Lecture-3

Date: 12.08.2014

- Lecture – 2 Review
- Microstrip Transmission Line
- Design Examples
- Lossless Transmission Line

Review – Lecture 2

- **Telegrapher's Equations:**

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

$$-\frac{\partial v(z,t)}{\partial z} = Ri(z,t) + L\frac{\partial i(z,t)}{\partial t}$$

- **The time-harmonic form of the telegrapher equations are:**

$$\mathrm{Re}\frac{\partial\left(f(z)e^{j\varphi(z)}e^{j\omega t}\right)}{\partial z} = -Re\left(R.g(z)e^{j\eta(z)}e^{j\omega t} + j\omega L.g(z)e^{j\eta(z)}e^{j\omega t}\right)$$

$$\mathrm{Re}\frac{\partial\left(g(z)e^{j\eta(z)}e^{j\omega t}\right)}{\partial z} = -Re\left(G.f(z)e^{j\varphi(z)}e^{j\omega t} + j\omega C.f(z)e^{j\varphi(z)}e^{j\omega t}\right)$$

- **In phasor form:**

$$\mathrm{Re}\frac{\partial\left(V(z)e^{j\omega t}\right)}{\partial z} = -Re\left(RI(z)e^{j\omega t} + j\omega LI(z)e^{j\omega t}\right)$$

$$\mathrm{Re}\frac{\partial\left(I(z)e^{j\omega t}\right)}{\partial z} = -Re\left(GV(z)e^{j\omega t} + j\omega CV(z)e^{j\omega t}\right)$$

Review – Lecture 2

- The equations can be further simplified as:

$$Re\frac{d\left(V(z)e^{j\omega t}\right)}{dz} = -Re\left(RI(z)e^{j\omega t} + j\omega LI(z)e^{j\omega t}\right)$$

$$Re\frac{d\left(I(z)e^{j\omega t}\right)}{dz} = -Re\left(GV(z)e^{j\omega t} + j\omega CV(z)e^{j\omega t}\right)$$

As $I(z)$ and $V(z)$ are function of only position

Further simplified as

$$\frac{d\,\mathrm{V(z)}}{dz} = -\left(R + j\omega L\right)\mathrm{I(z)}$$

$$\frac{d\,\mathrm{I(z)}}{dz} = -\left(G + j\omega C\right)\mathrm{V(z)}$$

Differentiating with respect to *z* gives

$$\frac{d^2\,\mathrm{V(z)}}{dz^2} - \gamma^2 V(z) = 0$$

$$\frac{d^2\,\mathrm{I(z)}}{dz^2} - \gamma^2 I(z) = 0$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Review – Lecture 2

- For a lossless transmission line the second order differential equation for phasors are:

$$\frac{d^2 V(z)}{dz^2} + \beta^2 V(z) = 0$$

$$\frac{d^2 I(z)}{dz^2} + \beta^2 I(z) = 0$$

$$\beta = \omega\sqrt{LC}$$

General Solution For Voltage Phasor

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

V_0^+ and V_0^- are complex constants

- Similarly the current phasor for a lossless line can be described:

$$\therefore I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$$

$$Z_0 = \frac{\omega L}{\beta} = \frac{\omega L}{\omega\sqrt{LC}} = \sqrt{\frac{L}{C}}$$

Review – Lecture 2

- For the simple case of V_0^+ **and** V_0^- being real, the voltage and current along the transmission line can be expressed as:

$$v(z,t) = V_0^+ \cos(\omega t - \beta z) + V_0^- \cos(\omega t + \beta z)$$

$$i(z,t) = \frac{V_0^+}{Z_0}\cos(\omega t - \beta z) - \frac{V_0^-}{Z_0}\cos(\omega t + \beta z)$$

- TL Parameters:

$$\beta = \frac{2\pi}{\lambda}$$

spatial frequency

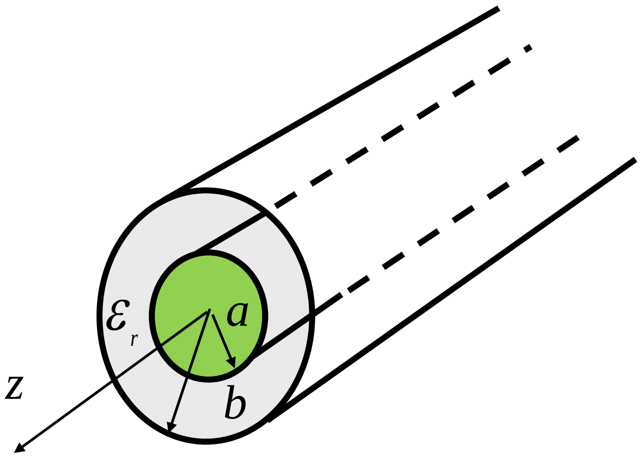
$$\beta = \omega\sqrt{LC}$$

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

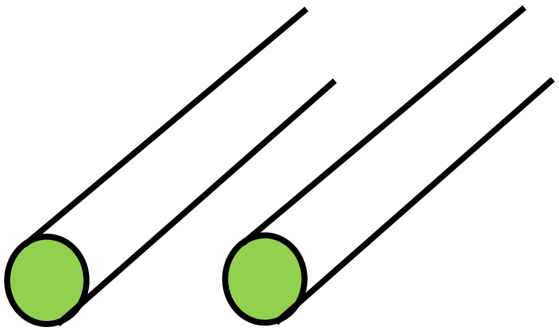
Phase Velocity

Example of Transmission Lines

Two common examples:



coaxial cable

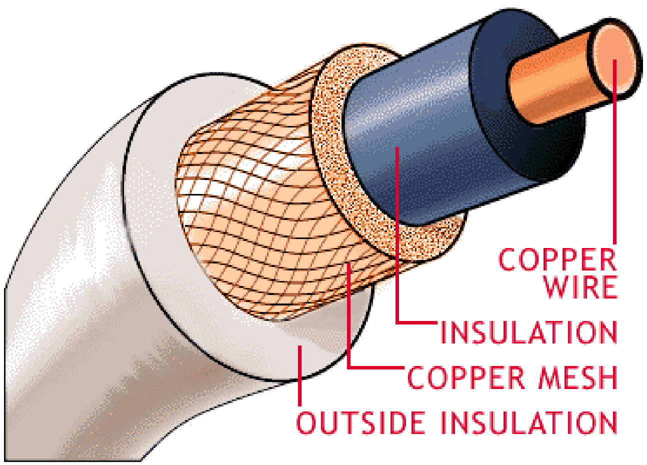


twin line

A transmission line is normally used in the balanced mode, meaning equal and opposite currents (and charges) on the two conductors.

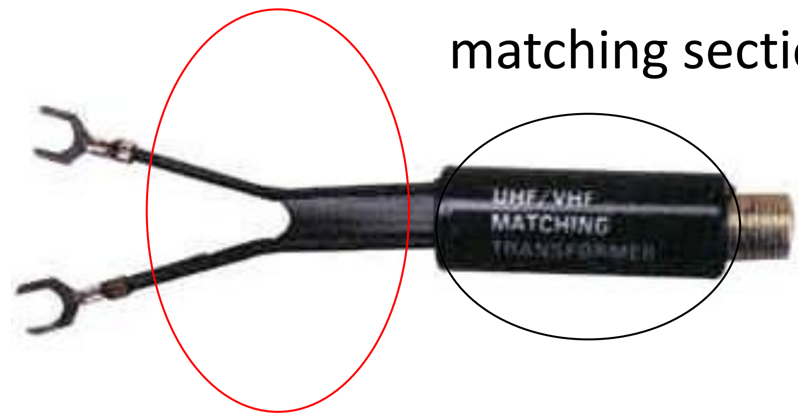
Example of Transmission Lines (contd.)

Here’s what they look like in real-life:



coaxial

coax to twin line matching section

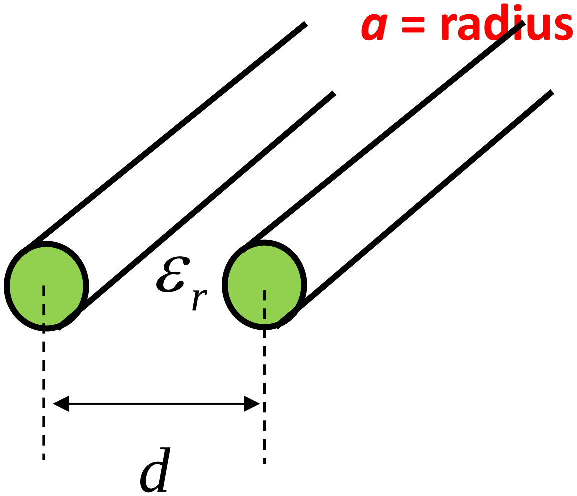


twin line

Example of Transmission Lines (contd.)

