Lecture – 22

Date: 13.11.2014

- High Frequency Filter
- Filter Phase Function
- The Linear Phase Filter
- The Insertion Loss Method

Filters

- A microwave **filter** → A two-port microwave network that allows source power to be transferred to a load as an explicit **<u>function of frequency</u>**.
- A RF/microwave **filter** is (typically) a passive, reciprocal, 2-port linear device.

If port 2 of this device is terminated in a **matched** load, then we can relate the incident and output power as:

$$P_{out} = |S_{21}|^2 P_{inc}$$

We define this power transmission through a filter in terms of the **power transmission coefficient T:**

$$T \doteq \frac{P_{out}}{P_{inc}} = |S_{21}|^2$$

As microwave filters are typically **passive** ⇒

$$0 \le T \le 1$$

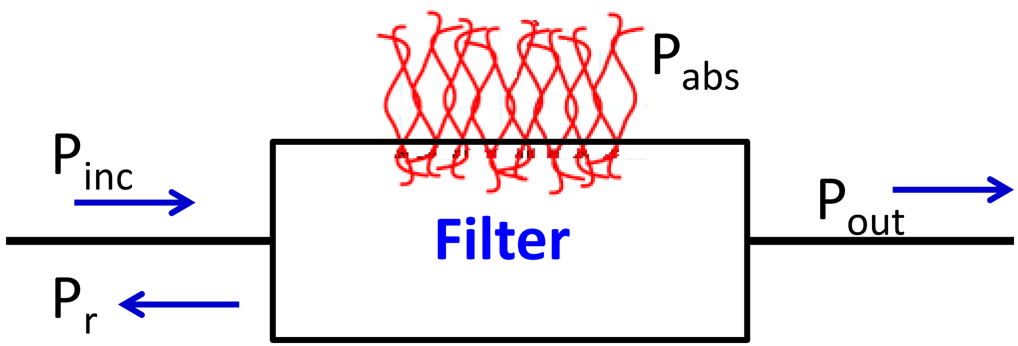
⇒

$$P_{out} \le P_{inc}$$

Filters (contd.)

Q: What happens to the "missing" power $P_{inc} - P_{out}$?

A: **Two** possibilities: the power is either **absorbed** (P_{abs}) by the filter (converted to heat), or is **reflected** (P_r) at the input port.



- Thus, by conservation of energy: $$P_{inc} = P_r + P_{abs} + P_{out}$$

- Now **ideally,** a microwave filter is lossless, therefore P_{abs} = 0 and:

$$P_{inc} = P_r + P_{out}$$

Filters (contd.)

- **Alternatively** we can write:

$$\frac{P_{inc}}{P_{inc}} = \frac{P_r + P_{out}}{P_{inc}} \Rightarrow 1 = \frac{P_r}{P_{inc}} + \frac{P_{out}}{P_{inc}} \Rightarrow 1 = \Gamma + T$$

lossless filter

Where:

$$T = \frac{P_{out}}{P_{inc}}$$ ← **Transmission Coefficient**

$$\Gamma \doteq \frac{P_r}{P_{inc}} = |S_{11}|^2$$ ← **Power Reflection Coefficient**

- **Therefore,** another way of saying a 2-port lossless device can be:

$$1 = |S_{11}|^2 + |S_{21}|^2$$

- Now, **here's** the important part! → For a microwave **filter**, the coefficients Γ and **T** are **functions of frequency**! i.e.,:

$$\Gamma(\omega) \qquad T(\omega)$$

← The **behavior** of a microwave filter is described by these **functions**!

Filters (contd.)

- We find that for most signal frequencies ω_s, these functions will have a value equal to one of **two** different **approximate** values.
- Either:

$$\Gamma(\omega = \omega_s) \approx 0$$

$$T(\omega = \omega_s) \approx 1$$

In **this** case, the signal frequency ω_s is said to lie in the **pass-band** of the filter. Almost all of the incident signal power will **pass through** the filter.

or

$$\Gamma(\omega = \omega_s) \approx 1$$

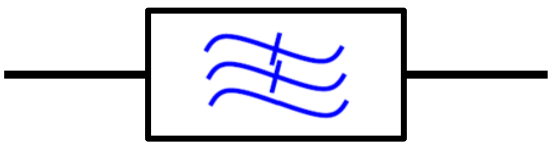
$$T(\omega = \omega_s) \approx 0$$

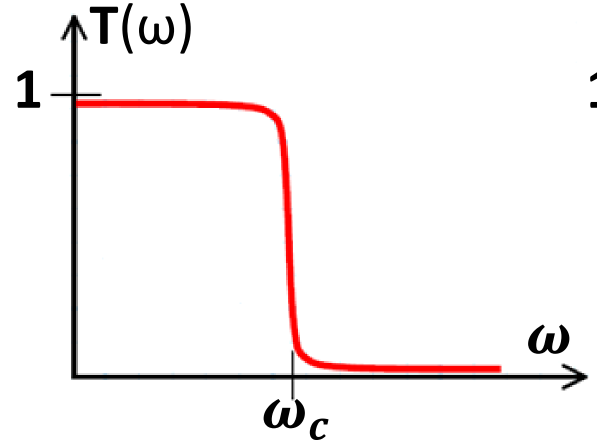
In **this** case, the signal frequency ω_s is said to lie in the **stop-band** of the filter. Almost all of the incident signal power will be **reflected** at the input—almost no power will appear at the filter output.

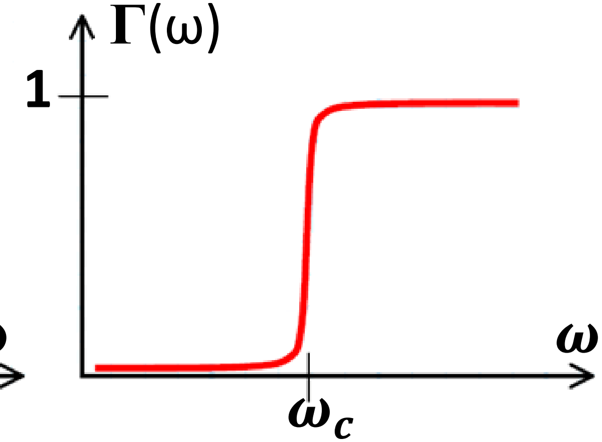
Filters (contd.)

- Consider then these **four types** of functions of $\Gamma(\omega)$ and **T**(ω):

1. Low Pass Filter







<u>Note for this filter:</u>

$$\mathbf{T}(\omega) = \begin{cases} \approx 1 & \omega < \omega_c \\ \approx 0 & \omega > \omega_c \end{cases} \qquad \Gamma(\omega) = \begin{cases} \approx 0 & \omega < \omega_c \\ \approx 1 & \omega > \omega_c \end{cases}$$

This filter is a **low-pass** type, as it **"passes"** signals with frequencies **less** than ω_c, while **"rejecting"** signals at frequencies **greater** than ω_c.

Filters (contd.)

Q: This frequency ω_c seems to be very important! What is it?

A: Frequency ω_c is a filter parameter known as the **cutoff frequency**; a value that **approximately** defines the frequency region where the filter pass-band **transitions** into the filter stop band.

