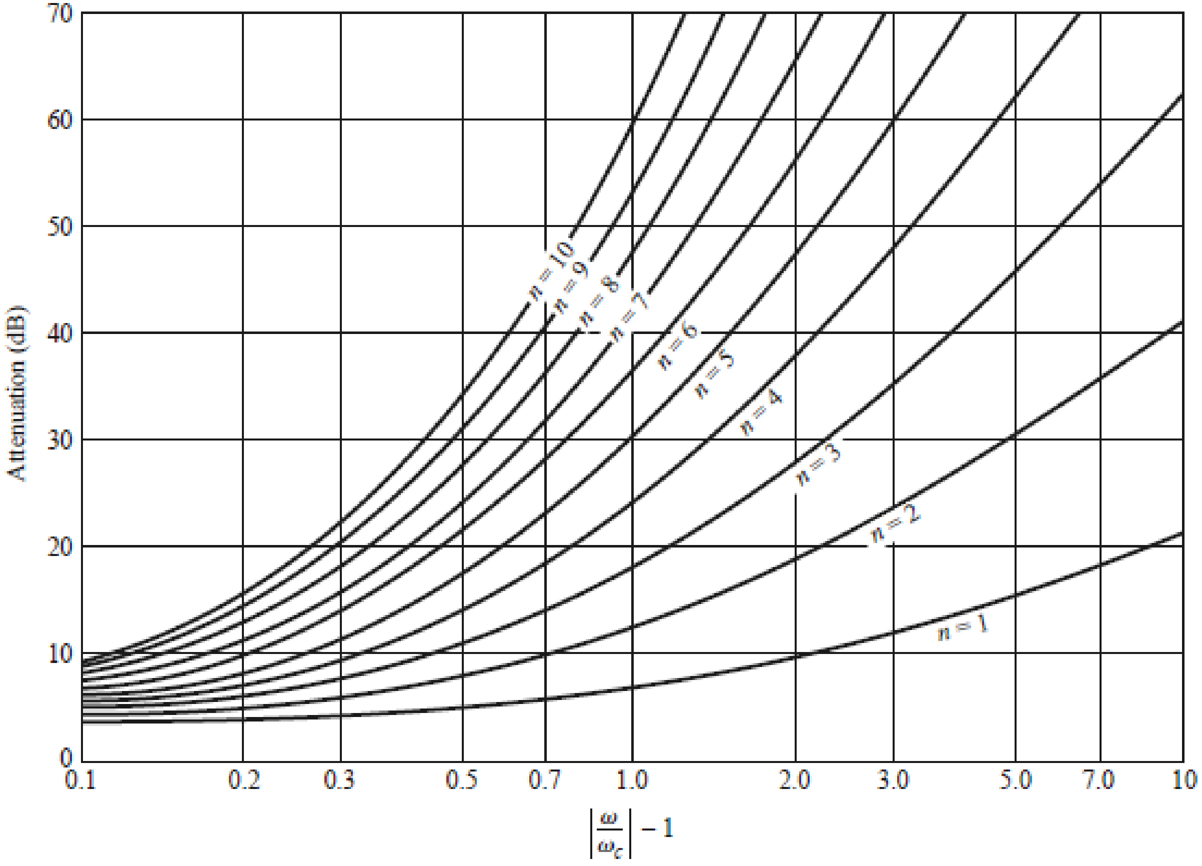
Lecture – 23 Date: 18.11.2014

- The Insertion Loss Method (contd.)
- Richard's Transformation
- Kuroda's Identities

Insertion Loss Method



Attenuation versus Normalized Frequency

Insertion Loss Method (contd.)

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

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Insertion Loss Method (contd.)

TABLE 8.4 Element Values for Equal-Ripple Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10, 0.5 dB and 3.0 dB ripple)

0.5 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

3.0 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

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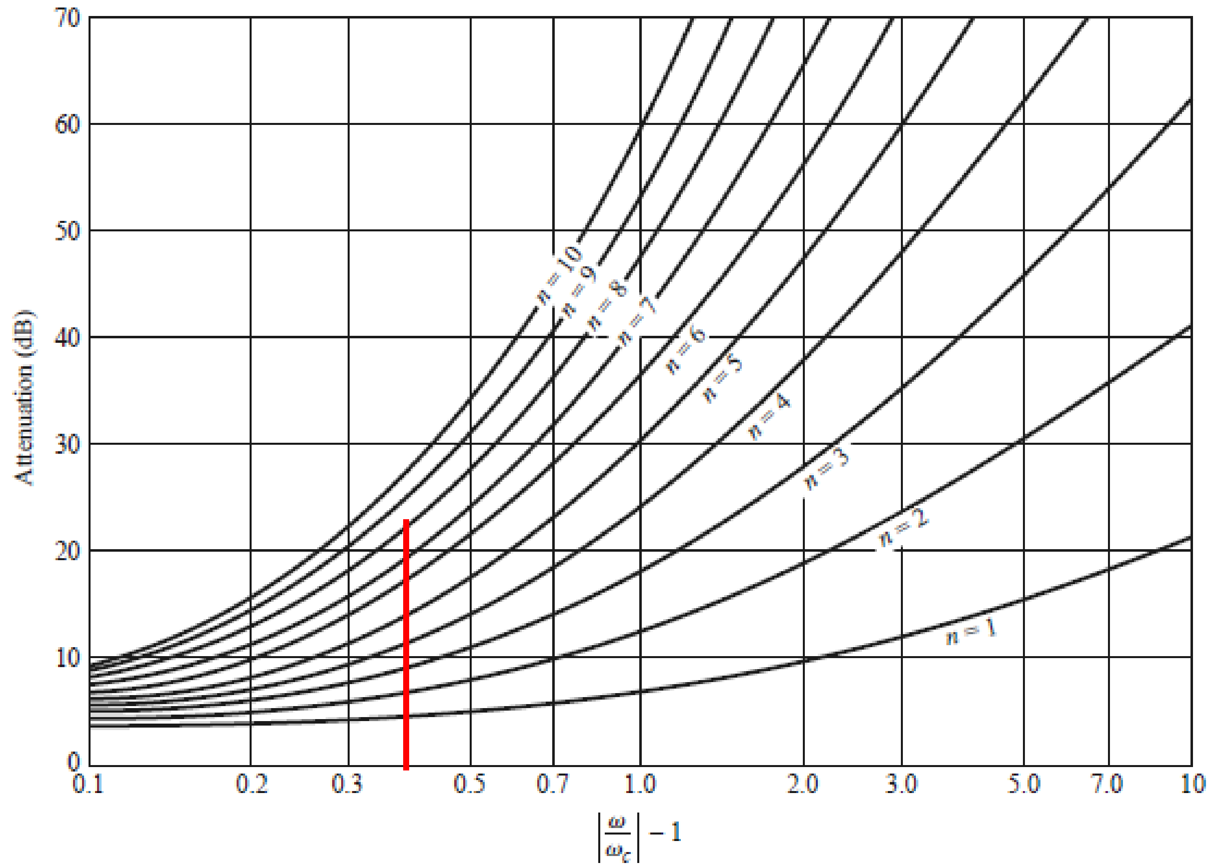
Example – 1

A maximally flat low-pass filter is to be designed with a cut-off frequency of 8GHz and a minimum attenuation of 20dB at 11GHz. How many filter elements are required?

