Lecture – 19 Date: 28.10.2014

- The Quadrature Hybrid

We began our discussion of dividers and couplers by considering important general properties of three- and four-port networks . This was followed by an analysis of three types of power dividers.

Let us now move on to (reciprocal) directional couplers, which are four-port networks. We will consider these specific types of couplers:

1. Quadrature Hybrid
2. 180^{o} Hybrid
3. Coupled Line

The Quadrature Hybrid

- There are **two** different types of ideal **4-port 3dB** couplers:
  - the **symmetric** solution
  - the **anti-symmetric** solution
- The **symmetric solution** is called the **Quadrature Hybrid**
- It is also called **90^0 Hybrid Coupler**, otherwise known as the **branch-line** coupler. Its scattering matrix (ideally) has the **symmetric** solution for a matched, lossless, reciprocal 4-port device:

$$\boldsymbol{S} = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

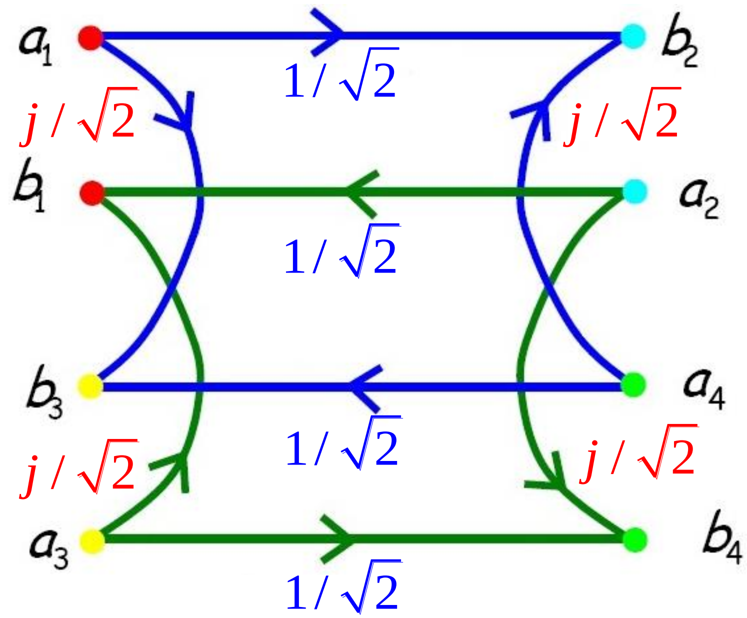
- However, for **this** coupler :

$$\alpha = \frac{-j}{\sqrt{2}} \qquad j\beta = \frac{-1}{\sqrt{2}}$$

The Quadrature Hybrid (contd.)

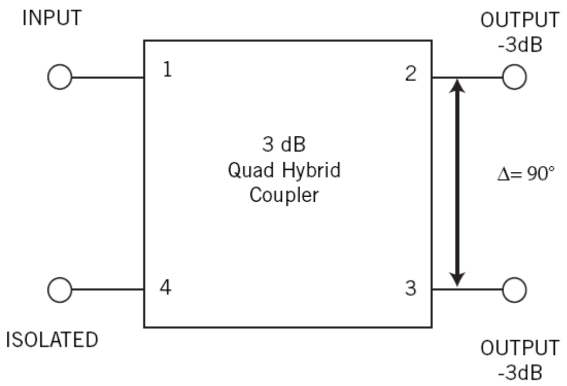
- Therefore, the **scattering matrix** and the corresponding **SFG** of a quadrature coupler is:

$$
\boldsymbol{S} = \begin{bmatrix} 0 & -j/\sqrt{2} & -1/\sqrt{2} & 0 \\ -j/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\ -1/\sqrt{2} & 0 & 0 & -j/\sqrt{2} \\ 0 & -1/\sqrt{2} & -j/\sqrt{2} & 0 \end{bmatrix}
$$



- It is evident that, just as with the directional coupler, the ports are **matched** and the device is **lossless**. Note also, that if a signal is incident on one port only, then there will be a port from which **no** power will exit (i.e., an **isolation** port).
- **Unlike** the directional coupler, the power that flows into the input port will be **evenly** divided between the two non-isolated ports.

The Quadrature Hybrid (contd.)



$$S = \begin{bmatrix} 0 & -j/\sqrt{2} & -1/\sqrt{2} & 0 \\ -j/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\ -1/\sqrt{2} & 0 & 0 & -j/\sqrt{2} \\ 0 & -1/\sqrt{2} & -j/\sqrt{2} & 0 \end{bmatrix}$$

No power exits port-4.

- -90° phase shift from **port-1** to **port-2**
- **one half** of the time average power input power on port-1 is delivered to port-2

- -180° phase shift from **port-1** to **port-3**
- **one half** of the time average power input power on port-1 is delivered to port-3

The Quadrature Hybrid (contd.)

- For example, if **10 mW is incident on port 1** (and all other ports are matched), then 5 mW will flow out of **both port 2** and **port 3**, while no power will exit port 4 (the isolated port).
- Note however, that although the **magnitudes** of the signals leaving ports 2 and 3 are **equal**, the relative **phase** of the two signals are separated by **90 degrees.**
- We find, therefore, that if in **real** terms the voltage out of port 2 is:

$$v_2(z,t) = \frac{|V_1^+|}{\sqrt{2}}\cos(\omega_0 t + \beta z)$$

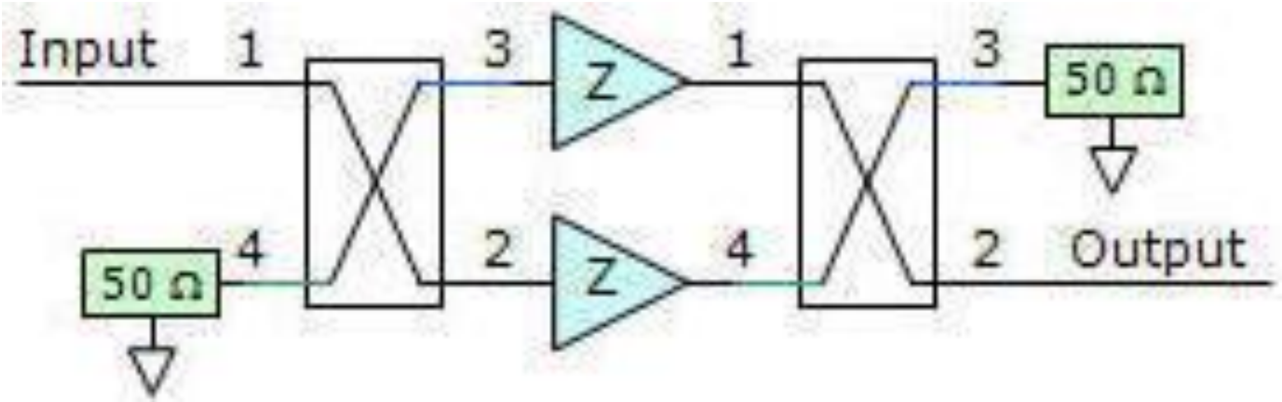
- then the output from port 3 will be:

$$v_3(z,t) = \frac{|V_1^+|}{\sqrt{2}}\sin(\omega_0 t + \beta z)$$

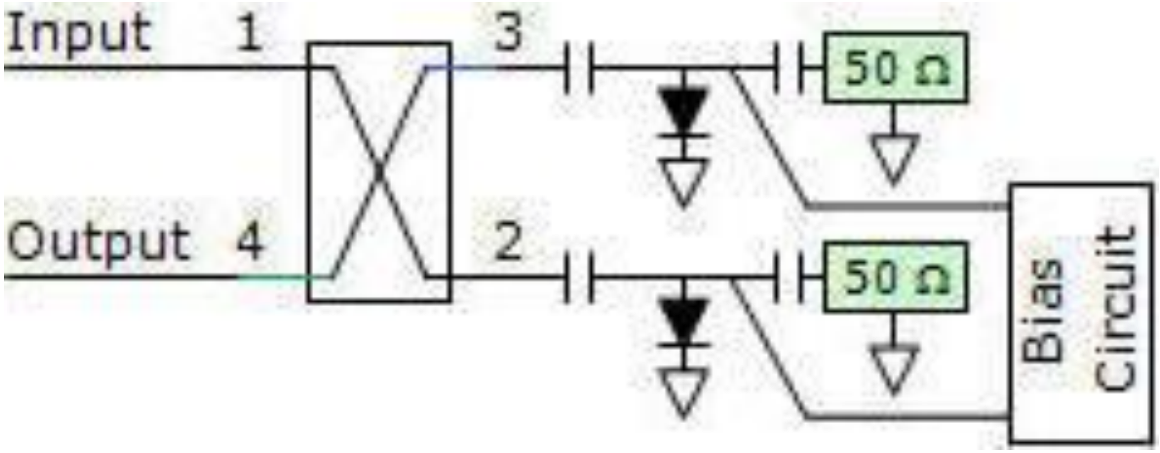
There are **many** useful applications where we require both the **sine** and **cosine** of a signal!

The Quadrature Hybrid (contd.)

Application – 1: high power balanced Amplifier



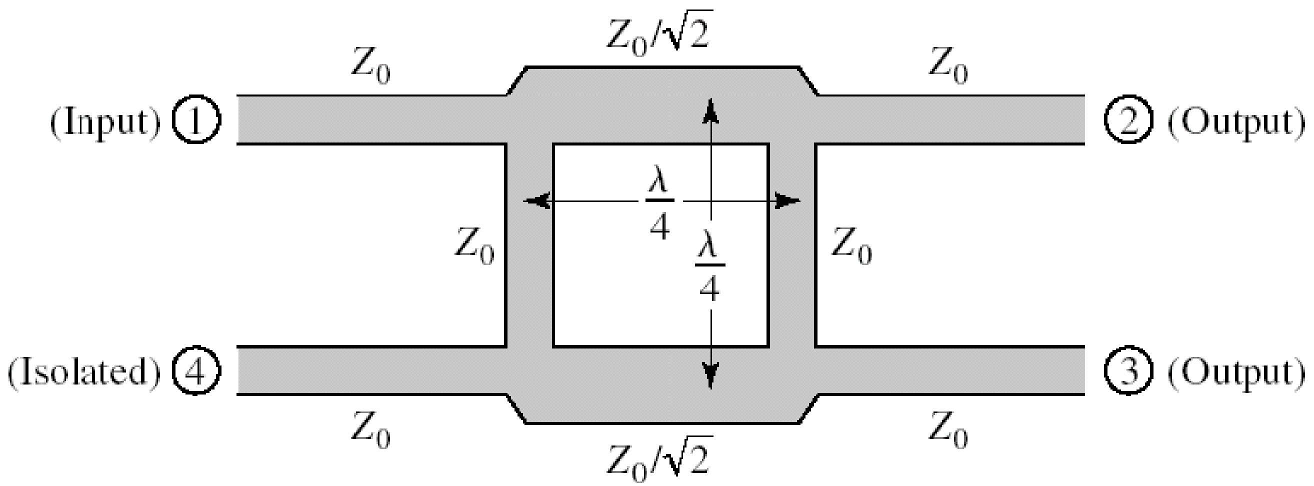
Application – 2: stepped attenuator



The Quadrature Hybrid (contd.)

Q: But how do we **construct** this device?