- Accordingly, this frequency is defined as the frequency where the power **transmission** coefficient is equal to 1/2:

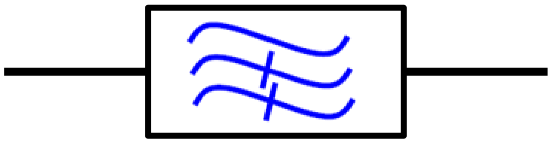
$$T(\omega = \omega_c) = 0.5$$

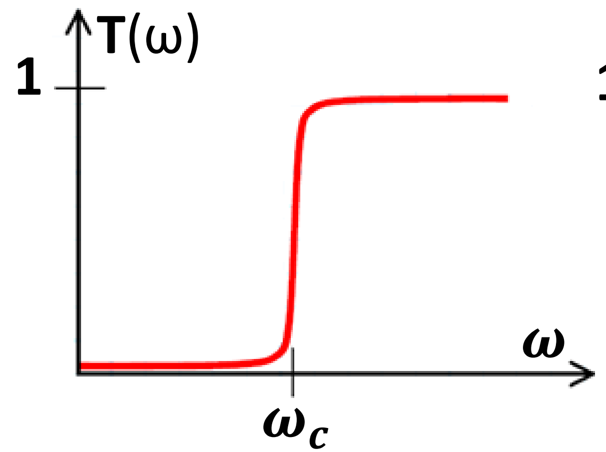
- Note for a lossless filter, the cutoff frequency is **likewise** the value where the power **reflection** coefficient is 1/2:

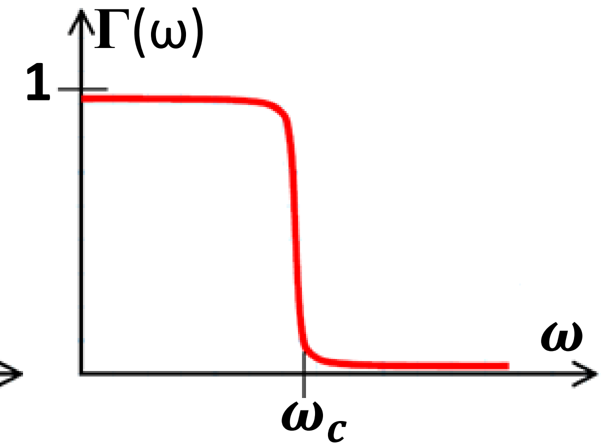
$$\Gamma(\omega = \omega_c) = 0.5$$

Filters (contd.)

2. High - Pass Filter







Note for this filter:

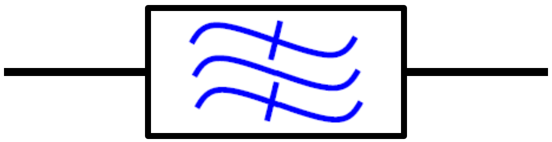
$$T(\omega) = \begin{cases} \approx 0 & \omega < \omega_c \\ \approx 1 & \omega > \omega_c \end{cases}$$

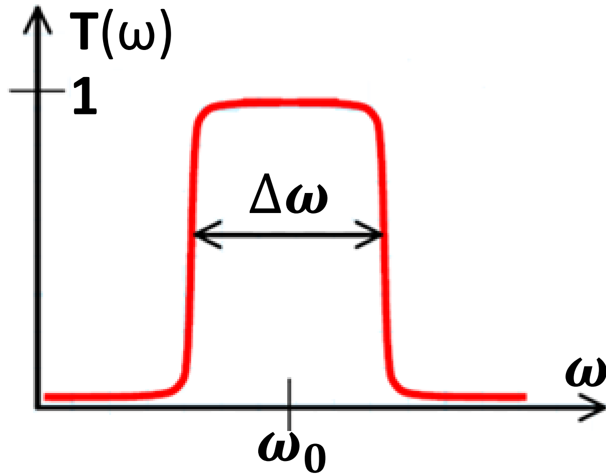
$$\Gamma(\omega) = \begin{cases} \approx 1 & \omega < \omega_c \\ \approx 0 & \omega > \omega_c \end{cases}$$

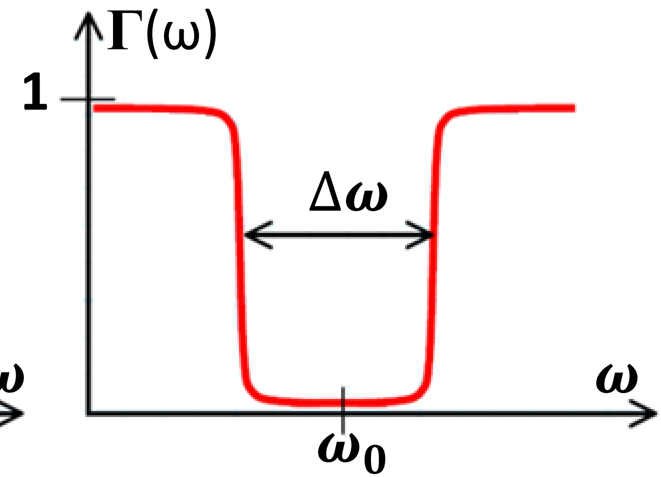
This filter is a **high-pass** type, as it **"passes"** signals with frequencies **greater** than ω_c , while **"rejecting"** signals at frequencies **less** than ω_c.

Filters (contd.)

3. Band - Pass Filter







Note for this filter:

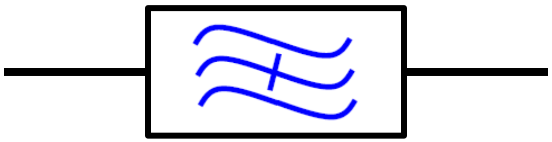
$$T(\omega)=\begin{cases}\approx 1 & |\omega-\omega_0|<\dfrac{\Delta\omega}{2}\\ \approx 0 & |\omega-\omega_0|>\dfrac{\Delta\omega}{2}\end{cases}$$

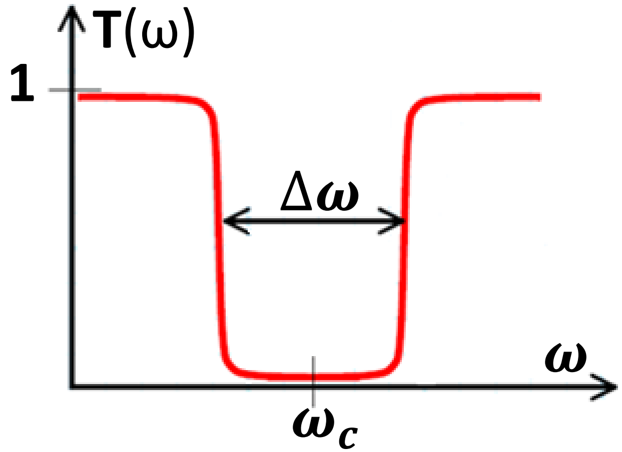
$$\Gamma(\omega)=\begin{cases}\approx 0 & |\omega-\omega_0|<\dfrac{\Delta\omega}{2}\\ \approx 1 & |\omega-\omega_0|>\dfrac{\Delta\omega}{2}\end{cases}$$

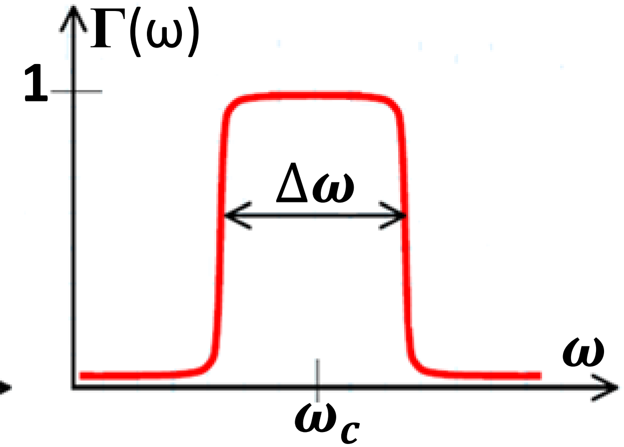
- This filter is a **band-pass** type, as it **"passes"** signals within a frequency bandwidth Δω, while **"rejecting"** signals at all frequencies **outside this bandwidth**.
- In addition to filter bandwidth Δω, a fundamental parameter of bandpass filters is ω_0, which defines the **center frequency** of the filter bandwidth.

Filters (contd.)

4. Band - Stop Filter







Note for this filter:

$$T(\omega) = \begin{cases} \approx 0 & |\omega - \omega_0| < \frac{\Delta\omega}{2} \\ \approx 1 & |\omega - \omega_0| > \frac{\Delta\omega}{2} \end{cases}$$

$$\Gamma(\omega) = \begin{cases} \approx 1 & |\omega - \omega_0| < \frac{\Delta\omega}{2} \\ \approx 0 & |\omega - \omega_0| > \frac{\Delta\omega}{2} \end{cases}$$

This filter is a band-stop type as it **"rejects"** signals within a frequency bandwidth Δω, while **"passing"** signals at all frequencies **outside this bandwidth.**

The Filter Phase Function

- Recall that the power transmission coefficient **T**(ω) can be determined from the **scattering parameter** $S_{21}(\omega)$:

$$T(\omega) = \left|S_{21}(\omega)\right|^2$$

Q: I see, we only care about the **magnitude** of complex function $S_{21}(\omega)$ when using microwave filters !?