We have:

$$\frac{\omega/2\pi}{\omega_c/2\pi} - 1 = \frac{11}{8} - 1 = 0.375$$

N=8



Example – 2

Design a maximally flat low-pass filter with a cut-off frequency of 2GHz, impedance of 50Ω and at least 15dB insertion loss at 3GHz.

- First, find the required order of the maximally flat filter to satisfy the insertion loss specification at 3GHz.
- We have:

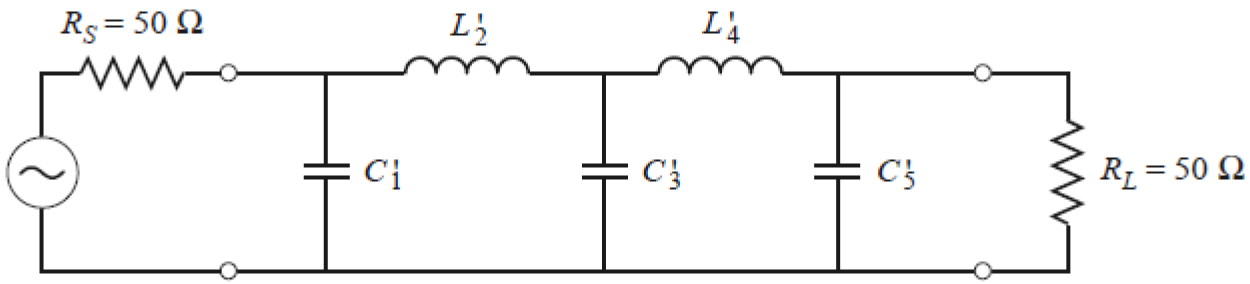
$$\frac{\omega/2\pi}{\omega_c/2\pi} - 1 = \frac{3}{2} - 1 = 0.5$$

- It is apparent that N =5 will be sufficient.
- From the table we get: $g_1 = 0.618$, $g_2 = 1.618$, $g_3 = 2.000$, $g_4 = 1.618$, $g_5 = 0.618$.

Example – 2 (contd.)

- The Analysis of N-element filters give: $L_n = g_n\left(\frac{R_s}{\omega_c}\right)$ $C_n = g_n\left(\frac{1}{R_s\omega_c}\right)$
- The elements are therefore:

$$C_1 = 0.984\,pF \quad L_2 = 6.438\,nH \quad C_3 = 3.183\,pF \quad L_4 = 6.438\,nH \quad C_5 = 0.984\,pF$$



Insertion Loss Method (contd.)

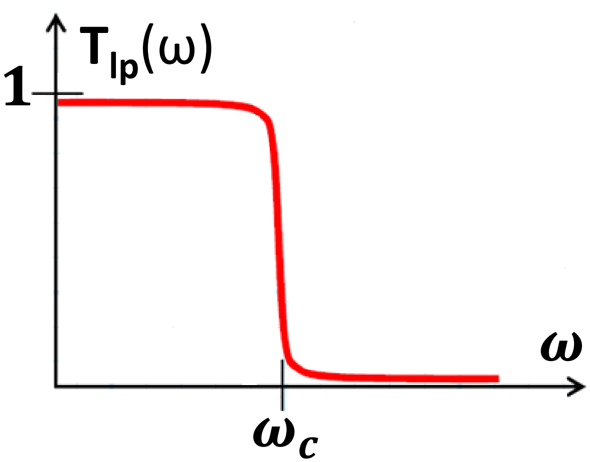
Q: OK, so we now have the solutions for **low-pass** filters. But what about high-pass, band-pass, or band-stop filters?

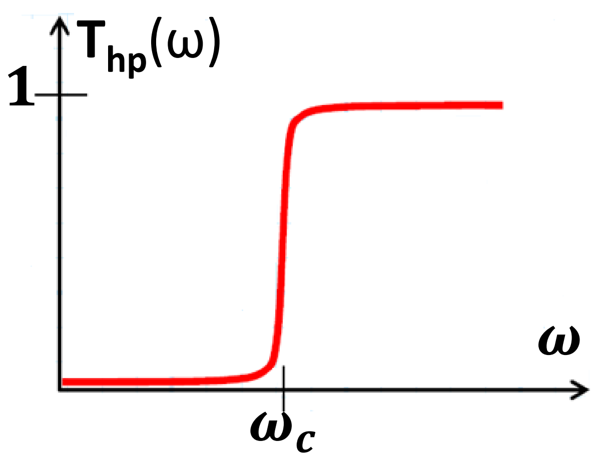
A: Surprisingly, the low-pass filter solutions **likewise** provide us with the solutions for **any** and **all** high-pass, band-pass and band-stop filters! All we need to do is apply **filter transformations**.

Filter Transformations

We can use the concept of **filter transformations** to determine the **new** filter designs from a low-pass design. As a result, we can construct a 3rd-order Butterworth **high-pass** filter or a 5th-order Chebychev **band-pass** filter!

It will be apparent that the mathematics for each filter design will be very **similar**. For example, the difference between a low-pass and high-pass filter is essentially an **inverse**—the frequencies below ω_c are mapped into frequencies above ω_c —and vice versa.





It is evident that:

$$T_{lp}(\omega=0)=T_{hp}(\omega=\infty)=1$$

$$T_{lp}(\omega=\infty)=T_{hp}(\omega=0)=0$$

Filter Transformations (contd.)

- However: $T_{lp}(\omega=\omega_c)=\mathrm{T}_{hp}(\omega=\omega_c)=0.5$
- Therefore, we can express: $T_{lp}(\omega=\alpha\omega_c)=\mathrm{T}_{hp}(\omega=\frac{1}{\alpha}\omega_c)$

  where α is some positive, real value (i.e., 0 <∝< ∞).
- For example, if ∝= 0.5, then: $T_{lp}(\omega=0.5\omega_c)=\mathrm{T}_{hp}(\omega=2\omega_c)$

In other words, the transmission through a low-pass filter at one half the cut-off frequency will be equal to the transmission through a (mathematically similar) high-pass filter at twice the cut-off frequency.

- Now, recall the loss-ratio functions for Butterworth and Chebychev low-pass filters: $P_{LR}^{lp}(\omega)=1+\left(\frac{\omega}{\omega_c}\right)^{2N}$ $P_{LR}^{lp}(\omega)=1+k^2T_N^2\left(\frac{\omega}{\omega_c}\right)$
- Note in each case that the argument of the function has the form: $\frac{\omega}{\omega_c}$

In other words, the frequency is **normalized** by the cut-off frequency.

Filter Transformations (contd.)

- Now Consider **this** mapping: $\frac{\omega}{\omega_c} \Rightarrow -\frac{\omega_c}{\omega}$
- This mapping **transforms** the low-pass filter response into a corresponding high pass filter response! i.e.:

$$P_{LR}^{hp}(\omega) = 1 + \left(-\frac{\omega_c}{\omega}\right)^{2N} = 1 + \left(\frac{\omega_c}{\omega}\right)^{2N}$$

$$P_{LR}^{hp}(\omega) = 1 + k^2 T_N^2\left(-\frac{\omega_c}{\omega}\right) = 1 + k^2 T_N^2\left(\frac{\omega_c}{\omega}\right)$$

Q: Yikes! Where did this mapping come from? Are you sure this works?

