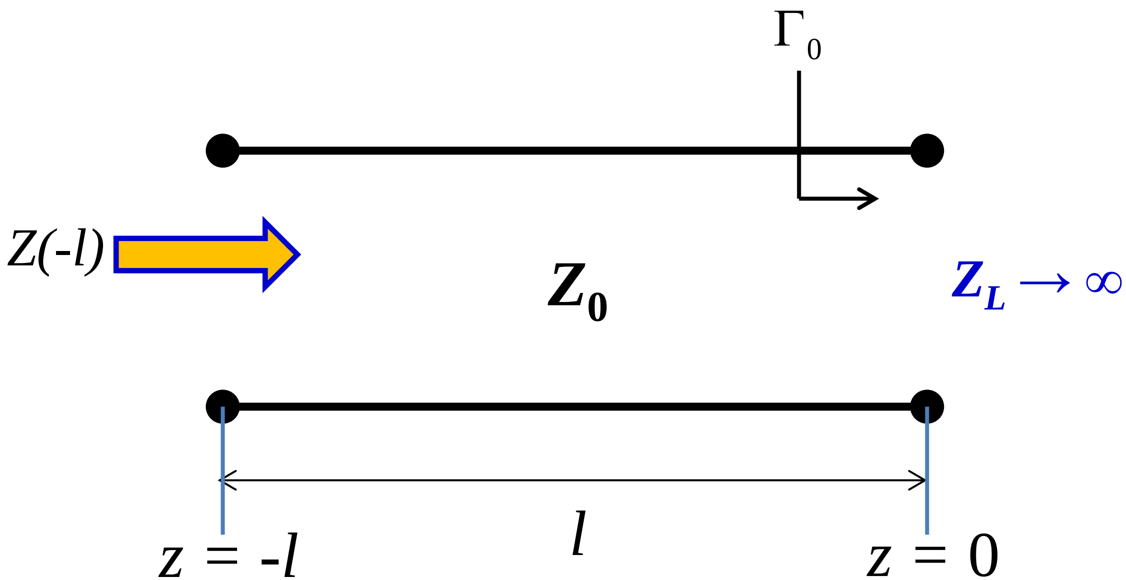




HW # 1 (part-1): plot these curves using MATLAB and ADS for frequency range of your choice.

# Review – Lecture 4

- **Open-Circuited Line**

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0} = 1$$




- The current and voltage along the TL is:

$$V(z) = V_0^+\left[e^{-j\beta z} + e^{+j\beta z}\right] = 2V_0^+\cos(\beta z)$$

$$I(z) = -j\frac{2V_0^+}{Z_0}\sin(\beta z)$$

- The line impedance is:

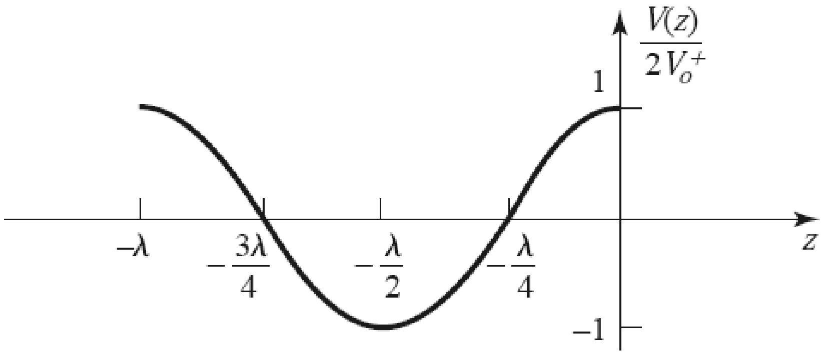
$$Z(-l) = -jZ_0\cot(\beta l)$$

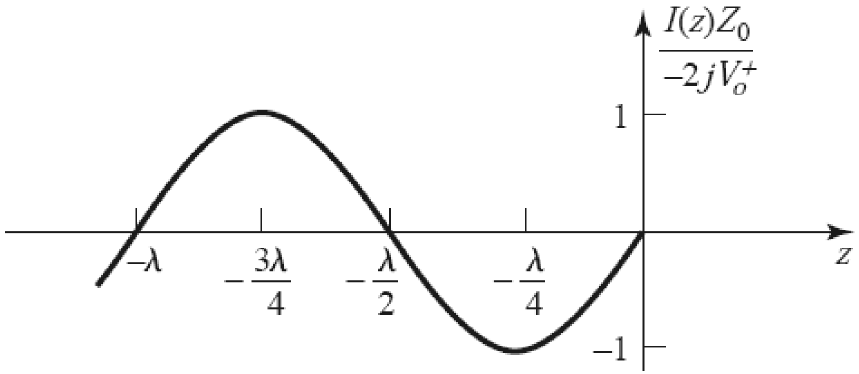
**Alternatively**

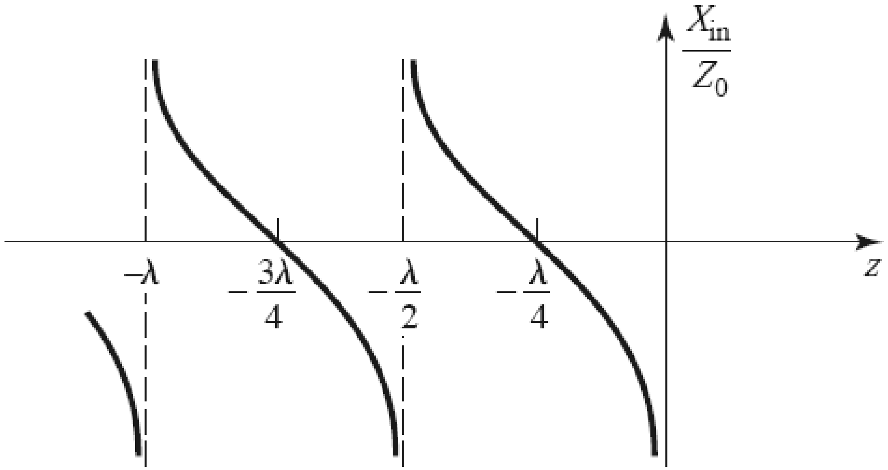
$$Z(-l) = -jZ_0\cot\left(\frac{2\pi l}{\lambda}\right)$$

# Review – Lecture 4

- <u>Open-Circuited Line</u>




**HW # 1 (part-2): plot these curves using MATLAB and ADS for frequency range of your choice.**

# Example – 1

A transistor has an input impedance of $Z_L$ = 25Ω → this needs to be matched to a 50Ω microstrip line at f = 500 MHZ by using a quarter-wave parallel-plate impedance transformer → Find the length, width and $Z_{line}$ (which also equals the characteristic impedance of the parallel-plate line) → The thickness of the dielectric is 1mm and relative dielectric constant of the material is 4. Use formulation for **inductance/m as μ*l*/w** and **capacitance/m as ε*l*/d**. Ignore R and G.

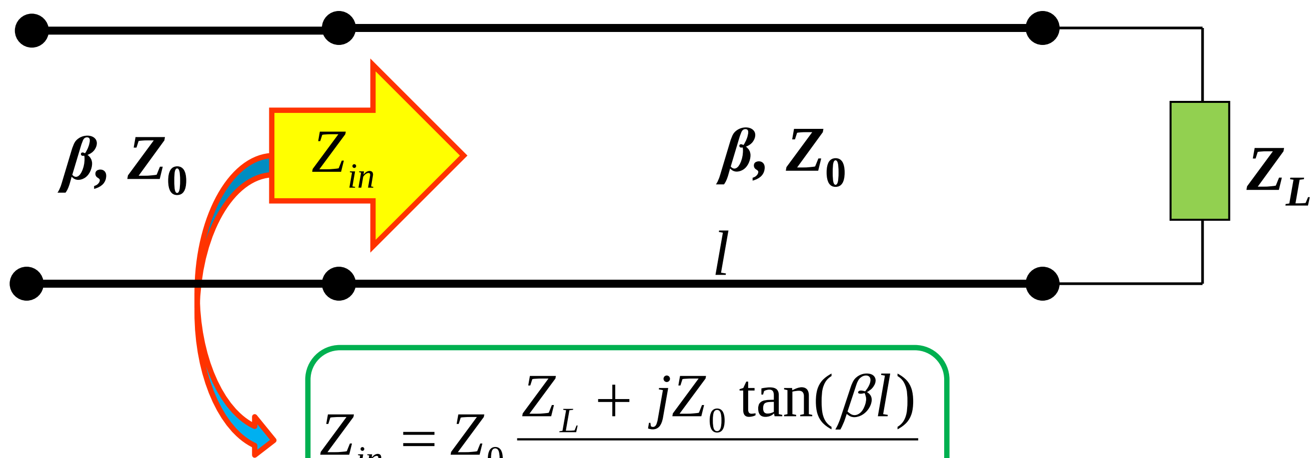
# Reflection Coefficient Transformation

- We know that the **load** at the end of some length of a transmission line (with characteristic impedance $Z_0$) can be specified in terms of its impedance $Z_L$ **or** its reflection coefficient $\Gamma_0$.
- Note **both** values are complex, and **either one** completely specifies the load—if you know **one**, you know the **other**!

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_L = Z_0\left(\frac{1+\Gamma_0}{1-\Gamma_0}\right)$$

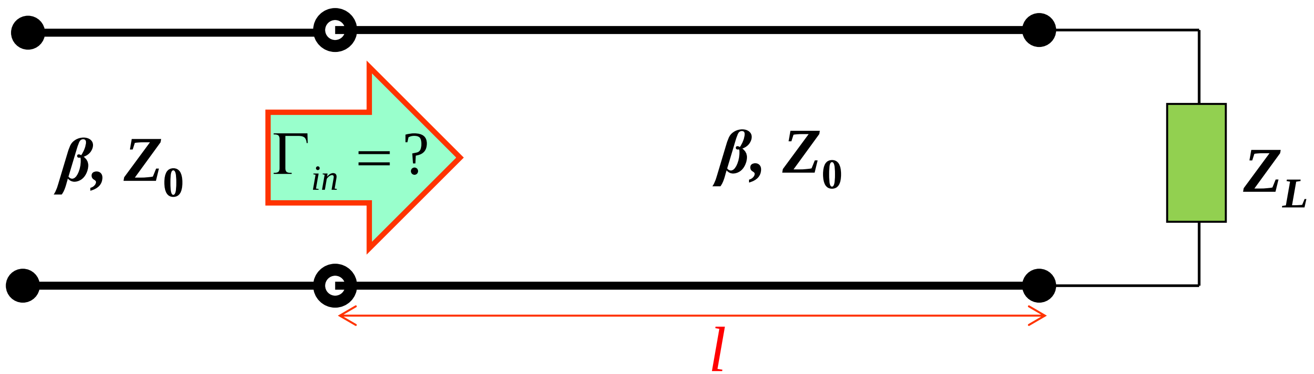
- Recall that we determined how a length of transmission line **transformed** the load **impedance** into an input **impedance** of a (generally) different value:




$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

# Reflection Coefficient Transformation (contd.)

**Q:** Say we know the load in terms of its **reflection coefficient**. How can we express the **input** impedance in terms its **reflection coefficient** (call this $\Gamma_{in}$)?




**A:** Well, we **could** execute these **three** steps:

1. Convert $\Gamma_0$ to $Z_L$:

$$Z_L = Z_0\left(\frac{1+\Gamma_0}{1-\Gamma_0}\right)$$

2. Transform $Z_L$ down the line to $Z_{in}$:

$$Z_{in} = Z_0 \frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}$$

3. Convert $Z_{in}$ to $\Gamma_{in}$:

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

# Reflection Coefficient Transformation (contd.)

**Q:** Yikes! This is a **ton** of complex arithmetic—isn't there an **easier** way?

**A:** Actually, there **is**!

- Recall that the input impedance of a transmission line length $l$, terminated with a load $\Gamma_0$, is:

$$Z_{in} = Z(z=-l) = \frac{V(z=-l)}{I(z=-l)} = Z_0\left(\frac{e^{j\beta l} + \Gamma_0 e^{-j\beta l}}{e^{j\beta l} - \Gamma_0 e^{-j\beta l}}\right)$$

Note this **directly** relates $\Gamma_0$ to $Z_{in}$ (steps 1 and 2 combined!).