A: Hardly! Since $S_{21}(\omega)$ is complex, it can be expressed in terms of its magnitude and **phase**:

$$S_{21}(\omega) = \mathrm{Re}\{S_{21}(\omega)\} + j\,\mathrm{Im}\{S_{21}(\omega)\} \Rightarrow S_{21}(\omega) = \left|S_{21}(\omega)\right| e^{j\angle S_{21}(\omega)}$$

<u>where</u> the phase is denoted as $\angle S_{21}(\omega)$:

$$\angle S_{21}(\omega) = \tan^{-1}\left[\frac{\mathrm{Im}\{S_{21}(\omega)\}}{\mathrm{Re}\{S_{21}(\omega)\}}\right]$$

We therefore care **very** much about this phase function!

The Filter Phase Function (contd.)

Q: Just what does this phase tell us?

A: It describes the relative phase **between** the wave incident on the input to the filter, and the wave exiting the output of the filter (given the output port is matched).

- In other words, if the **incident** wave is: $V_1^+(z_1) = V_{01}^+ e^{-j\beta z}$
- Then the exiting (output) wave will be:

$$V_2^-(z_1) = V_{02}^- e^{+j\beta z_2} \Rightarrow = S_{21} V_{01}^+ e^{+j\beta z_2} = |S_{21}| V_{01}^+ e^{+j(\beta z + \angle S_{21})}$$

We say that there has been a "**phase shift**" of $\angle S_{21}(\omega)$ between the input and output waves.

Q: What **causes** this phase shift?

A: Propagation **delay.** It takes some non-zero amount of **time** for signal energy to propagate from the input of the filter to the output.

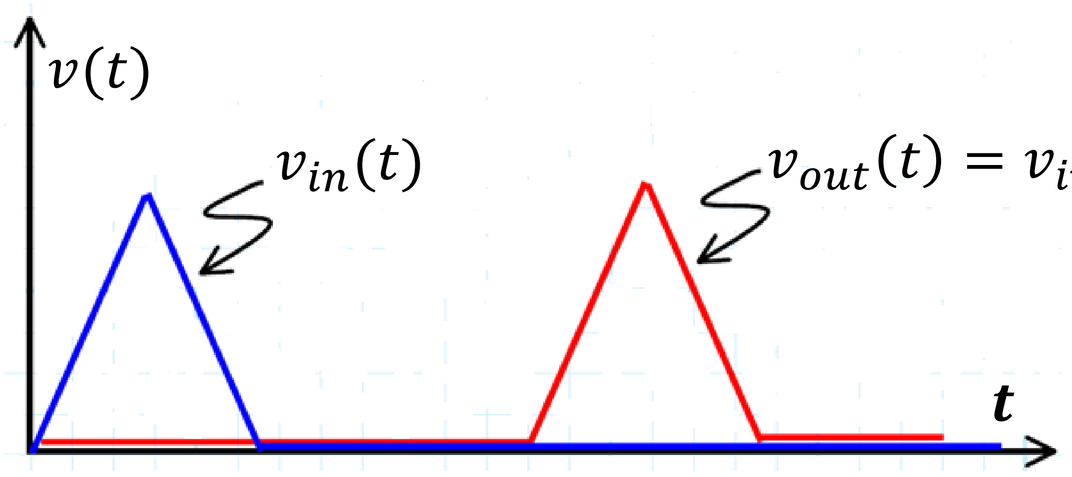
Q: Can we tell from $\angle S_{21}(\omega)$ how **long** this delay is?

A: Yes!

The Filter Phase Function (contd.)

- To see how, consider an **example** two-port network (filter) with the impulse response: $$h(t) = \delta(t - \tau)$$
- We just identified that this device would merely **delay** an input signal (say by some amount $\boldsymbol{\tau}$):

$$v_{out}(t) = \int_{-\infty}^{\infty} h(t - t')v_{in}(t')dt' \Rightarrow = \int_{-\infty}^{\infty} \delta(t - t' - \tau)v_{in}(t')dt' \Rightarrow = v_{in}(t - \tau)$$



The Filter Phase Function (contd.)

- Now if we take the **Fourier transform** of this impulse response, then **frequency response** of this two-port network is:

$$H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt \Rightarrow = \int_{-\infty}^{\infty} \delta(t-\tau) e^{-j\omega t} dt \Rightarrow = e^{-j\omega t}$$

- **In other words:** $|H(\omega)| = 1$ $\quad$ $\angle H(\omega) = -\omega\tau$

The interesting result here is the **phase** ∠H(ω). The result means that a delay of **τ** seconds results in an output "phase shift" of **−ωτ** radians!

Note that although the **delay** of device is a **constant τ**, the **phase shift** is a **function** of **ω** → in fact, it is directly proportional to frequency **ω**.

The Filter Phase Function (contd.)

- Note if the **input** signal for this device was of the form: $v_{in}(t) = \cos \omega t$
- Then the output would be:

$$v_{out}(t) = \cos \omega (t - \tau) \quad \Rightarrow \quad v_{out}(t) = \cos(\omega t - \omega \tau)$$

Thus, we could **either** view the signal $v_{in}(t) = \cos\omega t$ as being **delayed** by an amount **τ** seconds, **or phase shifted** by an amount **−ωτ** radians.

Q: Then by **measuring** the output signal phase shift ∠H(ω), we could determine the delay τ through the device with the equation:

$$\tau = -\frac{\angle H(\omega)}{\omega}$$

right?

A: Not exactly. The problem is that we cannot **unambiguously** determine the phase shift **∠H (ω) = −ωτ** by **looking** at the output signal!

The Filter Phase Function (contd.)

- The reason is that **cos(ωt + ∠H(ω)) = cos(ωt + ∠H(ω) + 2π) = cos(ωt + ∠H(ω) − 4π),** etc. More specifically:

$$\cos(\omega t + \angle H(\omega)) = \cos(\omega t + \angle H(\omega) + n2\pi)$$

where *n* is any integer —positive or negative. We can't tell **which** of these output signal we are looking at!

- Thus, any phase shift **measurement** has an inherent **ambiguity**. Typically, we interpret a phase measurement (in radians) such that:

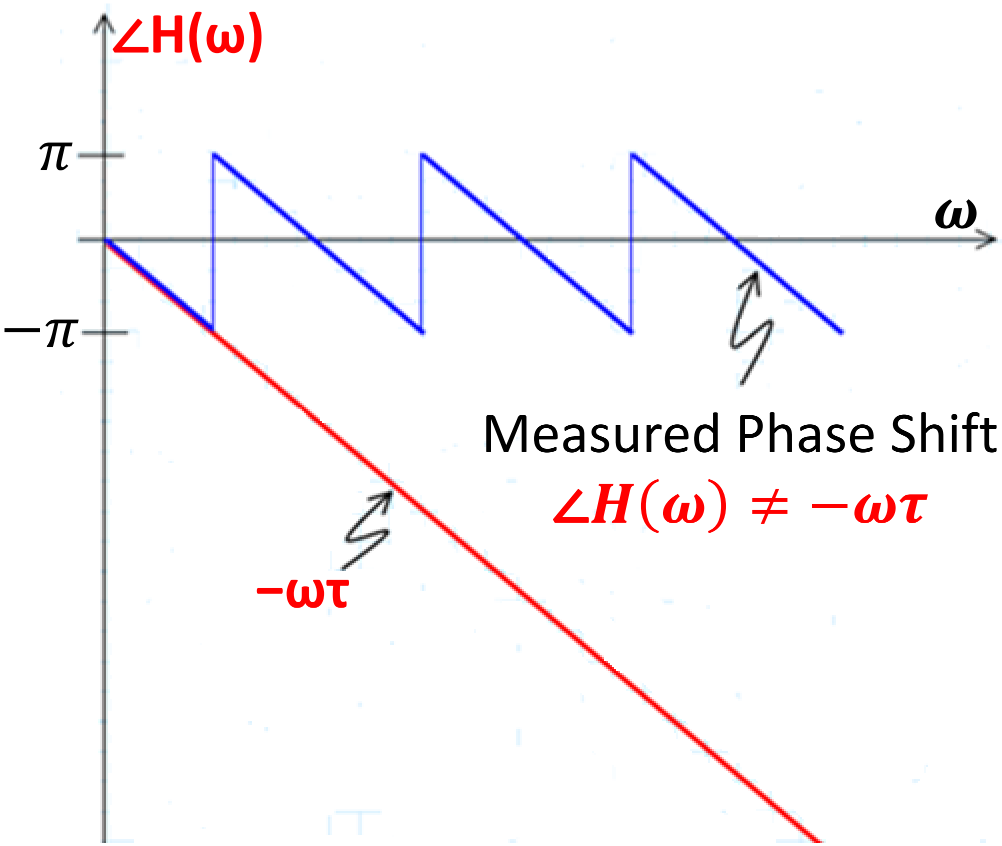
$$-\pi < \angle H(\omega) \le \pi$$

or

$$0 \le \angle H(\omega) < 2\pi$$

But almost certainly the actual value of **∠H(ω) = −ωτ** is **nowhere** near these interpretations!