Consider again the case where ω =αω$_c$; the low pass responses are:

$$P_{LR}^{lp}(\omega) = 1 + (\alpha)^{2N}$$

$$P_{LR}^{lp}(\omega) = 1 + k^2 T_N^2(\alpha)$$

- Now consider the high-pass responses where ω =ω$_c$/α:

$$P_{LR}^{hp}(\omega) = 1 + (\alpha)^{2N}$$

$$P_{LR}^{hp}(\omega) = 1 - k^2 T_N^2(\alpha)$$

- Thus, we can conclude from this mapping that: $P_{LR}^{lp}(\omega = \alpha\omega_c) = P_{LR}^{hp}(\omega = \omega_c / \alpha)$

Filter Transformations (contd.)

- And since $T = {P_{LR}}^{-1}$:

$$T_{lp}(\omega = \alpha\omega_c) = T_{hp}(\omega = \frac{1}{\alpha}\omega_c)$$

Exactly the result that we expected! Our mapping provides a method for transforming a low-pass filter into a high-pass filter!

Q: OK Poindexter, you have succeeded in providing another one of your "fascinating" mathematical insights, but does this "mapping" provide anything useful for us engineers?

A: Absolutely! We can apply this mapping one component element (capacitor or inductor) at a time to our low-pass schematic design, and the result will be a direct transformation into a high-pass filter schematic.

- Recall the reactance of an inductor element in a low-pass filter design is:

$$jX_n^{lp} = j\omega L_n^{lp} = j\omega g_n\left(\frac{R_s}{\omega_c}\right) = jg_n R_s\left(\frac{\omega}{\omega_c}\right)$$

- while that of a capacitor is:

$$jX_n^{lp} = \frac{1}{j\omega C_n^{lp}} = -j\frac{R_s}{g_n}\left(\frac{\omega_c}{\omega}\right)$$

- Now apply the mapping: $\frac{\omega}{\omega_c} \Rightarrow -\frac{\omega_c}{\omega}$

Filter Transformations (contd.)

- The inductor becomes:

$$jX_n^{hp} = jg_nR_s\left(-\frac{\omega_c}{\omega}\right) = -j\frac{g_nR_s\omega_c}{\omega} = \frac{1}{j\left(g_nR_s\omega_c\right)^{-1}\omega}$$

- and the capacitor:

$$jX_n^{hp} = -j\frac{R_s}{g_n}\left(-\frac{\omega_c}{\omega}\right) = j\omega\left(\frac{R_s}{g_n\omega_c}\right)$$

It is clear (do **you** see why?) that the transformation has converted a positive (i.e., inductive) reactance into a negative (i.e., capacitive) reactance—and vice versa.

- As a result, to transform a low-pass filter schematic into a high-pass filter schematic, we:

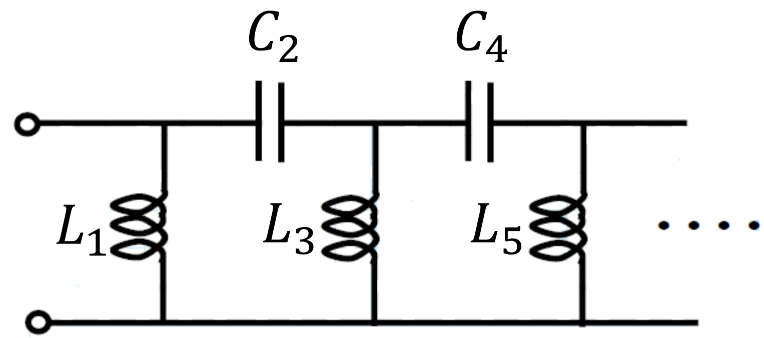
1. Replace each inductor with a capacitor of value:

$$C_n^{hp} = \frac{1}{g_nR_s\omega_c} = \frac{1}{\omega_c^2L_n^{lp}}$$

2. Replace each capacitor with an inductor of value:

$$L_n^{hp} = \frac{R_s}{g_n\omega_c} = \frac{1}{\omega_c^2C_n^{lp}}$$

Filter Transformations (contd.)



- Thus, a **high-pass ladder circuit** consists of **series capacitors** and **shunt inductors** (compare this to the low-pass) ladder circuit!).

Q: What about band-pass filters?

A: The difference between a low-pass and band-pass filter is simply a **shift** in the "center" frequency of the filter, where the center frequency of a low-pass filter is essentially $\omega = 0$.

- For this case, we find the **mapping**:

$$\frac{\omega}{\omega_c} \Rightarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

transforms a low-pass function into a **band-pass function**, where Δ is the **normalized bandwidth**:

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0}$$

ω_1 and ω_2 define the two **3dB frequencies** of the bandpass filter.

Filter Transformations (contd.)

- For example, the Butterworth **low-pass** function → becomes a Butterworth **band-pass** function:

$$P_{LR}^{lp}(\omega)=1+\left(\frac{\omega}{\omega_c}\right)^{2N} \quad \Rightarrow \quad P_{LR}^{bp}(\omega)=1+\frac{1}{\Delta^{2N}}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^{2N}$$

- Applying this transform to the **reactance** of a low-pass **inductive** element:

$$jX_n^{bp}=jg_nR_s\frac{1}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)=j\omega\left(\frac{g_nR_s}{\omega_0\Delta}\right)+\frac{1}{j\omega\left(\Delta/g_n\omega_0R_s\right)}$$

- Look what happened! The transformation turned the inductive reactance into an inductive reactance in series with a capacitive reactance.
- A similar analysis of the transformation of the low-pass capacitive reactance shows that it is transformed into an inductive reactance in parallel with an capacitive reactance.

Filter Transformations (contd.)

- As a result, to transform a low-pass filter schematic into a band-pass filter schematic, we:

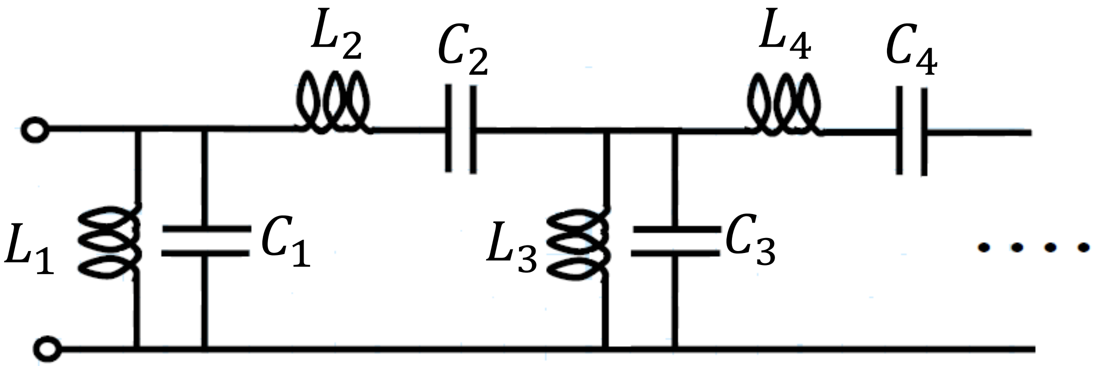
1. Replace each series inductor with a capacitor and inductor in series, with values:

$$L_n^{bp} = g_n \frac{R_s}{\omega_0 \Delta} \qquad C_n^{bp} = \frac{1}{g_n}\frac{\Delta}{\omega_0 R_s}$$

2. Replace each shunt capacitor with an inductor and capacitor in parallel, with values:

$$L_n^{bp} = \frac{1}{g_n}\frac{\Delta R_s}{\omega_0} \qquad C_n^{bp} = g_n \frac{1}{\omega_0 \Delta R_s}$$

- Thus, the ladder circuit for **band-pass circuit** is simply a ladder network of LC resonators, both series and parallel:



Filter Implementations

Q: So, we now know how to make any and all filters with **lumped** elements—but this is a **RF/microwave** engineering course!