Twin Line

a = radius of wires



$$C = \frac{\pi\varepsilon_0\varepsilon_r}{\cosh^{-1}\left(\frac{d}{2a}\right)} \quad [\text{F/m}]$$

$$L = \frac{\mu_0}{\pi}\cosh^{-1}\left(\frac{d}{2a}\right) \quad [\text{H/m}]$$

$$Z_0 = \frac{1}{\pi}\eta_0\frac{1}{\sqrt{\varepsilon_r}}\cosh^{-1}\left(\frac{d}{2a}\right) \quad [\Omega]$$

$$\cosh^{-1}x = \ln\left(x+\sqrt{x^2-1}\right) \xrightarrow{x\to\infty} \ln 2x$$

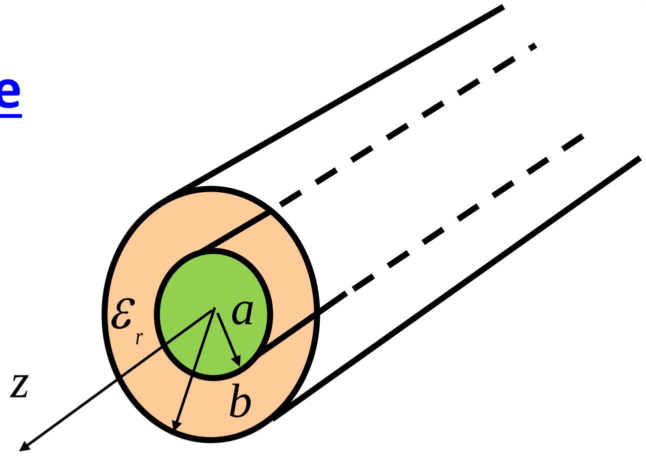
$$Z_0 \approx \frac{1}{\pi}\eta_0\frac{1}{\sqrt{\varepsilon_r}}\ln\left(\frac{d}{a}\right) \quad [\Omega]$$

$$a \ll d$$

HW # 0

Example of Transmission Lines (contd.)

Coaxial Cable



σ_d = conductivity of dielectric [S/m].

σ_m = conductivity of metal [S/m].

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)} \quad [\text{F/m}] \qquad G = \frac{2\pi\sigma_d}{\ln\left(\frac{b}{a}\right)} \quad [\text{S/m}]$$

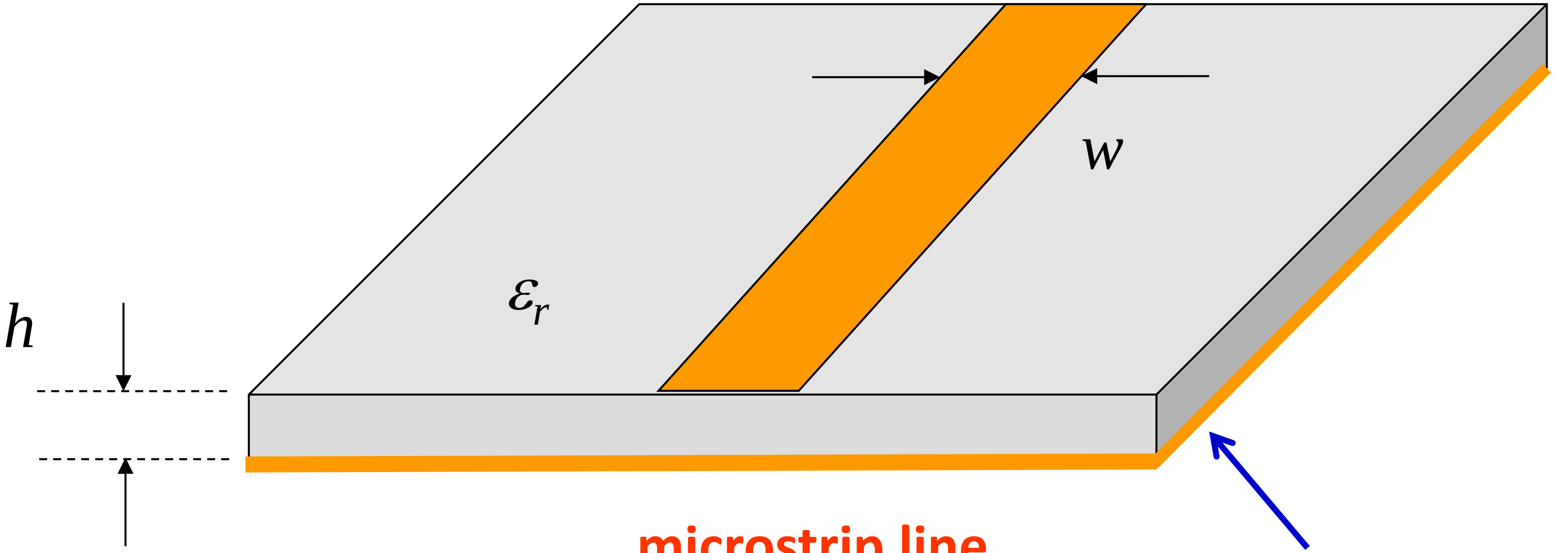
$$L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right) \quad [\text{H/m}] \qquad R = \frac{1}{\sigma_m\delta}\left(\frac{1}{2\pi a} + \frac{1}{2\pi b}\right) \quad [\Omega/\text{m}]$$

HW # 0

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma_m}}$$ (skin depth of metal)

Example of Transmission Lines (contd.)

Another common example (for printed circuit boards):

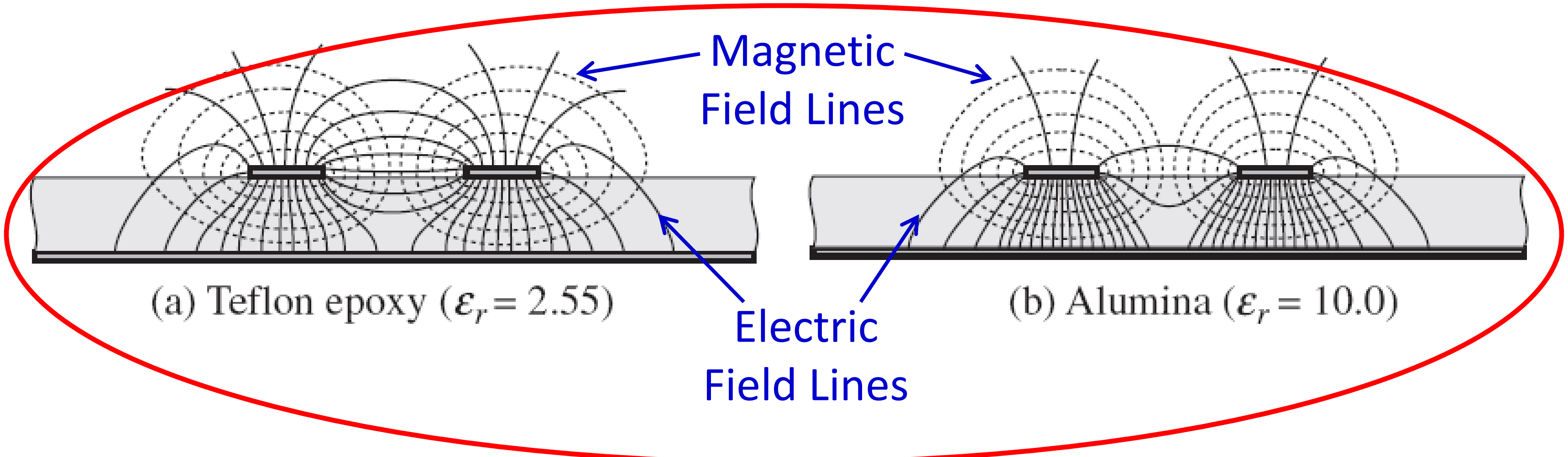


microstrip line

Ground plane helps in preventing the field leakage and thus reduces the radiation loss

Microstrip Line (contd.)