The Filter Phase Function (contd.)



Clearly using the equation:

$$\tau = -\frac{\angle H(\omega)}{\omega}$$

would **not** get us the correct result in this case—after all, there will be **several** frequencies ω with exactly the **same measured** phase ∠H (ω)!

Q: So determining the delay τ is **impossible**?

A: NO! It is **entirely** possible—we simply must find the correct **method**.

The Filter Phase Function (contd.)

Looking at the plot, this method should become **apparent**. Note that although the measured phase (blue curve) is definitely **not** equal to the phase function $-\omega\tau$ (red curve), the **slope** of the two are **identical** at every point!

Q: What good is knowing the **slope** of these functions?

A: Just look! Recall that we can determine the slope by taking the first **derivative**:

$$\frac{\partial(-\omega\tau)}{\partial\omega} = -\tau$$

The slope directly tells us the **propagation delay**!

The Filter Phase Function (contd.)

- Thus, we can determine the propagation delay of this device by:

$$\tau = -\frac{\partial \angle H(\omega)}{\partial \omega}$$

where ∠H(ω) can be the **measured** phase. Of course, the method requires us to **measure** ∠H(ω) as a **function** of frequency (i.e., to make measurements at **many** signal frequencies).

Q: Now I see! If we wish to **determine** the propagation delay τ through some **filter**, we simply need to take the derivative of $\angle S_{21}(\omega)$ with respect to frequency. **Right**?

A: Well, sort of!

- Recall for the **example** case that h(t) = δ(t −τ) and ∠H(ω) = −ωτ, where **τ** is a **constant**. For a microwave filter, **neither** of these conditions are true.
- Specifically, the phase function $\angle \mathbf{S_{21}(\omega)}$ will typically be some arbitrary **function** of frequency **($\angle S_{21}(\omega) \neq -\omega\tau$).**

The Filter Phase Function (contd.)

Q: How could this be true? I thought you said that phase shift was **due** to filter delay **τ**!

A: Phase shift **is** due to device delay, it's just that the propagation delay of most devices (such as filters) is **not a constant**, but instead depends on the **frequency** of the signal propagating through it!

In other words, the propagation delay of a filter is typically some arbitrary **function** of frequency (i.e., $\tau(\omega)$). That's why the phase $\angle S_{21}(\omega)$ is **likewise** an arbitrary function of frequency.

Q: Yikes! Is there **any** way to determine the relationship between these two arbitrary functions?

A: Yes there is! Just as before, the two can be related by:

$$\tau(\omega) = -\frac{\partial \angle S_{21}(\omega)}{\partial \omega}$$

This result $\tau(\omega)$ is also known as **phase delay**, and is **very** important function to consider when designing/specifying/selecting a **microwave filter**

The Filter Phase Function (contd.)

Q: why; what might happen if we don`t consider?

A: If you get a filter with wrong **τ(ω)**, your **output signal** could be **horribly distorted** – distorted by the evil effects of **signal dispersion**.

Filter Dispersion

Any signal that carries significant **information** must have some non-zero **bandwidth**. In other words, the signal energy (as well as the information it carries) is **spread** across many frequencies.

If the different frequencies that comprise a signal propagate at different velocities through a microwave filter (i.e., each signal frequency has a different delay τ), the output signal will be **distorted**. We call this phenomenon signal **dispersion.**

Filter Dispersion (contd.)

Q: I see! The phase delay τ(ω) of a filter **must** be a constant with respect to frequency—otherwise signal dispersion (and thus signal distortion) will result. Right?

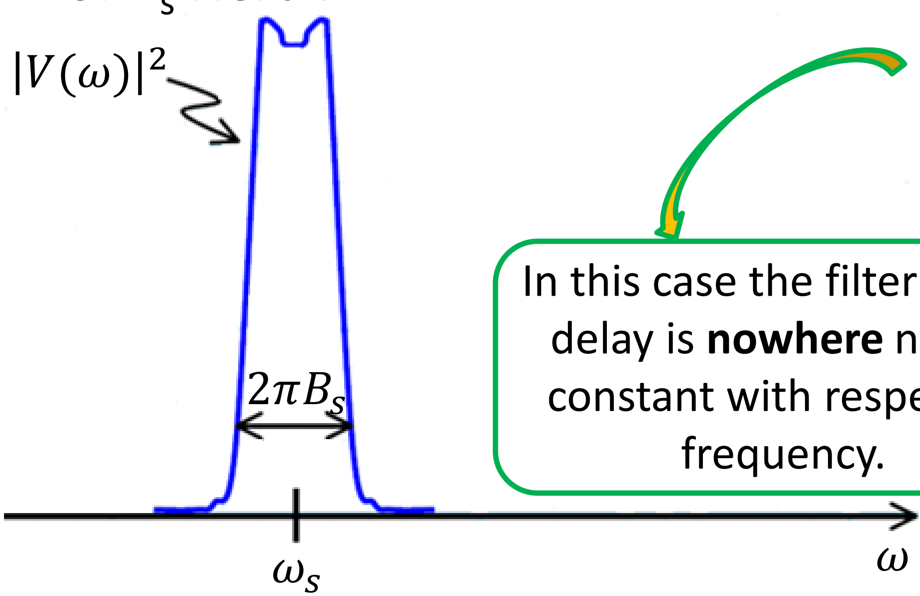
A: Not necessarily! Although a constant phase delay will **insure** that the output signal is not distorted, it is **not** strictly a requirement for that happy event to occur.

This is a **good** thing, for as we shall latter see, building a good filter with a constant phase delay is **very** difficult!

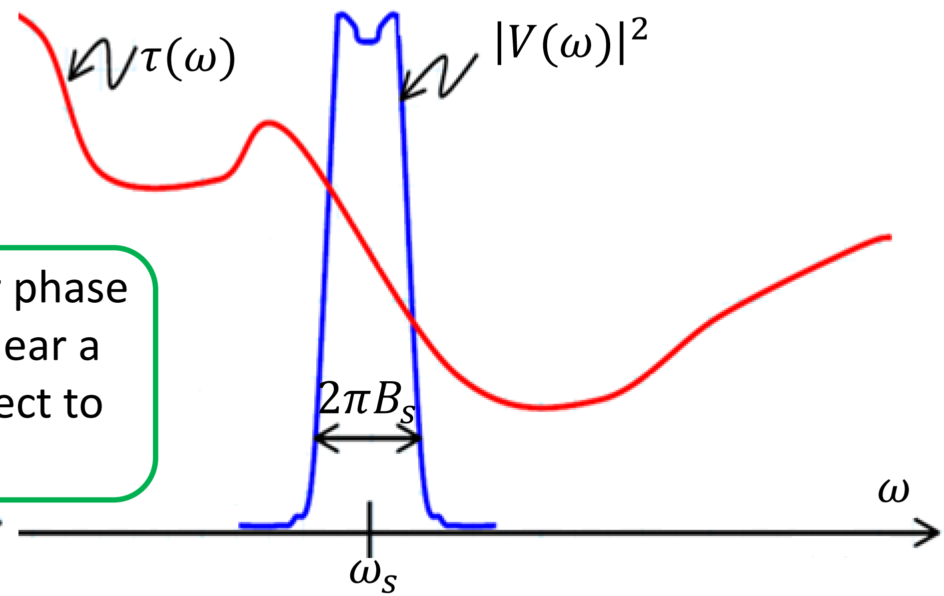
Filter Dispersion (contd.)

- For example, consider a modulated signal with the following frequency **spectrum**, exhibiting a **bandwidth** of B_s Hertz.