You said that lumped elements were difficult to make and implement at high frequencies. **You** said that distributed elements were used to make microwave components. So **how** do we make a filter with **distributed elements**!?!

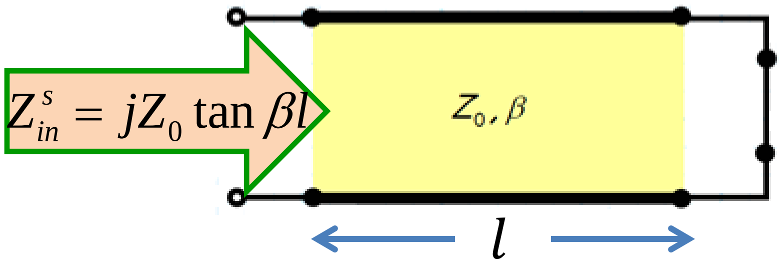
A: There are **many** ways to make RF/microwave filters with distributed elements. Perhaps the most straightforward is to "**realize**" each individual lumped element with transmission line sections, and then insert these **approximations** in our lumped element solutions.

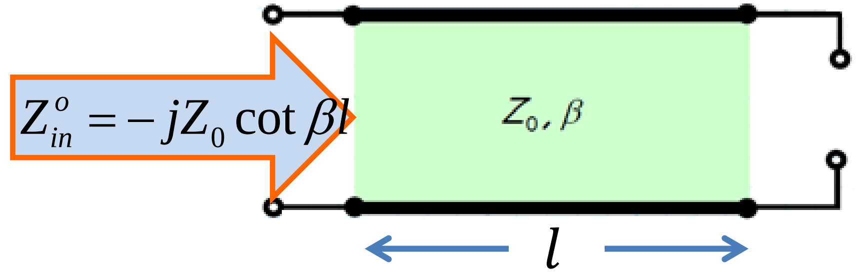
The **first** of these realizations is: Richard's Transformations

To easily **implement** Richard's Transforms in a microstrip or stripline circuit, we must apply one of **Kuroda's Identities**.

Richard’s Transformations

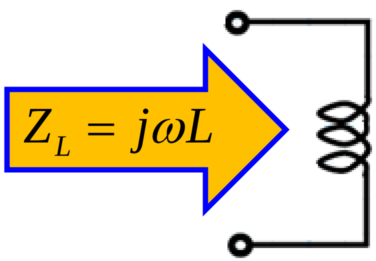
- Recall the input impedances of short-circuited and open-circuited transmission line **stubs**.

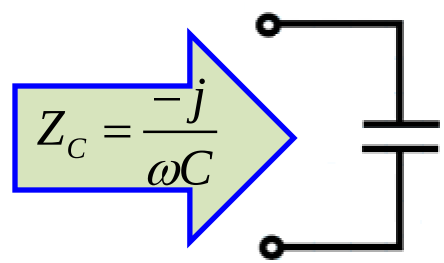




Note that the input impedances are purely **reactive**—just like **lumped** elements!

- However, the reactance of lumped inductors and capacitors have a **much** different mathematical form to that of transmission line stubs:





Richard's Transformations (contd.)

- In other words, the impedance of transmission line stubs and lumped elements (capacitors and inductors) are different functions with respect to **frequency**. Therefore, we can say in general that, for example:

$$Z_{in}^{s} \neq Z_L$$

$$Z_{in}^{o} \neq Z_C$$

However, for a given lumped element (L or C) and a given stub (with a given Z_0 and length l) the functions **will** be equal at precisely **one frequency**!

- For example, there is one frequency—let's call it ω_c —that satisfies **this** equation for a given L, Z_0, and l:

$$j\omega_c L = jZ_0 \tan \beta_c l = jZ_0 \tan\left[\frac{\omega_c}{v_p} l\right]$$

- Similarly:

$$\frac{-j}{\omega_c C} = -jZ_0 \cot \beta_c l = -jZ_0 \cot\left[\frac{\omega_c}{v_p} l\right]$$

- To make things easier, let's set the **length** of our transmission line stub to $\lambda_c/8$, where:

$$\lambda_c = \frac{v_p}{\omega_c} = \frac{2\pi}{\beta_c}$$

Richard's Transformations (contd.)

Q: Why $l = \lambda_c/8$?

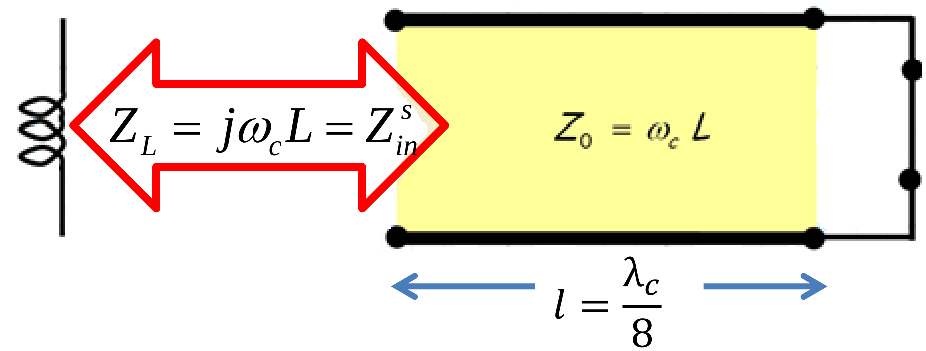
A: Well, for **one** reason, $\beta_c l = \pi/4$ and therefore $\tan(\pi/4) = 1.0$!

- This greatly **simplifies** our earlier results:

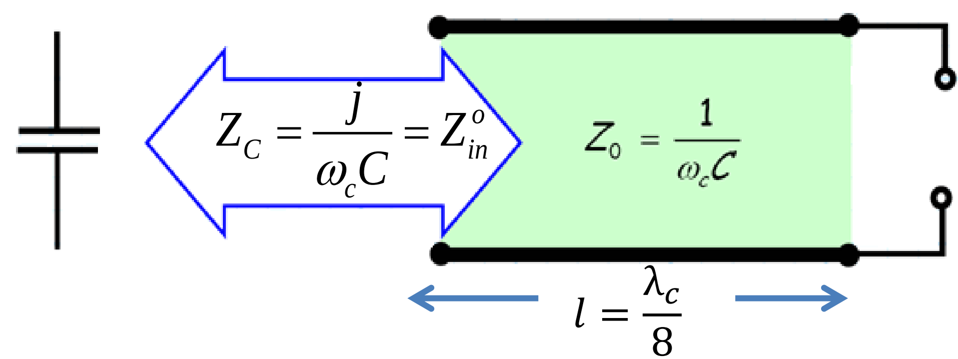
$$j\omega_c L = jZ_0 \tan\left(\frac{\pi}{4}\right) = jZ_0$$

$$\frac{-j}{\omega_c C} = -jZ_0 \cot\left(\frac{\pi}{4}\right) = -jZ_0$$

- Therefore, if we wish to build a **short-circuited** stub with the **same** impedance as an **inductor** L at frequency ω_c, we set the **characteristic impedance** of the stub transmission line to be $Z_0 = \omega_c L$:



- Similarly, if we wish to build **open-circuited** stub with the **same** impedance as a **capacitor** C at ω_c, we set the **characteristic impedance** of the stub transmission line to be $Z_0 = {}^{1}/_{\omega_c C}$:



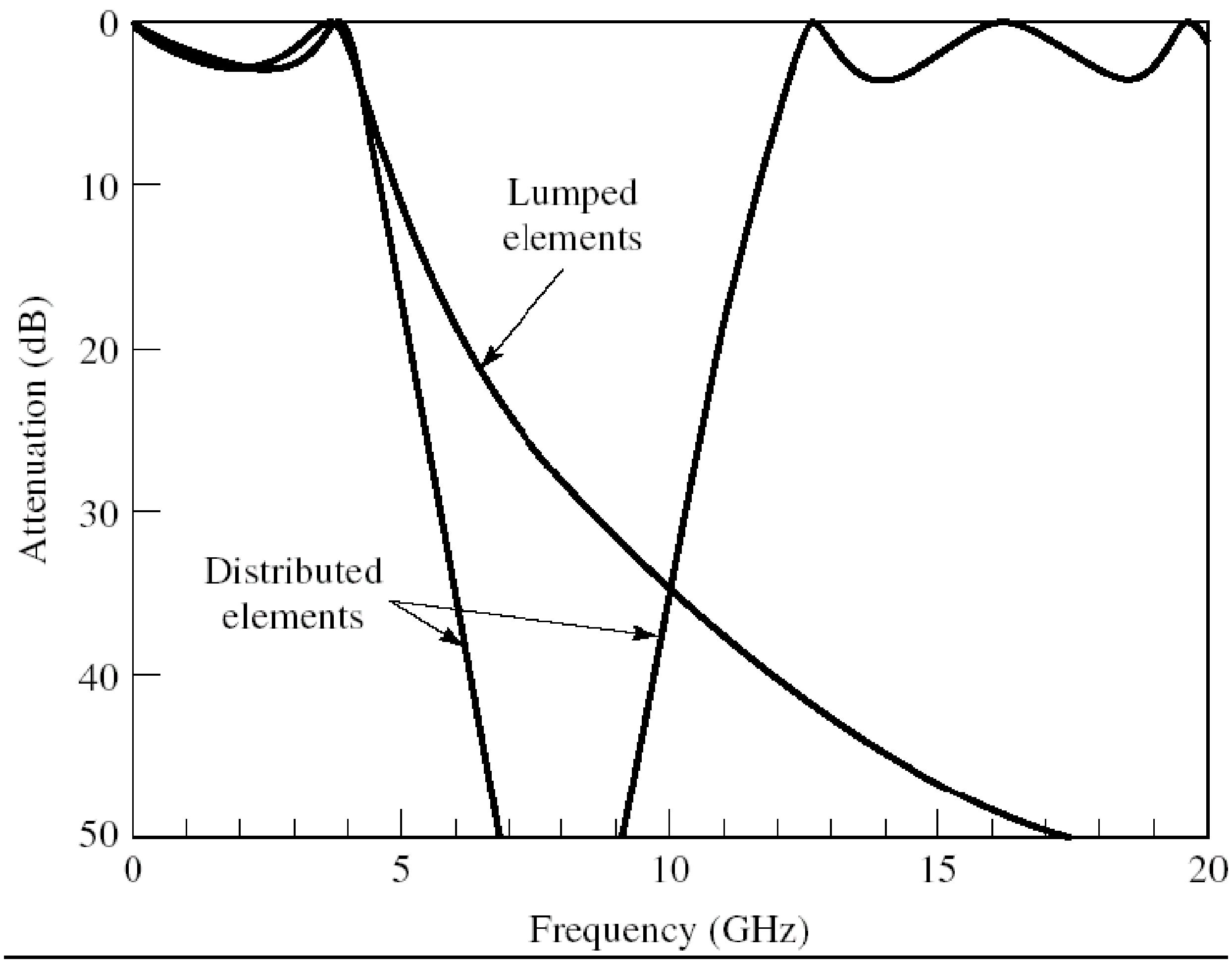
We call these two results as **Richard's Transformation.**

Richard’s Transformations (contd.)

However, it is important to remember that Richard’s Transformations do **not** result in **perfect** replacements for lumped elements—the stubs **do not** behave like capacitors and inductors!

- Instead, the transformation is **perfect**—the impedances are **equal**—at **only one frequency** (ω_c).
- We can use Richard’s transformations to replace the inductors and capacitors of a designed lumped element filter. In fact, for **low-pass filter design**, the frequency ω_c is the filter’s **cut-off frequency**.
- Using these stubs to **replace** inductors and capacitors will result in a filter response **similar** to that of the lumped element design—a low pass filter with cut-off frequency ω_c.
- However, the behavior of the filter in the **stop-band** will be **very** different from the lumped element design. For example, at the (high) frequencies where the stub length becomes a **multiple** of $\lambda/2$, the filter response will be that of $\omega = 0$—near perfect **transmission**!

Richard's Transformations (contd.)



Richard's Transformations (contd.)

Q: So **why** does the filter response match the lumped element response so **well** in the **pass-band**?

A: To see why, we first note that the **Taylor Series approximation** for $tan\varphi$ and $cot\varphi$ when φ is small (i.e., $\varphi \ll 1$) is:

$tan\varphi \approx \varphi$ **and** $cot\varphi \approx \dfrac{1}{\varphi}$ **for** $\boldsymbol{\varphi \ll 1}$

φ is expressed in **radians.**

- The **impedance** of Richard's transformation shorted stub at some **arbitrary frequency** ω is therefore:

$$Z_{in}^{s}(\omega) = jZ_0 \tan\left(\beta \frac{\lambda_c}{8}\right) = j(\omega_c L)\tan\left(\frac{\omega}{\omega_c}\frac{\pi}{4}\right)$$

- Therefore, when $\omega \ll \omega_c$(i.e., frequencies in the **pass-band** of a low-pass filter!), we can **approximate** this impedance as:

$$Z_{in}^{s}(\omega) = j(\omega_c L)\tan\left(\frac{\omega}{\omega_c}\frac{\pi}{4}\right) \approx j(\omega_c L)\left(\frac{\omega}{\omega_c}\frac{\pi}{4}\right) = j\omega L\left(\frac{\pi}{4}\right)$$

Richard's Transformations (contd.)

$$Z_{in}^{s}(\omega) = j(\omega_c L)\tan\left(\frac{\omega}{\omega_c}\frac{\pi}{4}\right) \approx j(\omega_c L)\left(\frac{\omega}{\omega_c}\frac{\pi}{4}\right) = j\omega L\left(\frac{\pi}{4}\right)$$

Compare this to a **lumped inductor** impedance

$$Z_L = j\omega L$$

Since the value $^{\pi}/_{4}$ is **relatively** close to one, we find that the Richard's Transformation shorted stub has an input impedance **very** close to the lumped element inductor for **all** frequencies **less than** ω_c (i.e., all frequencies of the low-pass filter pass-band)!