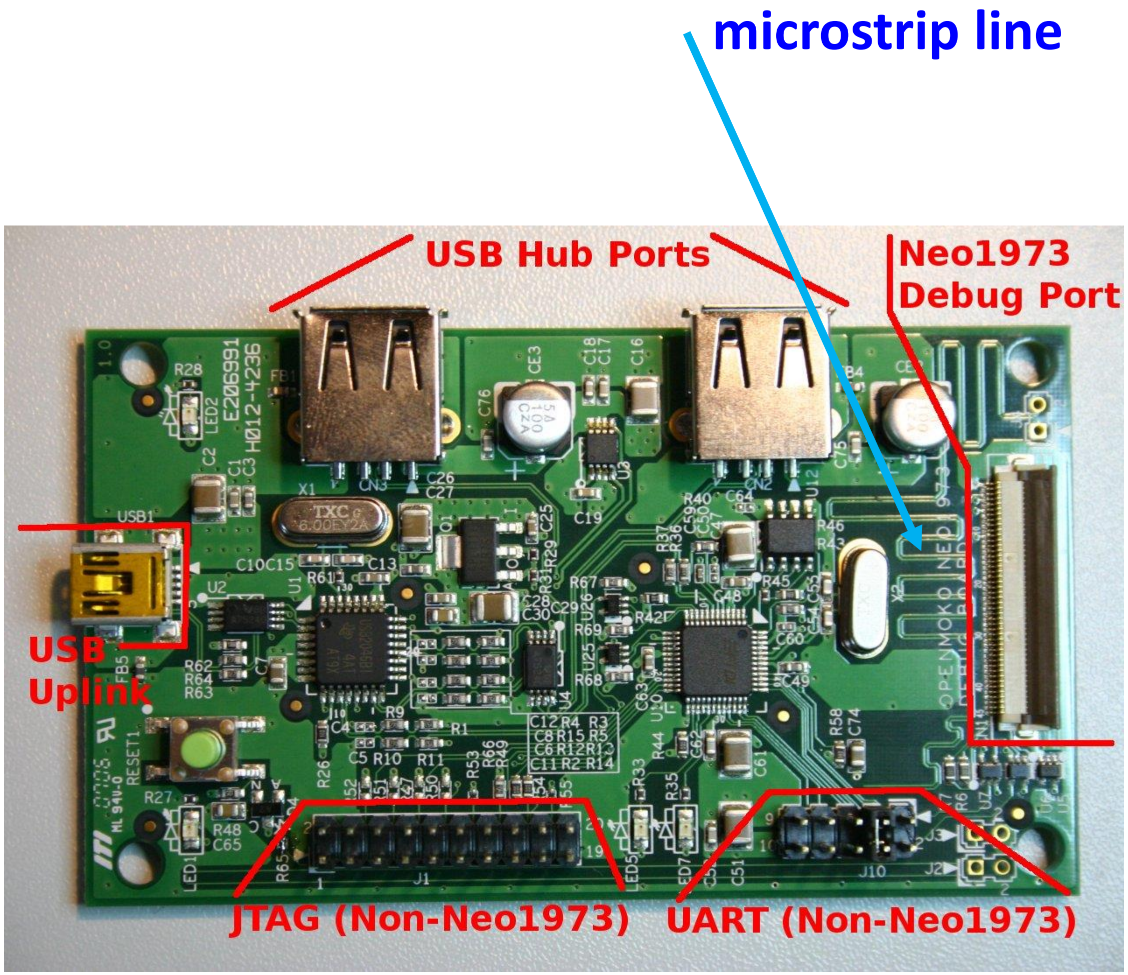
- The severity of field leakage also depends on the relative dielectric constants (ε_r).



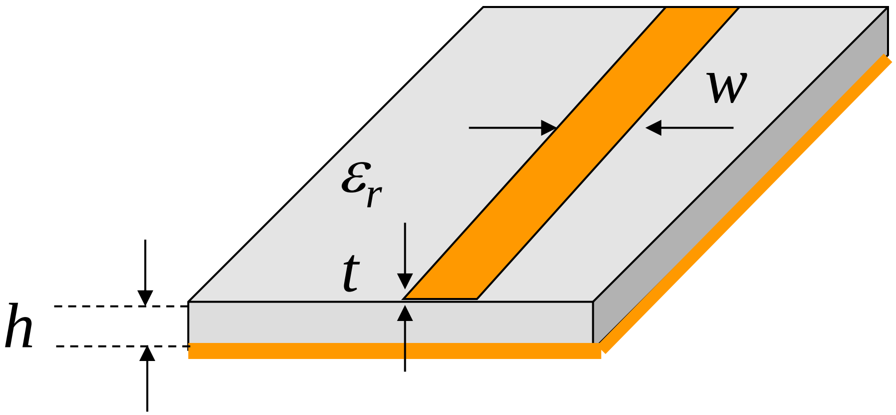
It is apparent that the radiation loss could be minimized by using substrates with high dielectric constants

- Alternative approaches to reduce radiation loss and interference are shielded microstrip line and multi-layer boards

Microstrip Line (contd.)



Microstrip Transmission Lines Design



- Simple parallel plate model can not accurately define this structure.
- Because, if the substrate thickness increases or the conductor width decreases then fringing filed become more prominent (and therefore need to be incorporated in the model).

Case-I: thickness (t) of the line is negligible

- **For narrow microstrips ($w/h \leq 1$):** $Z_0 = \frac{Z_f}{2\pi\sqrt{\varepsilon_{eff}}} \ln\left(8\frac{h}{w} + \frac{w}{4h}\right)$

Where, $Z_f = \sqrt{\mu_0/\varepsilon_0} = 377\Omega$ ← **wave impedance in free space**

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[\left(1 + 12\frac{h}{w}\right)^{-1/2} + 0.004\left(1 - \frac{w}{h}\right)^2\right]$$

← **Effective Dielectric Constant**

Microstrip Transmission Lines Design (contd.)

- For wide microstrips ($^w/_h \geq 1$):

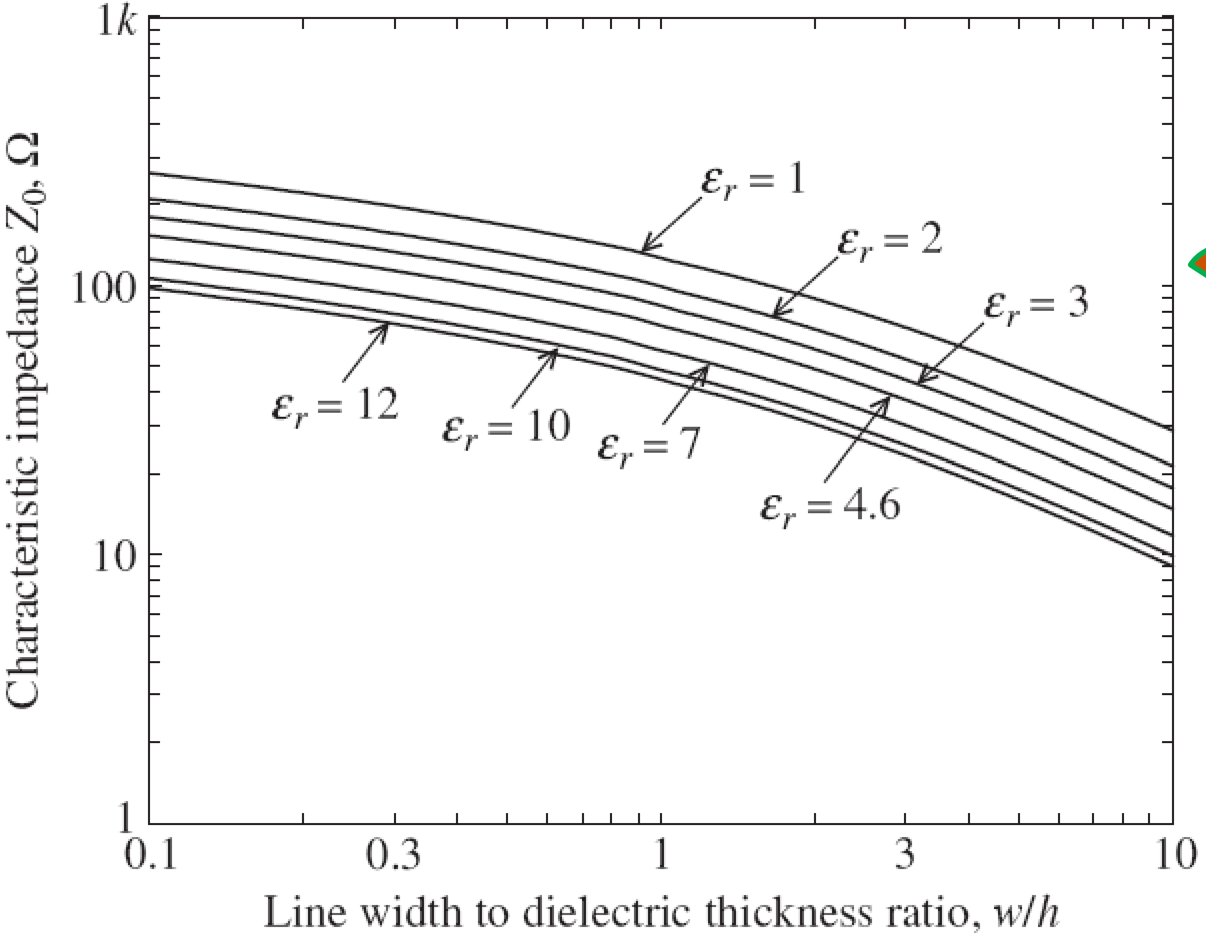
$$Z_0 = \frac{Z_f}{\sqrt{\varepsilon_{eff}}\left(1.393 + \frac{w}{h} + \frac{2}{3}\ln\left(\frac{w}{h} + 1.444\right)\right)}$$

- Where the effective dielectric constant is expressed as:

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + 12\frac{h}{w}\right)^{-1/2}$$

