Lecture – 20 Date: 11.11.2014

- The Coupled Line Coupler

The Coupled Line Coupler

Q: The "Quadrature Hybrid" or "Rat Race" are 3dB couplers. How do we build couplers with **less coupling**, say 10dB, 20dB, or 30 dB?

A: Such directional couplers are typically built using **coupled lines.**

Q: How can we **design** a coupled line couplers so that it is an **ideal** directional coupler with a **specific** coupling value?

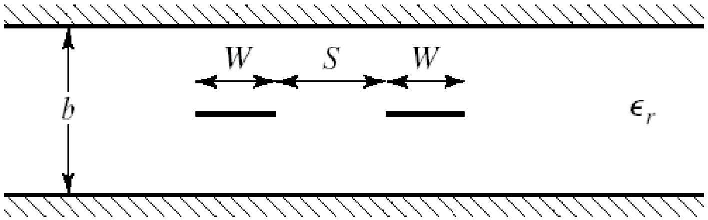
A: This lecture introduces the concept of such a design.

Q: Like all devices with quarter-wavelength sections, a coupled line coupler would seem to be inherently **narrow band**. Is there some way to **increase coupler bandwidth**?

A: Yes! add more coupled-line sections.

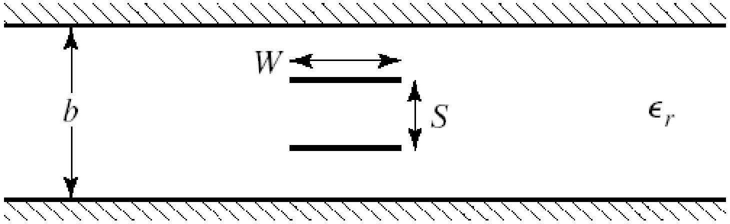
The Coupled Line Coupler (contd.)

- Two transmission lines in **proximity** to each other will **couple** power from one line into another.
- This proximity will **modify** the electromagnetic fields (and thus modify voltages and currents) of the propagating wave, and therefore **alter** the characteristic impedance of the transmission line!



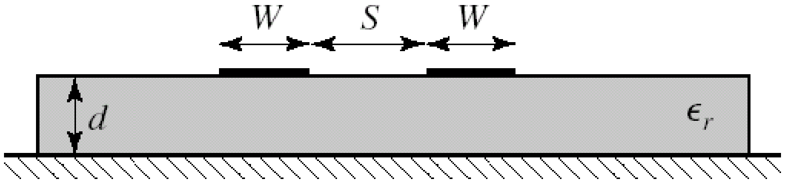
(a)

Coupled Stripline (edge-coupled or planar)



(b)

Coupled Stripline (broadside-coupled or stacked)

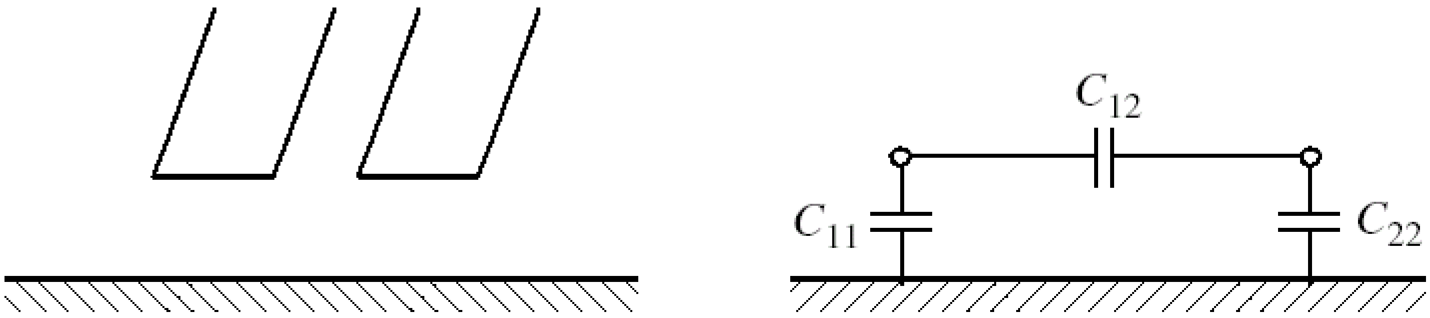


(c)

Coupled Microstrip

The Coupled Line Coupler (contd.)

- Generally, speaking, we find that these transmission lines are capacitively coupled (i.e., it appears that they are connected by a capacitor):

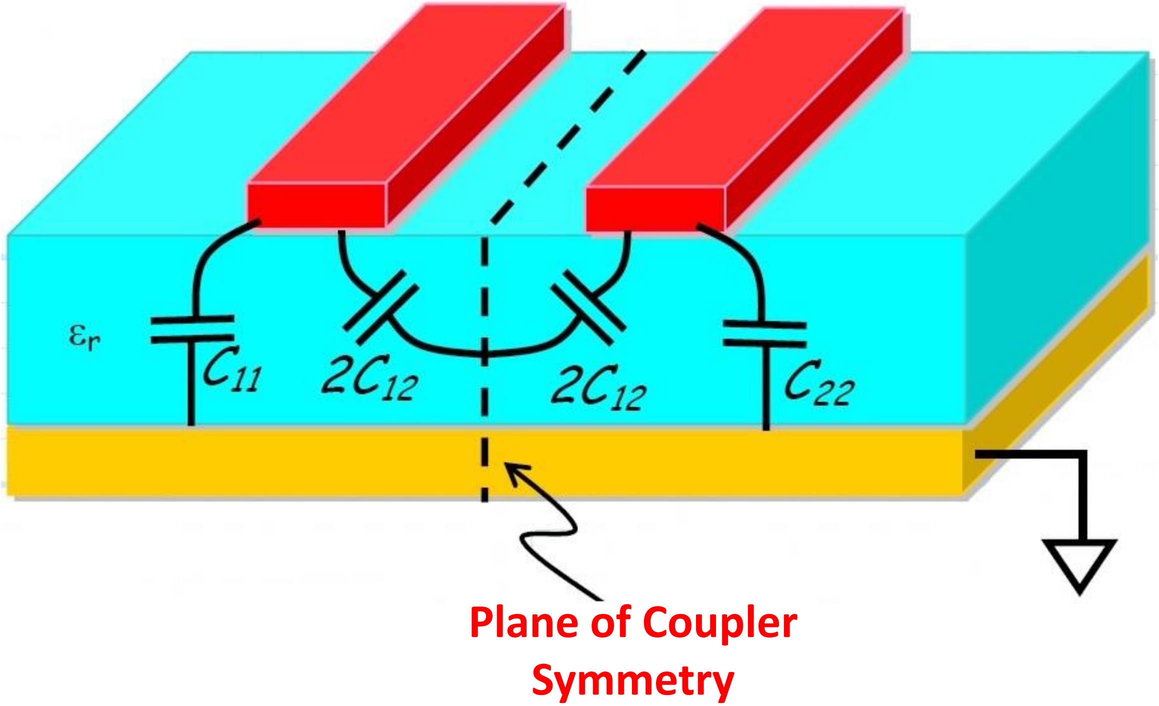


A three-wire coupled transmission line and its equivalent capacitance network

If the two transmission lines are **identical** (and they typically are), then $C_{11} = C_{22}$

The Coupled Line Coupler (contd.)

- Likewise, if the two transmission lines are identical, then a plane of circuit **symmetry** exists. As a result, we can analyze this circuit using **odd/even mode** analysis!