Directly insert this into:

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

**directly** relates $\Gamma_0$ to $\Gamma_{in}$.

$$\Gamma_{in} = \Gamma_0 e^{-j2\beta l}$$

**Q:** Hey! This result looks **familiar**.

**A:** Absolutely! Recall that we found the reflection coefficient **function** $\Gamma(z)$:

$$\Gamma(z) = \Gamma_0 e^{j2\beta z} \quad \Rightarrow \quad \Gamma(z=-l) = \Gamma_0 e^{-j2\beta l}$$

# Reflection Coefficient Transformation (contd.)

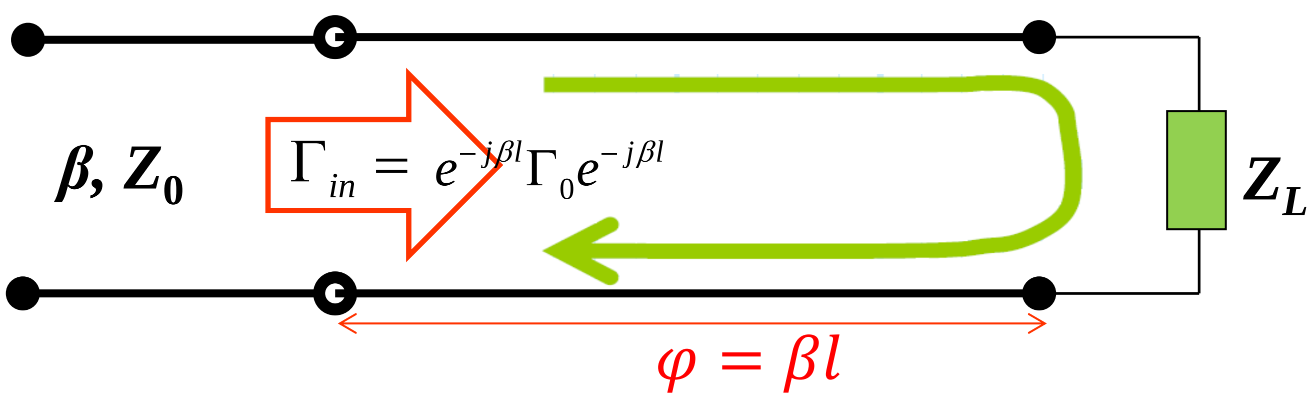
$$\Gamma_{in} = \Gamma_0 e^{-j2\beta l}$$

→ the **magnitude** of $\Gamma_{in}$ is the **same** as the magnitude of $\Gamma_0$!

→ $$|\Gamma_{in}| = |\Gamma_0 e^{-j2\beta l}| = |\Gamma_0|$$

The reflection coefficient at the input is simply related to $\Gamma_0$ by a **phase shift** of $2\beta l$.

Finally, the **phase shift** associated with transforming $\Gamma_0$ down a transmission line can be attributed to the phase shift associated with the wave propagating a length $l$ down the line, reflecting from load $Z_L$, and then propagating a length $l$ back up the line.



# Some Observations

$$\Rightarrow \Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

For Open Circuit Line: $Z_L \rightarrow \infty$

$$\Gamma_0 = 1$$

That means the wave gets fully reflected with the same polarity

For Short Circuit Line: $Z_L = 0$

$$\Gamma_0 = -1$$

That means the wave gets fully reflected with inverted amplitude

- **When, $Z_L = Z_0$** → the transmission line is matched that results in no reflection → $\Gamma_0 = 0$ → the incident voltage wave is completely absorbed by the load → a scenario that says a second transmission line with the same characteristic impedance but infinite length is attached at z = 0 → **VERY IMPORTANT CONCEPT**

# Some Observations (contd.)

- Let us revisit propagation constant and phase velocity

$$\gamma = \alpha + j\beta$$

For a lossless line →

$$\gamma = j\beta$$

Purely Imaginary

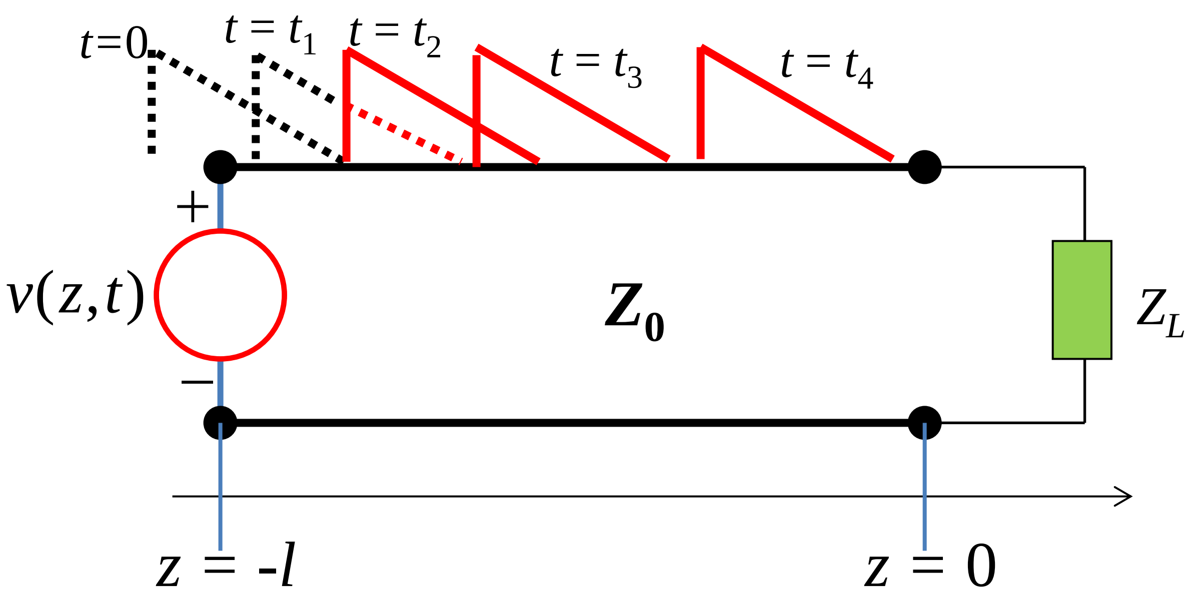
$$\beta = \omega\sqrt{LC}$$

Frequency Dependent

- We also know: $v_p = \dfrac{1}{\sqrt{LC}}$

- It is apparent that the phase velocity is independent of frequency (instead it is dependent on line parameters) → It means that if a saw tooth voltage signal propagates down a line then each frequency components of this sawtooth travels with same fixed velocity → means original pulse will appear at a different location without changing shape

# Some Observations (contd.)




- Wave is emerging from the source end of the line, traveling down the line, and then being absorbed by the matched load → it is known as dispersion-free transmission
- In practical situation, there is always frequency dependence on phase velocity → dispersion happens → signal distortion takes place → **This property can be used in the design of dual-band/multi-band circuits!!**

# Some Observations (contd.)

- A terminated transmission line experiences standing waves
- It is due to superposition of two waves of the same frequency propagating in opposite directions
- The effect of standing waves is presence of series of nodes (zero displacement) and anti-nodes (maximum displacement) at fixed points along the transmission line
- The failure of the line to transfer power at the standing wave frequency results in attenuation distortion → lossless transmission line is required!!
- Losses in transmission line doesn't allow a perfect reflection and a pure standing wave → that results into partial standing wave → superposition of standing and traveling wave → The degree to which the propagating wave resembles a pure traveling or standing wave is measured in terms of standing wave ratio (SWR)

# Some Observations (contd.)

**Which relationship to use:**

$V(z), I(z), Z(z)$

**or**

$V^{+}(z), V^{-}(z), \Gamma(z)$

# Some Observations (contd.)

Based on your **circuits** experience, you might well be **tempted** to always use **V(z)**, **I(z)** and **Z(z)**.

However, it is **useful (as well as simple)** to describe activity on a transmission line in terms of $V^{+}(z)$, $V^{-}(z)$ and $\Gamma(z)$

# Some Observations (contd.)

- The solution of Telegrapher equations (the equations defining the current and voltages along a TL) boils down to determination of complex coefficients $V^+$, $V^-$, $I^+$ and $I^-$. Once these are known, we can describe all the quantities along the TL.
- For example, the wave representations are:

$$V^+(z) = V_0^+ e^{-j\beta z}$$

$$V^-(z) = V_0^- e^{j\beta z}$$

$$\Gamma(z) = \frac{V_0^-(z)}{V_0^+(z)} = \frac{V_0^-}{V_0^+} e^{j2\beta z}$$

**Magnitudes**

$$\left|V^+(z)\right| = V_0^+$$

$$\left|V^-(z)\right| = V_0^-$$

$$\left|\Gamma(z)\right| = \left|\frac{V_0^-}{V_0^+}\right|$$

**Relative Phases**

$$\arg\left\{V^+(z)\right\} = -\beta z$$

$$\arg\left\{V^-(z)\right\} = +\beta z$$

$$\arg\left\{\Gamma(z)\right\} = +2\beta z$$

# Some Observations (contd.)

- Contrast the wave functions with complex voltage, current and impedance

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$I(z) = \frac{V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}}{Z_0}$$

$$Z(z) = Z_0 \frac{V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}}{V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}}$$

**Magnitudes**

$$|V(z)| = \left| V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \right| = ??$$

$$|I(z)| = \frac{\left| V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z} \right|}{Z_0} = ??$$

$$|Z(z)| = Z_0 \frac{\left| V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \right|}{\left| V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z} \right|} = ??$$

**Relative Phases**

$$\arg\{V(z)\} = \arg\left\{ V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \right\} = ??$$

$$\arg\{I(z)\} = \arg\left\{ V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z} \right\} = ??$$

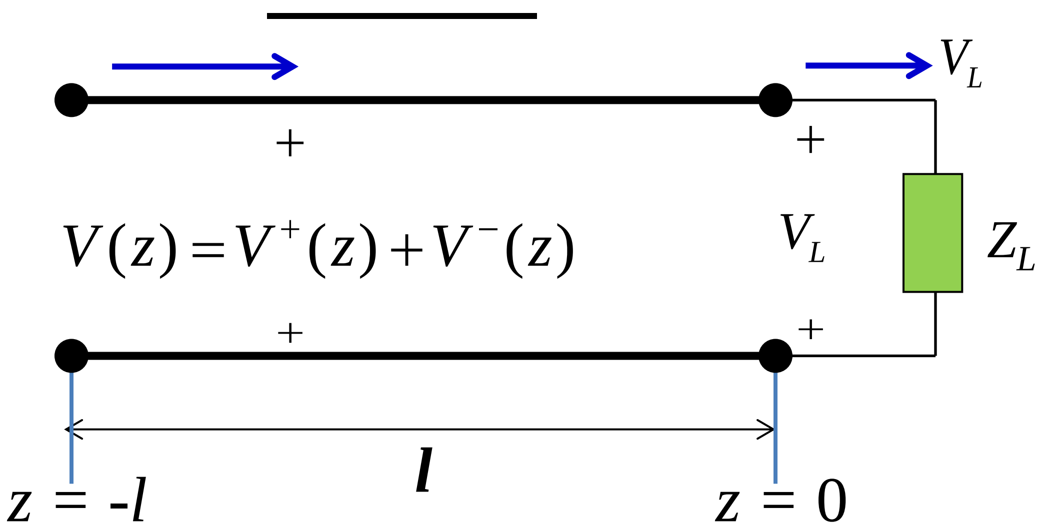
$$\arg\{Z(z)\} = \arg\left\{ V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \right\} - \arg\left\{ V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z} \right\} = ??$$

# Some Observations (contd.)

- It is thus apparent that the description of quantities along a transmission line — as a function of **position** z — is much **easier** and more **straightforward** to use the **wave** representation.
- However, this does **not** mean that we **never** determine V(z ), I(z ), or Z(z ); these quantities are still **fundamental** and very important—particularly at each **end** of the transmission line!