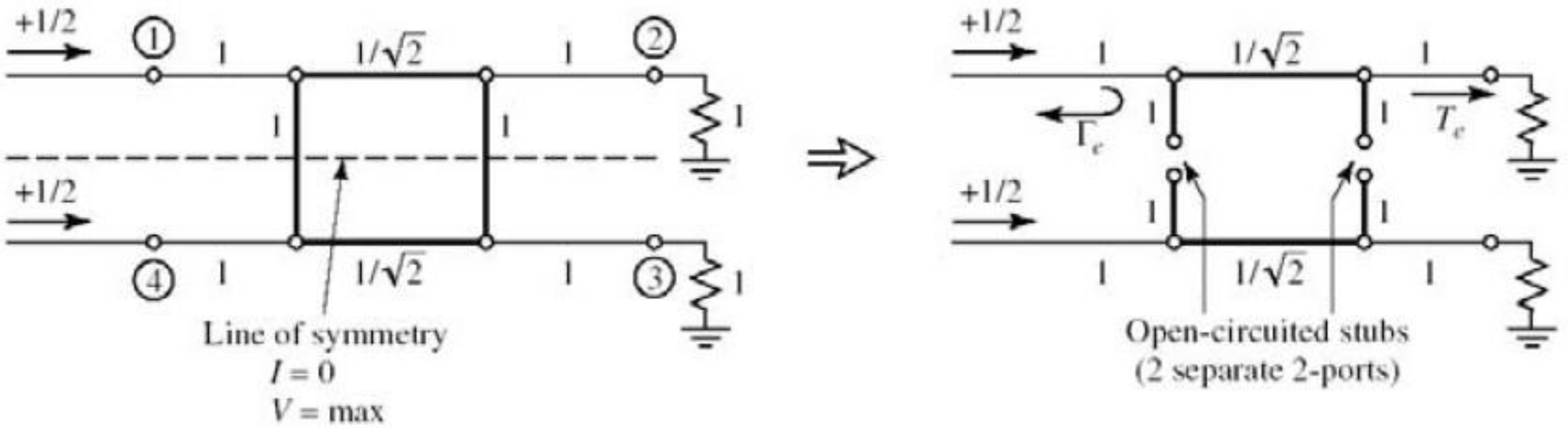
A: Similar to the Wilkinson power divider, we construct a quadrature hybrid with **quarter-wavelength** sections of transmission lines.



Q: Wow! How can we **analyze** such a complex circuit?

The Quadrature Hybrid (contd.)

A: Note that this circuit is **symmetric**—we can use **odd/even mode** analysis!



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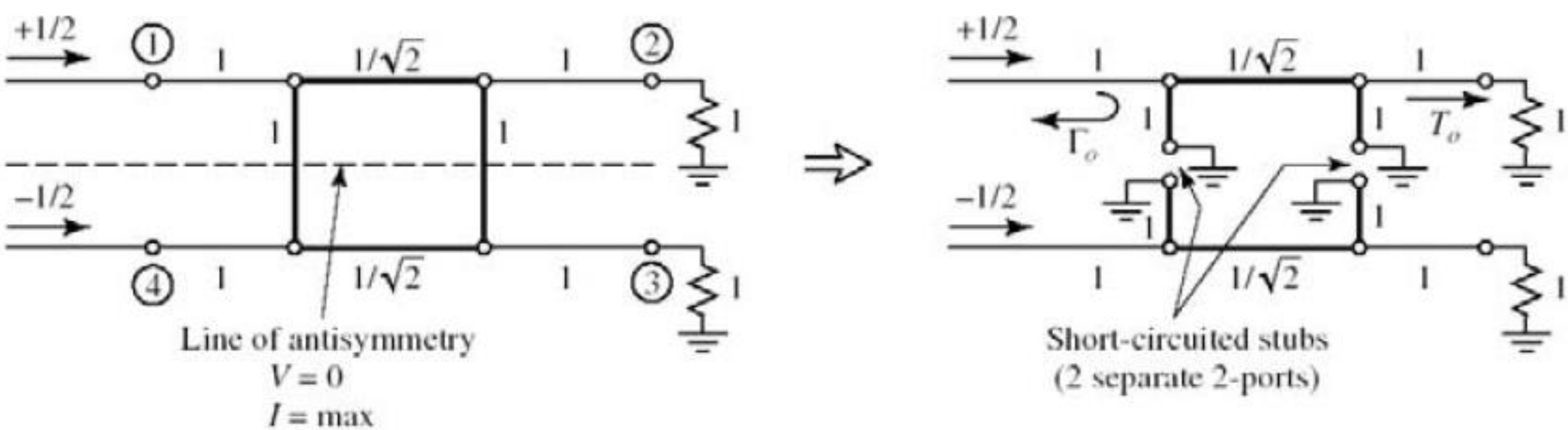

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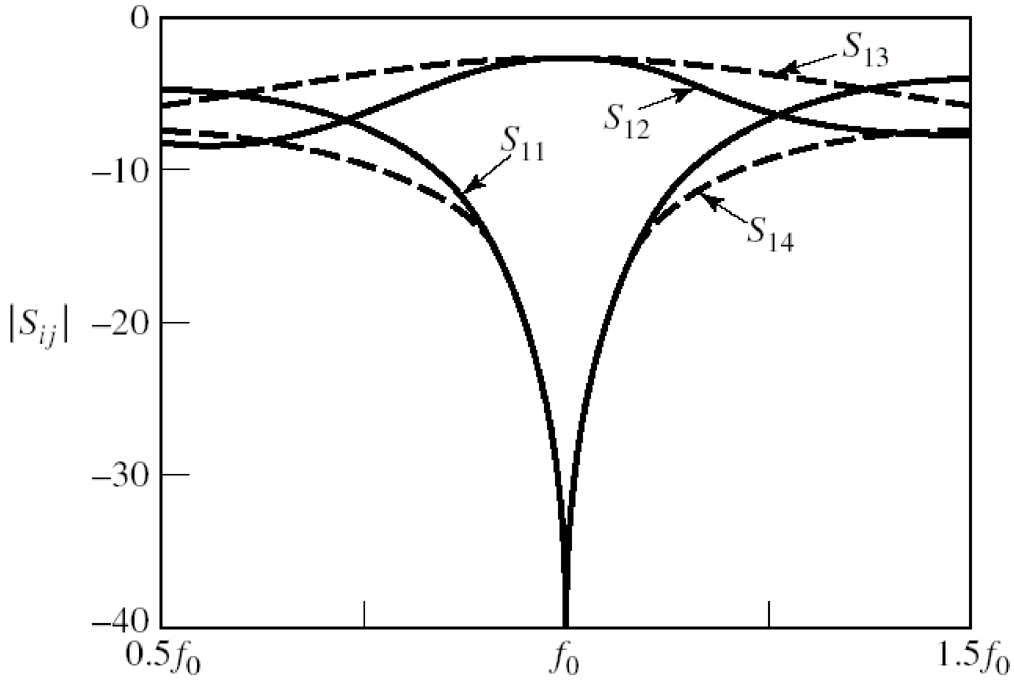
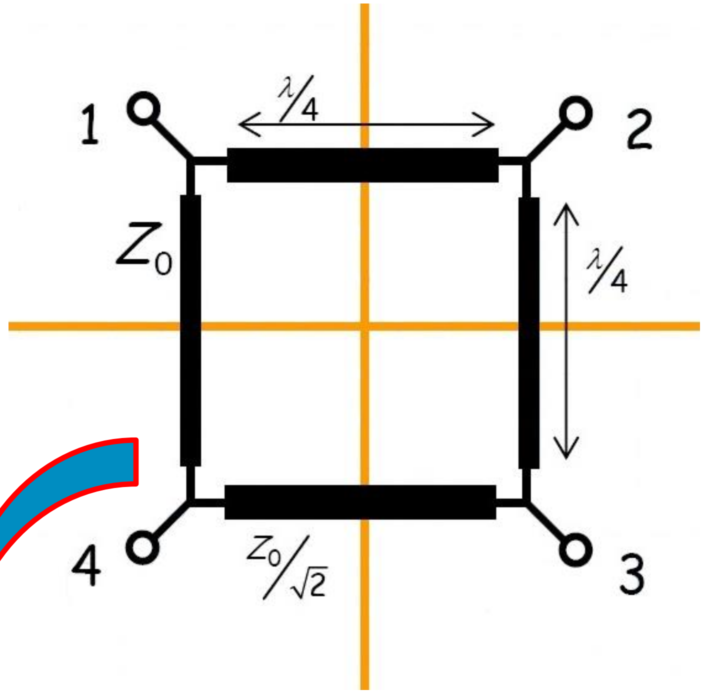
The Quadrature Hybrid (contd.)