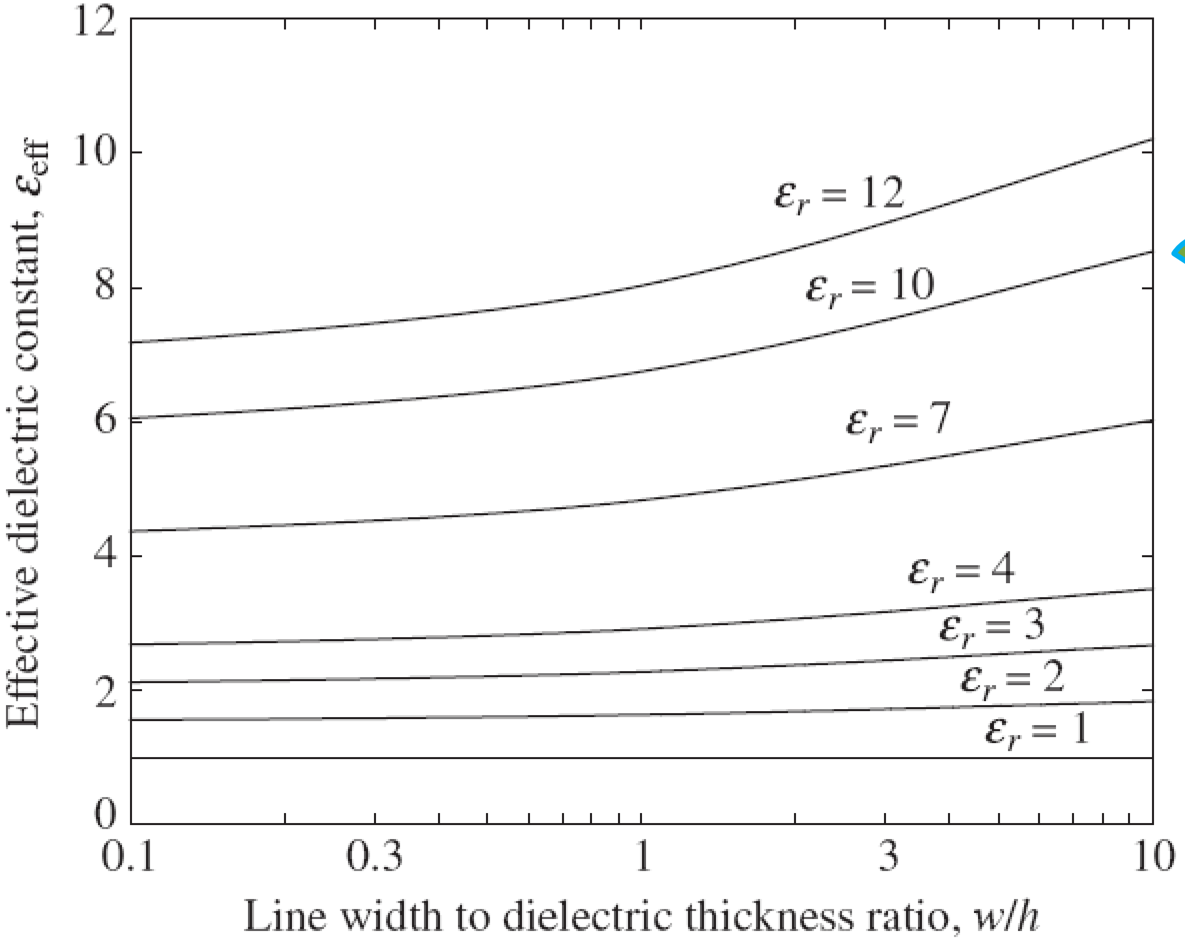
- The two distinct expressions give approximate values of characteristic impedance and effective dielectric constant for narrow and wide strip microstrip lines → these can be used to plot Z_0 and ε_{eff} as a function of $^w/_h$.

Microstrip Transmission Lines Design (contd.)



For a desired characteristic impedance using known substrate, the dimension w/h can be identified from this curve

Microstrip Transmission Lines Design (contd.)



Once the line dimensions are known, the effective dielectric constant can be identified

Microstrip Transmission Lines Design (contd.)

- The effective dielectric constant (ε_{eff}) is viewed as the dielectric constant of a homogenous material that fills the entire space around the line. Therefore:

Speed of Light

Free Space Wavelength

$$\lambda = \frac{v_p}{f} = \frac{c}{f\sqrt{\varepsilon_{eff}}} = \frac{\lambda_0}{\sqrt{\varepsilon_{eff}}}$$

- The wavelength in the microstrip line for $W/h \geq 0.6$ is:

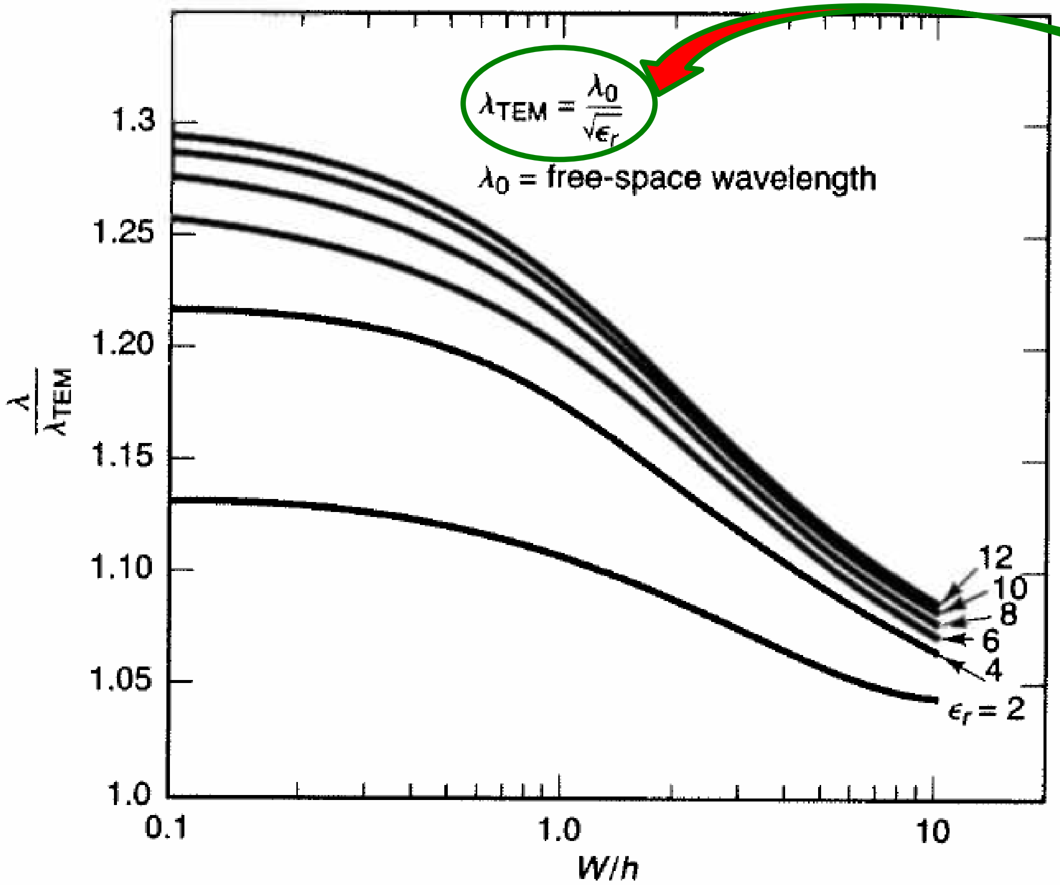
$$\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_r}}\left[\frac{\varepsilon_r}{1+0.63(\varepsilon_r-1)(W/h)^{0.1255}}\right]^{1/2}$$

- The wavelength in the microstrip line for $W/h \leq 0.6$ is:

$$\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_r}}\left[\frac{\varepsilon_r}{1+0.6(\varepsilon_r-1)(W/h)^{0.0297}}\right]^{1/2}$$

Microstrip Transmission Lines Design (contd.)

- In some specifications, wavelength is known. In that case following curve can be used to identify the *w/h* ratio.



It is a good approximation at lower microwave frequencies. However, at higher microwave frequencies this assumption is no more valid.

Microstrip Transmission Lines Design (contd.)

- If Z_0 and ε_r is specified or known, following expression can be used to determine w/h:

For w/h≤2:

$$\frac{w}{h} = \frac{8e^{A}}{e^{2A} - 2}$$

Where:

$$A = 2\pi \frac{Z_0}{Z_f}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right)$$

For w/h≥2:

$$\frac{w}{h} = \frac{2}{\pi}\left(B - 1 - \ln(2B-1) + \frac{\varepsilon_r - 1}{2\varepsilon_r}\left[\ln(B-1) + 0.39 - \frac{0.61}{\varepsilon_r}\right]\right)$$

Where:

$$B = \frac{Z_f \pi}{2Z_0\sqrt{\varepsilon_r}}$$

Microstrip Transmission Lines Design (contd.)

Case-II: thickness (t) of the line is not negligible → in this scenario all the formulas are valid with the assumption that the effective width of the line increases as:

$$w_{eff} = w + \frac{t}{\pi}\left(1 + \ln\frac{2x}{t}\right)$$

Where $x = h \;\; if \;\; w > h/2\pi$ or $x = 2\pi w \;\; if \;\; h/2\pi > w > 2t$

Example – 1

A microstrip material with $\varepsilon_r = 10$ and $h = 1.016\ mm$ is used to build a narrow transmission line. Determine the width for the microstrip transmission line to have a characteristic impedance of 50Ω. Also determine the wavelength and the effective relative dielectric constant of the microstrip line.