# Power Considerations on a TL

- We have discovered that **two waves propagate** along a transmission line, one in each direction $(V^+(z)\ and\ \ V^-(z))$.




The result is that electromagnetic energy flows along the transmission line at a given rate (i.e., **power**).

**Q:** How much power flows along a transmission line, and where does that power go?

**A:** We can answer that question by determining the power **absorbed** by the **load**!

# Power Considerations on a TL (contd.)

- Expression for Time-Averaged Power Absorbed by load $Z_L$ is:

$$P_{abs} = \frac{1}{2}\mathrm{Re}\left(V_L I_L^*\right) = \frac{1}{2}\mathrm{Re}\left(V(0)I(0)^*\right) = \frac{\left|V_0^+\right|^2}{2Z_0}\left(1-\left|\Gamma_0\right|^2\right)$$

$$P_{abs} = \frac{\left|V_0^+\right|^2}{2Z_0} - \frac{\left|V_0^+\Gamma_0\right|^2}{2Z_0} = \frac{\left|V_0^+\right|^2}{2Z_0} - \frac{\left|V_0^-\right|^2}{2Z_0}$$

Incident Power, $P_{inc}$

Reflected Power, $P_{ref}$

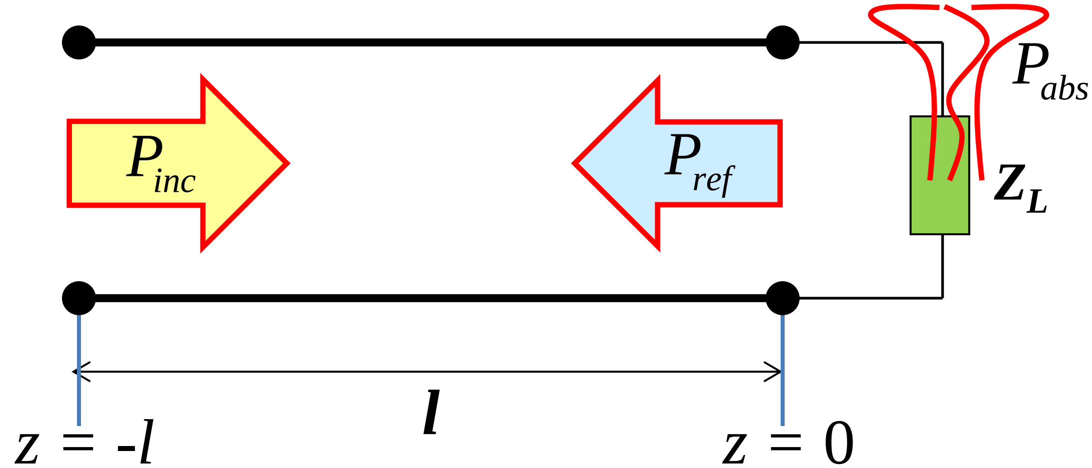
$$P_{ref} = \frac{\left|V_0^+\Gamma_0\right|^2}{2Z_0} = \left|\Gamma_0\right|^2 P_{inc}$$

$$\therefore P_{abs} = P_{inc} - P_{ref} \qquad \Rightarrow P_{inc} = P_{abs} + P_{ref}$$

Conservation of Energy

# Power Considerations on a TL (contd.)

- It is thus apparent that the power flowing towards the load ($P_{inc}$) is either absorbed by the load ($P_{abs}$) or reflected back from the load ($P_{ref}$)



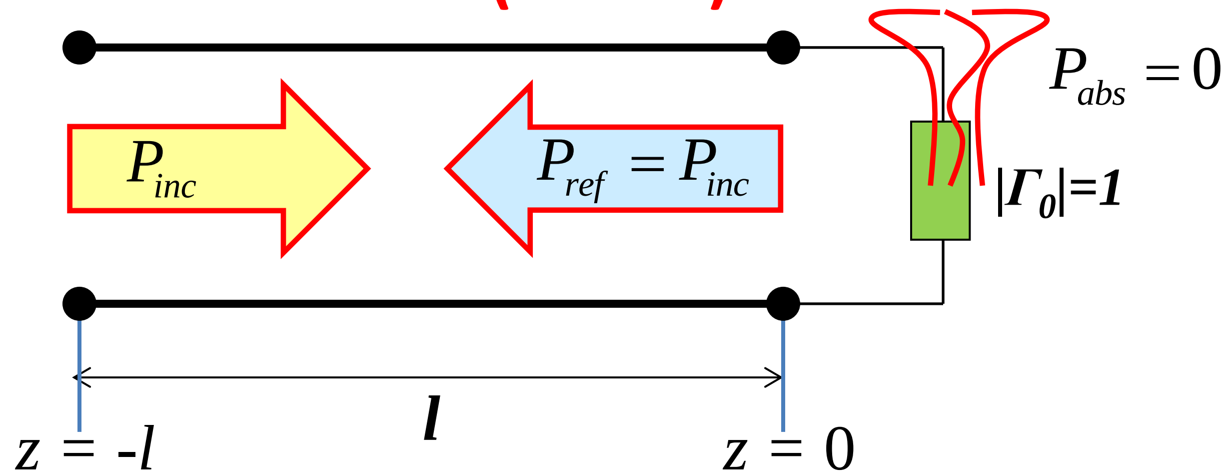

Now let us consider some special cases:

**1.** $|\Gamma_0| = 1$ **In this case:** $P_{ref} = |\Gamma_0|^2 P_{inc} = P_{inc} \quad \Rightarrow P_{abs} = 0$

**There is no power absorbed by the load → all the incident power is reflected**

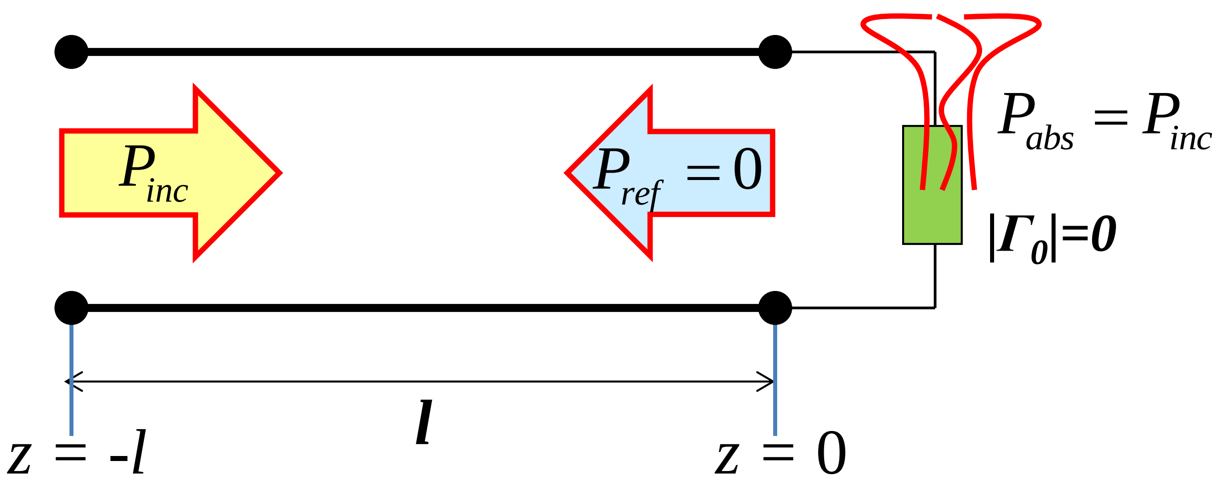
# Power Considerations on a TL (contd.)

**1.** $|\Gamma_0| = 1$




**2.** $|\Gamma_0| = 0$

**In this case:** $P_{ref} = |\Gamma_0|^2 P_{inc} = 0 \quad \Rightarrow P_{abs} = P_{inc}$




**all the incident power is absorbed by the load**

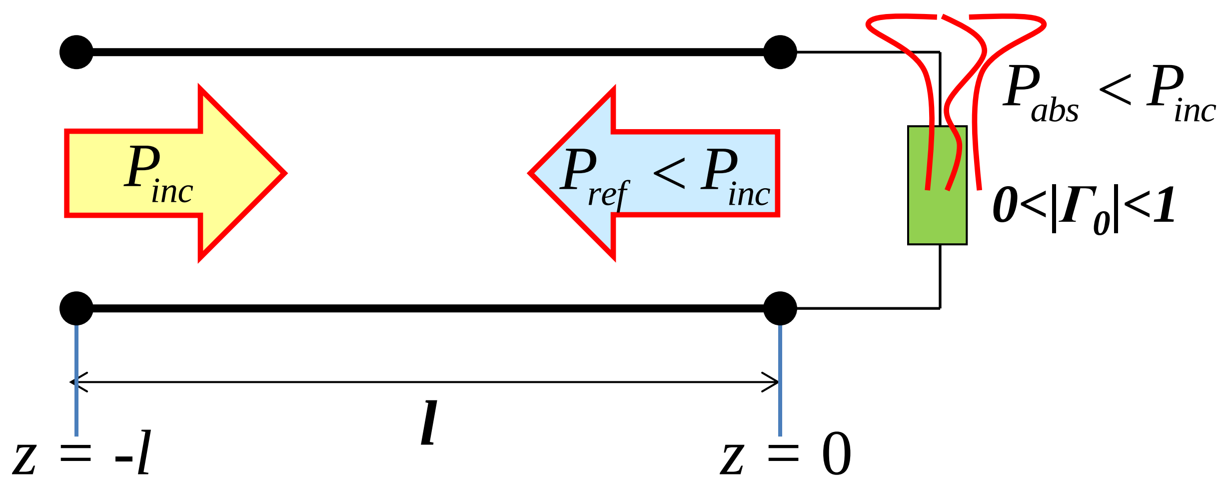
**None** of the incident power is **reflected**

# Power Considerations on a TL (contd.)

**3.** $0 < \left|\Gamma_0\right| < 1$

**In this case:** $0 < P_{ref} = \left|\Gamma_0\right|^2 P_{inc} < 0 \qquad \Rightarrow 0 < P_{abs} = P_{inc}\left(1 - \left|\Gamma_0\right|^2\right) < P_{inc}$

**In this case the incident power is divided → some of the incident power is absorbed by the loads whereas the remainder is reflected from the load**



# Power Considerations on a TL (contd.)

**4.** $|\Gamma_0| > 1$

Power Absorbed is Negative

**In this case:** $P_{ref} = |\Gamma_0|^2 P_{inc} > P_{inc}$ $\Rightarrow P_{abs} = P_{inc}\left(1 - |\Gamma_0|^2\right) < 0$

Alternatively, we can say that the load creates extra power → i.e, acts as a power source and not a sink!

What type of load it could be?

Definitely not a passive load → A passive device can't produce power

Therefore: $|\Gamma_0| \leq 1$ For all passive loads

# Power Considerations on a TL (contd.)

**Q:** Can $\Gamma_0$ every be **greater** than one?

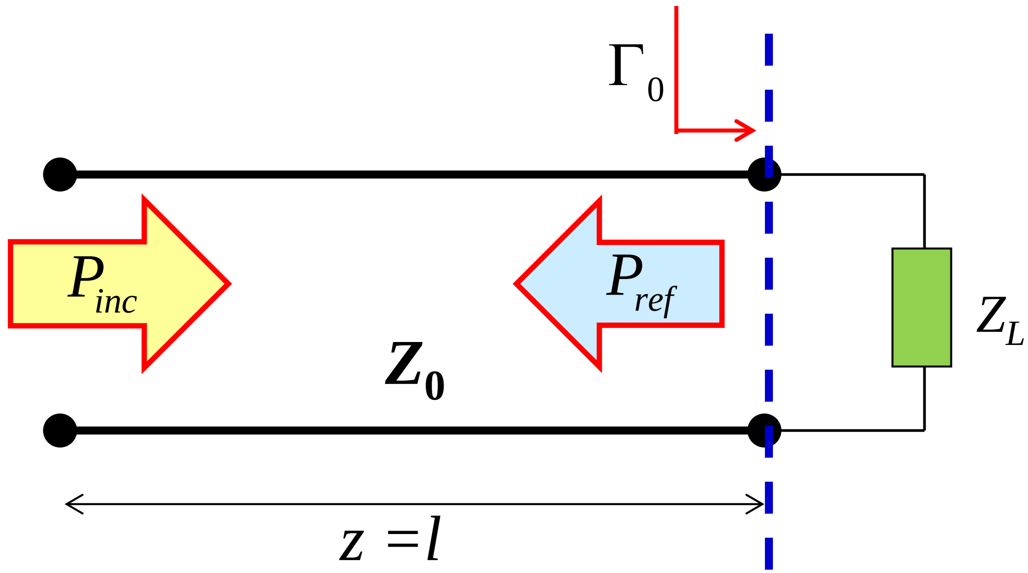
**A:** Sure, if the "load" is an **active** device. In other words, the load must have some **external power** source connected to it.