- Now, let's likewise plot the **phase delay** function τ(ω) of some filter:



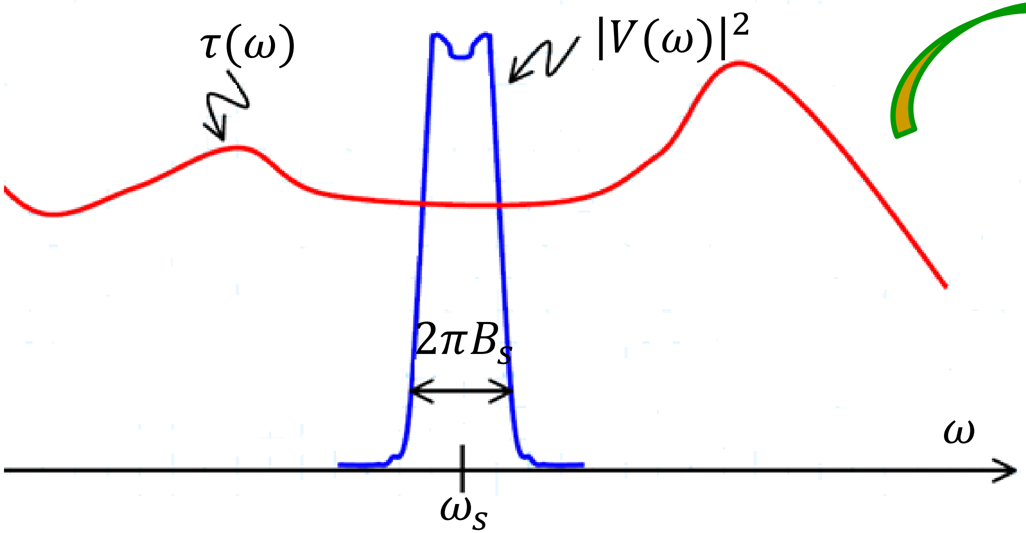
In this case the filter phase delay is **nowhere** near a constant with respect to frequency.



However, this fact alone does **not** necessarily mean that our signal would suffer from **dispersion** if it passed through this filter. Indeed, the signal in this case **would** be distorted, but **only** because the phase delay τ(ω) changes significantly across the **bandwidth** B_s of the signal.

Filter Dispersion (contd.)

- Conversely, consider this **phase delay**:

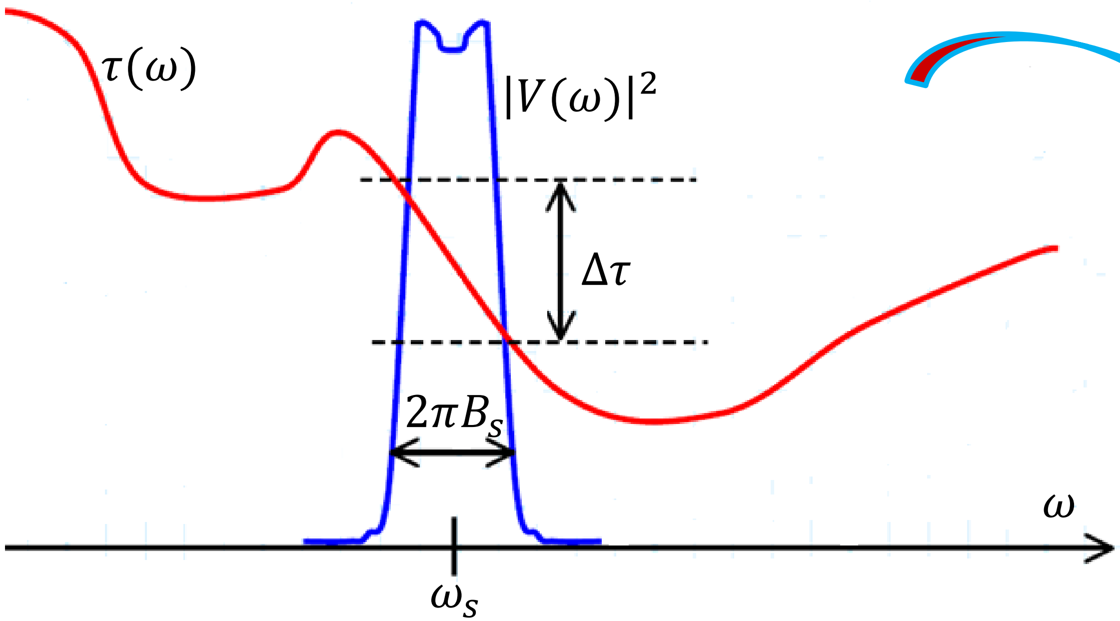


As with the previous case, the phase delay of the filter is **not** a constant. Yet, if this signal were to pass through this filter, it would **not** be distorted!

The reason for this is that the phase delay across the **signal bandwidth** is approximately constant—each frequency component of the **signal** will be delayed by the **same** amount.

Filter Dispersion (contd.)

- Compare this to the **previous** case, where the phase delay changes by a precipitous value Δτ across signal bandwidth B_s:



Now **this** is a case where dispersion **will** result!

Q: So does $\Delta\tau$ need to be **precisely** zero for no signal distortion to occur, or is there some **minimum** amount $\Delta\tau$ that is acceptable?

A: Mathematically, we find that dispersion will be **insignificant** if:

$$\omega_s \Delta\tau \leq 1$$

Filter Dispersion (contd.)

- A more specific (but **subjective**) "rule of thumb" is: $$\omega_s \Delta\tau \leq \frac{\pi}{5}$$
- Or, using $\omega_s = 2\pi f_s$: $$f_s \Delta\tau \leq 0.1$$

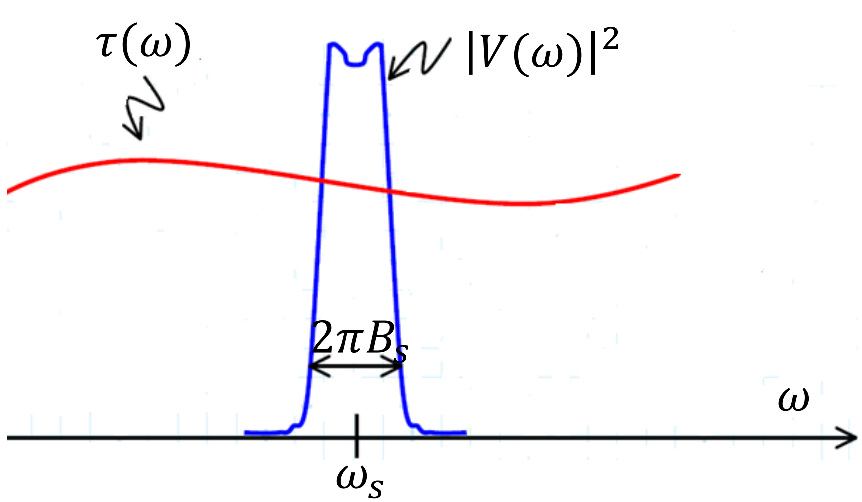
Generally speaking, we find for **wideband** filters—where filter bandwidth B is much greater than the signal bandwidth (i.e.,$B >> B_s$)—the above criteria is **easily** satisfied. In other words, signal dispersion is **not** typically a problem for wide band filters (e.g., pre-select filters).

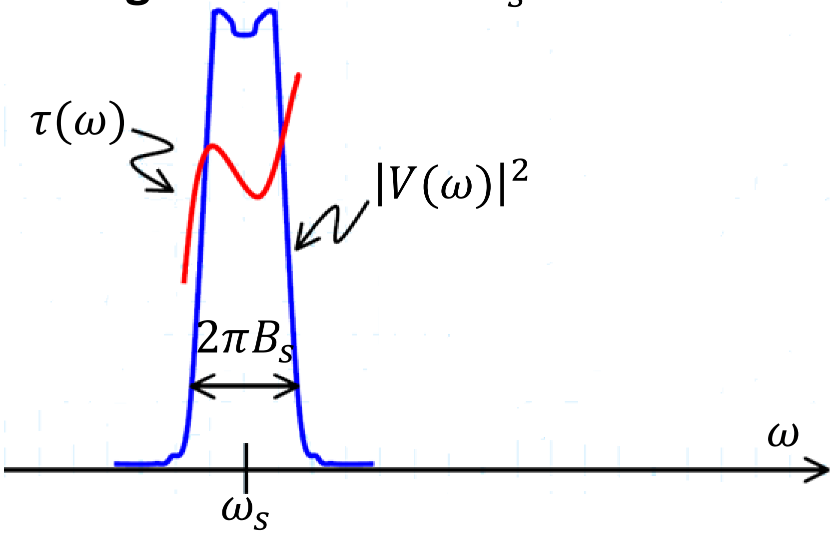
This is **not** to say that τ(ω) is a constant for wide band filters. Instead, the phase delay can change **significantly** across the wide **filter** bandwith.