Using the Formulas:

Let us consider the first formula: $$\frac{w}{h} = \frac{8e^{A}}{e^{2A} - 2}$$

$$A = 2\pi \frac{Z_0}{Z_f}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right) = 2\pi \frac{50}{377}\sqrt{\frac{10+1}{2}} + \frac{10-1}{10+1}\left(0.23 + \frac{0.11}{10}\right)$$

$$\Rightarrow A = 2.1515$$

Therefore: $$\frac{w}{h} = \frac{8e^{2.1515}}{e^{2(2.1515)} - 2} = 0.9563$$

Now: h = 1.016 mm = 0.1016 cm = 0.1016(1000/2.54) mils = 40 mils

$$\therefore w = 0.9563 * 40 mils = 38.2 mils$$

Example – 1 (contd.)

$$\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_r}}\left[\frac{\varepsilon_r}{1+0.63(\varepsilon_r-1)(w/h)^{0.1255}}\right]^{1/2}$$

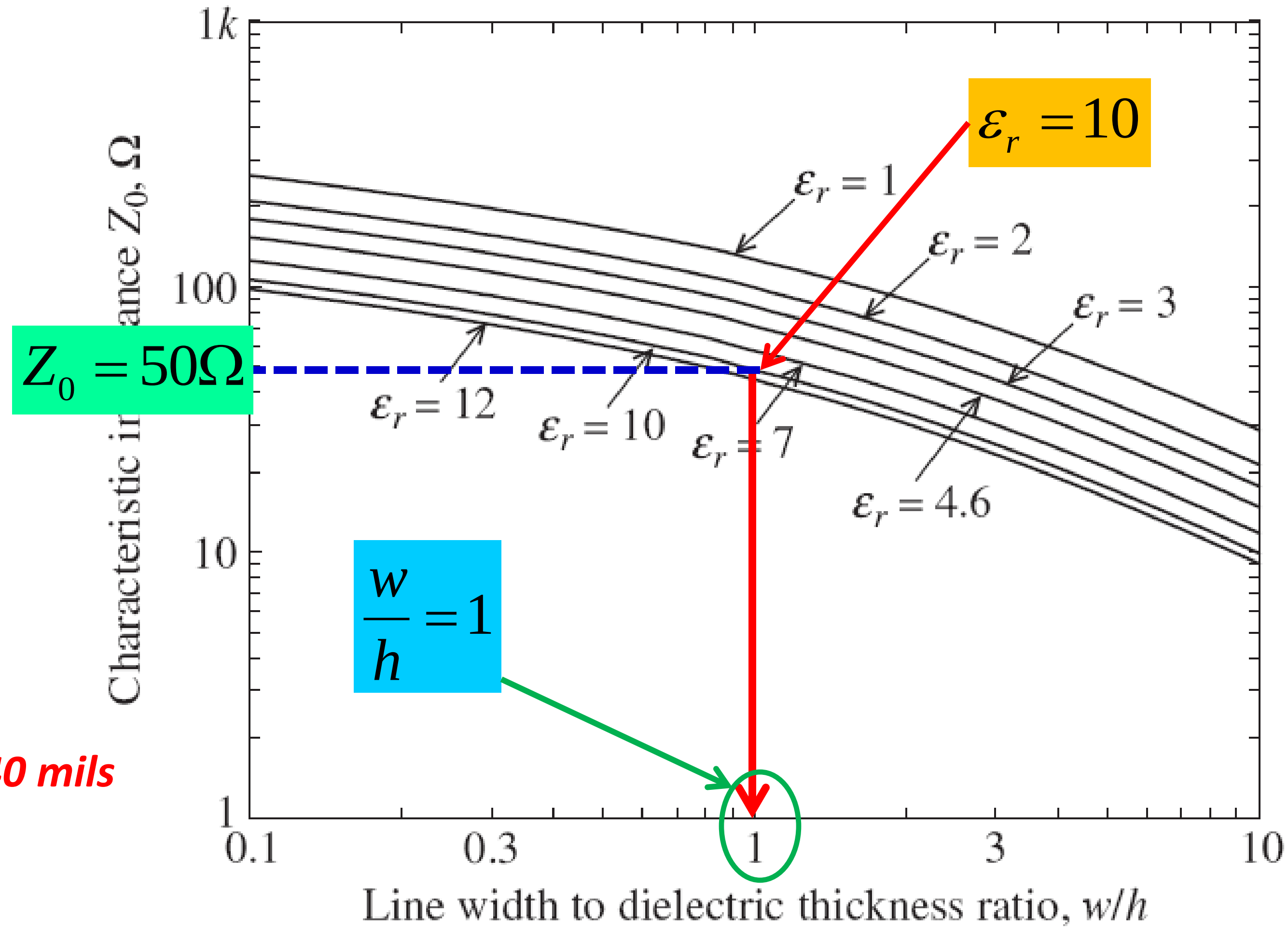
$$\therefore \lambda = \frac{\lambda_0}{\sqrt{10}}\left[\frac{10}{1+0.63(10-1)(0.9563)^{0.1255}}\right]^{1/2} = 0.387\lambda_0$$

$$\lambda = \frac{v_p}{f} = \frac{c}{f\sqrt{\varepsilon_{eff}}} = \frac{\lambda_0}{\sqrt{\varepsilon_{eff}}} \quad \Rightarrow \varepsilon_{eff} = \left(\frac{\lambda_0}{\lambda}\right)^2$$

$$\therefore \varepsilon_{eff} = \left(\frac{1}{0.387}\right)^2 = 6.68$$

Example – 1 (contd.)

Using the Design Curves

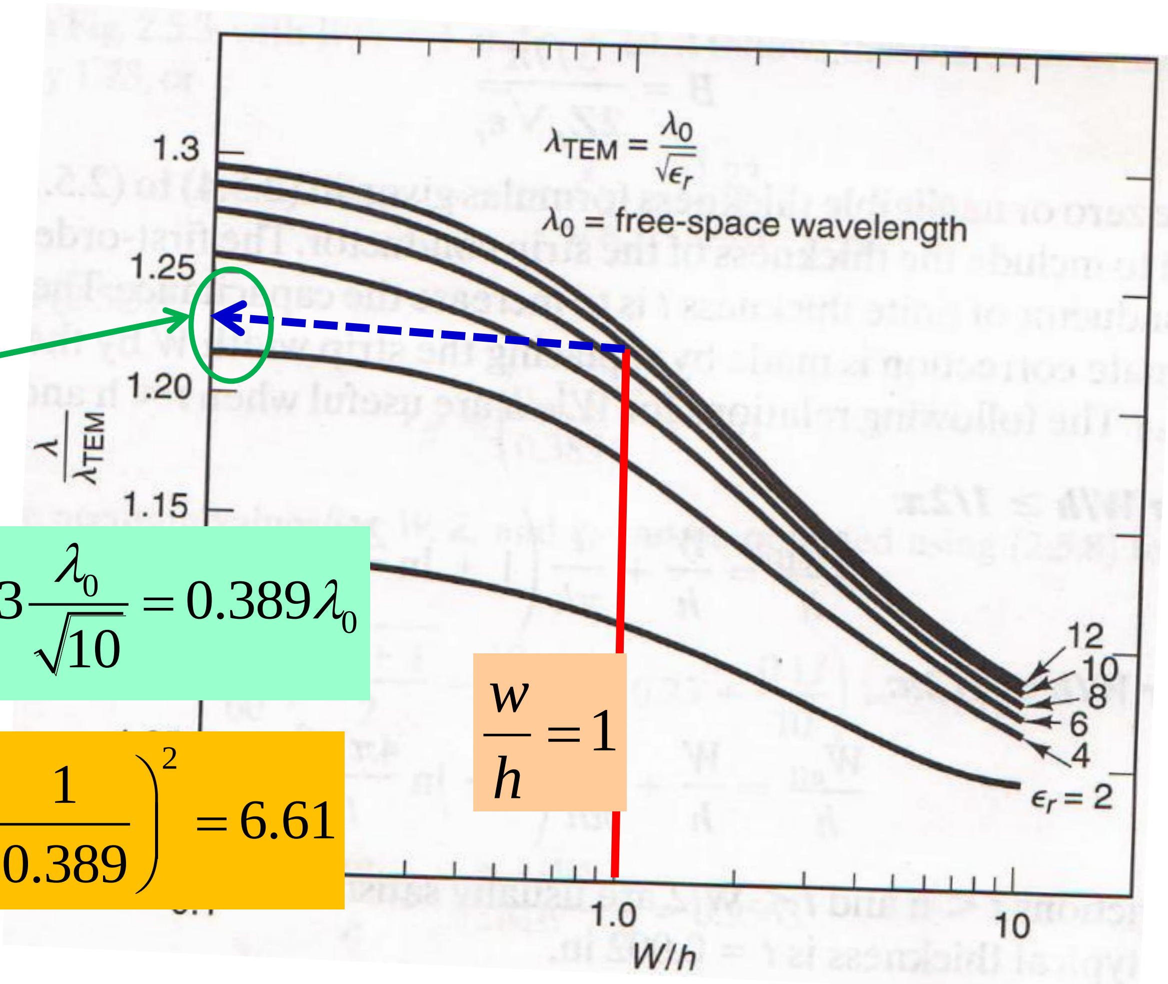


h = 1.016 mm = 40 mils

=> w = 40 mils

Example – 1 (contd.)