- Similarly, we find that the Richard's transformation **open-circuit** stub, when $\omega \ll \omega_c$, has an input impedance of **approximately**:

$$Z_{in}^{o}(\omega) = \frac{-j}{\omega_c C}\cot\left(\frac{\omega}{\omega_c}\frac{\pi}{4}\right) \approx \frac{-j}{\omega_c C}\left(\frac{\omega_c}{\omega}\frac{4}{\pi}\right) = \frac{1}{j\omega C}\left(\frac{4}{\pi}\right)$$

Compare this to a **lumped capacitor** impedance

$$Z_C = \frac{1}{j\omega C}$$

we find that results are approximately the **same** for all pass-band frequencies (i.e., when $\omega \ll \omega_c$).

Kuroda's Identities

- We will find that **Kuroda's Identities** can be very useful in making the implementation of Richard's transformations more **practicable.**
- Kuroda's Identities essentially provide a list of **equivalent** two port networks. By equivalent, we mean that they have **precisely** the same scattering/impedance/admittance/transmission matrices.
- In other words, we can **replace** one two-port network with its equivalent in a circuit, and the behavior and characteristics (e.g., its scattering matrix) of the circuit will **not** change!

Q: Why would we want to do this?

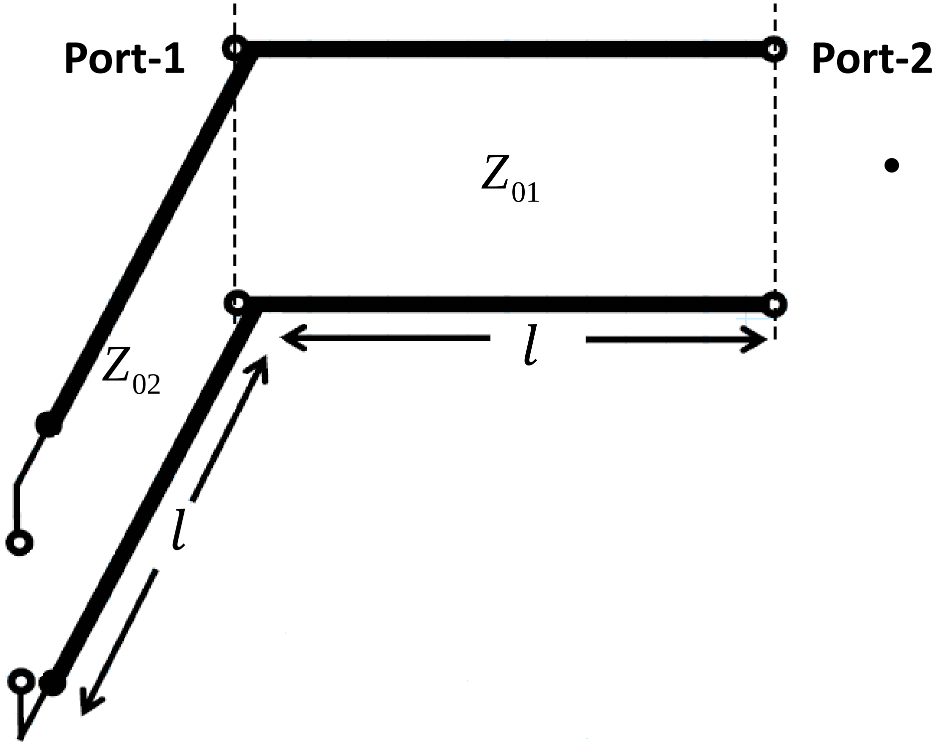
A: Because one of the equivalent may be more **practical** to implement!

For example, we can use Kuroda's Identities to:

1. Physically **separate** transmission line stubs.
2. Transform series stubs into **shunt** stubs.
3. Change impractical **characteristic impedances** into more realizable ones.

Kuroda's Identities (contd.)

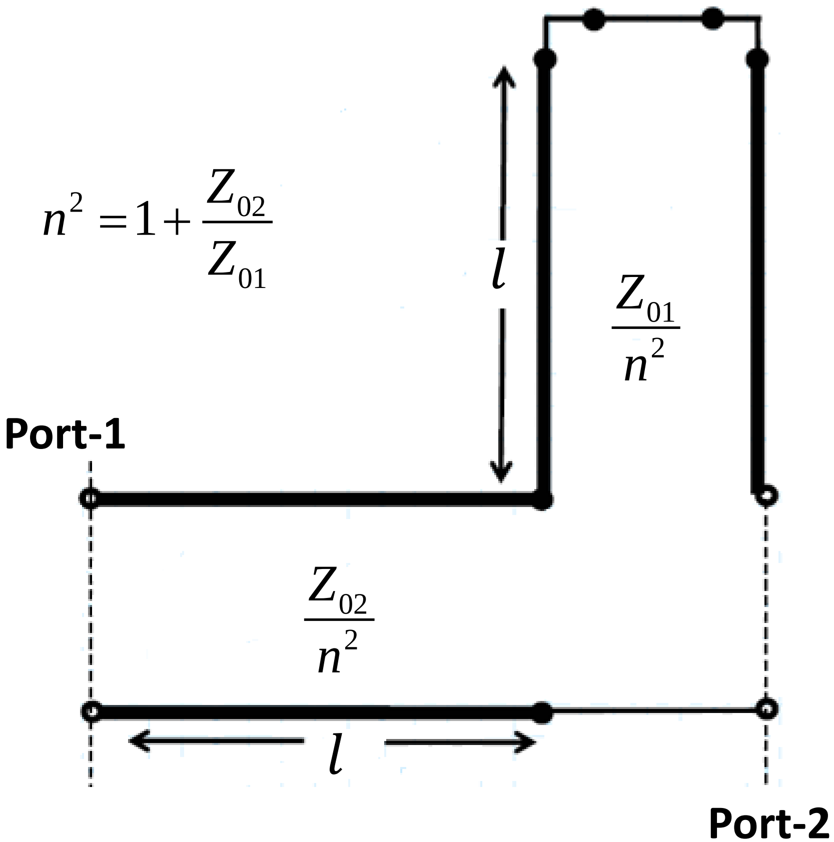
- **Four** Kuroda's identities are provided in a very **ambiguous** and confusing table (Table 8.7) in your **book**. We will find the **first two** identities to be the most useful.
- Consider the following two-port network, constructed with a length of transmission line, and an **open**-circuit **shunt** stub:



- Note that the **length** of the stub and the transmission line are **identical**, but the characteristic **impedance** of each are **different**.

Kuroda's Identities (contd.)