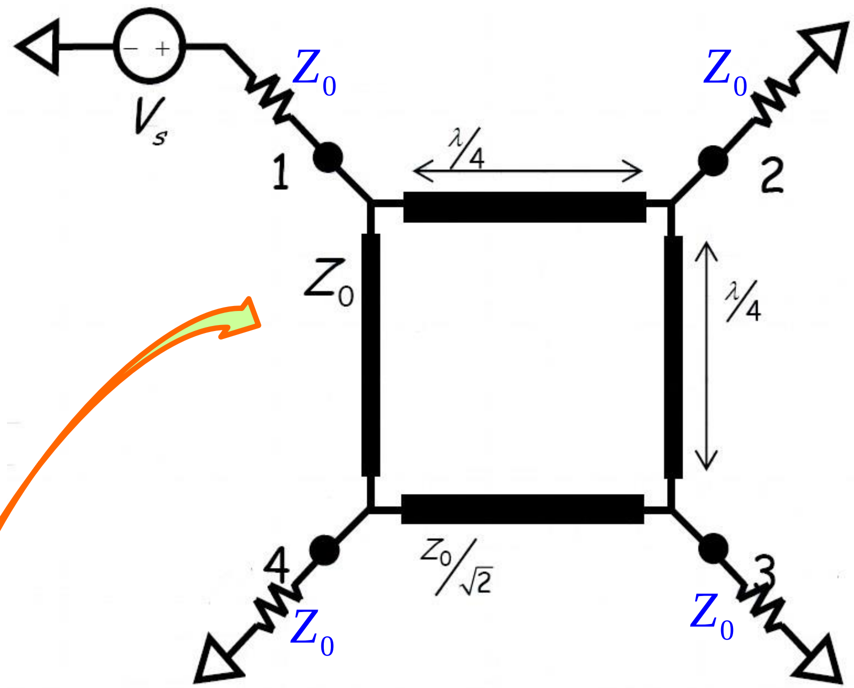
- Please go through Pozar for the detailed odd-mode/even-mode analysis of quadrature hybrid.
- Note that the λ/4 structures make the quadrature hybrid an inherently **narrow-band** device.



Quad-mode Analysis

- The quadrature hybrid is a matched, lossless, reciprocal four-port network that possesses two planes of bilateral symmetry