Using the Design Curves



$$\frac{\lambda}{\lambda_{TEM}} = 1.23$$

$$\frac{w}{h} = 1$$

$$\lambda = 1.23\frac{\lambda_0}{\sqrt{\varepsilon_r}} = 1.23\frac{\lambda_0}{\sqrt{10}} = 0.389\lambda_0$$

$$\Rightarrow \varepsilon_{eff} = \left(\frac{\lambda_0}{\lambda}\right)^2 = \left(\frac{1}{0.389}\right)^2 = 6.61$$

Example – 2

a. Using the design curves, calculate W, λ, and ε_{eff} for a characteristic impedance of 50Ω using RT/Duroid with $\varepsilon_r = 2.23$ and $h = 0.7874\ mm$.

b. Use design equations to show that for RT/Duroid with $\varepsilon_r = 2.23$ and $h = 0.7874\ mm$, a 50Ω-characteristic impedance is obtained with $W/_h = 3.073$. Also show, $\varepsilon_{eff} = 1.91$ and $\lambda = 0.7236\lambda_0$.

Lossless Transmission Line

- For a lossless transmission line the second order differential equation for phasors are:

$$\frac{d^2 V(z)}{dz^2} + \beta^2 V(z) = 0$$

$$\frac{d^2 I(z)}{dz^2} + \beta^2 I(z) = 0$$

$$\beta = \omega\sqrt{LC}$$

General Solution For Voltage Phasor

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

V_0^+ and V_0^- are complex constants

- Similarly the current phasor for a lossless line can be described:

$$\therefore I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$$

$$Z_0 = \frac{\omega L}{\beta} = \frac{\omega L}{\omega\sqrt{LC}} = \sqrt{\frac{L}{C}}$$

Lossless Transmission Line (contd.)

We now know that a **lossless** transmission line is **completely** characterized by **real** constants Z_0 and β.

Likewise, the **2 waves** propagating on a transmission line are **completely** characterized by **complex** constants ${V_0}^+ \; and \; {V_0}^-$.

Q: Z_0 and β are determined from L, C, and ω. How do we find ${V_0}^+ \; and \; {V_0}^-$?

A: Apply **Boundary Conditions**!

Every transmission line has **2** "boundaries":

1) At one end of the transmission line.

2) At the **other** end of the trans line!

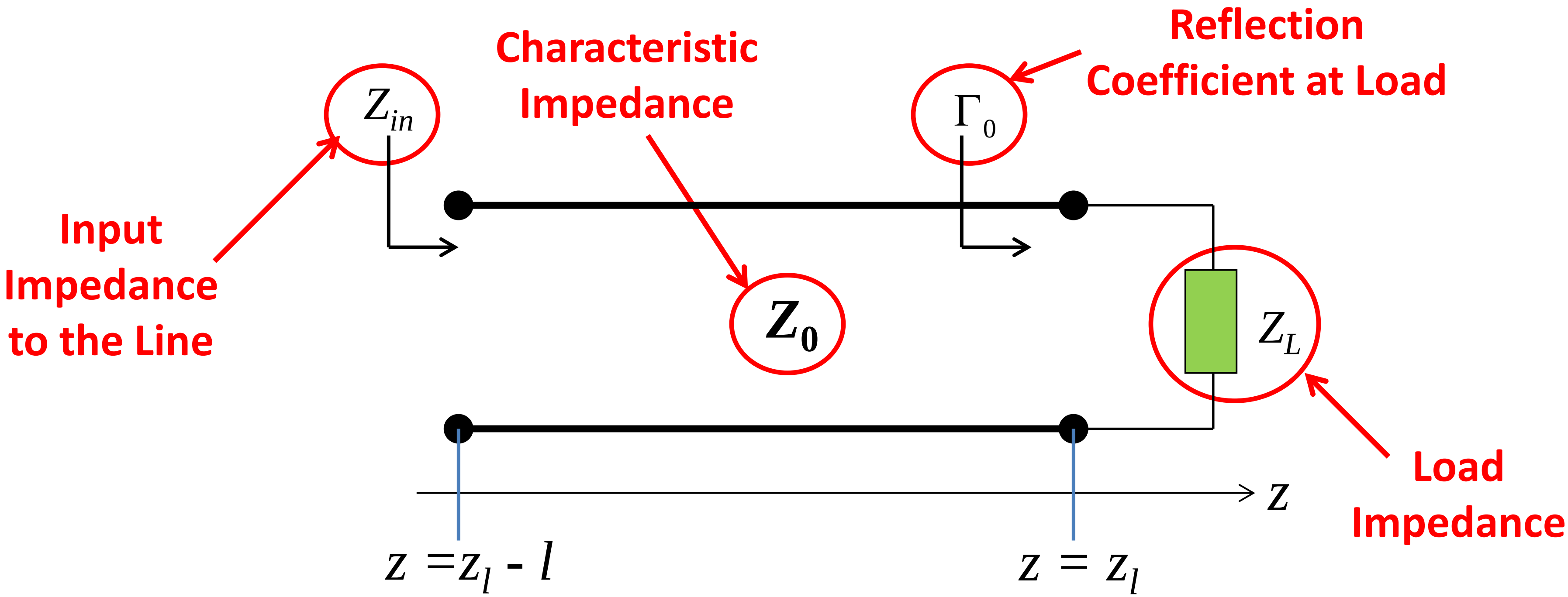
Typically, there is a **source** at one end of the line, and a **load** at the other.

→ The purpose of the transmission line is to get power **from** the source, **to** the load!

Let's apply the **load** boundary condition!

Terminated Lossless Transmission Line

- Now let's **attach** something to our transmission line. Consider a **lossless** line, length l, terminated with a **load** Z_l.

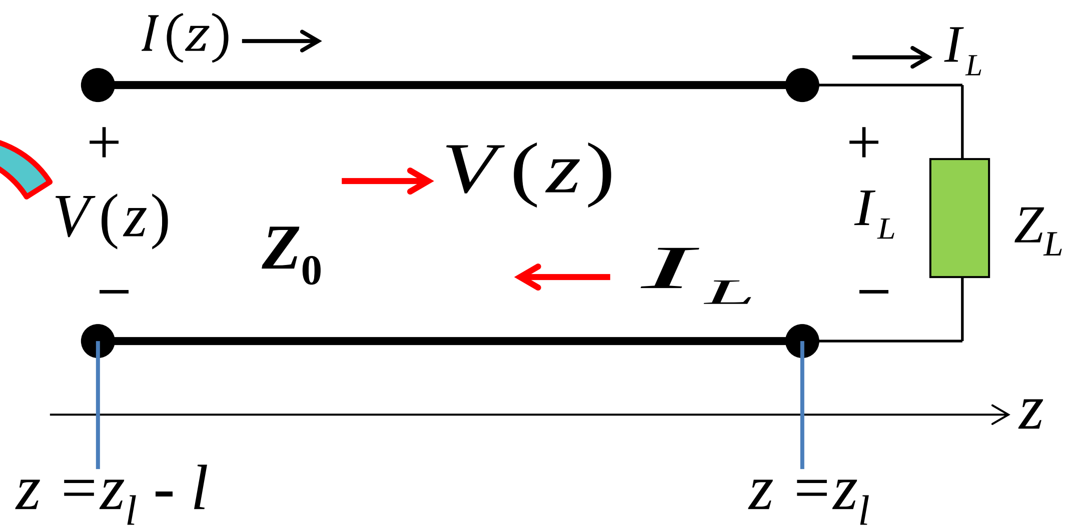


Q: What is the **current** and **voltage** at each and **every** point on the transmission line (i.e., what is $I(z)$ and $V(z)$ for **all** points z where $z_l - l < z < z_l$.

A: To find out, we must apply **boundary conditions**!

Terminated Lossless Transmission Line (contd.)

- The load is assumed at $z = z_l$
- The voltage wave couples into the line at $z = z_l - l$



$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

Incident Wave   Reflected Wave

- **At the load:** the voltage and current must be consistent with a valid **transmission line solution**:

$$V(z = z_l) = V^+(z = z_l) + V^-(z = z_l) = V_0^+ e^{-j\beta z_l} + V_0^- e^{j\beta z_l}$$

Terminated Lossless Transmission Line (contd.)

$$I(z=z_l)=\frac{V^+(z=z_l)}{Z_0}-\frac{V^-(z=z_l)}{Z_0}=\frac{V_0^+}{Z_0}e^{-j\beta z_l}-\frac{V_0^-}{Z_0}e^{j\beta z_l}$$

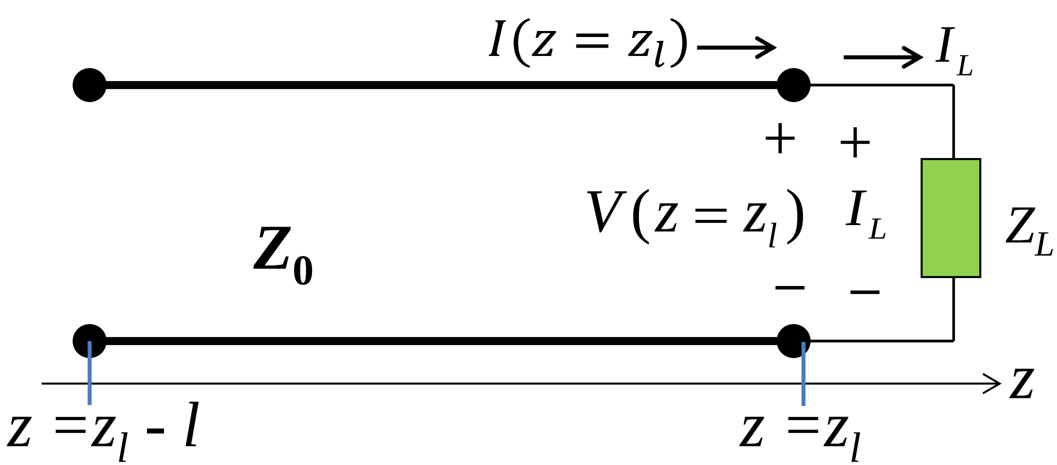
- Furthermore, the load voltage and current must be related by **Ohm's law**:

$$V_L = Z_L I_L$$

- Most importantly, we recognize that the values $I(z=z_l), V(z=z_l)$ and I_L, V_L are **not** independent, but in fact are strictly related by **Kirchoff's Laws**!

$$V(z=z_l)=V_L$$

$$I(z=z_l)=I_L$$



Terminated Lossless Transmission Line (contd.)