Filter Dispersion (contd.)

- What we typically find however, is that the function τ(ω) does not change very **rapidly** across the wide filter bandwidth. As a result, the phase delay will be **approximately** constant across the relatively narrow signal bandwidth Bs.

- Conversely, a **narrowband** filter – where filter bandwidth B is approximately **equal** to the signal bandwidth (i.e., $B_s = B$) – can (if we are not careful!) exhibit a phase delay which changes **significantly** over **filter** bandwidth B. This means that the delay also changes significantly over the **signal** bandwidth B_s.





Thus, a **narrowband** filter (e.g., IF Filter) must exhibit a near constant phase delay τ(ω) in order to avoid distortion due to signal dispersion.

The Linear Phase Filter

Q: So, narrowband filters should exhibit a **constant** phase delay τ(ω). What should the phase function $\angle S_{21}(\omega)$ be for this **dispersionless** case?

A: We can express this problem mathematically as:

$$\tau(\omega) = \tau_c$$

where τ_c is some **constant**.

- Recall that the definition of **phase delay** is:

$$\tau(\omega) = -\frac{\partial \angle S_{21}(\omega)}{\partial \omega}$$

- Thus **combining** these two equations, we find ourselves with a **differential equation**:

$$-\frac{\partial \angle S_{21}(\omega)}{\partial \omega} = \tau_c$$

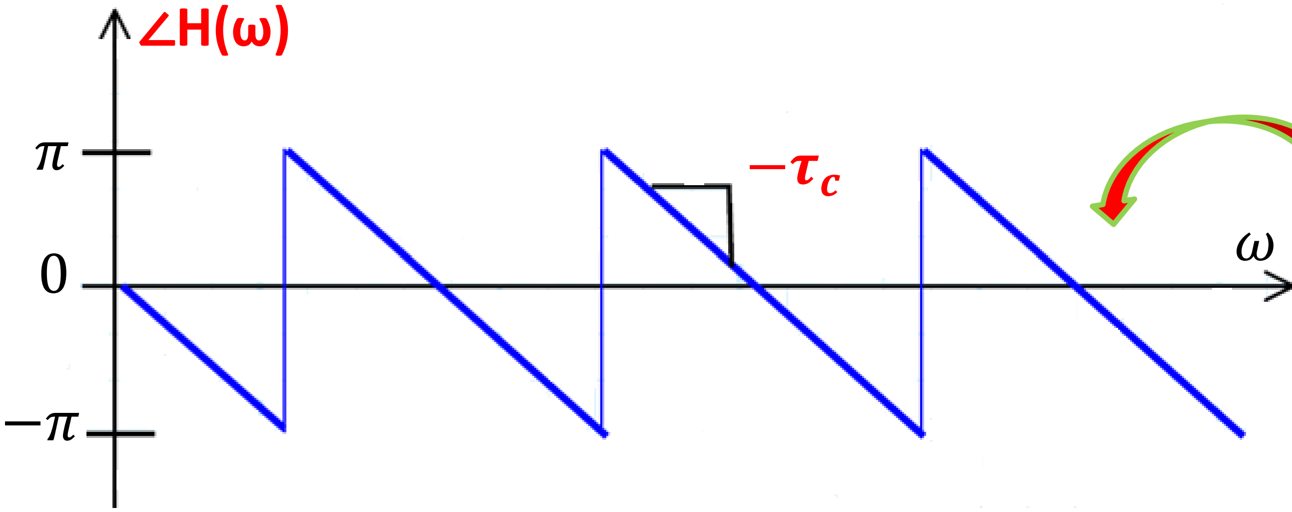
The **solution** to this differential equation provides us with the necessary phase function $\angle S_{21}(\omega)$ for a **constant** phase delay τ_c.

Fortunately, this differential equation is **easily** solved!

The Linear Phase Filter (contd.)

- The solution is: $\angle S_{21}(\omega) = -\omega\tau_c + \phi_c$ where ϕ_c is an arbitrary **constant**.

- Plotting this phase function (with ϕ_c =0):



As you rightly expected, this phase function is linear, such that it has constant slope ($-\tau_c$)

Filters with such phase response are called linear phase filters, and have the desirable trait that cause no dispersion distortion.

The Insertion Loss Method

- Recall that a **lossless** filter can be described in terms of either its power transmission coefficient **T**(ω) **or** its power reflection coefficient **Γ**(ω), as the two values are completely **dependent**:

$$\Gamma(\omega) = 1 - T(\omega)$$

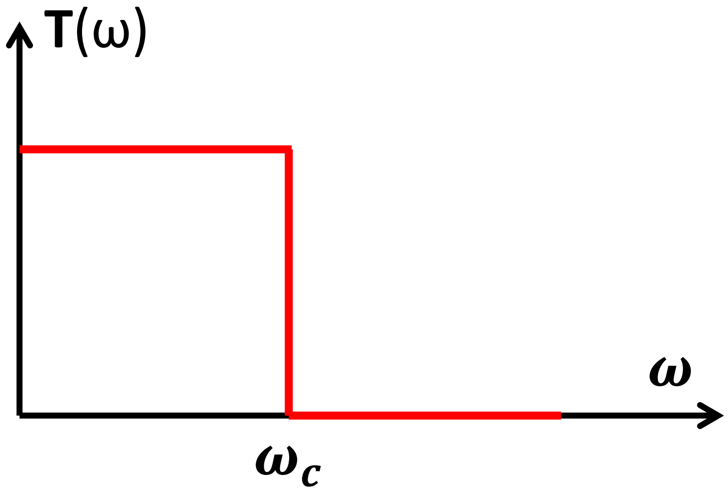
- **Ideally**, these functions would be quite **simple**:

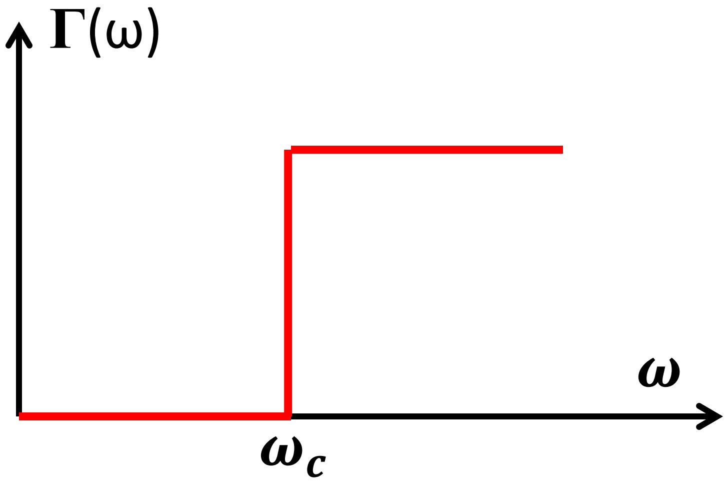
1. T(ω) = 1 and **Γ**(ω) = 0 for **all** frequencies within the **pass**band.

2. T (ω) = 0 and **Γ** (ω) = 1 for **all** frequencies within the **stop**band.

The Insertion Loss Method (contd.)

- For example, the **ideal** low-pass filter would be:





- Add to this a **linear phase** response, and you have the **perfect** microwave filter!
- There's just one small problem with this **perfect** filter → It's **impossible** to build!

The Insertion Loss Method (contd.)

- Now, if we consider only possible (i.e., **realizable**) filters, we must limit ourselves to filter functions that can be expressed as **finite polynomials** of the form:

$$T(\omega) = \frac{a_o + a_1\omega + a_2\omega^2 + \ldots}{b_o + b_1\omega + b_2\omega^2 + \ldots + b_N\omega^{2N}}$$

The **order** N of the (denominator) polynomial is likewise the **order** of the filter.

- Instead of the power transmission coefficient, we often use an equivalent function (assuming lossless) called the **power loss ratio** P_{LR}:

$$P_{LR} = \frac{P_1^+}{P_2^-} = \frac{1}{1-\Gamma(\omega)}$$

Note, $P_{LR} = \infty$ when $\Gamma(\omega) = 1$, and $P_{LR} = 1$ when $\Gamma(\omega) = 0$.