**Q:** What about the case where $|\Gamma_0| < 0$, shouldn't we examine **that** situation as well?

**A:** That would be just plain **silly**; do **you** see why?

# Return Loss

- The **ratio** of the reflected power from a load, to the incident power on that load, is known as **return loss**. Typically, return loss is expressed in **dB:**




**Return Loss (R.L.):** $RL[dB] = -10\log\left(\frac{P_{ref}}{P_{inc}}\right) = -10\log\left(\left|\Gamma_0\right|^2\right)$

# Return Loss (contd.)

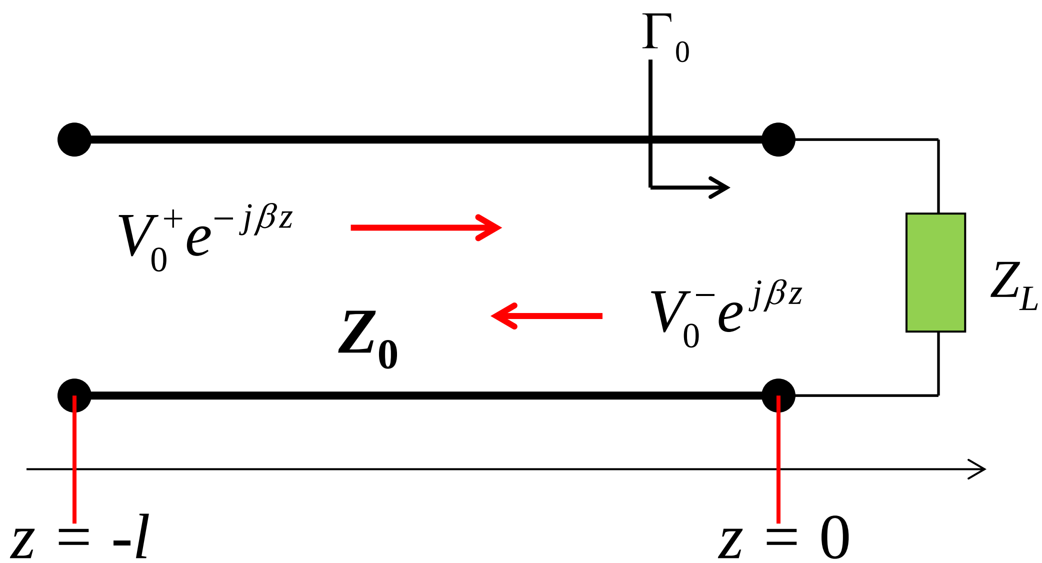
## Summary

- The return loss tells us the percentage of the incident power reflected at the point of mismatch
- For example, if the return loss is **10dB**, then **10%** of the power is **reflected** while the **90%** is **absorbed/transmitted** → i.e, we lose 10% of the incident power
- For the **return loss of 30dB**, the **reflected power is 0.1%** of the incident power → **we lose only 0.1% of the incident power**
- A larger numeric value of return loss actually indicates smaller lost power → **An ideal return loss would be ∞** → **matched condition**
- A **return loss** of **0dB** indicates that **reflection coefficient** is **ONE** → reactive termination
- Return Loss **(RL)** is very helpful as it provides **real-valued** measures of mismatch (unlike the **complex-valued** $Z_L$ and $\Gamma_0$)

A match is good if the return loss is high. A high return loss is desirable and results in a lower insertion loss.

# Standing Wave and Standing Wave Ratio

- Another traditional real-valued measure of load match is **Voltage Standing Wave Ratio** (VSWR). Consider again the **voltage** along a terminated transmission line, as a function of **position** $z$.




$$V(z) = V_0^+ \left[ e^{-j\beta z} + \Gamma_0 e^{+j\beta z} \right]$$

$$V(-l) = V_0^+ \left[ e^{j\beta l} + \Gamma_0 e^{-j\beta l} \right]$$

- For a short circuited line: $\Gamma_0 = -1$ ⇨ $V(-l) = V_0^+ \left( e^{+j\beta l} - e^{-j\beta l} \right)$

$2j\sin(\beta l)$

# Standing Wave and Standing Wave Ratio (contd.)

$$v(-l,t) = \text{Re}\left(V(-l)e^{j\omega t}\right) = \text{Re}\left(2jV_0^+(z)\sin(\beta l)e^{j\omega t}\right)$$

$$\therefore v(-l,t) = 2V_0^+\sin(\beta l)\cos(\omega t + (\pi/2))$$

**Definitely not a traveling wave!!**

**Always zero for $-l$=0 i.e., the point of short-circuit**

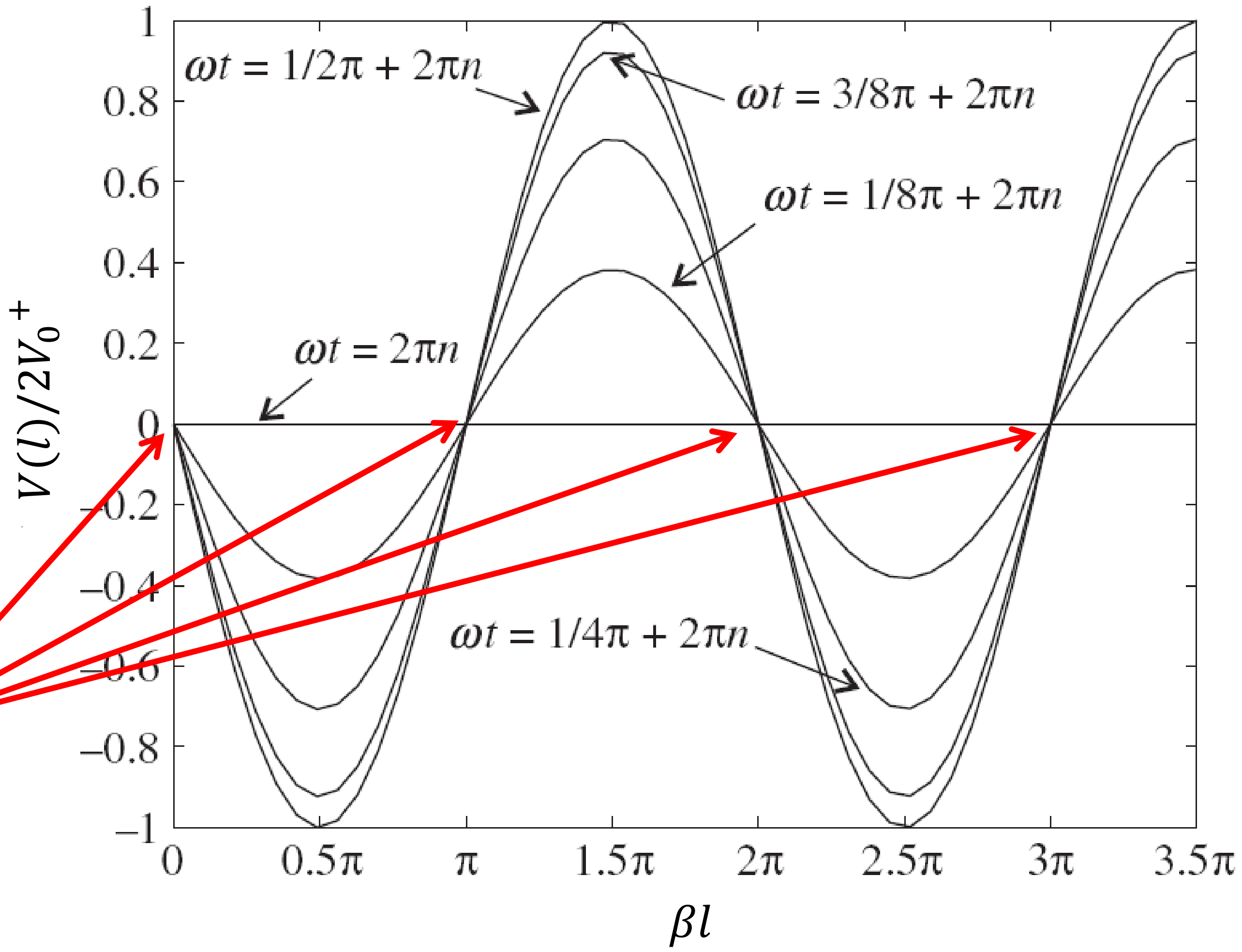
**Where has the traveling wave V(z) gone?**

- **As the time and space are decoupled → No wave propagation takes place**
- **The incident wave is 180° out of phase with the reflected wave → gives rise to zero crossings of the wave at 0, λ/2, λ, 3λ/2, and so on → standing wave pattern!!!**

# Standing Wave and Standing Wave Ratio (contd.)

**Corresponding Electrical Length ($\beta l$):**
**$0, \pi, 2\pi, 3\pi$**

**Spatial Location:**
**$0, \lambda/2, \lambda, 3\lambda/2$**




**Standing Wave Pattern for Various Instances of Time**

# Standing Wave and Standing Wave Ratio (contd.)

→ **for arbitrarily terminated line:**

$$V(-l) = V_0^+\left(e^{+j\beta l} + \Gamma_0 e^{-j\beta l}\right) = V_0^+ e^{+j\beta l}\left(1 + \Gamma_0 e^{-j2\beta l}\right)$$

A(-*l*) — $V_0^+ e^{+j\beta l}$

Γ(-*l*) — $\Gamma_0 e^{-j2\beta l}$

$$\Rightarrow V(-l) = A(-l)\left(1+\Gamma(-l)\right)$$

**Valid anywhere on the line**

**Similarly:** $$I(-l) = \frac{A(-l)}{Z_0}\left(1-\Gamma(-l)\right)$$

**Valid anywhere on the line**

- Under the matched condition, $\Gamma_0 = 0$ and therefore $\Gamma(-l) = 0$ → as expected, only positive traveling wave exists.
- For other arbitrary impedance loads: Standing Wave Ratio (SWR) or Voltage Standing Wave Ratio (VSWR) is the measure of mismatch.

# Standing Wave and Standing Wave Ratio (contd.)

- SWR is defined as the ratio of maximum voltage (or current) amplitude and the minimum voltage (or current) amplitude along a line → therefore, for an arbitrarily terminated line:

$$VSWR = ISWR = SWR = \left|\frac{V(-l)_{\max}}{V(-l)_{\min}}\right| = \left|\frac{I(-l)_{\max}}{I(-l)_{\min}}\right|$$

**We have:** $V(-l) = V_0^+ e^{+j\beta l}\left(1 + \Gamma_0 e^{-j2\beta l}\right)$

Recall this is a **complex** function, the magnitude of which expresses the **magnitude** of the **sinusoidal signal** at position $z$, while the phase of the complex value represents the relative **phase** of the sinusoidal signal.

- Therefore two possibilities for extreme values:

$$\Gamma_0 e^{-j\beta l} = 1 \qquad \Gamma_0 e^{-j\beta l} = -1$$

# Standing Wave and Standing Wave Ratio (contd.)

**Max. voltage:** $|V(-l)|_{max} = |V_0^+|(1+|\Gamma_0|)$ **Min. voltage:** $|V(-l)|_{min} = |V_0^+|(1-|\Gamma_0|)$

$$\therefore VSWR = \frac{1+|\Gamma_0|}{1-|\Gamma_0|}$$

**Apparently:** $0 \le \Gamma_0 \le 1$ → $\therefore 1 \le VSWR < \infty$

- Note if $|\Gamma_0| = 0$ (i.e., $Z_L = Z_0$), then VSWR = 1. We find for this case:

$$|V(z)|_{max} = |V(z)|_{min} = |V_0^+|$$

In other words, the voltage magnitude is a **constant** with respect to position $z$.