So now we have the **boundary conditions** for **this** particular problem.

Careful! Different transmission line problems lead to **different** boundary conditions—**you** must assess each problem **individually** and **independently**!

- **Combining** these equations and boundary conditions, we find that:

$$V(z=z_l)=V_L=Z_L I_L=Z_L I(z=z_l)$$

$$V^+(z=z_l)+V^-(z=z_l)=\frac{Z_L}{Z_0}\left(V^+(z=z_l)-V^-(z=z_l)\right)$$

- Rearranging, we can conclude: $$\frac{V^-(z=z_l)}{V^+(z=z_l)}=\frac{Z_L-Z_0}{Z_L+Z_0}$$

Terminated Lossless Transmission Line (contd.)

$$\frac{V^-(z=z_l)}{V^+(z=z_l)} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

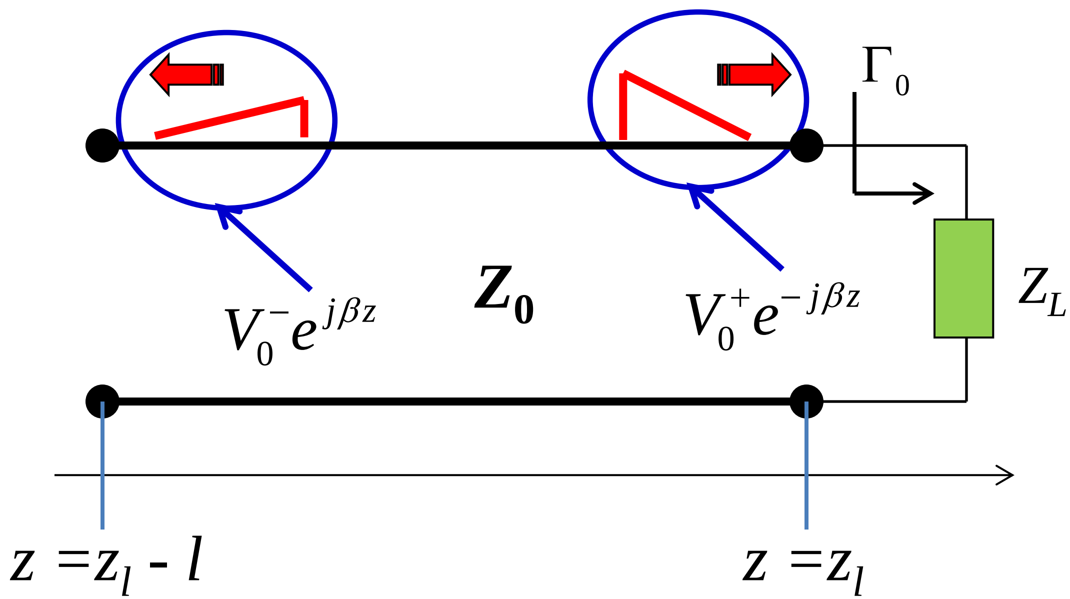
ratio of reflected to incident voltage wave

also holds true for current wave but with opposite sign

Voltage Reflection Coefficient $\Gamma(z = z_l)$

This value is of **fundamental** importance for the terminated transmission line problem, so we provide it with its **own** special symbol (Γ_0)!

$$\Gamma_0 = \frac{V^-(z=z_l)}{V^+(z=z_l)}$$



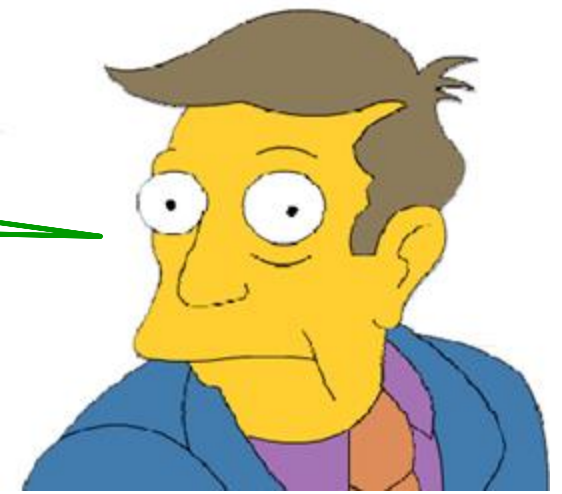
Terminated Lossless Transmission Line (contd.)

- Therefore:

$$\Gamma_0 = \frac{V^-(z=z_l)}{V^+(z=z_l)} \quad \Rightarrow \quad \Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

More useful representation as it involves known circuit/system quantities

Q: I'm confused! Just what are we trying to accomplish in this handout?



A: We are trying to find V(z) and I(z) when a lossless transmission line is terminated by a load Z_L!

Terminated Lossless Transmission Line (contd.)

- We can express the reflected voltage wave as:

$$\Gamma_0 = \frac{V^-(z=z_l)}{V^+(z=z_l)} = \frac{V_0^- e^{+j\beta z_l}}{V_0^+ e^{-j\beta z_l}} \Rightarrow V_0^- = \Gamma_0 V_0^+ e^{-j2\beta z_l}$$

- Therefore:

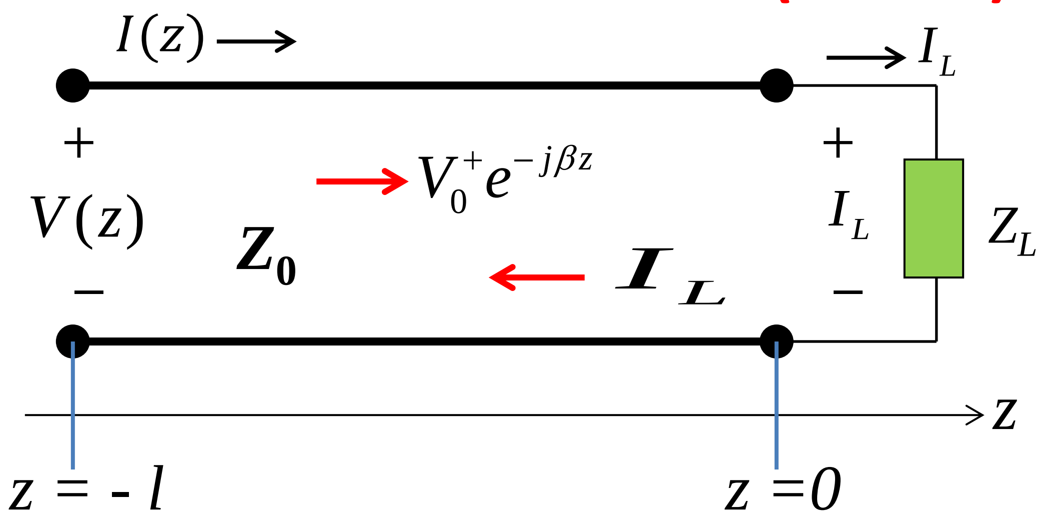
$$V^-(z) = \left(\Gamma_0 V_0^+ e^{-j2\beta z_l}\right) e^{+j\beta z}$$

$$V(z) = V^+(z) + V^-(z) = V_0^+ \left[e^{-j\beta z} + \left(\Gamma_0 e^{-j2\beta z_l}\right) e^{+j\beta z} \right]$$

$$I(z) = \frac{V^+(z) - V^-(z)}{Z_0} = \frac{V_0^+}{Z_0} \left[e^{-j\beta z} - \left(\Gamma_0 e^{-j2\beta z_l}\right) e^{+j\beta z} \right]$$

we can further **simplify** our analysis by **arbitrarily** assigning the end point z_l a **zero** value (i.e., $z_l = 0$)

Terminated Lossless Transmission Line (contd.)



$$V(z=0) = V^+(z=0) + V^-(z=0) = V_0^+ e^{-j\beta(0)} + V_0^- e^{+j\beta(0)} = V_0^+ + V_0^-$$

$$I(z=0) = \frac{V_0^+ - V_0^-}{Z_0}$$

$$Z(z=0) = \frac{V(z=0)}{I(z=0)} = Z_0 \left[\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \right] = Z_L$$

- The current and voltage along the line in this case are:

$$V(z) = V_0^+ \left[e^{-j\beta z} + \Gamma_0 e^{+j\beta z} \right]$$

$$I(z) = \frac{V_0^+}{Z_0} \left[e^{-j\beta z} - \Gamma_0 e^{+j\beta z} \right]$$

Terminated Lossless Transmission Line (contd.)