- We likewise note that, for a lossless filter: $P_{LR} = \dfrac{1}{T(\omega)}$

- Therefore P_{LR}(dB) is: $P_{LR}(dB) = 10\log_{10} P_{LR} = -10\log_{10} T(\omega)$

$\rightarrow$ ***Insertion Loss***

The Insertion Loss Method (contd.)

The power loss ratio in dB is simply the insertion loss of a lossless filter, and thus filter design using the power loss ratio is also called the Insertion Loss Method.

- We find that realizable filters will have a power loss ratio of the form:

$$P_{LR}(\omega) = 1 + \frac{M(\omega^2)}{N(\omega^2)}$$

where $M(\omega^2)$ and $N(\omega^2)$ are polynomials with terms $\omega^2, \omega^4, \omega^6$, etc.

By specifying these polynomials, we specify the frequency behavior of a realizable filter. Our job is to first choose a desirable polynomial!

- There are many different **types** of polynomials that result in good filter responses, and each type has its own set of **characteristics**.
- The type of **polynomial** likewise describes the type of microwave **filter**. Let's consider **three** of the most popular types:

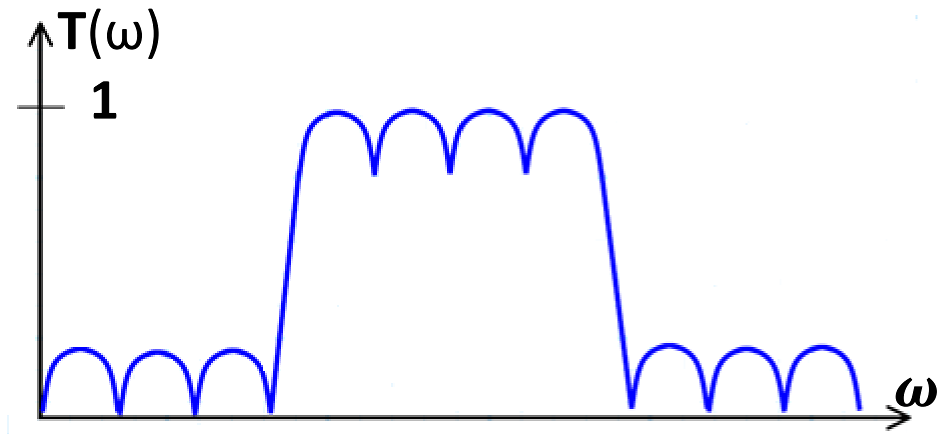
The Insertion Loss Method (contd.)

1. Elliptical: These filters have three primary characteristics:

a) They exhibit very **steep** "**roll-off**", meaning that the transition from pass-band to stop-band is very rapid.

b) They exhibit **ripple** in the **pass**-band, meaning that the value of **T** will vary slightly within the pass-band.

c) They exhibit ripple in the **stop**-band, meaning that the value of **T** will vary slightly within the stop-band.

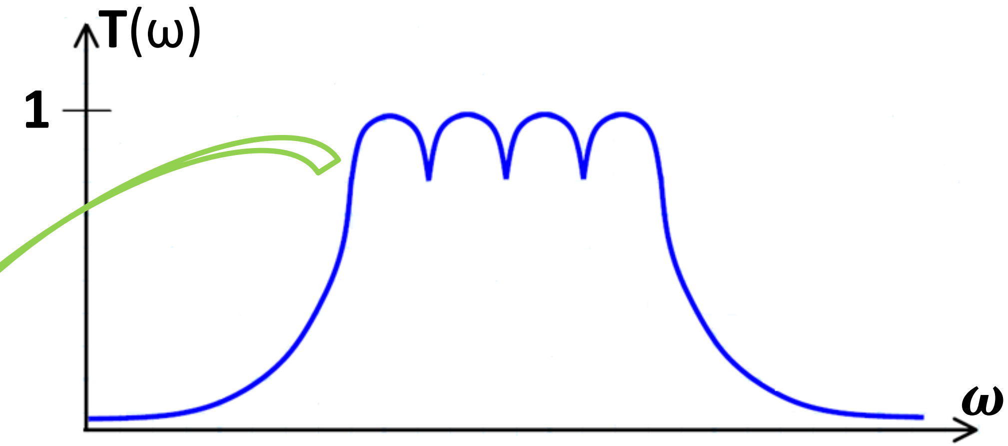


We can make the roll-off **steeper** by accepting more **ripple**.

The Insertion Loss Method (contd.)

2. Chebychev: These filters are also known as **equal-ripple** filters, and have two primary characteristics

a) **Steep** roll-off (but not as steep as Elliptical).
b) Pass-band **ripple** (but not stop-band ripple).



We likewise find that the roll-off can be made steeper by **accepting** more ripple.

- The Chebychev **low-pass** filters have a power loss ratio equal to:

$$P_{LR}(\omega) = 1 + k^2 T_N^2\left(\frac{\omega}{\omega_c}\right)$$

where ***k*** specifies the passband **ripple,** $T_N(x)$ is a Chebychev polynomial of **order** N, and ω_c is the low-pass **cutoff frequency**.

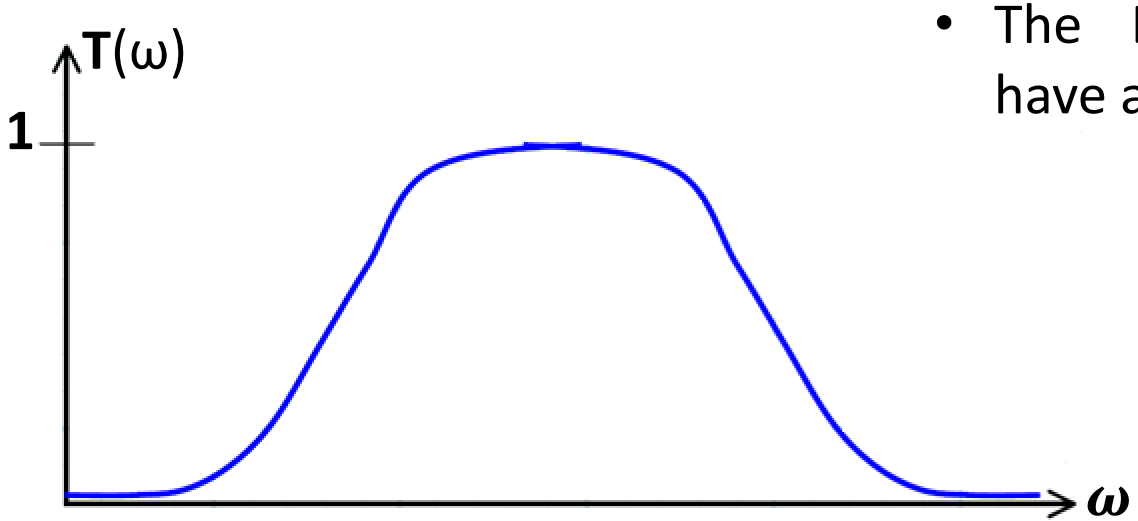
The Insertion Loss Method (contd.)

3. Butterworth

Also known as **maximally flat** filters, they have two primary characteristics

a) **Gradual** roll-off

b) **No ripple**—not anywhere.



- The Butterworth **low-pass** filters have a power loss ratio equal to:

$$P_{LR}(\omega) = 1 + \left(\frac{\omega}{\omega_c}\right)^{2N}$$

where ω_c is the low-pass **cutoff frequency**, and N specifies the **order** of the filter.

The Insertion Loss Method (contd.)

Q: So we always choose **elliptical** filters; since they have the steepest roll-off, they are **closest** to ideal—**right**?

A: Ooops! I forgot to talk about the **phase response** $\angle S_{21}(\omega)$ of these filters. Let's examine $\angle S_{21}(\omega)$ for each filter type **before** we pass judgment.

Butterworth $\angle S_{21}(\omega)$ → **Close** to linear phase
Chebyshev $\angle S_{21}(\omega)$ → **Not** very linear
Elliptical $\angle S_{21}(\omega)$ → A big non-linear **mess!**

- Thus, it is apparent that as a filter roll-off **improves**, the phase response gets **worse** (watch out for **dispersion!**).

  → There is **no** such thing as the "**best**" filter type!

Q: So, a filter with **perfectly** linear phase is impossible to construct?