- Conversely, if $|\Gamma_0| = 0$ (i.e., $Z_L = Z_0$), then VSWR = ∞. We find for **this** case:

$$|V(z)|_{max} = |V(z)|_{min} = |V_0^+|$$

$$|V(z)|_{max} = |V(z)|_{min} = |V_0^+|$$

In other words, the voltage magnitude varies **greatly** with respect to position $z$.

# Standing Wave and Standing Wave Ratio (contd.)

- Similarly,

We have: $$I(-l) = \frac{V^+}{Z_0}\left(e^{+j\beta l} + \Gamma_0 e^{-j\beta l}\right)$$

$$\left|I(d)\right|_{max} = \left(\frac{V^+}{Z_0}\right)\left(1 + \left|\Gamma_0\right|\right)$$ **and** $$\left|I(d)\right|_{min} = \left(\frac{V^+}{Z_0}\right)\left(1 - \left|\Gamma_0\right|\right)$$

$$\therefore ISWR = \frac{1+\left|\Gamma_0\right|}{1-\left|\Gamma_0\right|}$$

$$\therefore 1 \le ISWR < \infty$$

**Thus: VSWR=ISWR=SWR**

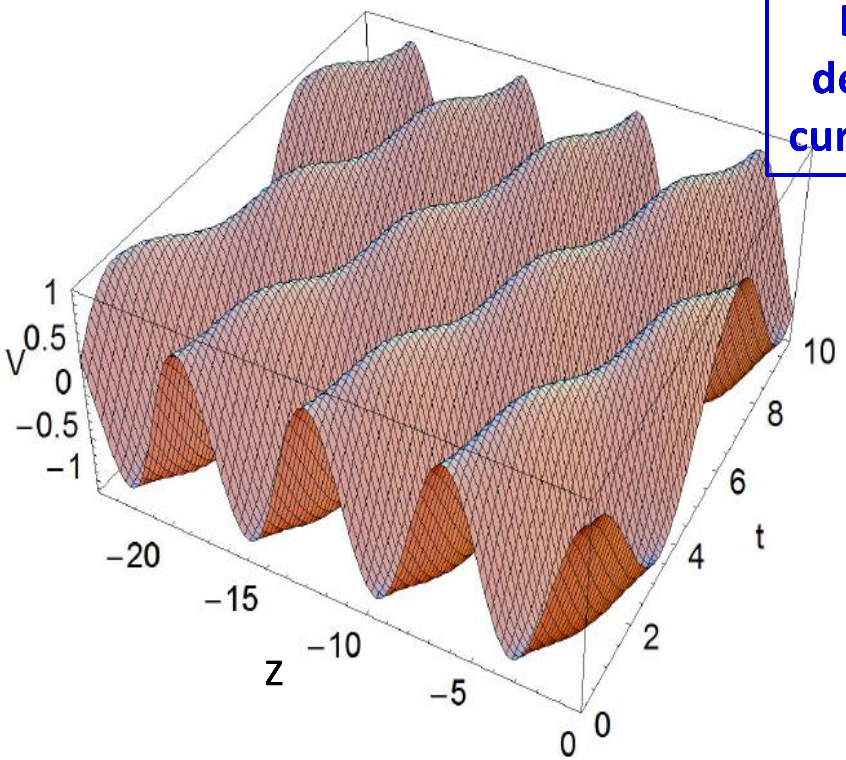
In our course we will mention both as VSWR

As with **return loss**, VSWR is dependent on the **magnitude** of $|\Gamma_0|$ (i.e, $|\Gamma_0|$) **only** !

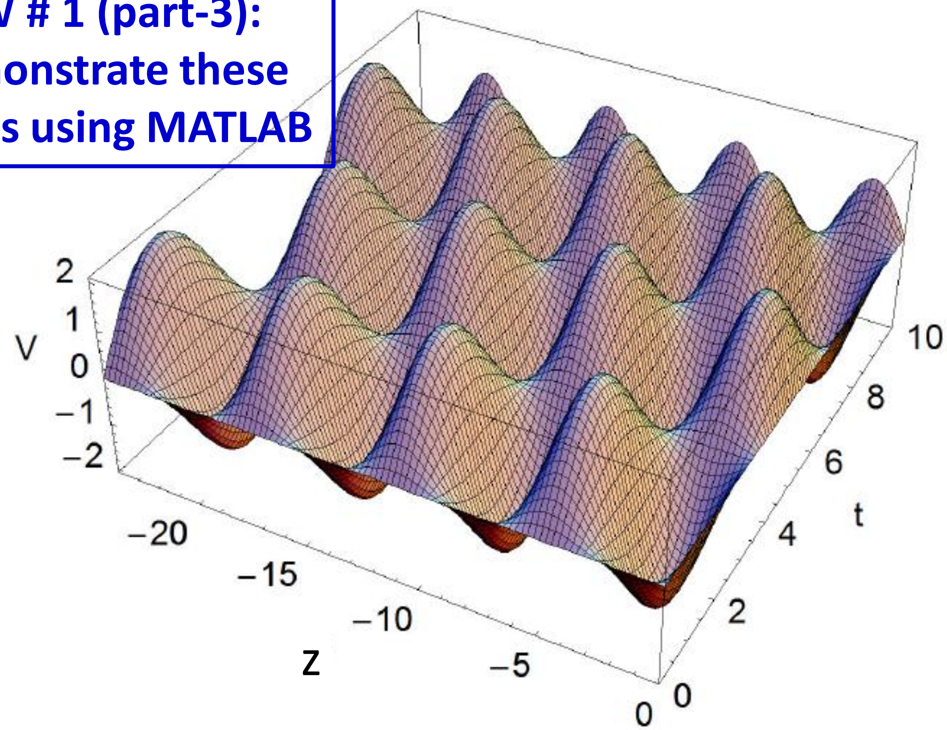
**In practice, SWR can only be defined for lossless line as the SWR equation is not valid for attenuating voltage and current**

# Standing Wave and Standing Wave Ratio (contd.)

**HW # 1 (part-3): demonstrate these curves using MATLAB**




**Standing Wave Pattern at $\Gamma_0$=0.1**




**Standing Wave Pattern at $\Gamma_0$=1**

- **It is apparent that the maximum and minimum repeats periodically and its values can be used to identify the degree of mismatch by calculating the Standing Wave Ratio**

# Insertion Loss

- This is another parameter to address the mismatch problem and is defined as:

$$IL[dB] = -10\log\left(\frac{P_{transmitted}}{P_{incident}}\right) = -10\log\left(\frac{P_{incident} - P_{reflected}}{P_{incident}}\right) - 10\log\left(1 - |\Gamma_{in}|^2\right)$$

**For open- and short-circuit conditions**

$$IL \to \infty$$

**For perfectly matched conditions**

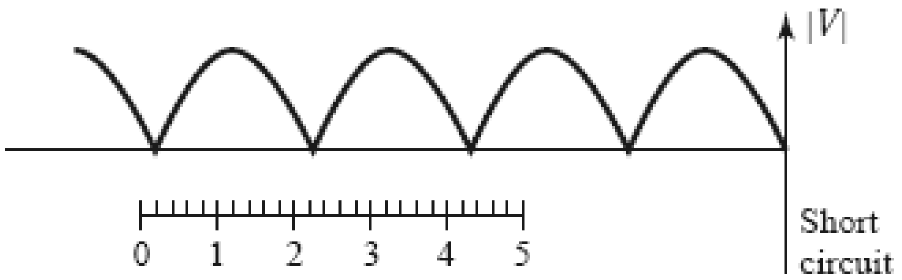
$$IL = 0$$

**insertion loss** signifies the loss of signal power resulting from the insertion of a device in a transmission line.

# Example – 2

- The following two-step procedure has been carried out with a $50\Omega$ coaxial slotted line to determine an unknown load impedance:

1. short circuit is placed at the load plane, resulting in a standing wave on the line with infinite SWR and sharply defined voltage minima, as shown in Figure.

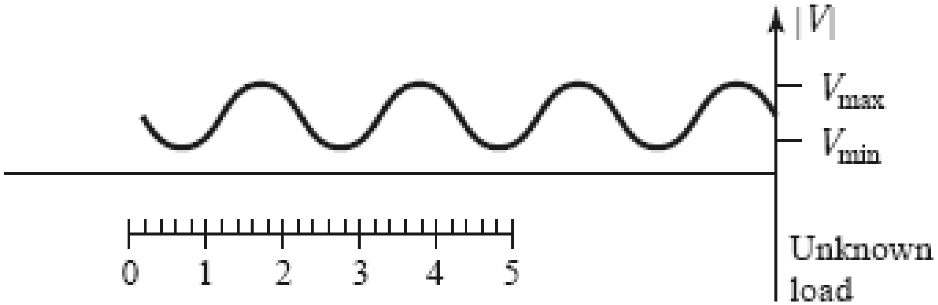



On the arbitrarily positioned scale on the slotted line, voltage minima are recorded at:

$$z = 0.2cm, \qquad 2.2cm, \qquad 4.2cm$$

# Example – 2 (contd.)

2. The short circuit is removed and replaced with the unknown load. The standing wave ratio is measured as SWR = 1.5, and voltage minima, which are not as sharply defined as those in step 1, are recorded at:




$$z = 0.72cm, \qquad 2.72cm, \qquad 4.72cm$$

**Find the load impedance.**

# Example – 2 (contd.)

- Knowing that voltage minima repeat every $\lambda/2$, we have from the data of step 1 that $\lambda$ = 4.0 cm.
- In addition, because the reflection coefficient and input impedance also repeat every $\lambda/2$, we can consider the load terminals to be effectively located at any of the voltage minima locations listed in step 1.
- Thus, if we say the load is at $4.2cm$, then the data from step 2 show that the next voltage minimum away from the load occurs at $2.72cm$.
- It gives:

$$l_{min} = 4.2 - 2.72 = 1.48cm = 0.37\lambda$$

- Now: $$|\Gamma_0| = \frac{SWR-1}{SWR+1} \Rightarrow |\Gamma_0| = \frac{1.5-1}{1.5+1} = 0.2$$

$$\theta_\Gamma = \pi + 2\beta l_{\min} \Rightarrow \theta_\Gamma = \pi + \left(2 \times \frac{2\pi}{\lambda} l_{\min}\right) = 86.4^\circ$$

# Example – 2 (contd.)

- Therefore:

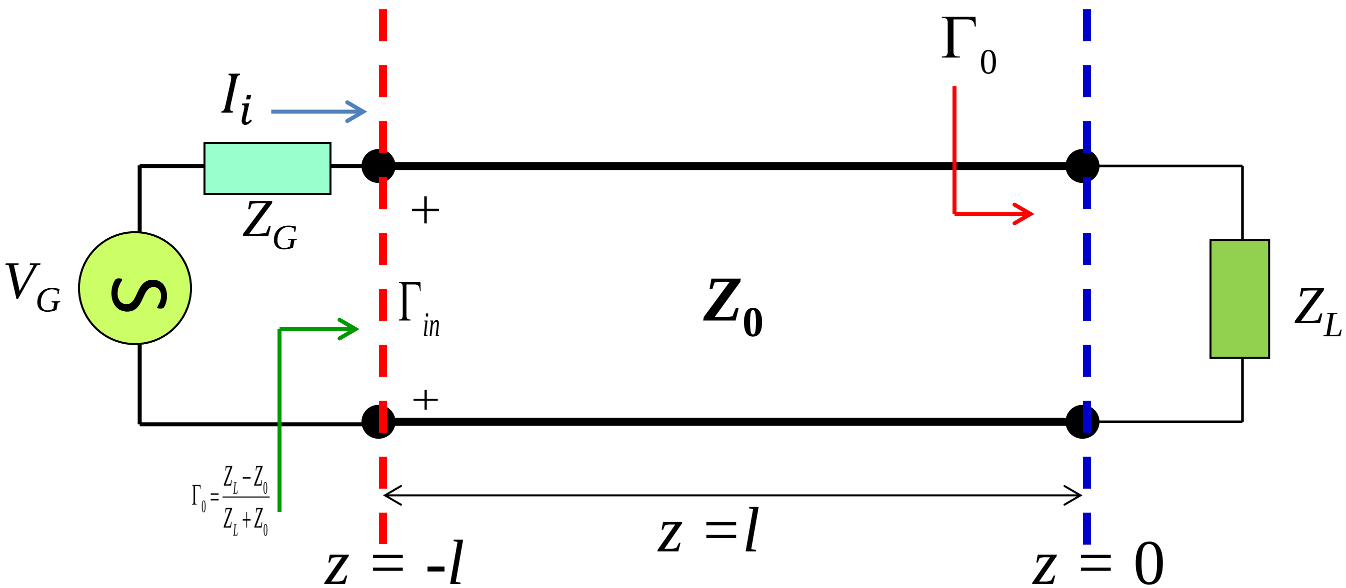
$$\Gamma_0 = 0.2e^{j86.4^\circ} = 0.0126 + j0.1996$$