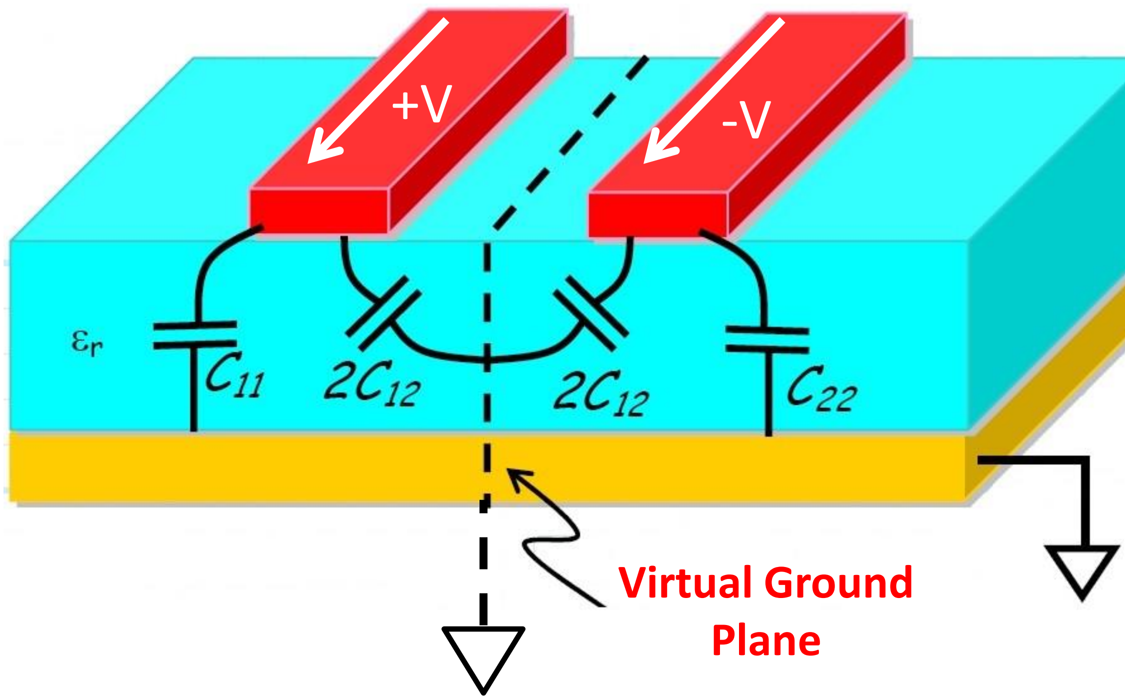


Note the capacitor C_{12} has been divided into **two series** capacitors, each with a value of $2C_{12}$

The Coupled Line Coupler (contd.)

Odd Mode

- If the incident wave along the two transmission lines are **opposite** (i.e., equal magnitude but 180^o out of phase), then a virtual **ground plane** is created at the plane of circuit symmetry.



- Thus, the capacitance per unit length of each transmission line, in the **odd** mode, is:

$$C_o = C_{11} + 2C_{12} = C_{22} + 2C_{12}$$

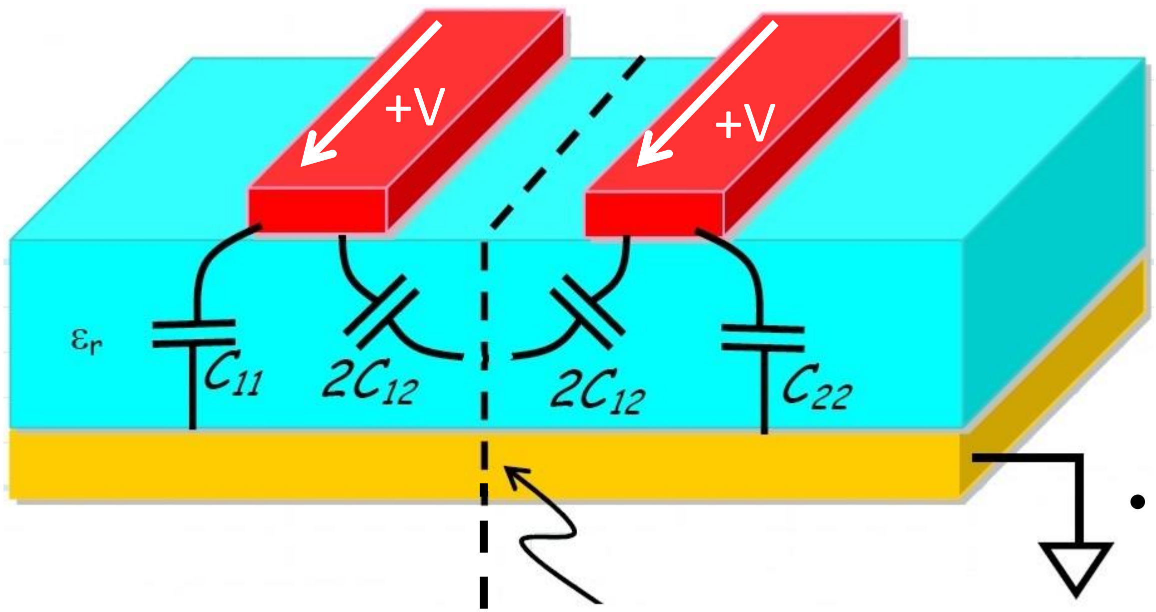
- Therefore the corresponding characteristic impedance is:

$$Z_0^o = \sqrt{\frac{L}{C_o}}$$

The Coupled Line Coupler (contd.)

Even Mode

- If the incident wave along the two transmission lines are **equal** (i.e., equal magnitude and phase), then a **virtual open** plane is created at the plane of circuit symmetry.



- Note the $2C_{12}$ capacitors have been "**disconnected**", and thus the capacitance per unit length of each transmission line, in the **even** mode, is:

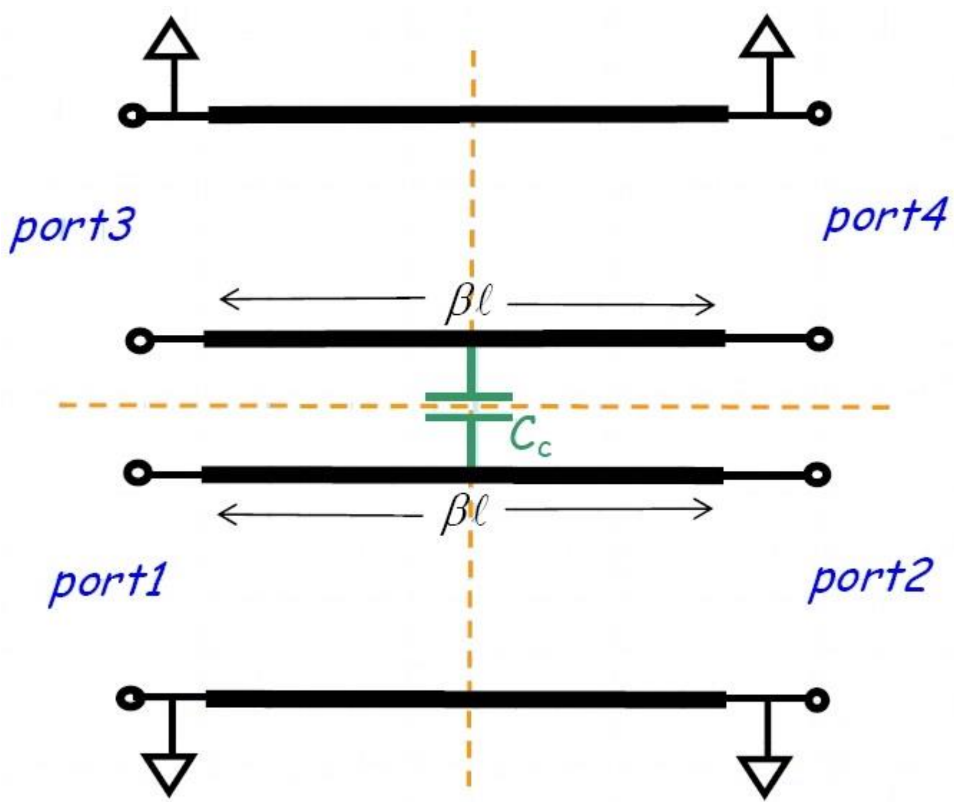
$$C_e = C_{11} = C_{22}$$

- Therefore the corresponding characteristic impedance is:

$$Z_0^e = \sqrt{\frac{L}{C_e}}$$

Analysis and Design

- A pair of coupled lines form a **4-port** device with **two** planes of reflection symmetry.