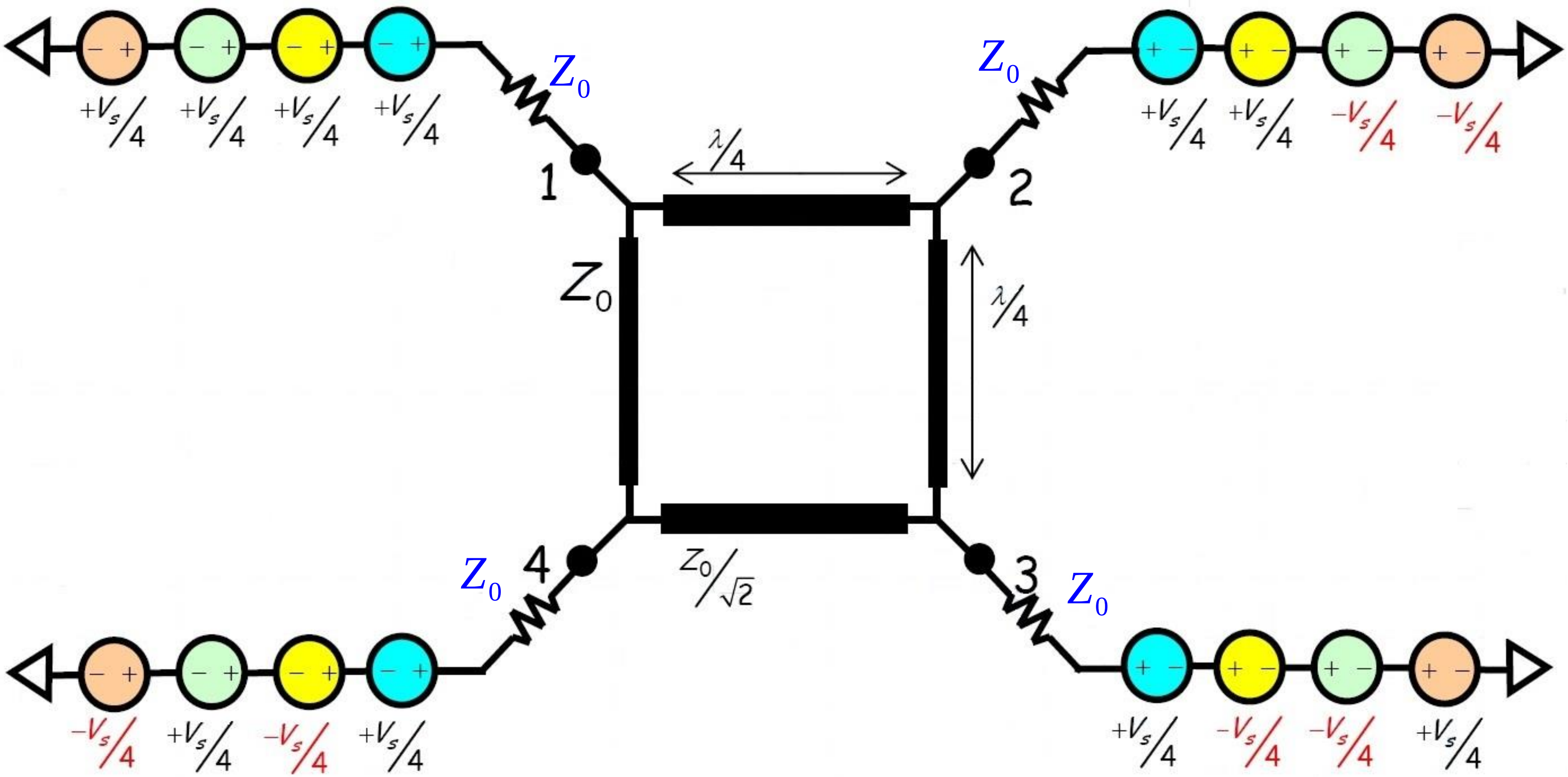




To determine the scattering parameters S_{11}, S_{21}, S_{31}, S_{41} of this network, a matched source is placed on port 1, while matched loads terminate the other 3 ports.

Quad-mode Analysis (contd.)

- The placement of source at port 1 destroys both planes of bilateral symmetry in the circuit. We can however recast this circuit with a precisely equivalent circuit:

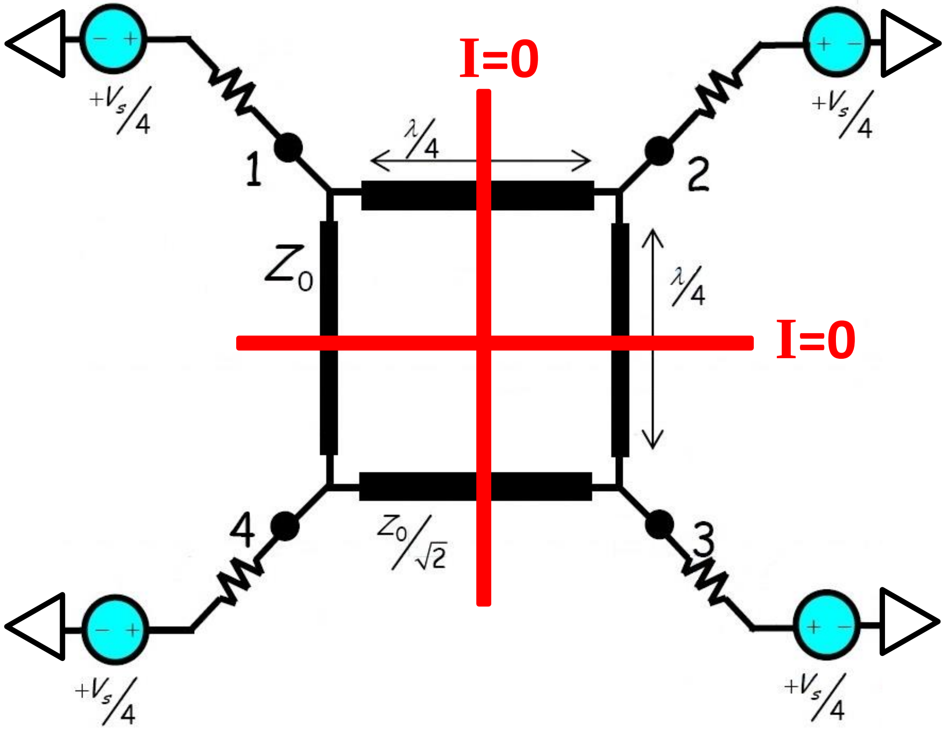


Quad-mode Analysis (contd.)

- Note that the four series voltage sources on **port 1** add to the **original value of V_s,** while the series source at the **other three ports** add to a value of **zero**—thus providing short circuit from the passive load Z_0 to ground.
- This circuit can now be analyzed by applying superposition:
  - Sequentially turn off all but one source at each of the 4 ports.
  - it provides us with **four "modes".**
  - Each of these four modes can be analyzed individually.
  - The final circuit response is simply a coherent summation of the results of each of the four modes!
  - The benefit of this procedure is that each of the four modes preserve circuit symmetry. As a result, the planes of bilateral symmetry become virtual shorts and/or virtual opens.

Quad-mode Analysis (contd.)

Mode A



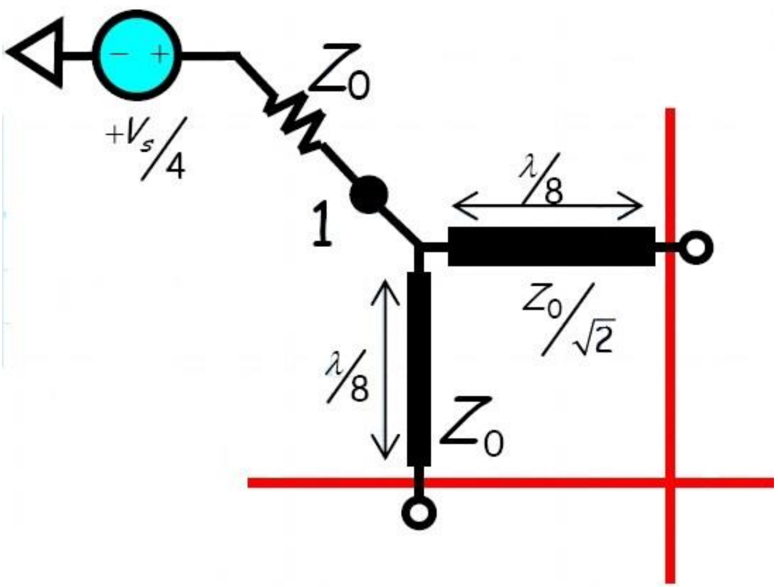
The even symmetry of this circuit is now restored, so the voltages at each port are identical:

$$V_{1a}^{+} = V_{2a}^{+} = V_{3a}^{+} = V_{4a}^{+} = \frac{V_s}{8}$$

$$V_{1a}^{-} = V_{2a}^{-} = V_{3a}^{-} = V_{4a}^{-} = ?$$

Quad-mode Analysis (contd.)