- The unknown impedance is then:

$$Z_L = Z_0\left(\frac{1+\Gamma_0}{1-\Gamma_0}\right) \Rightarrow Z_L = 50\left(\frac{1+\Gamma_0}{1-\Gamma_0}\right) = 47.3 + j19.7\Omega$$

# Sourced and Loaded Transmission Line

- Thus far, we have discussed a TL with terminated load impedance → Let us now consider a TL with terminated load impedance and a source at the input (with line-to-source mismatch)




- At $z = 0$

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

- The current and voltage along the TL is:

$$V(z) = V_0^+ \left[ e^{-j\beta z} + \Gamma_0 e^{+j\beta z} \right]$$

$$I(z) = \frac{V_0^+}{Z_0} \left[ e^{-j\beta z} - \Gamma_0 e^{+j\beta z} \right]$$

# Sourced and Loaded Transmission Line (contd.)

- We are left with the **question**: just what is the **value** of complex constant $V_0^+$?
- **This** constant depends on the signal **source**! To determine its exact **value**, we must now apply **boundary conditions** at $z = -l$.
- We know that at the **beginning** of the transmission line:

$$V(z=-l) = V_0^+\left[e^{+j\beta l} + \Gamma_0 e^{-j\beta l}\right]$$

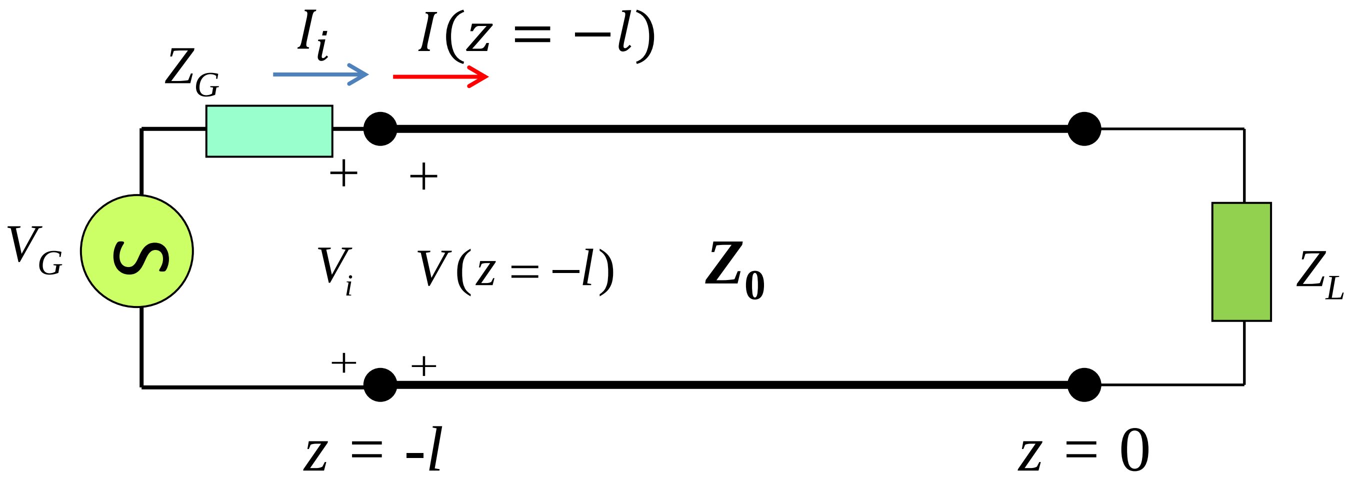
$$I(z=-l) = \frac{V_0^+}{Z_0}\left[e^{+j\beta l} - \Gamma_0 e^{-j\beta l}\right]$$

- Likewise, we know that the **source** must satisfy:

$$V_G = V_i + Z_G I_i$$

To relate these **three** expressions, we need to apply **boundary conditions** at $z = -l$.

# Sourced and Loaded Transmission Line (contd.)

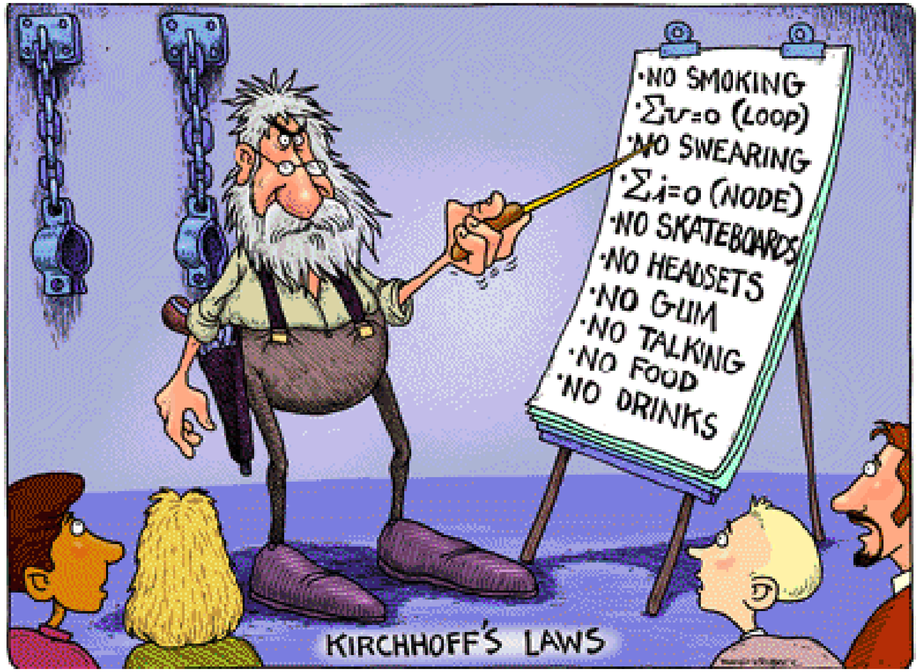



- From **KVL** we find:

$$V_i = V(z=-l)$$

- From **KCL** we find:

$$I_i = I(z=-l)$$



# Sourced and Loaded Transmission Line (contd.)

- **Combining** these equations, we find:

$$V_G = V_0^+\left[e^{+j\beta l} + \Gamma_0 e^{-j\beta l}\right] + Z_G \frac{V_0^+}{Z_0}\left[e^{+j\beta l} - \Gamma_0 e^{-j\beta l}\right]$$

**One** equation → **one** unknown ($V_0^+$)!!

- **Solving**, we find the value of $V_0^+$:

$$V_0^+ = V_G e^{-j\beta l}\frac{Z_0}{Z_0(1+\Gamma_{in}) + Z_G(1-\Gamma_{in})}$$

$$\Gamma_{in} = \Gamma(z=-l) = \Gamma_0 e^{-j\beta l}$$

- Note this result looks different than the equation in your book (Pozar):

$$V_0^+ = V_G \frac{Z_0}{Z_0 + Z_G}\frac{e^{-j\beta l}}{\left(1 - \Gamma_0 \Gamma_G e^{-j\beta l}\right)}$$

$$\Gamma_G = \frac{Z_G - Z_0}{Z_G + Z_0}$$

**I like the first expression better.**

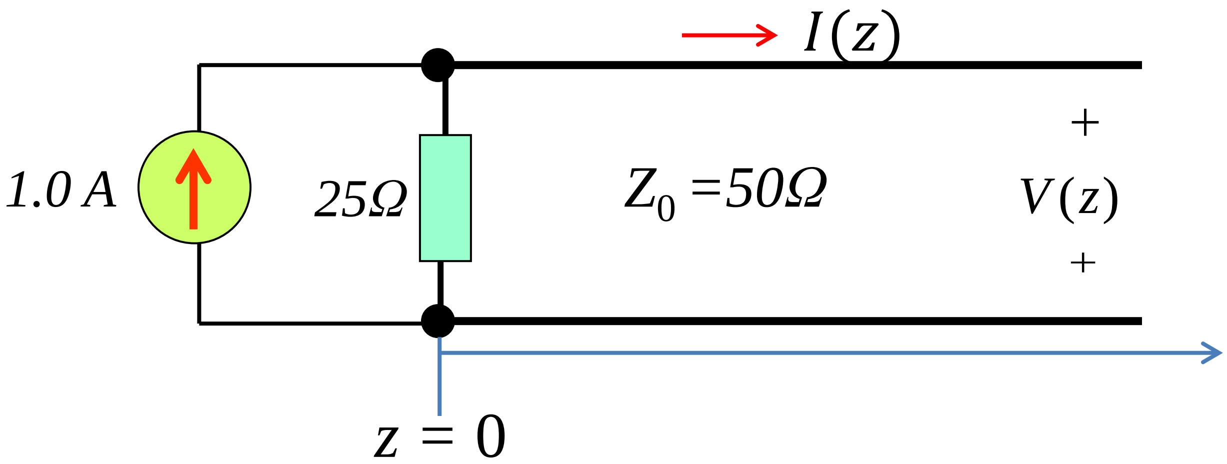
# Sourced and Loaded Transmission Line (contd.)

Although the two equations are equivalent, **first** expression is explicitly written in terms of $\Gamma_{in} = \Gamma(z = -l)$ (a very **useful**, **precise**, and **unambiguous** value), while the book's expression is written in terms of this **so-called** "source reflection coefficient" $\Gamma_G$(a **misleading**, **confusing**, **ambiguous**, and mostly **useless** value).

Specifically, we might be **tempted** to equate $\Gamma_G$ with the value $\Gamma_{in} = \Gamma(z = -l)$, but it is **not** $\Gamma_G \neq \Gamma(z = -l)$!

# Example – 3

- Consider the circuit below:




- It is known that the **current** along the transmission line is:

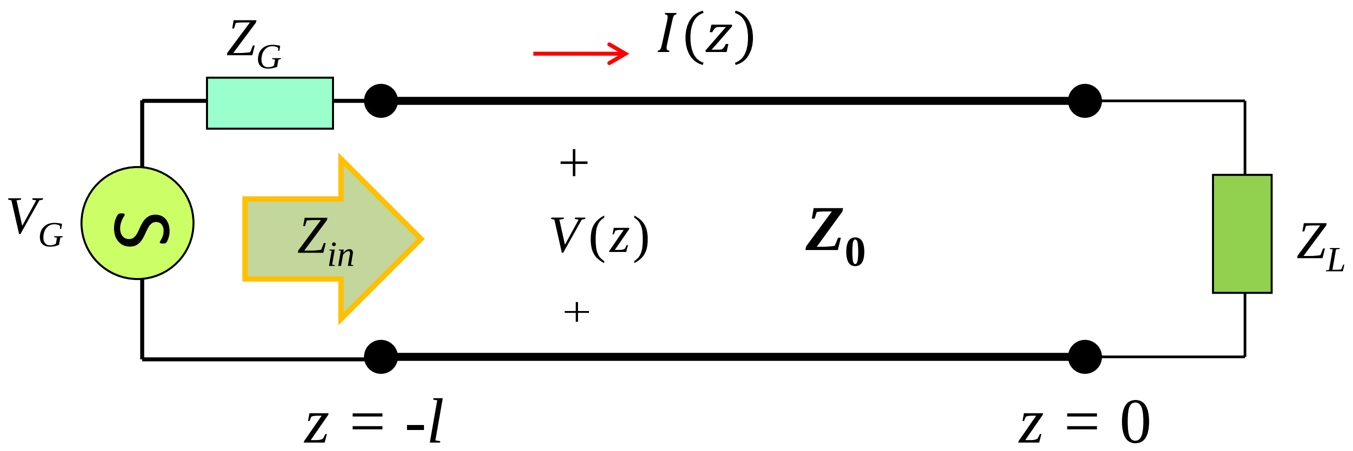
$$I(z) = 0.4e^{-j\beta z} - Be^{+j\beta z} \quad Amp \quad for\ z > 0$$

where B is some unknown complex value.