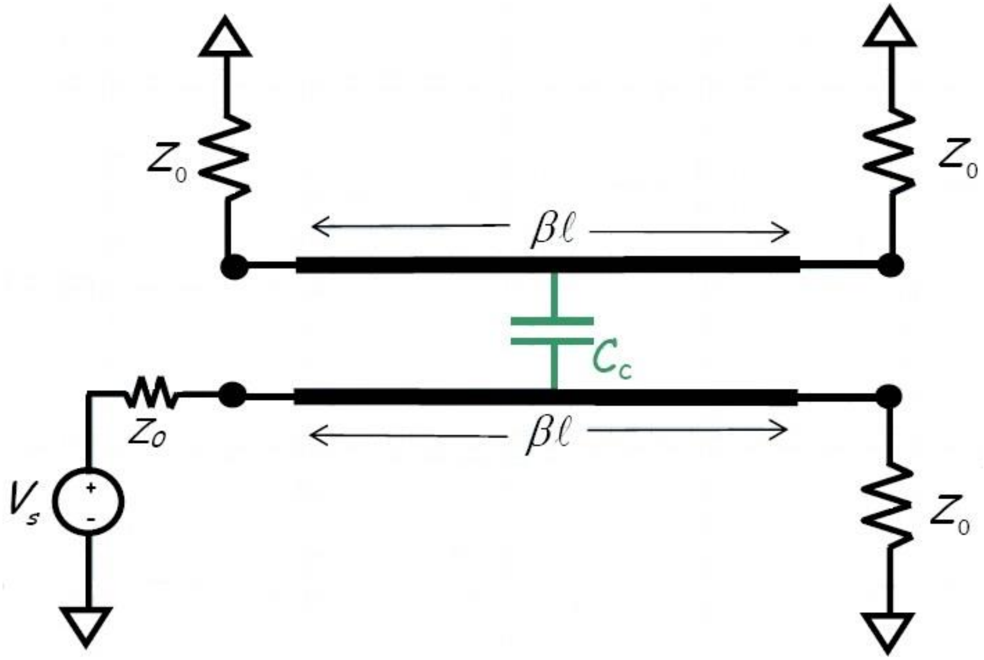


- As a result, we know that the **scattering matrix** of this four-port device has just **4 independent** elements:

$$\mathbf{S} = \begin{bmatrix} S_{11} & S_{21} & S_{31} & S_{41} \\ S_{21} & S_{11} & S_{41} & S_{31} \\ S_{31} & S_{41} & S_{11} & S_{21} \\ S_{41} & S_{31} & S_{21} & S_{11} \end{bmatrix}$$

Analysis and Design (contd.)

- To determine these four elements, we can apply a **source to port 1** and then **terminate** all other ports:

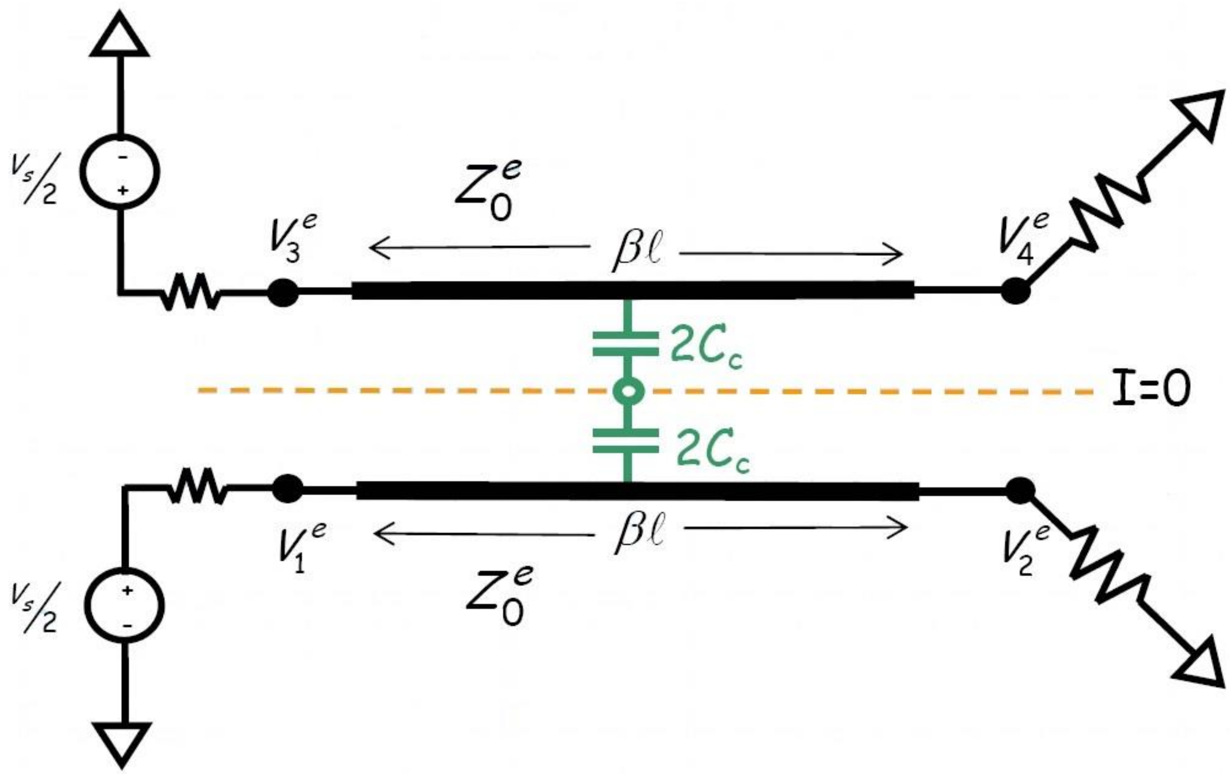


Typically, a coupled-line coupler schematic is drawn **without** explicitly showing the **ground conductors** (i.e., without the ground plane)

To analyze this circuit, we must apply **odd/even mode analysis**

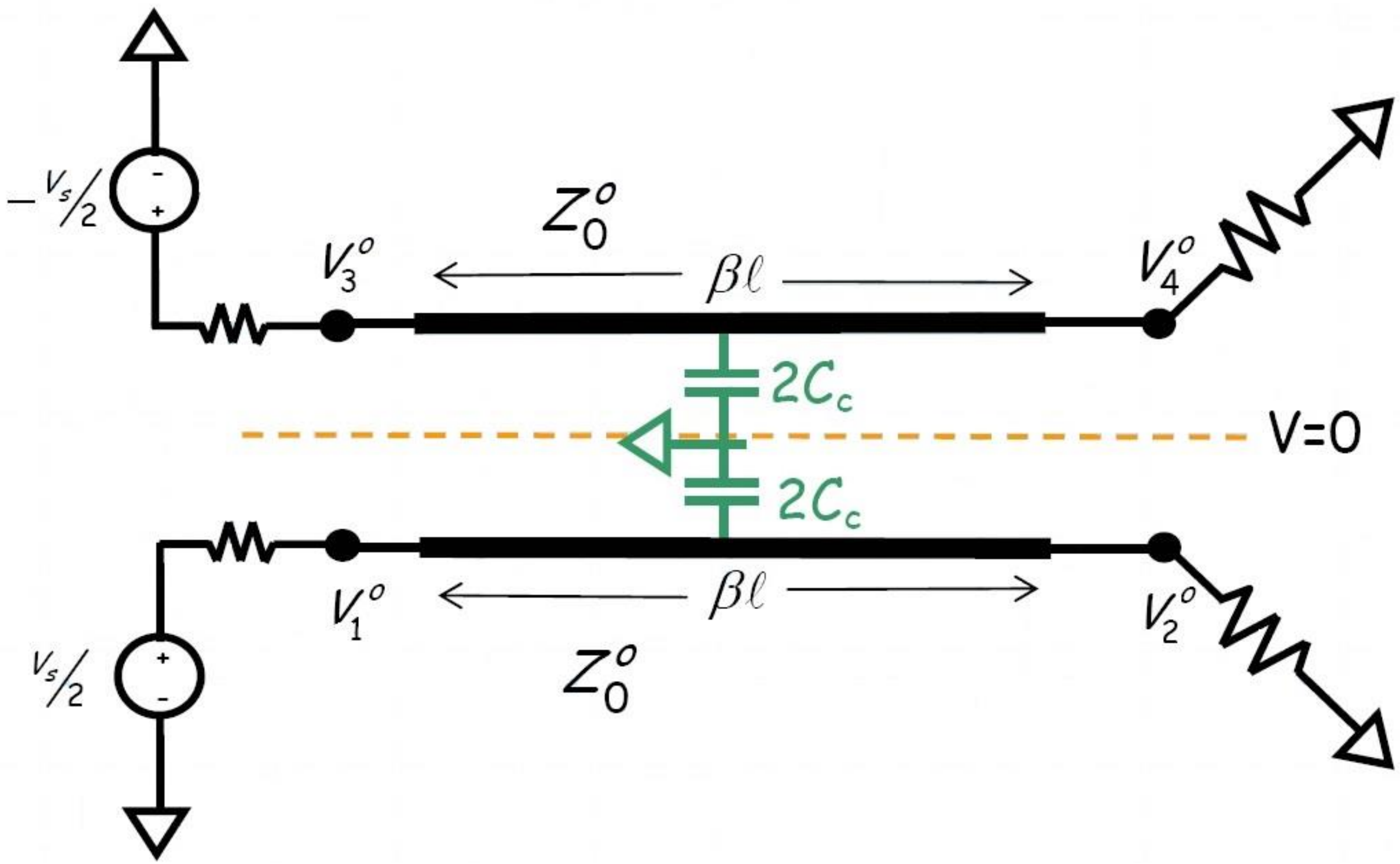
Analysis and Design (contd.)