- The **first Kuroda identity** states that the considered two-port network is **precisely** the same two-port network as **this** one:

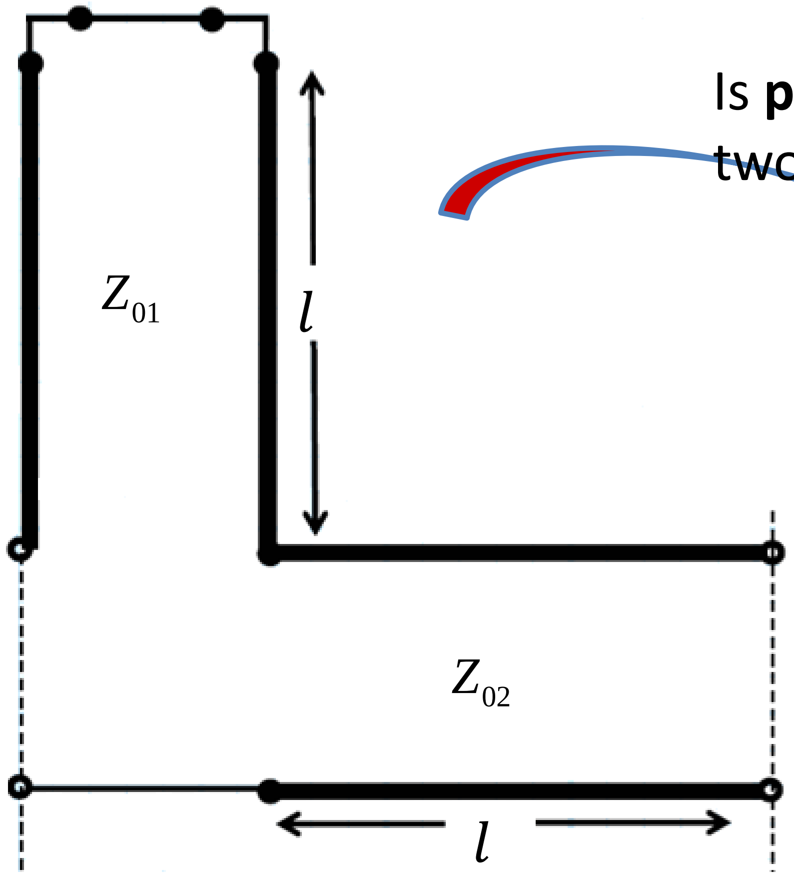


- Thus, we can **replace** the first structure in some circuit with the one above, and the behavior of that circuit will **not change** in the least!

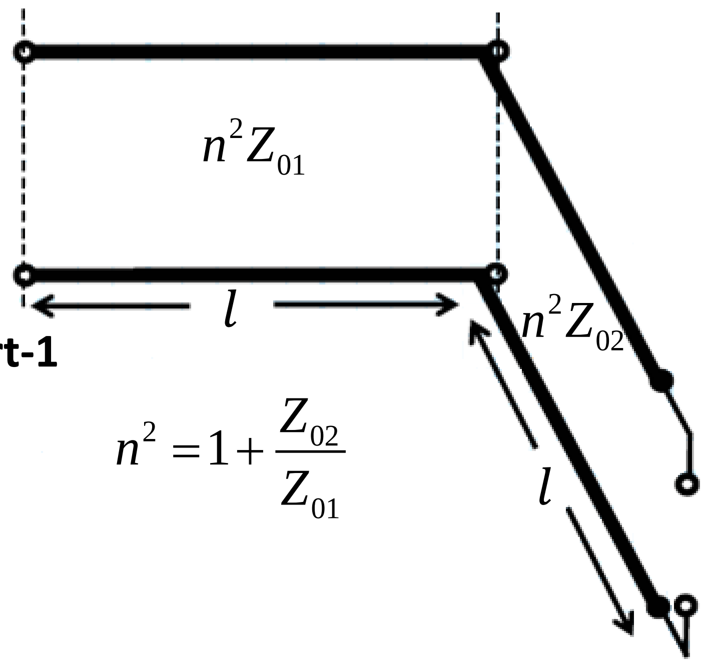
- Note this equivalent circuit uses a **short**-circuited **series** stub.

Kuroda's Identities (contd.)

- The **second** of Kuroda's Identities states that this two port network:



Is **precisely identical** to this two-port network

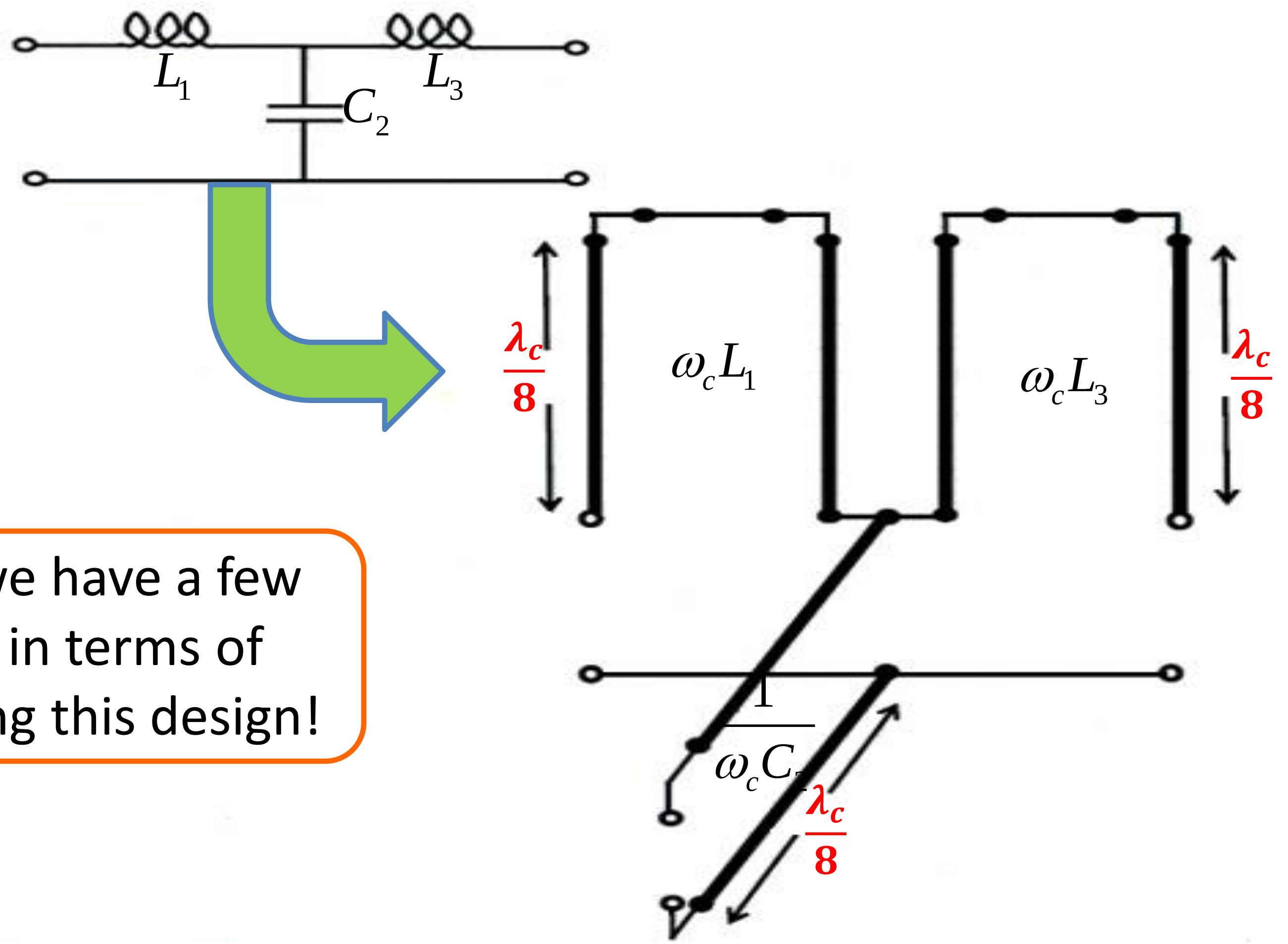


$$n^2 = 1 + \frac{Z_{02}}{Z_{01}}$$

With regard to **Richard's Transformation**, these identities are useful when we replace the series **inductors** with **shorted stubs**.

Kuroda's Identities (contd.)