**Determine the value of B.**

# Sourced and Loaded Transmission Line (contd.)

**Q:** If the purpose of a transmission line is to transfer **power** from a source to a load, then exactly how much power is **delivered** to $Z_L$ for the circuit shown below ??




**A:** We of course **could** determine $V_0^+$ and $V_0^-$, and then determine the power absorbed by the load ($P_{abs}$) as:

$$V(z)$$

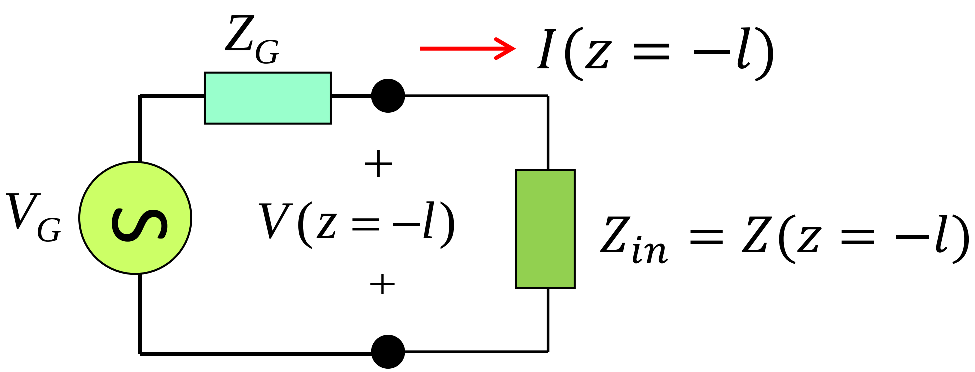
# Sourced and Loaded Transmission Line (contd.)

- However, if the transmission line is **lossless**, then we know that the power delivered to the load must be **equal** to the power "delivered" to the **input** ($P_{in}$) of the transmission line:

$$P_{abs} = P_{in} = \frac{1}{2}\text{Re}\left\{V(z=-l)I^*(z=-l)\right\}$$

- However, we can determine this power **without** having to solve for $V_0^+$ and $V_0^-$ (i.e., V(z) and I(z)). We can simply use our knowledge of **circuit theory**!

- We can **transform** load $Z_L$ to the beginning of the transmission line, so that we can replace the transmission line with its **input impedance** $Z_{in}$:



# Sourced and Loaded Transmission Line (contd.)

- Note by **voltage division** we can determine: $V(z=-l) = V_G \frac{Z_{in}}{Z_G + Z_{in}}$

- And from **Ohm's Law** we conclude: $I(z=-l) = \frac{V_G}{Z_G + Z_{in}}$

- And thus, the **power** $P_{in}$ delivered to $Z_{in}$ (and thus the **power** $P_{abs}$ delivered to the load $Z_L$) is:

$$P_{abs} = P_{in} = \frac{1}{2}\mathrm{Re}\left\{V(z=-l)I^*(z=-l)\right\} = \frac{1}{2}\mathrm{Re}\left\{V_G \frac{Z_{in}}{Z_G + Z_{in}} \frac{V_G^*}{\left(Z_G + Z_{in}\right)^*}\right\}$$

$$\Rightarrow P_{abs} = P_{in} = \frac{1}{2}\frac{\left|V_G\right|^2}{\left|Z_G + Z_{in}\right|^2}\mathrm{Re}\left\{Z_{in}\right\} = \frac{1}{2}\left|V_G\right|^2 \frac{\left|Z_{in}\right|^2}{\left|Z_G + Z_{in}\right|^2}\mathrm{Re}\left\{Y_{in}\right\}$$

# Sourced and Loaded Transmission Line (contd.)

- Note that we could **also** determine $P_{abs}$ from our **earlier** expression:

$$P_{abs} = \frac{1}{2}\text{Re}\left\{V(z=0)I^*(z=0)\right\} = \frac{\left|V_0^+\right|^2}{2Z_0}\left(1-\left|\Gamma_0\right|^2\right)$$

But we would of course have to **first** determine $V_0^+$(! ):

$$V_0^+ = V_G e^{-j\beta l}\frac{Z_0}{Z_0\left(1+\Gamma_{in}\right)+Z_G\left(1-\Gamma_{in}\right)}$$

- Let's look at **specific cases** of $Z_G$ and $Z_L$, and determine how they affect $V_0^+$ and $P_{abs}$.

$\boldsymbol{Z_G = Z_0}$

- For this case, we find that $V_0^+$ **simplifies** greatly:

$$V_0^+ = \frac{1}{2}V_G e^{-j\beta l}$$

# Sourced and Loaded Transmission Line (contd.)

$$V_0^+ = \frac{1}{2} V_G e^{-j\beta l}$$

It says that the incident wave in this case is **independent** of the load attached at the other end!

Thus, for the **one** case $\boldsymbol{Z_G = Z_0}$ , we in fact can consider $V^+(z)$ as being the source wave, and then the reflected wave $V^-(z)$ as being the result of this stimulus.

- Remember, the complex value $V_0{}^+$ is the value of the incident wave evaluated at the end of the transmission line ($V_0{}^+ = V^+(z=0)$). We can likewise determine the value of the incident wave at the **beginning** of the transmission line (i.e. $V^+(z=-l)$). For this case, where $\boldsymbol{Z_G = Z_0}$, we find that this value can be very simply stated (!):

$$V^+(z=-l) = V_0^+ e^{-j\beta(z=-l)} = \left(\frac{1}{2} V_G e^{-j\beta l}\right) e^{+j\beta l} = \frac{V_G}{2}$$

# Sourced and Loaded Transmission Line (contd.)

- Likewise, we find that the delivered power for this case can be simply stated as:

$$P_{abs} = \frac{\left|V_0^+\right|^2}{2Z_0}\left(1-\left|\Gamma_0\right|^2\right) = \frac{\left|V_G\right|^2}{8Z_0}\left(1-\left|\Gamma_0\right|^2\right)$$

$\boldsymbol{Z_L = Z_0}$

- In this case, we find that $\Gamma_0 = 0$, and thus $\Gamma_{in} = 0$. As a result:

$$V_0^+ = V_G e^{-j\beta l}\frac{Z_0}{Z_0 + Z_G}$$

- Likewise, we find that:

$$P_{abs} = \frac{\left|V_0^+\right|^2}{2Z_0}\left(1-\left|\Gamma_0\right|^2\right) = \frac{\left|V_0^+\right|^2}{2Z_0}$$

Here the delivered power $P_{abs}$ is simply that of the incident wave ($P^+$), as the matched condition causes the reflected power to be zero ($P^- = 0$)!

# Sourced and Loaded Transmission Line (contd.)

- Inserting the value of $V_0^+$, we find:

$$P_{abs} = \frac{\left|V_0^+\right|^2}{2Z_0} = \frac{\left|V_G\right|^2}{2}\frac{Z_0}{\left|Z_0 + Z_G\right|^2}$$

Note that this result can likewise be found by recognizing that $Z_{in} = Z_0$ when $Z_L = Z_0$.

$$Z_{in} = Z_G^*$$

For this case, we find $Z_L$ takes on whatever value required to make $Z_{in} = Z_G^*$. This is a **very** important case!

- First, we can express:

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{Z_G^* - Z_0}{Z_G^* + Z_0}$$

We can show that (trust me!):

$$V_0^+ = V_G e^{-j\beta l}\frac{Z_G^* + Z_0}{4\,\mathrm{Re}\{Z_G\}}$$

# Sourced and Loaded Transmission Line (contd.)

- let's look at the absorbed power:

$$P_{abs} = \frac{1}{2}\frac{|V_G|^2}{|Z_G + Z_{in}|^2}\mathrm{Re}\{Z_{in}\} \Rightarrow P_{abs} = \frac{1}{2}\frac{|V_G|^2}{|Z_G + Z_G^*|^2}\mathrm{Re}\{Z_G^*\}$$

$$\therefore P_{abs} = \frac{1}{2}|V_G|^2\frac{1}{4\mathrm{Re}\{Z_G^*\}} \doteq P_{avl}$$

**HW #1 (part-4)**

It can be shown that—for a **given** $V_G$ and $Z_G$—the value of input impedance $Z_{in}$ that will absorb the **largest possible** amount of power is the value $Z_{in} = Z_G^*$.

This case is known as the **conjugate match**, and is essentially the goal of every transmission line problem—to deliver the largest possible power to $Z_{in}$, and thus to $Z_L$ as well! → This power is known as the **available power** ($P_{avl}$) of the source.

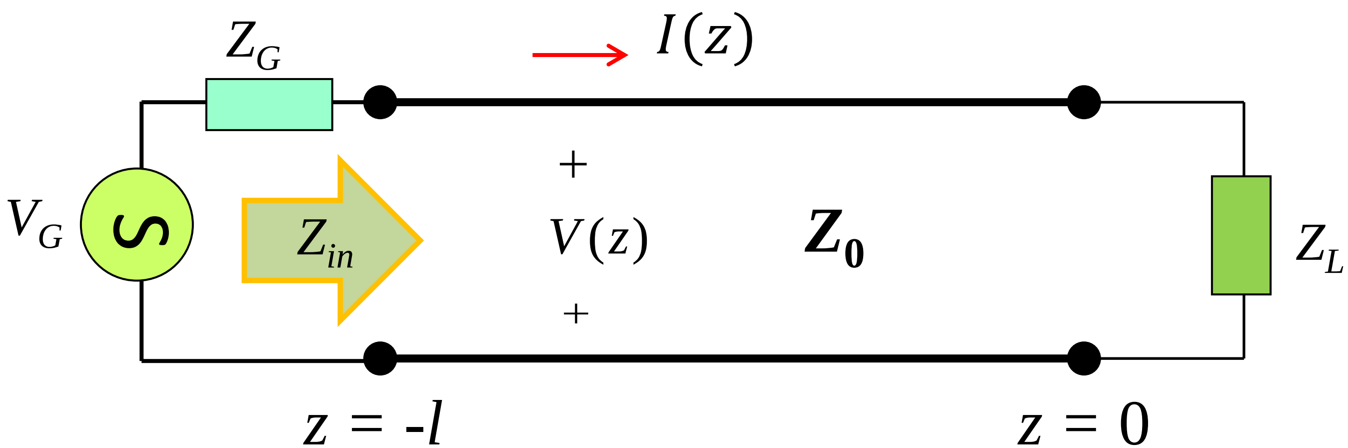
# Sourced and Loaded Transmission Line (contd.)

$$P_{abs} = \frac{1}{2}|V_G|^2 \frac{1}{4\text{Re}\{Z_G^*\}} \doteq P_{avl}$$

There are **two** very important things to understand about this result!

## Very Important Thing #1

- Consider again the terminated transmission line:




- Recall that if $Z_L = Z_0$, the **reflected** wave will be **zero**, and the absorbed power will be:

$$P_{abs} = \frac{|V_G|^2}{2} \frac{Z_0}{|Z_0 + Z_G|^2} \leq P_{avl}$$

# Sourced and Loaded Transmission Line (contd.)

- But note if $Z_L = Z_0$, the input impedance $Z_{in} = Z_0$ —but then $Z_{in} \neq {Z_G}^*$ (generally)! In other words, $Z_L = Z_0$ does **not** (generally) result in a **conjugate match**, and thus setting $Z_L = Z_0$ does **not** result in maximum power absorption!