Even Mode Circuit



Analysis and Design (contd.)

Odd Mode Circuit



Analysis and Design (contd.)

Note that the **capacitive coupling** associated with these modes are different, resulting in a **different** characteristic impedance of the lines for the two cases (i.e., Z_0^e, Z_0^o)

Q: So what?

A: Consider what would happen if the characteristic impedance of each line were **identical** for **each mode**:

$$Z_0 = Z_0^e = Z_0^o$$

- In such a situation we can find that:

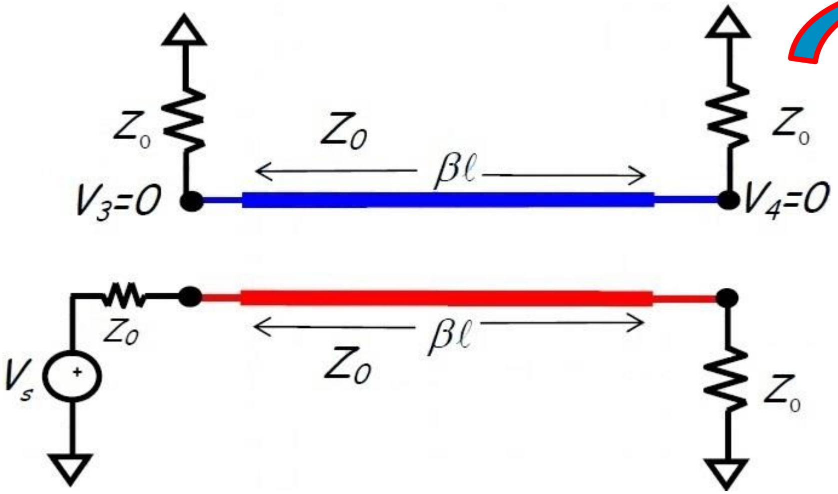
$$V_3^e = -V_3^o \qquad V_4^e = -V_4^o$$

- Therefore from **superposition**:

$$V_3 = V_3^e + V_3^o = 0 \qquad V_4 = V_4^e + V_4^o = 0$$

Analysis and Design (contd.)

- This indicates that **no power is coupled** from the "**energized**" transmission line onto the "**non-energized**" transmission line.



This makes sense! After all, **if** no coupling occurs, then the characteristic impedance of each line is **unaltered** by the presence of the other—their characteristic impedance is Z_0 **regardless** of "mode".

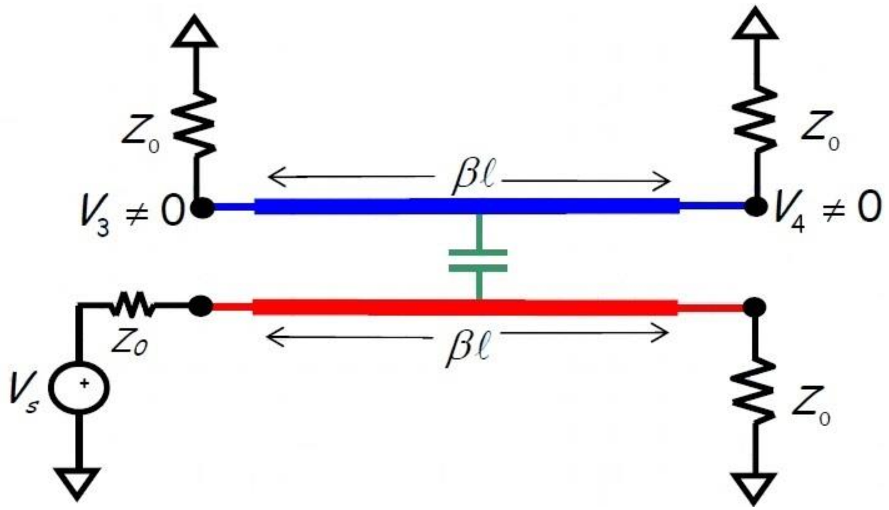
- However, if coupling **does** occur, then $\mathbf{Z_0^e \neq Z_0^o}$, meaning in general:

$$V_3^e \neq -V_3^o \qquad V_4^e \neq -V_4^o$$

- and thus in general: $V_3 = V_3^e + V_3^o \neq 0$ $\qquad$ $V_4 = V_4^e + V_4^o \neq 0$

Analysis and Design (contd.)

- The odd/even mode analysis thus reveals the amount of **coupling from** the ***energized*** section **onto** the ***non-energized*** section!



- Now, our **first step** in performing the odd/even mode analysis will be to determine scattering parameter S_{11}. To accomplish this, we will need to determine voltage V_1:

$$V_1 = V_1^e + V_1^o$$

- The analysis is a bit complicated, so it won't be presented here. However, a pertinent question we might ask is, what value **should S_{11}** be?

A: For the device to be a **matched** device, it must be **zero**!

Analysis and Design (contd.)

- From the value of S_{11} derived from our odd/even analysis, it can be shown that S_{11} will be equal to zero **if** the odd and even mode characteristic impedances are related as:

$$\sqrt{Z_0^e Z_0^o} = Z_0$$

- In other words, we should design our coupled line coupler such that the **geometric mean** of the even and odd mode impedances is **equal to Z_0**.
- Now, assuming this design rule has been implemented, we also find (from odd/even mode analysis) that the scattering parameter **S_{31}** is:

$$S_{31} = \frac{j\left(Z_0^e - Z_0^o\right)}{2Z_0 \cot(\beta l) + j\left(Z_0^e + Z_0^o\right)}$$

Thus, it can be seen that **unless $Z_0^e = Z_0^o$**, power must be coupled from port 1 to port 3!

Analysis and Design (contd.)

Q: But what is the value of line **electrical length** βl ?

A: The **electrical length** of the coupled transmission lines is also a **design parameter**. Assuming that we want to **maximize** the coupling onto port 3, we find from the S_{31} expression that this is accomplished if we set βl such that:

$$\cot(\beta l) = 0 \;\Rightarrow\; \beta l = \frac{\pi}{2} \;\Rightarrow\; l = \frac{\lambda}{4}$$

Once again, our design rule is to set the transmission line length to a value equal to **one-quarter wavelength** (at the design frequency).

- Implementing these **two** design rules, we find that (at the design frequency):

$$S_{31} = \frac{\left(Z_0^e - Z_0^o\right)}{\left(Z_0^e + Z_0^o\right)}$$

Analysis and Design (contd.)

- The value of S_{31} is a **very** important one with respect to coupler performance. Specifically, it is the **coupling coefficient** c !

$$c = \frac{\left(Z_0^e - Z_0^o\right)}{\left(Z_0^e + Z_0^o\right)}$$

- Given this definition, we can **rewrite** the scattering parameter S_{31} as:

$$S_{31} = \frac{jc\tan(\beta l)}{\sqrt{1-c^2} + j\tan(\beta l)}$$

- **Similarly,** the odd/even mode analysis gives (given that $\sqrt{Z^e{}_0\, Z^o{}_0} = Z_0$):

$$S_{21} = \frac{\sqrt{1-c^2}}{\sqrt{1-c^2}\cos(\beta l) + j\sin(\beta l)}$$

- so at our **design frequency**, where $\beta l = \pi/2$, we find:

$$S_{21} = -j\sqrt{1-c^2}$$

Analysis and Design (contd.)

- **Finally**, the odd/even analysis also gives (at the design frequency):

$$S_{41} = 0$$

- **Combining** these results, at the design frequency, the **scattering matrix** of coupled-line coupler is:

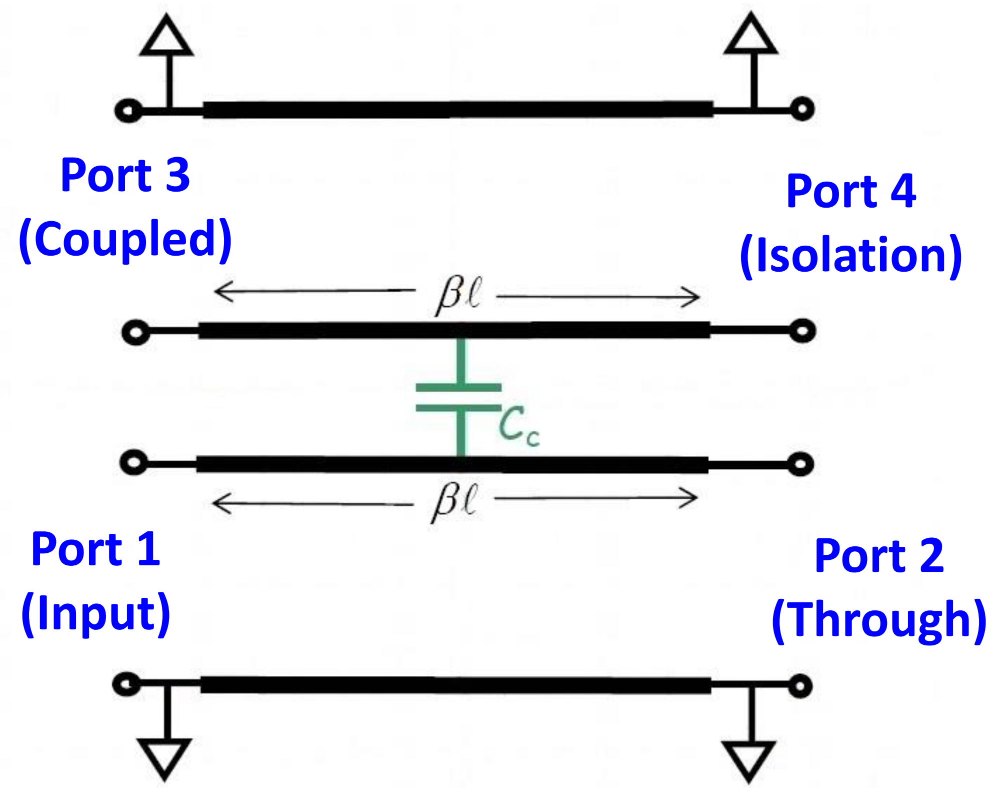
$$\boldsymbol{S} = \begin{bmatrix} 0 & -j\sqrt{1-c^2} & c & 0 \\ -j\sqrt{1-c^2} & 0 & 0 & c \\ c & 0 & 0 & -j\sqrt{1-c^2} \\ 0 & c & -j\sqrt{1-c^2} & 0 \end{bmatrix}$$

Q: Hey! Isn't this the same scattering matrix as the **ideal symmetric directional coupler ?**

A: The very same! The coupled-line coupler—**if** our design rules are followed—results in an "ideal" directional coupler.

Analysis and Design (contd.)

- If the **input** is port 1, then the **through** port is port 2, the **coupled** port is port 3, and the **isolation** port is port 4!



Analysis and Design (contd.)

Q: But, how do we **design** a coupled-line coupler with a **specific** coupling coefficient c?

A: We know the **two design constraints**:

$$\sqrt{Z_0^e Z_0^o} = Z_0$$

$$c = \frac{Z_0^e - Z_0^o}{Z_0^e + Z_0^o}$$

We can **rearrange** these two expressions to find **solutions** for our odd and even mode impedances:

$$Z_0^e = Z_0 \sqrt{\frac{1+c}{1-c}}$$

$$Z_0^o = Z_0 \sqrt{\frac{1-c}{1+c}}$$

Therefore, **given** the desired values Z_0 and **c**, we can determine the proper values of Z_0^e and Z_0^o for an ideal directional coupler

Analysis and Design (contd.)

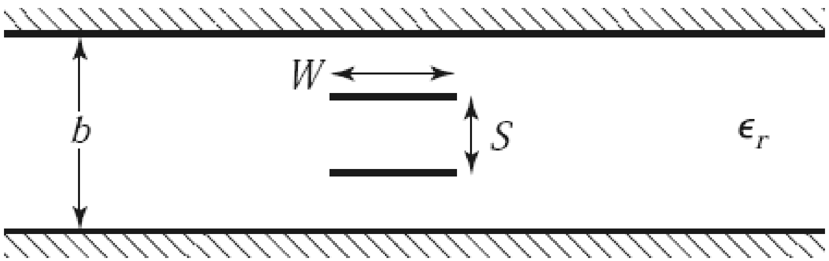
Q: Yes, but the odd and even mode impedance depends on the **physical structure** of the coupled lines, such as substrate dielectric ε_r, substrate thickness, conductor width, and separation distance. How do we determine **these** physical design parameters for desired values of Z_0^e and Z_0^o ?

A: That's a much more difficult question to answer! Recall that there is **no** direct formulation relating microstrip and stripline parameters to **characteristic impedance** (There are numerically derived **approximations**).

- So it's no surprise that there is **no direct formulation** relating odd and even mode characteristic impedances to the specific physical parameters of microstrip and stripline coupled lines.
- Instead, there are again numerically derived **approximations** that allow us to determine (approximately) the required microstrip and stripline parameters, or one can always use a **microwave CAD package** (such as ADS!).
- For details please follow chapter-7 (Pozar)

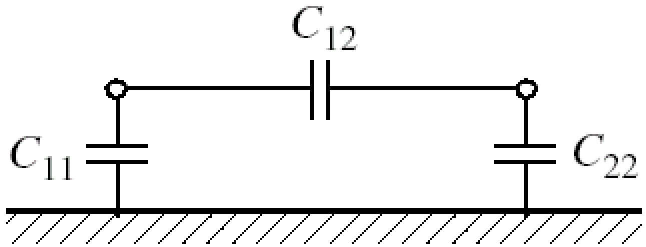
Example – 1

For the broadside coupled stripline geometry of shown below, assume $W \gg S$ and $W \gg b$, so that fringing fields can be ignored. Determine the even- and odd-mode characteristic impedances.



Solution:

The equivalent circuit is:



- First determine the equivalent network capacitances, C_{11} and C_{12}.
- The capacitance per unit length of broadside parallel lines with width W and separation S is:

$$C = \frac{\epsilon W}{S} \quad F/m$$

Ignores the fringing field

Example – 1 (contd.)

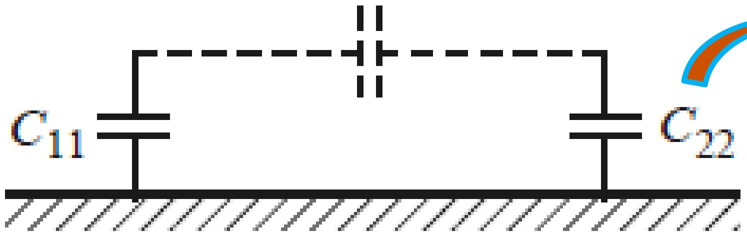
- C_{11} and C_{22} are formed by the capacitance of one strip to the ground planes. Thus the capacitance per unit length is:

$$C_{11} = C_{22} = \frac{2\epsilon_0\epsilon_r W}{b - S} \quad F/m$$

- The capacitance per unit length between the strips is:

$$C_{12} = \frac{\epsilon_0\epsilon_r W}{S} \quad F/m$$

- For the **even mode**, the electric field has even symmetry about the center line, and no current flows between the two strip conductors. This leads to the equivalent circuit shown, where C_{12} is effectively open-circuited.



The resulting capacitance of either line to ground for the even mode is:

$$C_e = C_{11} = C_{22} = \frac{2\epsilon_0\epsilon_r W}{b - S} \quad F/m$$

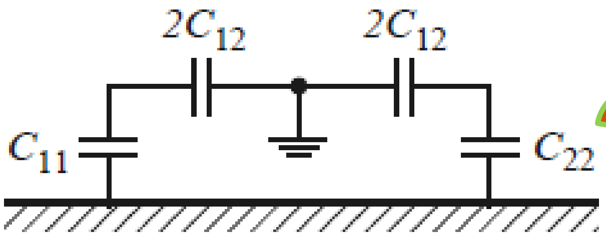
Example – 1 (contd.)

- Therefore:

$$Z_{0e} = \frac{1}{v_p C_e} = \eta_0 \frac{b - S}{2W\sqrt{\epsilon_r}}$$

$$v_p = c/\sqrt{\epsilon_r}$$

- For the **odd mode**, the electric field lines have an odd symmetry about the center line, and a voltage null exists between the two strip conductors. We can imagine this as a ground plane through the middle of C_{12}, which leads to the equivalent circuit shown.



the effective capacitance between either strip conductor and ground is:

$$C_o = C_{11} + 2C_{12} = C_{11} + 2C_{12}$$

Example – 1 (contd.)