- The two virtual **"open condition"** segments this circuit into 4 identical sections. To determine the amplitude V_{1a}^-, we need to only analyze **one** of these sections:



- The circuit has simplified to a 1-port device consisting of the parallel combination of two λ/8 open-circuited stubs. The admittance of a λ/8 open-circuit stub is:

$$Y_{stub}^{OC} = jY_0 \cot(\beta l) = jY_0 \cot(\lambda/8) = jY_0$$

- As a result, the input admittance of this circuit segment is:

$$Y_{in}^{a} = j\sqrt{2}Y_0 + jY_0 = jY_0\left(\sqrt{2}+1\right)$$

Quad-mode Analysis (contd.)

- The corresponding reflection coefficient is:

$$\Gamma_a = \frac{Y_0 - Y_{in}^a}{Y_0 + Y_{in}^a} = \frac{Y_0 - jY_0\left(\sqrt{2}+1\right)}{Y_0 + jY_0\left(\sqrt{2}+1\right)} = \frac{1 - j\left(\sqrt{2}+1\right)}{1 + j\left(\sqrt{2}+1\right)}$$

$$\Gamma_a = \frac{-1-j}{\sqrt{2}} = 1 * e^{-j(3\pi/4)}$$

- Therefore the reflected wave at port 1 is:

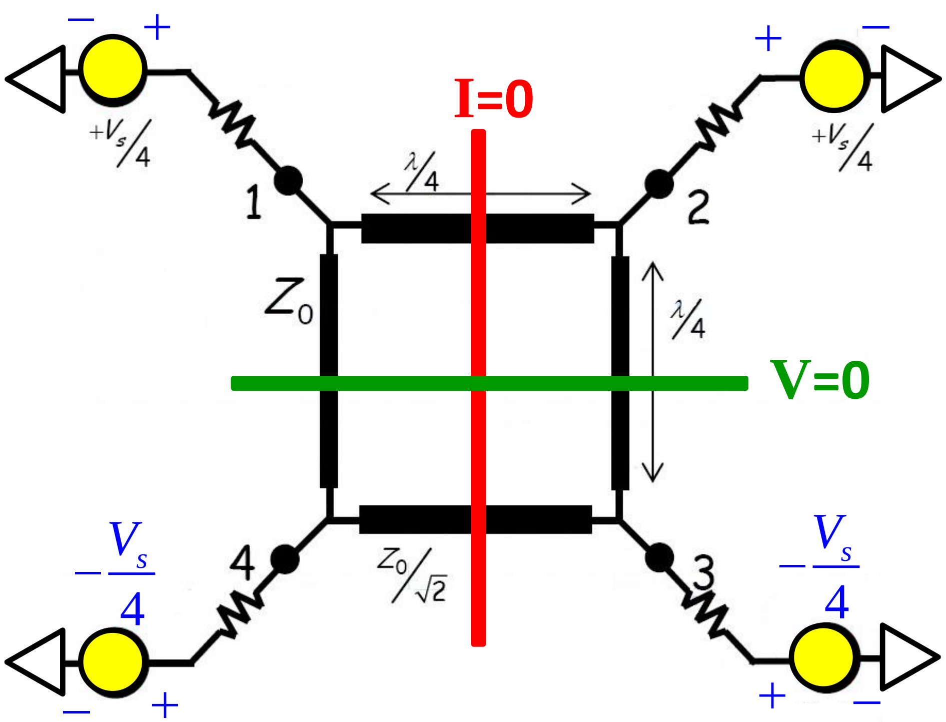
$$V_{1a}^- = V_{1a}^+ \Gamma_a = \frac{V_s}{8} e^{-j(3\pi/4)}$$

- And so from the even symmetry of **mode A** we conclude:

$$V_{1a}^- = V_{2a}^- = V_{3a}^- = V_{4a}^- = \frac{V_s}{8} e^{-j(3\pi/4)}$$

Quad-mode Analysis (contd.)

Mode B



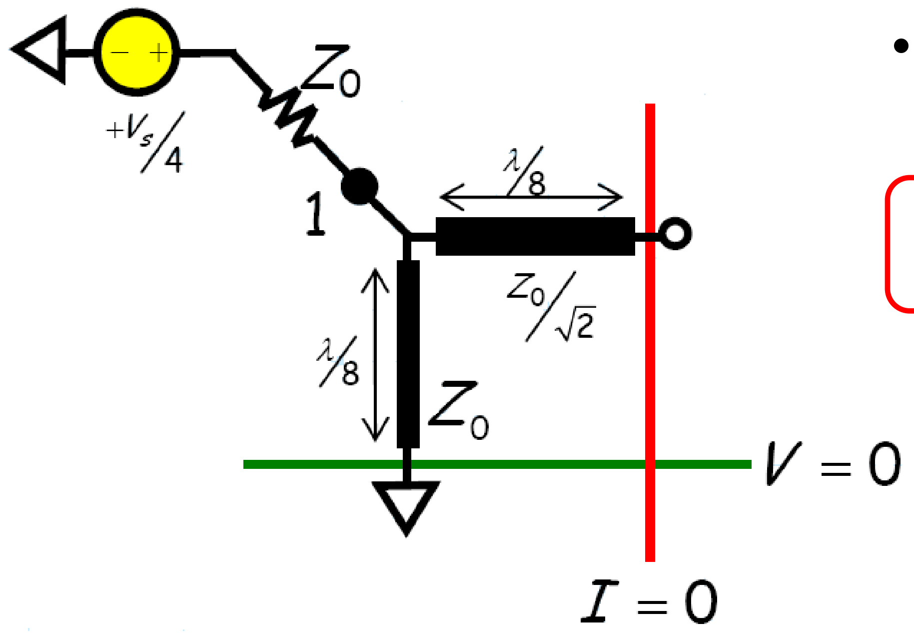
- For mode B, the even symmetry exists about the vertical circuit plane, while odd symmetry occurs across the horizontal plane.

$$V_{1b}^{+} = V_{2b}^{+} = -V_{3b}^{+} = -V_{4b}^{+} = \frac{V_s}{8}$$

$$V_{1b}^{-} = V_{2b}^{-} = -V_{3b}^{-} = -V_{4b}^{-} = ?$$

Quad-mode Analysis (contd.)