**Q:** Huh!? This makes **no** sense! A load value of $Z_L = Z_0$ will **minimize** the reflected wave $(P^- = 0)$—**all** of the incident power will be absorbed.

- Any other value of $Z_L$ will result in **some** of the incident wave being reflected—how in the world could this **increase** absorbed power?
- After all, just **look** at the expression for absorbed power:

$$P_{abs} = \frac{\left|V_0^+\right|^2}{2Z_0}\left(1 - \left|\Gamma_0\right|^2\right)$$

**Clearly**, this value is maximized when $\Gamma_0 = 0$ (i.e., when $Z_L = Z_0$)

# Sourced and Loaded Transmission Line (contd.)

**A:** You are forgetting one very important fact! Although it **is** true that the load impedance $\boldsymbol{Z_L}$ affects the **reflected** wave power $P^-$, the value of $\boldsymbol{Z_L}$ — as we have shown— **likewise** helps determine the value of the **incident** wave (i.e., the value of $P^+$) as well.

- Thus, the value of $\boldsymbol{Z_L}$ that minimizes $P^-$ will **not** generally maximize $P^+$!
- **Likewise** the value of $\boldsymbol{Z_L}$ that maximizes $P^+$ will not generally minimize $P^-$.
- Instead, the value of $\boldsymbol{Z_L}$ that maximizes the **absorbed** power $P_{abs}$ is, by definition, the value that maximizes the **difference** $P^+ - P^-$.
- We find that this impedance $\boldsymbol{Z_L}$ is the value that results in the **ideal** case of $\boldsymbol{Z_{in}} = \boldsymbol{Z_G}^*$.

# Sourced and Loaded Transmission Line (contd.)

**Q:** Yes, but what about the case where $Z_G = Z_0$? For **that** case, we determined that the incident wave **is** independent of $Z_L$. Thus, it would seem that at least for that case, the **delivered** power would be maximized when the **reflected** power was minimized (i.e., $Z_L = Z_0$).

**A: True!** But think about what the **input** impedance would be in that case—$Z_{in} = Z_0$. Oh by the way, that provides a **conjugate match** ($Z_{in} = Z_0 = {Z_G}^*$).

- Thus, in some ways, the case $Z_G = Z_0 = Z_L$ (i.e., **both** source and load impedances are numerically equal to $Z_0$) is **ideal**. A **conjugate match** occurs, the incident wave is **independent** of $Z_L$, there is **no** reflected wave, and all the math **simplifies** quite nicely:

$$V_0^+ = \frac{1}{2} V_G e^{-j\beta l}$$

$$P_{abs} = P_{avl} = \frac{|V_G|^2}{8Z_0}$$

# Sourced and Loaded Transmission Line (contd.)

## Very Important Thing #2

Note the conjugate match criteria **says**:

**Given** source impedance $\boldsymbol{Z_G}$, maximum power transfer occurs when the input impedance is set at value $\boldsymbol{Z_{in}} = \boldsymbol{Z_G}^*$.

It does **NOT** say:

**Given** input impedance $\boldsymbol{Z_{in}}$, maximum power transfer occurs when the source impedance is set at value $\boldsymbol{Z_G} = \boldsymbol{Z_{in}}^*$.

**This last statement is in fact false!**

A **factual** statement is this:

**Given** input impedance $\boldsymbol{Z_{in}}$, maximum power transfer occurs when the source impedance is set at value $\boldsymbol{Z_G} = 0 - jX_{in}$(i.e., $\boldsymbol{R_G} = 0$).

**Q:** Huh??

# Sourced and Loaded Transmission Line (contd.)

**A:** Remember, the value of source impedance $Z_G$ affects the available power $P_{avl}$ of the source. To maximize $P_{avl}$, the real (resistive) component of the source impedance should be as small as possible (regardless of $Z_{in}$!), a fact that is **evident** when observing the expression for **available power**:

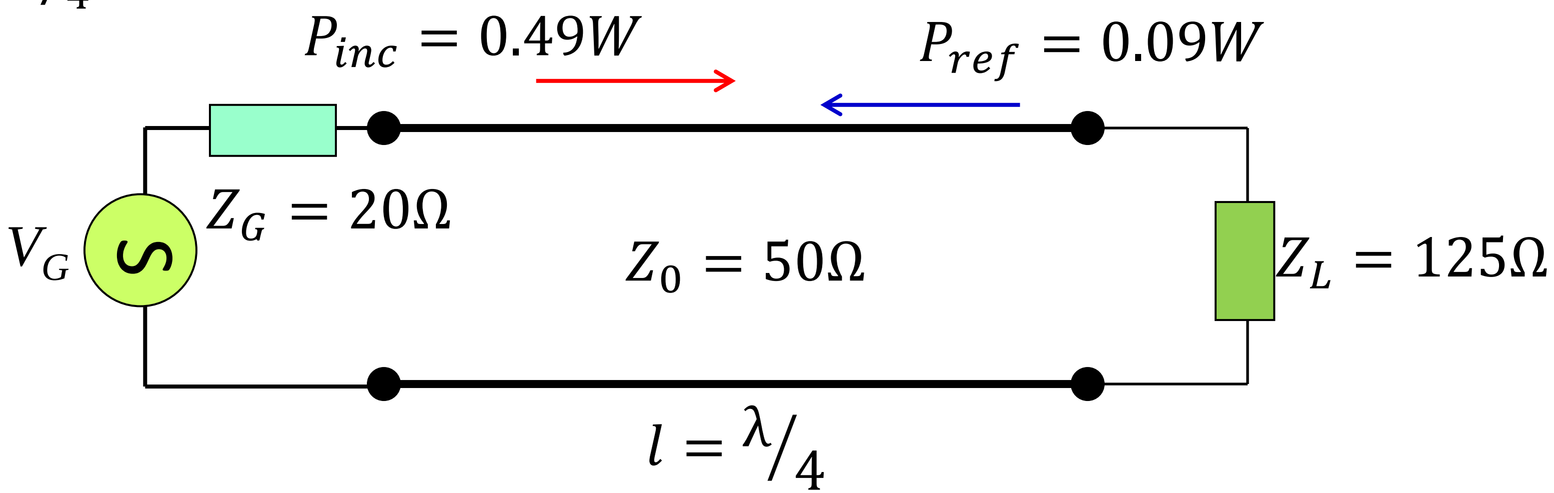
$$P_{avl} = \frac{1}{2}|V_G|^2 \frac{1}{4\text{Re}\{Z_G^*\}} = \frac{|V_G|^2}{8R_G}$$

- Thus, **maximizing** the power delivered **to** a load ($P_{abs}$), **from** a source, has **two** components:

  1. Maximize the **power available** ($P_{avl}$) from a source (e.g., minimize $R_G$).
  2. **Extract** all of this available power by setting the input impedance $Z_{in}$ to a value $Z_{in} = Z_G^*$ (thus $P_{abs} = P_{avl}$).

# Example – 4

- Consider this circuit, where the transmission line is **lossless** and has length $l = {}^{\lambda}/_{4}$ :




**Determine the magnitude of source voltage $V_G$ (i.e., determine $|V_G|$).**

**Hint:** This is **not** a boundary condition problem. Do **not** attempt to find $V(z)$ and/or $I(z)$!

# Lossy Transmission Lines

- Recall that we have been **approximating** low-loss transmission lines as lossless (R =G = 0):

$$\alpha = 0 \qquad \beta = \omega\sqrt{LC}$$

- But, **long** low-loss lines require a **better** approximation:

$$\alpha = \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right) \qquad \beta = \omega\sqrt{LC}$$

- Now, if we have **really long** transmission lines (e.g., long distance communications), we can apply **no** approximations at all:

$$\alpha = \mathrm{Re}\{\gamma\} \qquad \beta = \mathrm{Im}\{\gamma\}$$

For these **very** long transmission lines, we find that $\beta = Im\{\gamma\}$ is a **function** of signal **frequency** ω . This results in an extremely serious problem—signal **dispersion**.

# Lossy Transmission Lines (contd.)

- Recall that the **phase velocity** $\boldsymbol{v_p}$ (i.e., propagation velocity) of a wave in a transmission line is:

$$v_p = \frac{\omega}{\beta}$$

$$\beta = \text{Im}\{\gamma\} = \text{Im}\left\{\sqrt{(R + j\omega L)(G + j\omega C)}\right\}$$

Thus, for a lossy line, the phase velocity $\boldsymbol{v_p}$ is a function of frequency ω (i.e., $\boldsymbol{v_p(\omega)}$)—this is **bad**!

- Any signal that carries significant **information** must has some non-zero **bandwidth**. In other words, the signal energy (as well as the information it carries) is **spread** across many frequencies.
- If the different frequencies that comprise a signal travel at different velocities, that signal will arrive at the end of a transmission line **distorted**. We call this phenomenon signal **dispersion.**
- Recall for **lossless** lines, however, the phase velocity is **independent** of frequency—**no** dispersion will occur!

# Lossy Transmission Lines (contd.)

- For lossless line: $v_p = \dfrac{1}{\sqrt{LC}}$

however, a perfectly lossless line is impossible, but we find phase velocity is **approximately** constant if the line is low-loss.

Therefore, dispersion distortion on low-loss lines is **most often** not a problem.

**Q**: You say "**most** often" not a problem—that phrase seems to imply that dispersion sometimes **is** a problem!

# Lossy Transmission Lines (contd.)

**A:** Even for low-loss transmission lines, dispersion can be a problem **if** the lines are **very** long—just a small difference in phase velocity can result in significant differences in propagation delay **if** the line is very long!

- Modern examples of long transmission lines include phone lines and cable TV. However, the **original** long transmission line problem occurred with the **telegraph**, a device invented and implemented in the 19$^{th}$ century.
- Telegraphy was the essentially the **first** electrical engineering technology ever implemented, and as a result, led to the first ever **electrical engineers**!
- Early telegraph "engineers" discovered that if they made their telegraph lines **too long**, the dots and dashes characterizing Morse code turned into a muddled, indecipherable **mess**. Although they did not realize it, they had fallen victim to the heinous effects of **dispersion**!
- Thus, to send messages over long distances, they were forced to implement a series of intermediate "**repeater**" stations, wherein a human operator received and then **retransmitted** a message on to the next station. This **really** slowed things down!

# Lossy Transmission Lines (contd.)

**Q:** Is there any way to **prevent** dispersion from occurring?

**A:** You bet! **Oliver Heaviside** figured out how in the **19th** Century!

- Heaviside found that a transmission line would be distortionless (i.e., no dispersion) **if** the line parameters exhibited the following **ratio**:

$$\frac{R}{L} = \frac{G}{C}$$

- Let's see **why** this works. Note the complex propagation constant $\gamma$ can be expressed as:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{LC(R/L + j\omega)(G/C + j\omega)}$$

# Lossy Transmission Lines (contd.)

- Then **IF**: $$\frac{R}{L} = \frac{G}{C}$$
- we find:

$$\gamma = \sqrt{LC\left(R/L + j\omega\right)\left(R/L + j\omega\right)} = \left(R/L + j\omega\right)\sqrt{LC} = R\sqrt{\frac{C}{L}} + j\omega\sqrt{LC}$$

- Thus: $$\alpha = \text{Re}\{\gamma\} = R\sqrt{\frac{C}{L}}$$ $$\beta = \text{Im}\{\gamma\} = \omega\sqrt{LC}$$
- The propagation **velocity** of the wave is thus: $$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

The propagation velocity is **independent** of frequency! This lossy transmission line is **not** dispersive!

# Lossy Transmission Lines (contd.)

**Q:** Right. All the transmission lines I use have the property that $R/_{L} > G/_{C}$. I've **never** found a transmission line with this **ideal** property $R/_{L} = G/_{C}$!

**A:** It is true that typically $R/_{L} > G/_{C}$. But, we can reduce the ratio $R/_{L}$(until it is equal to $G/_{C}$) by adding series **inductors** periodically along the transmission line.

This was **Heaviside's** solution—and it worked! **Long** distance transmission lines were made possible.

**Q:** Why don't we increase G instead?

**A:**