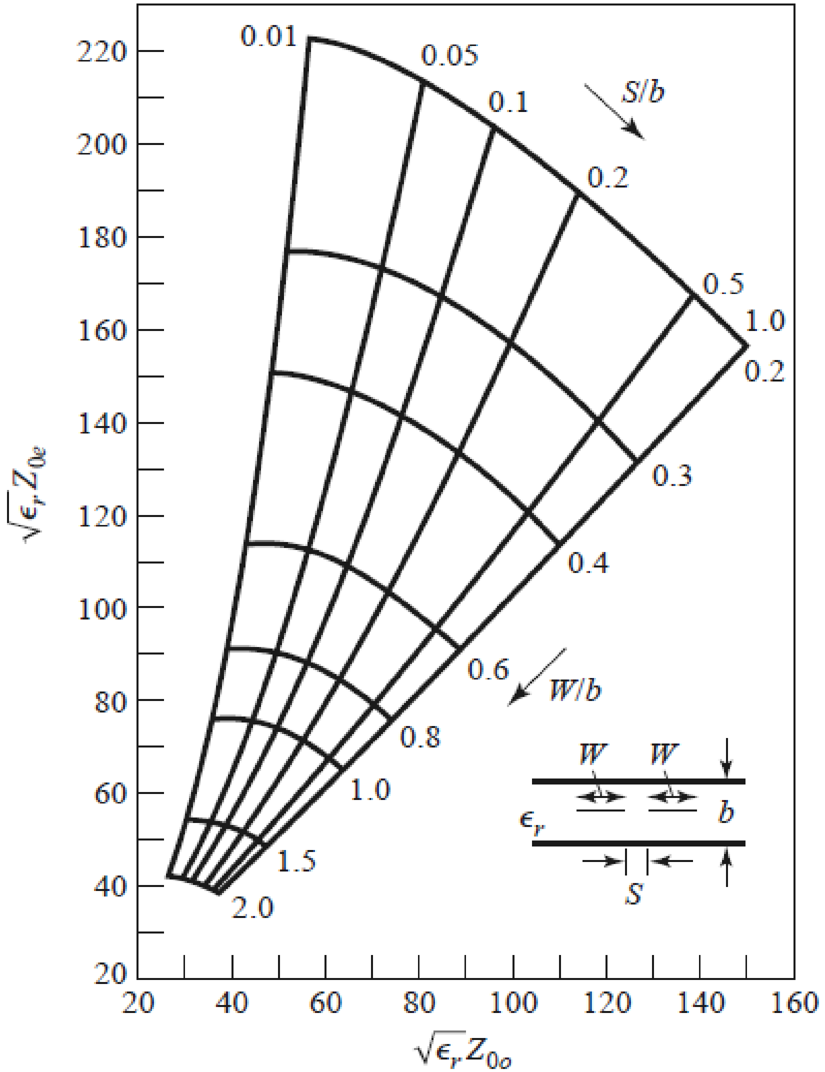
- Therefore:

$$Z_{0o} = \frac{1}{v_p C_o} = \eta_0 \frac{1}{2W\sqrt{\epsilon_r}\left[\frac{1}{(b-S)} + \frac{1}{S}\right]}$$

$$v_p = c/\sqrt{\epsilon_r}$$

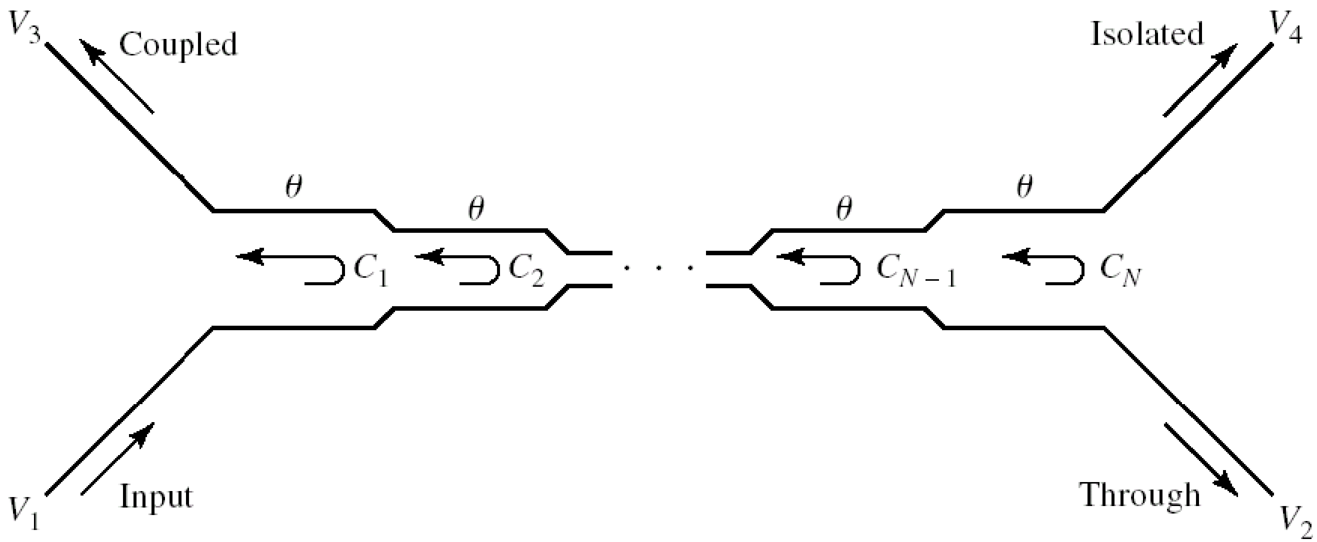
Example – 2

Design a 20 dB single-section coupled line coupler in stripline with a ground plane spacing of 0.32 cm, a dielectric constant of 2.2, a characteristic impedance of 50, and a center frequency of 3 GHz. Plot the coupling and directivity from 1 to 5 GHz. Include the effect of losses by assuming a loss tangent of 0.05 for the dielectric material and copper conductors of 2 mil thickness.



Multi-section Coupled-Line Couplers

- We can add **multiple** coupled lines in series to increase coupler bandwidth.



- The couplers are typically designed such that they are **symmetric**, i.e.:

$C_1 = C_N$ $C_2 = C_{N-1}$ $C_3 = C_{N-2}$ **etc.** where N is odd.

Because the phase characteristics are usually better

Multi-section Coupled-Line Couplers (contd.)

Q: What is the coupling of this device as a function of **frequency**?

A: To analyze this structure, we make some **approximations:**

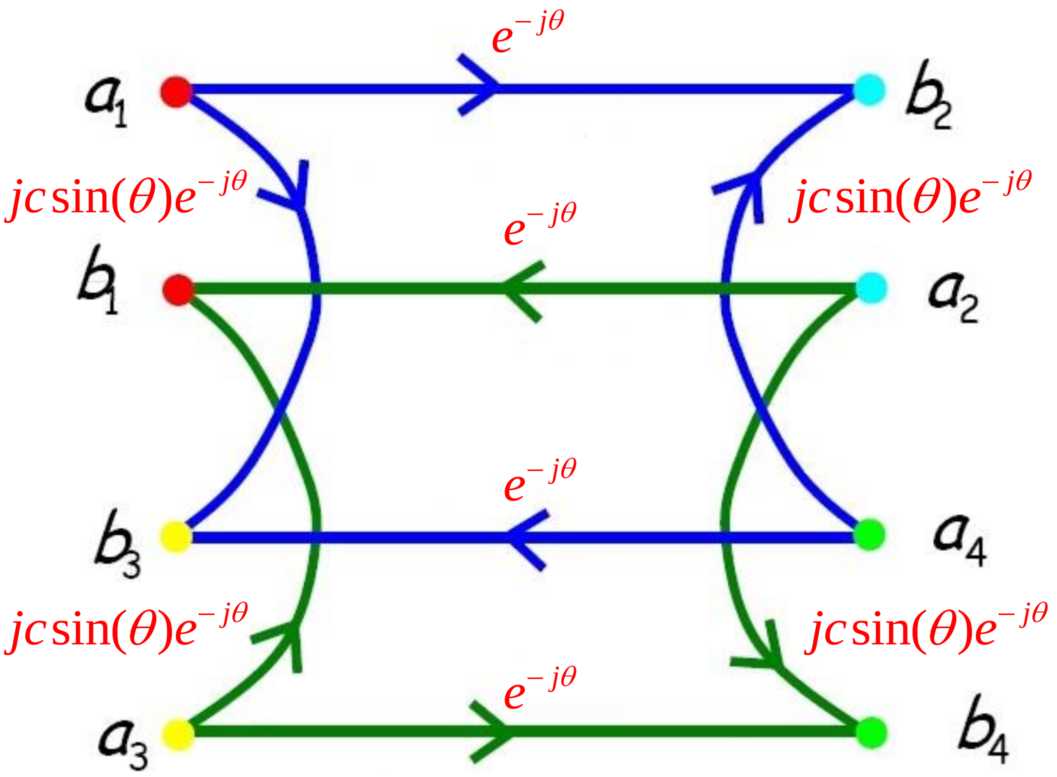
$$S_{31}(\theta) = \frac{jc\tan(\theta)}{\sqrt{1-c^2} + \tan(\theta)} \approx \frac{jc\tan(\theta)}{1 + j\tan(\theta)} = jc\sin(\theta)e^{-j\theta}$$