- The circuit can again be segmented into four sections, with each section consisting of a shorted λ/8 stub and an open-circuited λ/8 stub in parallel.



- The admittance of a λ/8 short-circuit stub is:

$$Y_{stub}^{SC} = -jY_0 \tan(\beta l) = -jY_0 \tan(\lambda/8) = -jY_0$$

- As a result, the input admittance of this circuit segment is:

$$Y_{in}^{b} = j\sqrt{2}Y_0 - jY_0 = jY_0\left(\sqrt{2}-1\right)$$

Quad-mode Analysis (contd.)

- The corresponding reflection coefficient is:

$$\Gamma_b = \frac{Y_0 - Y_{in}^b}{Y_0 + Y_{in}^b} = \frac{Y_0 - jY_0\left(\sqrt{2}-1\right)}{Y_0 + jY_0\left(\sqrt{2}-1\right)} = \frac{1 - j\left(\sqrt{2}+1\right)}{1 + j\left(\sqrt{2}+1\right)}$$

$$\Gamma_b = \frac{1-j}{\sqrt{2}} = 1 * e^{-j(\pi/4)}$$

- Therefore the reflected wave at port 1 is:

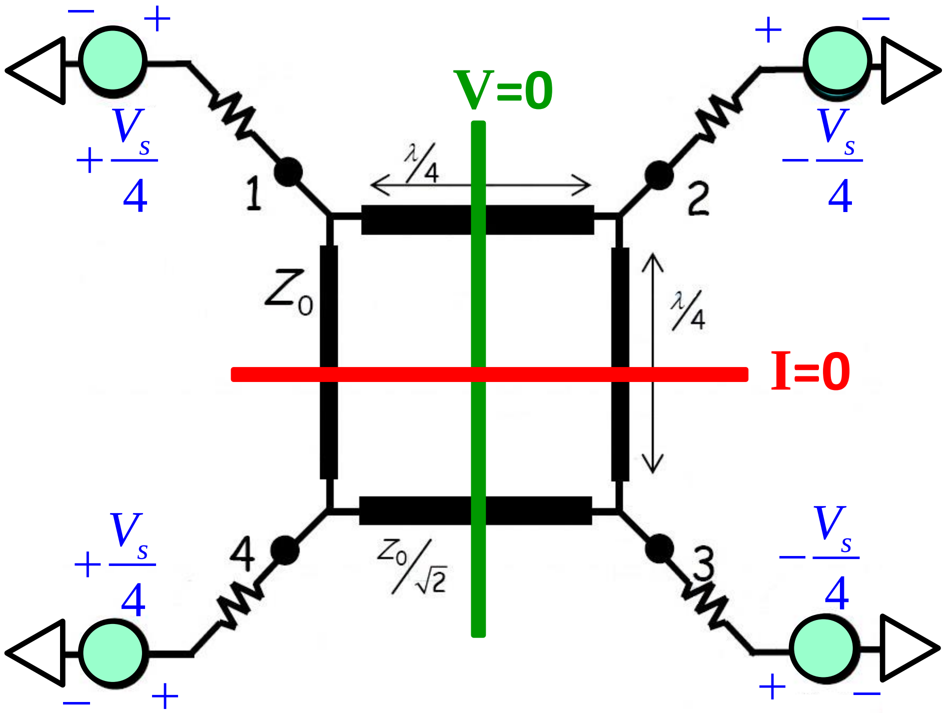
$$V_{1b}^- = V_{1b}^+ \Gamma_b = \frac{V_s}{8} e^{-j(\pi/4)}$$

- And so from the odd and even symmetry of mode B we conclude:

$$V_{1b}^- = V_{2b}^- = -V_{3b}^- = -V_{4b}^- = \frac{V_s}{8} e^{-j(\pi/4)}$$

Quad-mode Analysis (contd.)

Mode C



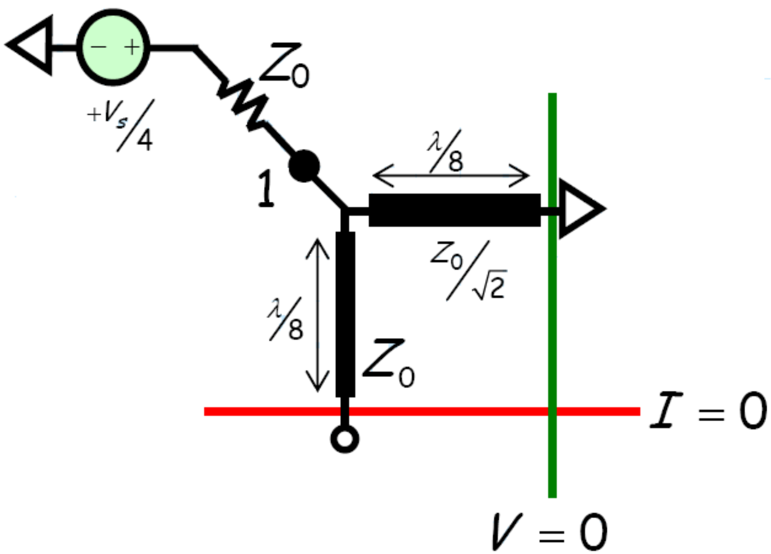
- For mode C, odd symmetry exists about the vertical circuit plane, while even symmetry occurs across the horizontal plane.

$$V_{1c}^{+} = -V_{2c}^{+} = -V_{3c}^{+} = V_{4c}^{+} = \frac{V_s}{8}$$

$$V_{1c}^{-} = -V_{2c}^{-} = -V_{3c}^{-} = V_{4c}^{-} = ?$$

Quad-mode Analysis (contd.)