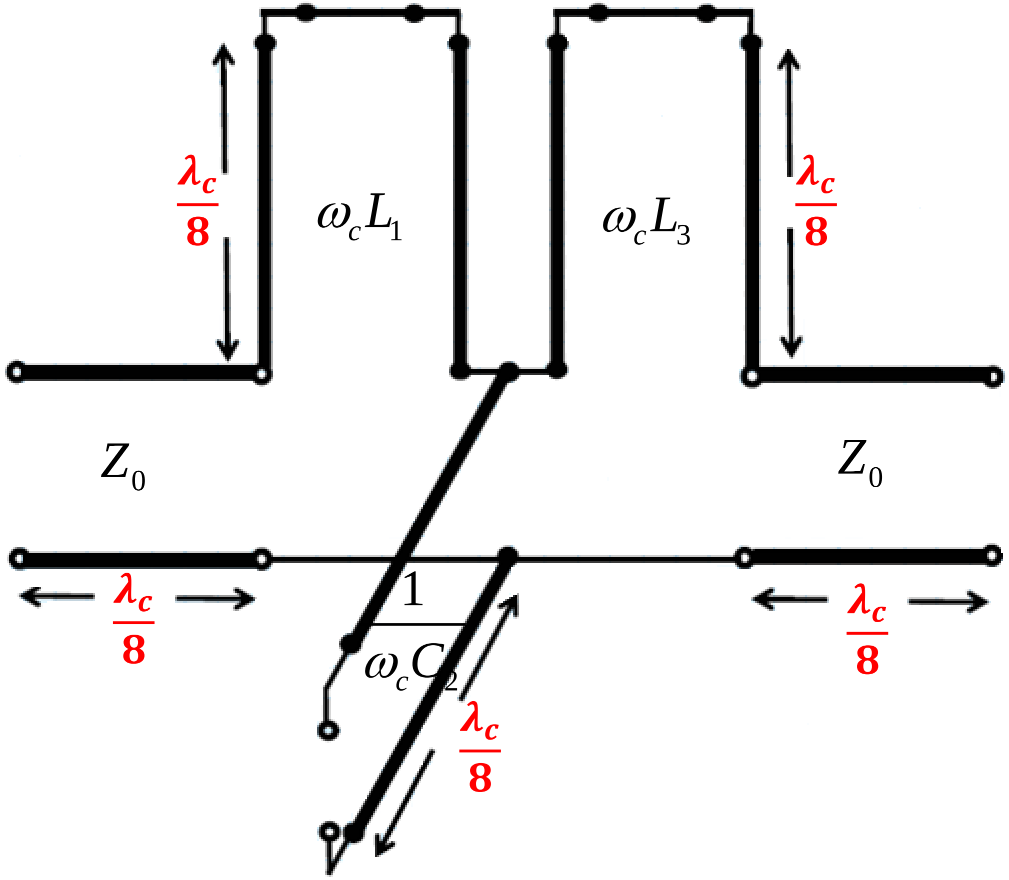
- To see **why** this is useful when implementing a **lowpass filter** with distributed elements, consider this third order filter example, realized using Richard's Transformations:



Note that we have a few **problems** in terms of implementing this design!

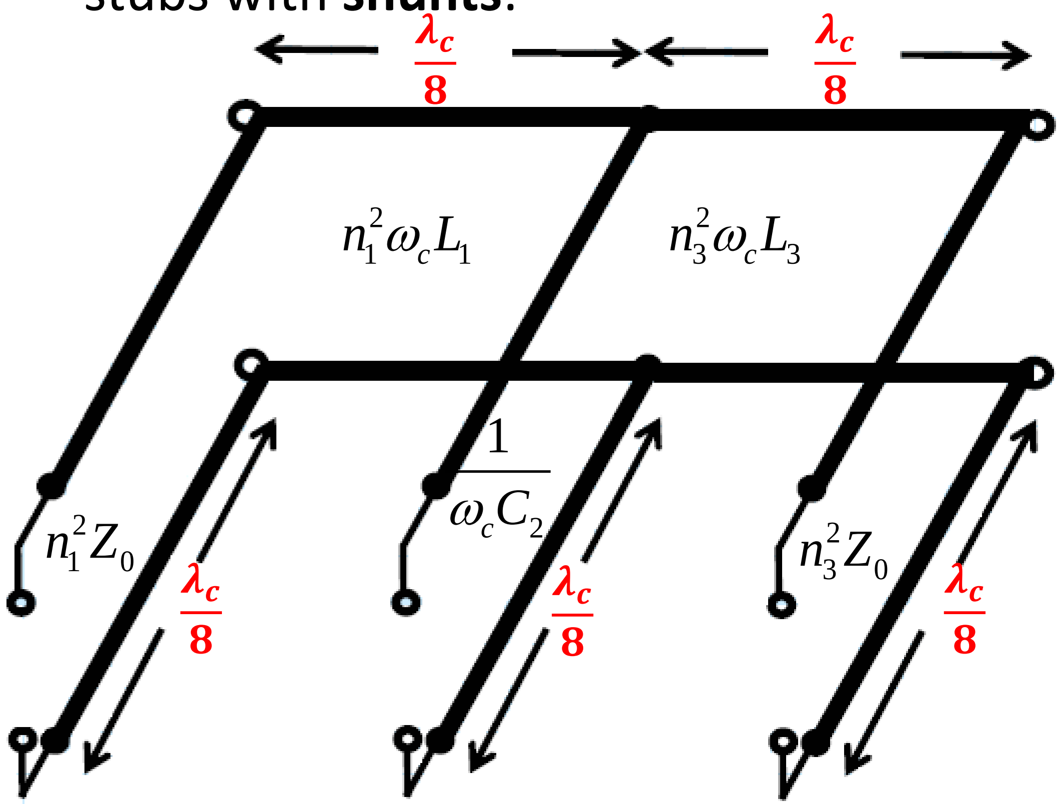
Kuroda's Identities (contd.)

- First of all the stubs are ideally **infinitely close** to each other— how do we build that? We could physically **separate** them, but this would introduce some transmission **line length** between them that would **mess up** our filter response!
- Secondly, **series** stubs are difficult to construct in microstrip/ stripline—we like **shunt** stubs **much** better!
- To solve these problems, we first **add** a short length of transmission line (Z_0 and $l = \lambda_c/8$) to the **beginning** and **end** of the filter:



Kuroda's Identities (contd.)

- Note adding these lengths only results in a **phase shift** in the filter response—the transmission and reflection functions will remain **unchanged**.
- Now we can use the second of **Kuroda's Identities** to replace the **series** stubs with **shunts**:



$$n_1^2 = 1 + \frac{Z_0}{\omega_c L_1} \qquad n_3^2 = 1 + \frac{Z_0}{\omega_c L_3}$$

Now **this** is a realizable filter! Note the **three stubs** are separated, and they are all **shunt** stubs.

Stepped-Impedance Low-Pass Filters

Q: Are there **other** methods for building microwave filters?

A: There are a **bundle** of them!

All distributed elements (e.g., transmission lines, coupled lines, resonators, stubs) exhibit **some** frequency dependency. If we are clever, we can construct these structures in a way that their frequency dependency (i.e., $S_{21}(\omega)$) conforms to a desirable function of ω.

Another distributed element realization of a lumped element low-pass filter design is the **stepped-impedance** low-pass filter.

Q: Won't you **ever** stop talking??

A: Yup. I'm all done.