$$S_{21}(\theta) = \frac{\sqrt{1-c^2}\, jc\tan(\theta)}{\sqrt{1-c^2}\cos(\theta) + j\sin(\theta)} \approx \frac{1}{\cos(\theta) + j\sin(\theta)} = e^{-j\theta}$$

where obviously, $\theta = \beta l = \omega T$, and $T = l/v_p$

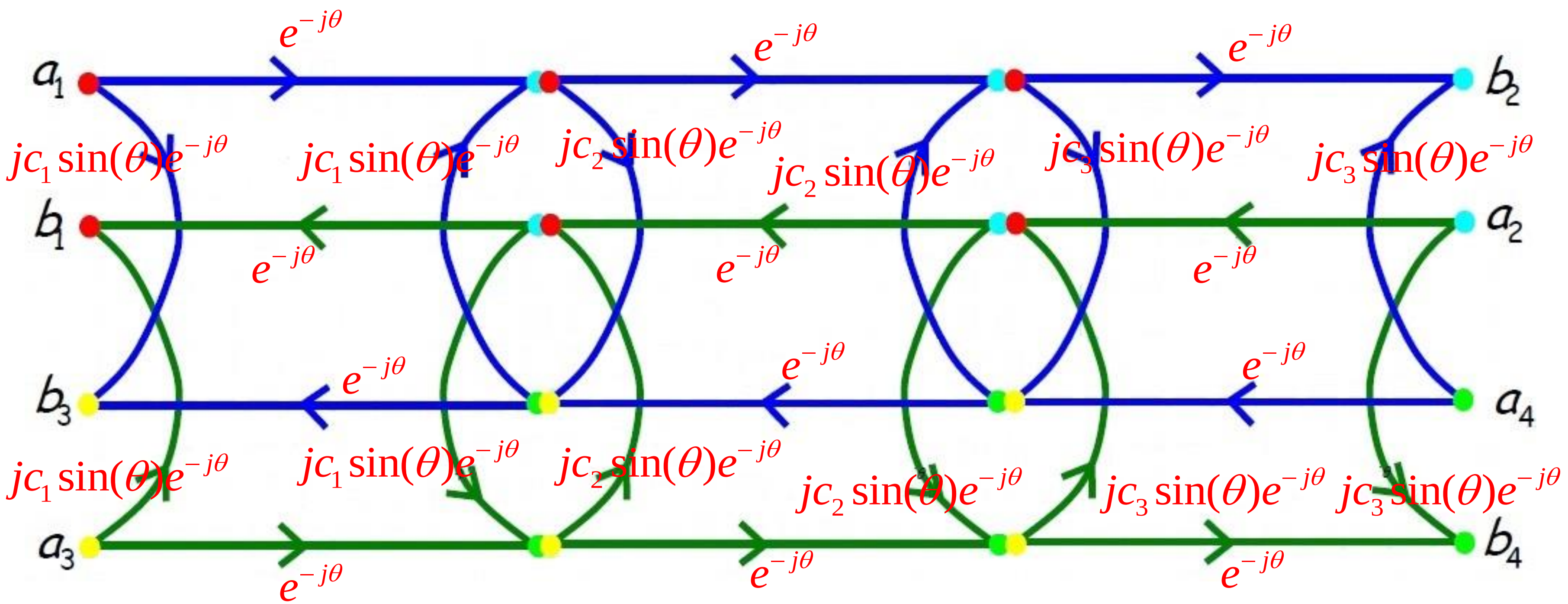
Multi-section Coupled-Line Couplers (contd.)

- We can use these approximations to construct a **signal flow graph** of a **single**-section coupler:



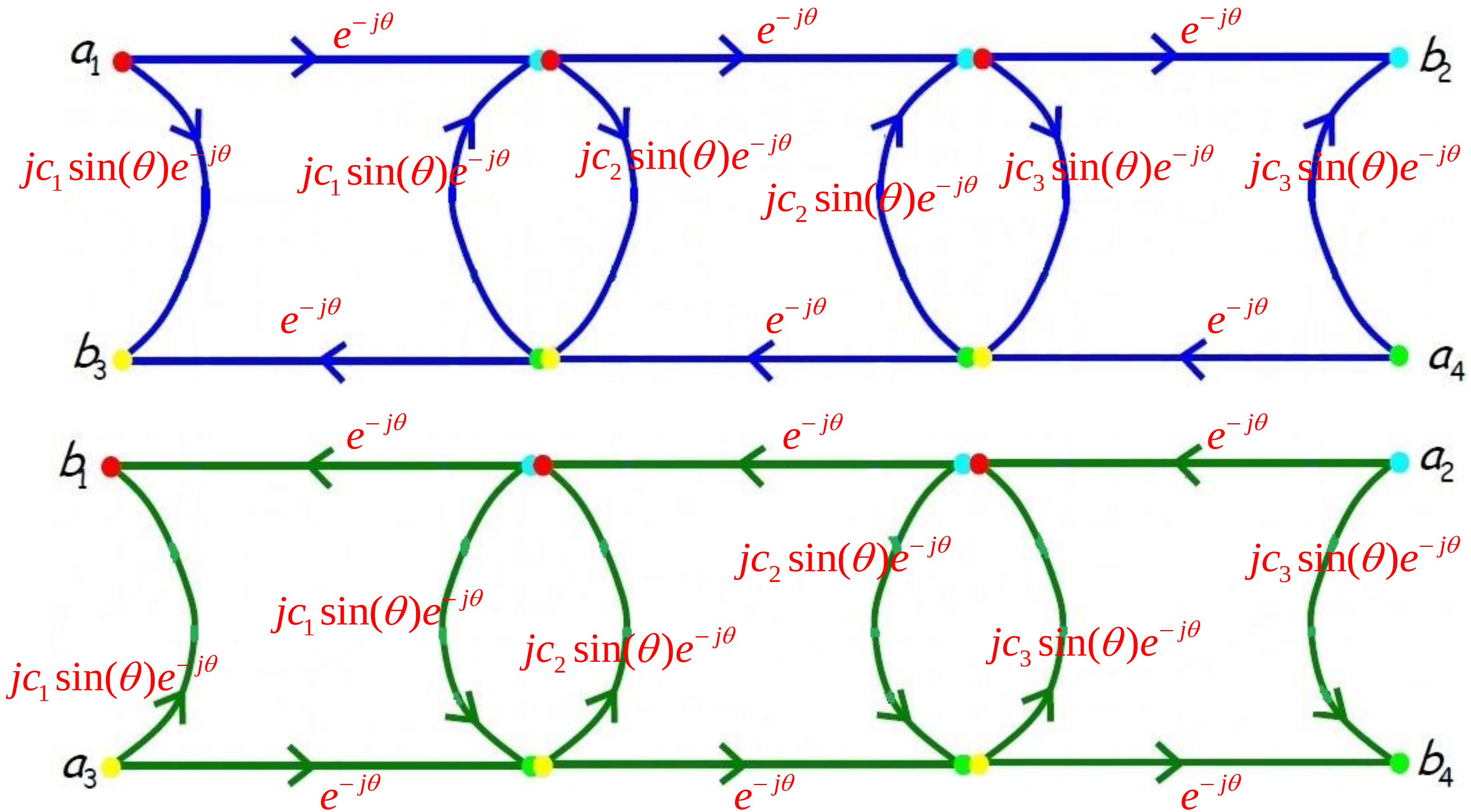
Multi-section Coupled-Line Couplers (contd.)