A: No, it **is** possible to construct a filter with **near** perfect linear phase—but it will exhibit a **horribly** poor roll-off!

The Insertion Loss Method (contd.)

- Now, for any **type** of filter, we can **improve** roll-off (i.e., increase stop-band attenuation) by **increasing the filter order** N. However, be aware that increasing the filter order likewise has these **deleterious** effects:

    1. It makes **phase response** $\angle S_{21}(\omega)$ worse (i.e., more nonlinear).
    2. It increases filter **cost**, **weight**, and **size**.
    3. It increases filter **insertion loss** (this is bad).
    4. It makes filter performance more **sensitive** to temperature, aging, etc.

From a **practical** viewpoint, the **order** of a filter should typically be kept to **N** < **10**.

The Insertion Loss Method (contd.)

Q: So how do we take these polynomials and make real filters

A: Similar to matching networks and couplers, we:

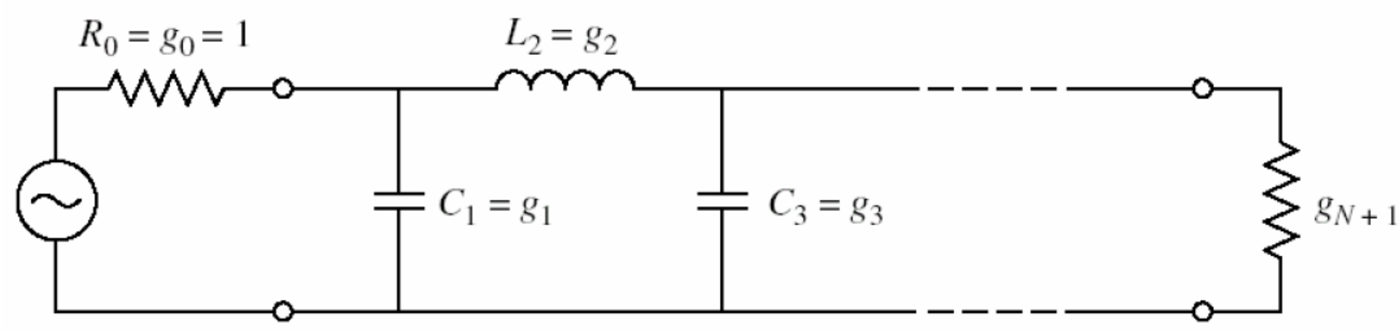
1. Form a general circuit structure with **several** degrees of design freedom.
2. Determine the **general form** of the power loss ratio for these circuits.
3. Use the degrees of design freedom to equate terms in the general form to the terms of the **desired** power loss ratio polynomial.

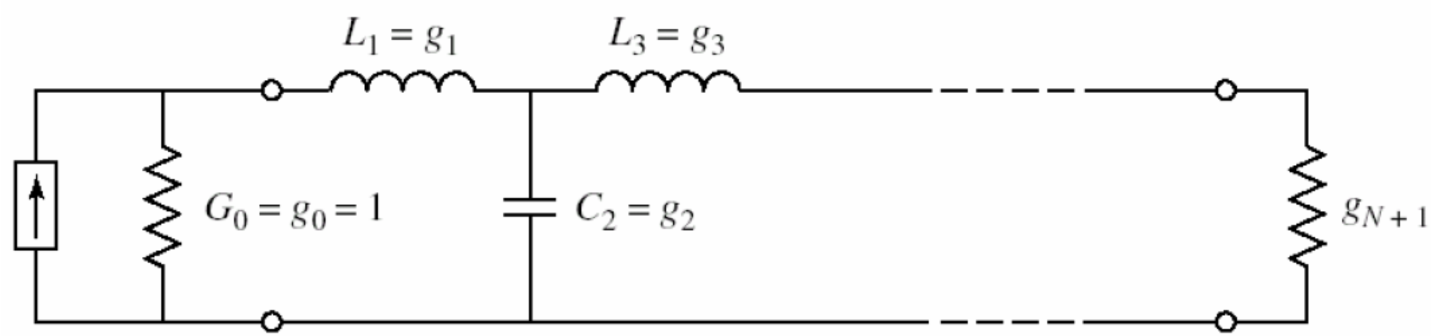
Filter Realizations Using Lumped Elements

- Our **first** filter circuit will be "**realized**" with lumped elements.
- **Lumped elements**—we mean inductors L and capacitors C !
- Since each of these elements are (ideally) perfectly **reactive**, the resulting filter will be **lossless** (ideally).

Filter Realizations Using Lumped Elements (contd.)

- Let us first consider two configurations of a **ladder circuit**:





Note that these two structures provide a **low-pass** filter response (evaluate the circuits at ω = 0 and ω = ∞!).

Moreover, these structures have N different **reactive elements** (i.e., N degrees of design freedom) and thus can be used to realize an **N-order** power loss ratio.

Filter Realizations Using Lumped Elements (contd.)

- For example, consider the **Butterworth** power loss ratio function:

$$P_{LR}(\omega) = 1 + \left(\frac{\omega}{\omega_c}\right)^{2N}$$

- Recall this is a **low-pass** function, as $P_{LR} = 1$ at ω = 0, and $P_{LR} = \infty$ at $\omega = \infty$. Note also that at $\omega_c = \omega$:

$$P_{LR}(\omega = \omega_c) = 1 + \left(\frac{\omega_c}{\omega_c}\right)^{2N} = 2$$

Thus →

$$\Gamma(\omega = \omega_c) = T(\omega = \omega_c) = 1/2$$

In other words, ω_c defines the 3dB bandwidth of the low-pass filter.

Filter Realizations Using Lumped Elements (contd.)

- Likewise, we find that this Butterworth function is **maximally flat** at ω = 0:

$$P_{LR}(\omega=0)=1+\left(\frac{0}{\omega_c}\right)^{2N}=1$$

and:

$$\frac{d^n P_{LR}(\omega)}{d\omega^n}\Big|_{\omega=0}=0$$

For all n

- Now, we can determine the function $P_{LR}(\omega)$ for a lumped element ladder circuit of N elements using our knowledge of **complex circuit theory**.
- Then, we can **equate** the resulting polynomial to the **maximally flat** function above. In this manner, we can determine the appropriate **values** of all inductors L and capacitors C!
- Finding these L an C requires little bit of complex algebra.
- Pozar provides tables of complete Butterworth and Chebychev low-pass solutions.

Filter Realizations Using Lumped Elements (contd.)

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

Filter Realizations Using Lumped Elements (contd.)

TABLE 8.4 Element Values for Equal-Ripple Low-Pass Filter Prototypes ($g_0 = 1, \omega_c = 1, N = 1$ to 10, 0.5 dB and 3.0 dB ripple)

0.5 dB Ripple											
N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841
3.0 dB Ripple											
N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass.,1980, with permission.

Filter Realizations Using Lumped Elements (contd.)

Q: What?! What the heck do these values g_n mean?

A: We can use the values g_n to find the values of inductors and capacitors required for a given **cutoff frequency** ω_c and source resistance R_s (Z_0).

- Specifically, we use the values of g_n to find ladder circuit **inductor** and **capacitor** values as:

$$L_n = g_n \left(\frac{R_s}{\omega_c} \right) \qquad C_n = g_n \left(\frac{1}{R_s \omega_c} \right) \qquad \text{where } n = 1,2,\ldots,N$$

- Likewise, the value g_{N+1} describes the **load impedance**. Specifically, we find that **if** the **last** reactive element (i.e., g_N) of the ladder circuit is a **shunt capacitor**, then:

$$R_L = g_{N+1} R_s$$

- Whereas, **if** the **last** reactive element (i.e., g_N) of the ladder circuit is a **series inductor**, then:

$$R_L = \frac{R_s}{g_{N+1}}$$

Filter Realizations Using Lumped Elements (contd.)

- Note, however, for the **Butterworth** solutions **(in Table 8.3)** we find that g_{N+1}=1 **always**, and therefore:

$$R_L = R_s$$

← Regardless of the last element

- Moreover, we note (in Table 8.4) that this (i.e., g_{N+1}=1) is likewise true for the Chebyshev solutions – provided that **N is odd**.
- Thus, since we typically desire a filter where:

$$R_L = R_s = Z_0$$

→ We can use **any** order of **Butterworth** filter, or an **odd** order of **Chebyshev.**

In other words, avoid even order Chebyshev filters!