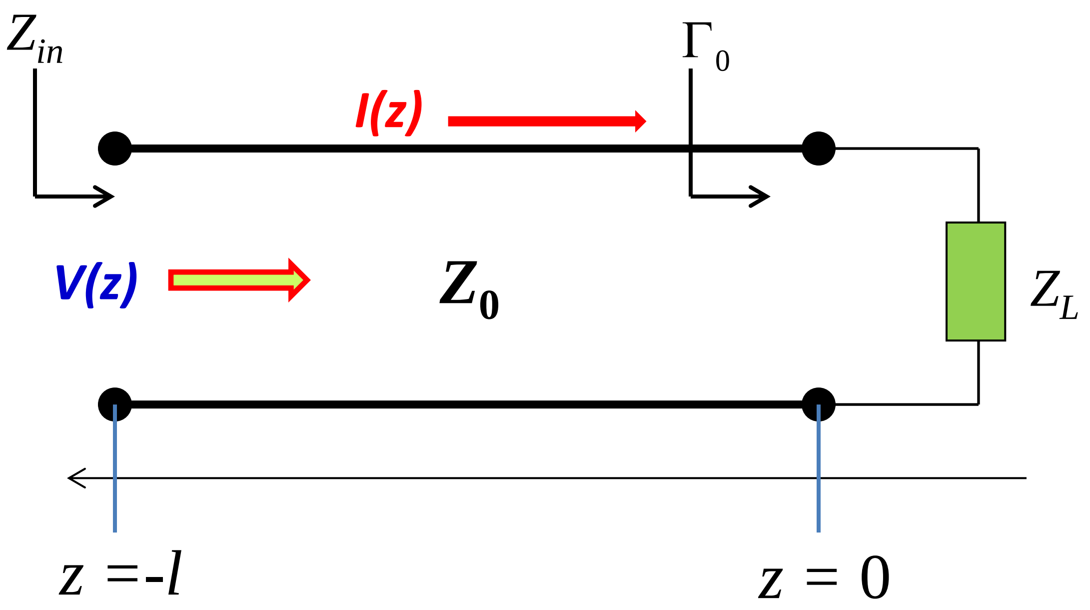
Q: But, how do we determine V_0^+??

A: We require a **second** boundary condition to determine V_0^+. The only boundary left is at the **other end** of the transmission line. Typically, a **source** of some sort is located there. This makes physical sense, as something must generate the **incident** wave !

Now let us consider the **Special Values of Load impedances**

Special Termination Conditions

- Let us once again consider a generic TL terminated in arbitrary impedance Z_L



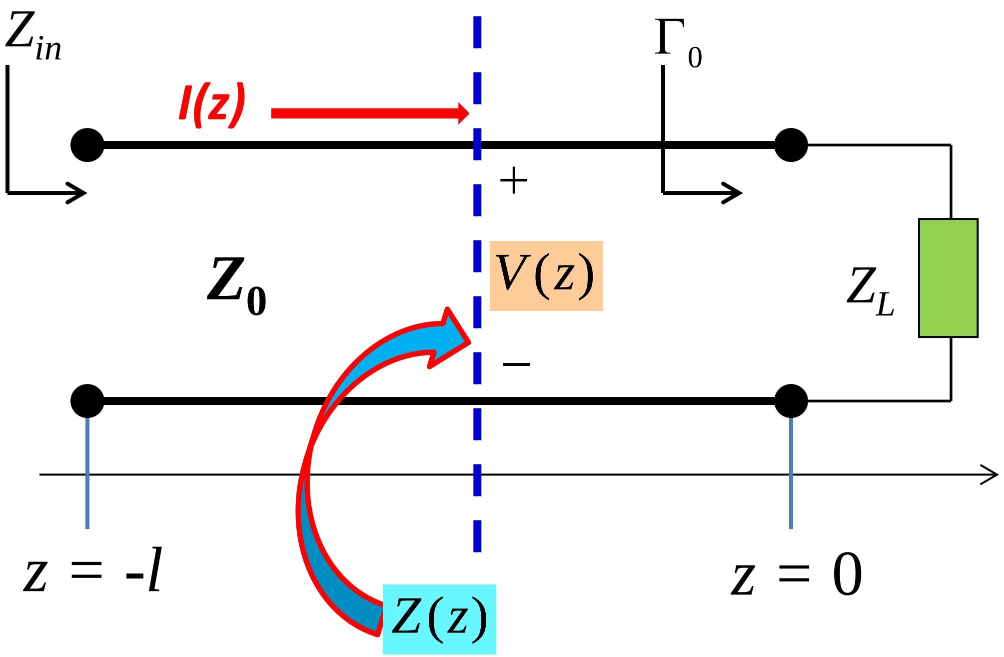
It's interesting to note that the load $\mathbf{Z_L}$ enforces a boundary condition that explicitly determines neither **V(z)** nor **I(z)**—but **completely** specifies **line impedance** Z(z)!

Special Termination Conditions (contd.)

- We define the generalized impedance at any point on the line as:

$$Z(z) = \frac{V(z)}{I(z)}$$

This is the impedance we would measure if we cut the line at *z* and measured its impedance there.



$$Z(z) = \frac{V(z)}{I(z)} = \frac{V^+ e^{+j\beta z}\left(1 + \Gamma_0 e^{-j(2\beta z)}\right)}{\frac{V^+ e^{+j\beta z}}{Z_0}\left(1 - \Gamma_0 e^{-j(2\beta z)}\right)}$$

Special Termination Conditions (contd.)

$$Z(z) = Z_0 \frac{e^{-j\beta z} + \Gamma_0 e^{+j\beta z}}{e^{-j\beta z} - \Gamma_0 e^{+j\beta z}} = Z_0 \frac{Z_L \cos(\beta z) - jZ_0 \sin(\beta z)}{Z_0 \cos(\beta z) - jZ_L \sin(\beta z)}$$

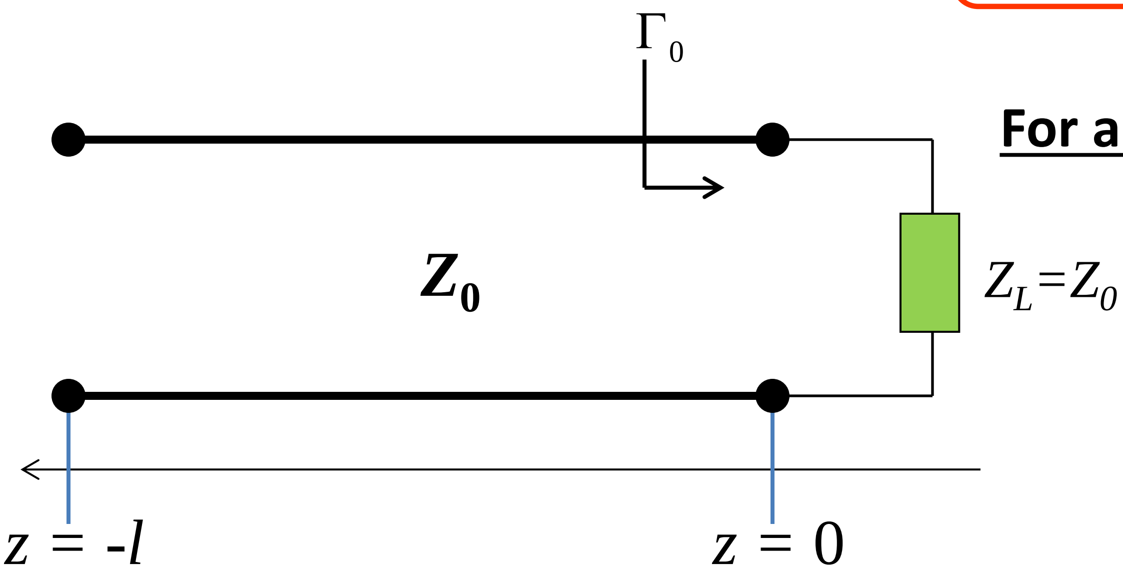
- Likewise, the load boundary condition leaves $V^+(z)$ and $V^-(z)$ undetermined, but **completely** determines **reflection coefficient function** $\Gamma(\boldsymbol{z})$!

$$\Gamma(z) = \frac{V^-(z)}{V^+(z)} = \Gamma_0 e^{+j2\beta z} = \frac{Z_L - Z_0}{Z_L - Z_0} e^{+j2\beta z}$$

Let's look at some **specific** values of load impedance $Z_L = R_L + jX_L$ and see what functions $Z(z)$ and $\Gamma(\boldsymbol{z})$ result!

Special Termination Conditions (contd.)

- $Z_L = Z_0$ ⇦ **Matched Line** ⇨ the load impedance equals the characteristic impedance of the TL



For a lossless TL: $R_L = Z_0$, $X_L = 0$ ← **Purely Real**

The load reflection coefficient:

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0} = 0$$

means no reflected wave $V^-(z)$

reflection coefficient is zero at all points along the line

The impedance at position *z*:

$$Z(z) = Z_0$$

The line impedance equals Z_0 → matched condition