- Now, say we cascade **three** coupled line pairs, to form a **three section** coupled line coupler. The signal flow graph would thus be:



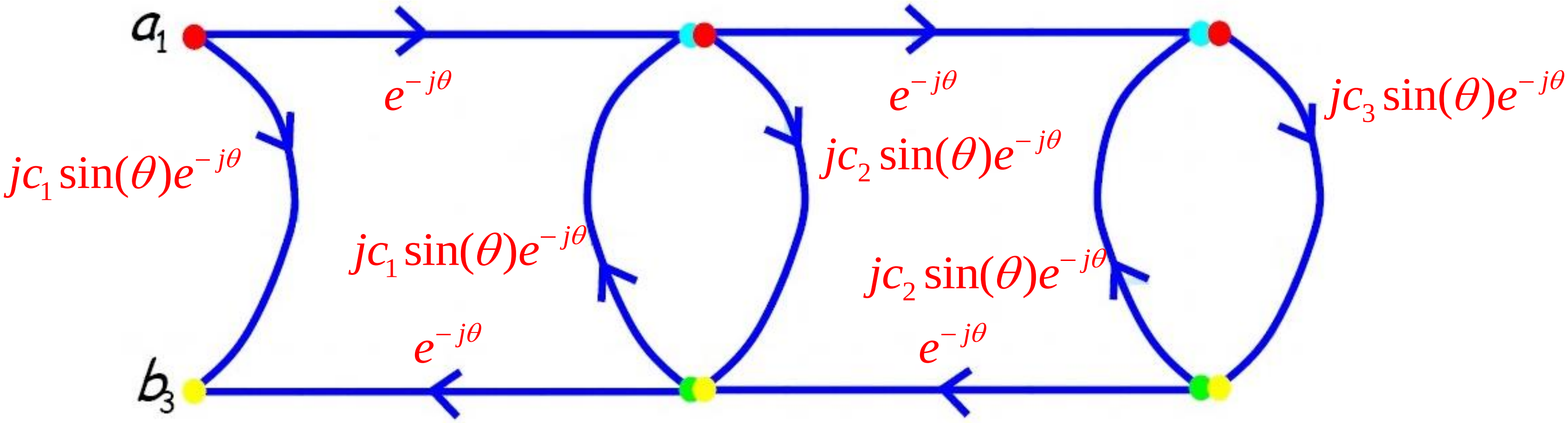
Multi-section Coupled-Line Couplers (contd.)

- Note that this signal flow graph **decouples** into two separate graphs (i.e., the blue graph and the green graph).



Multi-section Coupled-Line Couplers (contd.)

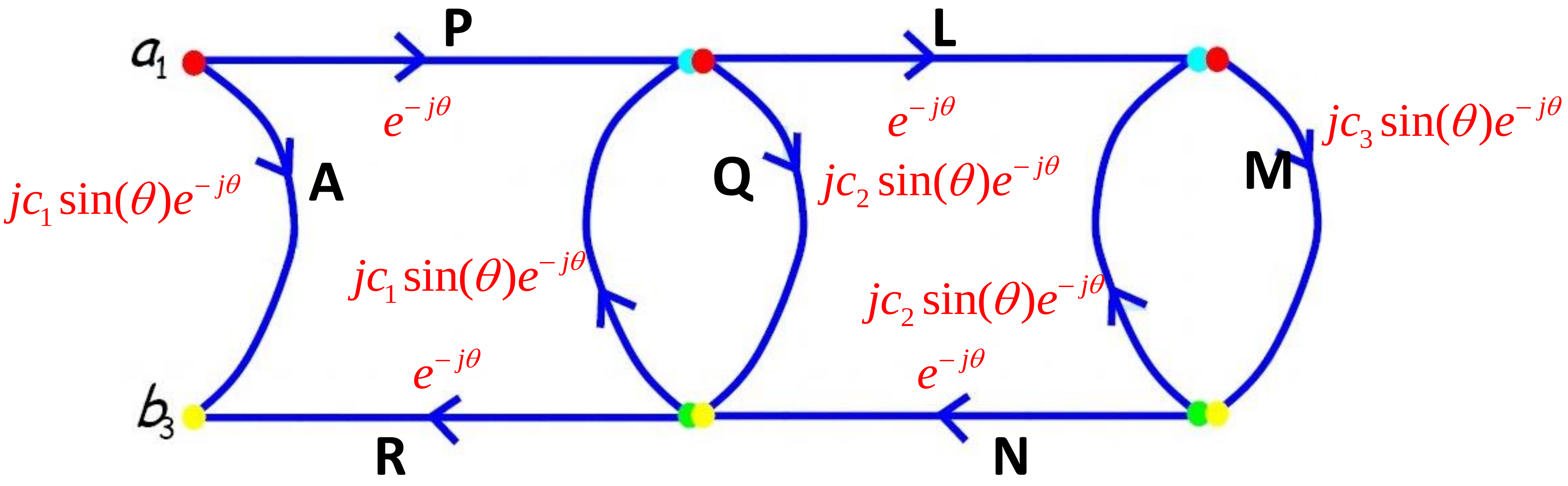
- Note that these two graphs are essentially **identical**, and emphasize the **symmetric** structure of the coupled-line coupler.
- Now, we are interested in describing the **coupled output** (i.e., b_3) in terms of the incident wave (i.e., a_1). Assuming ports 2, 3 and 4 are **matched** (i.e., $a_4 = 0$), we can reduce the graph to simply:



Now, we **could** reduce this signal flow graph even further—**or** we can apply the **multiple reflection viewpoint** explicitly to each propagation term! **(follow Microwave Engineering *by* Collins)**

Multi-section Coupled-Line Couplers (contd.)

- As per **theory of multiple reflection small reflections**, one can only consider the propagation paths where **one coupling** is involved—i.e., the signal propagates **across** a coupled-line pair only **once**!
- In our example, there are **three** propagation paths, corresponding to the coupling across each of the **three** separate coupled line pairs:



Here the propagation paths are: A PQR PLMNR

Multi-section Coupled-Line Couplers (contd.)

$$b_3 = \left(jc_1 \sin(\theta) e^{-j\theta} + e^{-j\theta} jc_2 \sin(\theta) e^{-j\theta} e^{-j\theta} + e^{-j2\theta} jc_3 \sin(\theta) e^{-j\theta} e^{-j2\theta} \right) a_1$$

$$b_3 = \left(jc_1 \sin(\theta) e^{-j\theta} + jc_2 \sin(\theta) e^{-j3\theta} + jc_3 \sin(\theta) e^{-j5\theta} \right) a_1$$

Therefore, according to this **approximation**:

$$S_{31}(\theta) = \frac{V_3^-}{V_1^+}(\theta) = \frac{b_3}{a_1}(\theta) = jc_1 \sin(\theta) e^{-j\theta} + jc_2 \sin(\theta) e^{-j3\theta} + jc_3 \sin(\theta) e^{-j5\theta}$$

Furthermore, for a **multi**-section coupler with N sections, we can write:

$$S_{31}(\theta) = \frac{V_3^-}{V_1^+}(\theta) = \frac{b_3}{a_1}(\theta) = jc_1 \sin(\theta) e^{-j\theta} + jc_2 \sin(\theta) e^{-j3\theta} + jc_3 \sin(\theta) e^{-j5\theta} + \ldots.$$

$$\ldots.. + jc_N \sin(\theta) e^{-j(2N-1)\theta}$$

Multi-section Coupled-Line Couplers (contd.)

- And for **symmetric** couplers with an **odd** value N , we find:

$$S_{31}(\theta) = j2\sin(\theta)e^{-jN\theta}\left[c_1\cos(N-1)\theta + c_2\cos(N-3)\theta + c_3\cos(N-5)\theta + \ldots + \frac{1}{2}c_M\right]$$

where M=(N+1) /2. Note M is an **even integer**, as N is an **odd** number

- Thus, we find the coupling **magnitude** as a function of frequency:

$$|c(\theta)| = |S_{31}(\theta)| = c_1 2\sin(\theta)\cos(N-1)\theta + c_2 2\sin(\theta)\cos(N-3)\theta + c_3 2\sin(\theta)\cos(N-5)\theta + \ldots + c_M 2\sin(\theta)$$

- Therefore, the **coupling in dB** is: $C(\theta) = 10\log_{10}|c(\theta)|^2$

Multi-section Coupled-Line Couplers (contd.)

- Now, our design goals are to **select** the coupling values c_1, c_2, c_N such that:
  1. The coupling value $C(\theta)$ is a specific, **desired** value at our design frequency.
  2. The coupling **bandwidth** is as **large** as possible.
- For the first condition, recall that the at the **design frequency**:

$$\theta = \beta l = \pi / 2$$

i.e., the section lengths are a **quarter-wavelength** at our design frequency

- Thus, we find our **first** design equation:

$$\left|c(\theta)\right|_{\theta=\pi/2} = \left|S_{31}(\theta)\right| = c_1 2\cos\{(N-1)\pi/2\} + c_2 2\cos\{(N-3)\pi/2\} + c_3 2\cos\{(N-5)\pi/2\} + ... + c_M$$

where we have used the fact that **sin(π/2) = 1**.

Multi-section Coupled-Line Couplers (contd.)

- Note the value $|c(\theta)|_{\theta=\pi/2}$ is set to the value necessary to achieve the **desired** coupling value. This equation thus provides **one** design constraint—we have **M-1** degrees of design freedom left to accomplish our **second** goal!
- To **maximize bandwidth**, we typically impose the **maximally flat** condition:

$$\left(\frac{d^m |c(\theta)|}{d\theta^m} \right)_{\theta=\pi/2} = 0$$

m = 1, 2, M-1

Be careful! Remember to perform the derivative **first**, and **then** evaluate the result at θ = π/2.