- The circuit can again be segmented into four sections, with each section consisting of a shorted λ/8 stub and an open-circuited λ/8 stub in parallel



- As a result, the input admittance of this circuit segment is:

$$Y_{in}^{c} = -j\sqrt{2}Y_0 + jY_0 = jY_0\left(1-\sqrt{2}\right)$$

- The corresponding reflection coefficient is:

$$\Gamma_c = \frac{Y_0 - Y_{in}^{c}}{Y_0 + Y_{in}^{c}} = \frac{Y_0 - jY_0\left(1-\sqrt{2}\right)}{Y_0 + jY_0\left(1-\sqrt{2}\right)} = \frac{1-j\left(1-\sqrt{2}\right)}{1+j\left(1-\sqrt{2}\right)}$$

$$\Gamma_c = \frac{1+j}{\sqrt{2}} = 1 * e^{+j(\pi/4)}$$

Quad-mode Analysis (contd.)

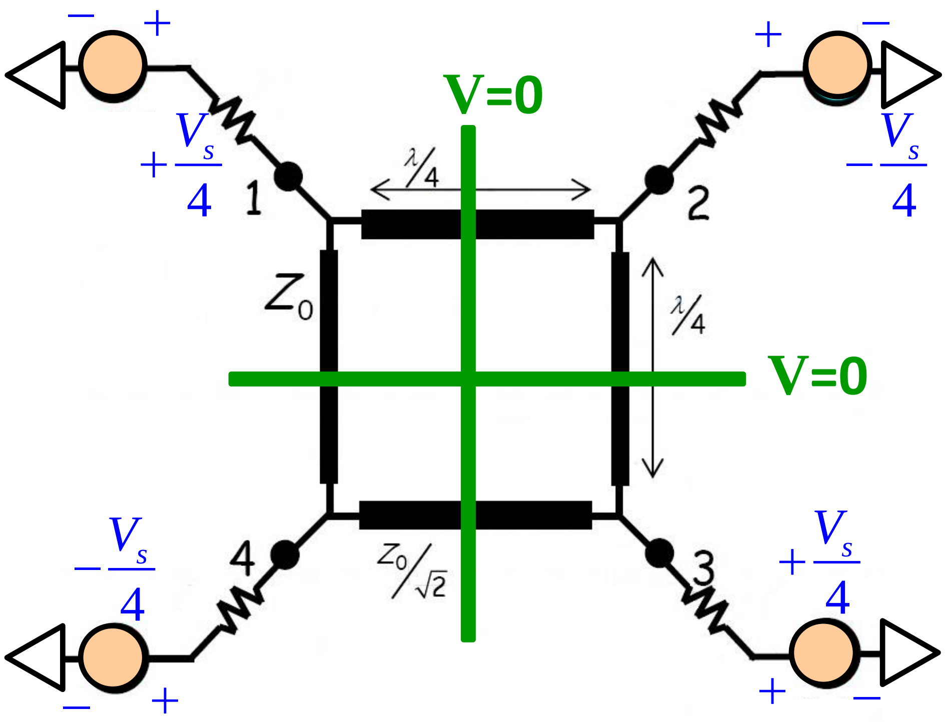
- Therefore the reflected wave at port 1 is:

$$V_{1c}^{-} = V_{1c}^{+}\Gamma_{c} = \frac{V_s}{8}e^{+j(\pi/4)}$$

- And so from the odd and even symmetry of mode C we conclude:

$$V_{1c}^{-} = -V_{2c}^{-} = -V_{3c}^{-} = V_{4c}^{-} = \frac{V_s}{8}e^{+j(\pi/4)}$$

Mode D



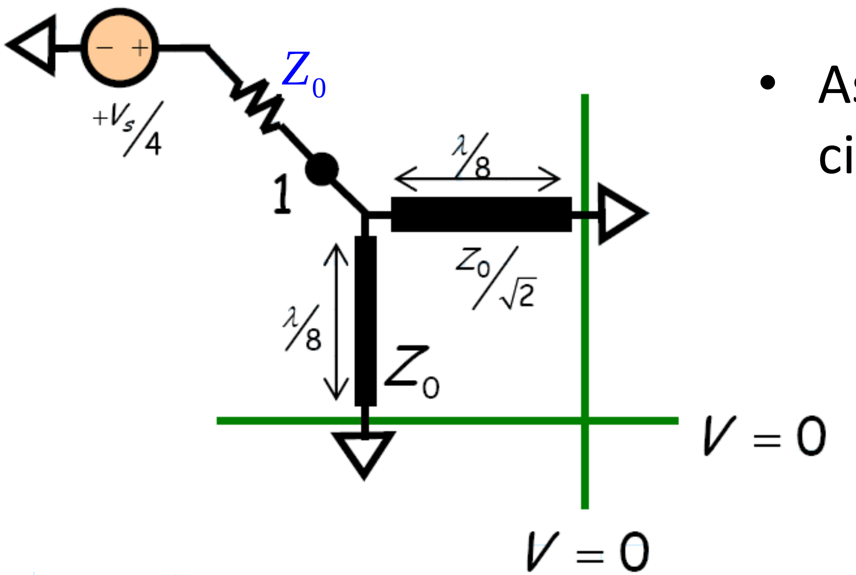
- For mode D, odd symmetry exists about both vertical and horizontal planes

$$V_{1d}^{+} = -V_{2d}^{+} = V_{3d}^{+} = -V_{4d}^{+} = \frac{V_s}{8}$$

$$V_{1d}^{-} = -V_{2d}^{-} = V_{3d}^{-} = -V_{4d}^{-} = ?$$

Quad-mode Analysis (contd.)

- The circuit can again be segmented into four sections, with each section consisting two short-circuited λ/8 stubs in parallel.



- As a result, the input admittance of this circuit segment is:

$$Y_{in}^{d} = -j\sqrt{2}Y_0 - jY_0 = -jY_0\left(1+\sqrt{2}\right)$$

- The corresponding reflection coefficient is:

$$\Gamma_d = \frac{Y_0 - Y_{in}^{d}}{Y_0 + Y_{in}^{d}} = \frac{Y_0 + jY_0\left(1+\sqrt{2}\right)}{Y_0 - jY_0\left(1+\sqrt{2}\right)} = \frac{1+j\left(1+\sqrt{2}\right)}{1-j\left(1+\sqrt{2}\right)}$$

$$\Gamma_d = \frac{1+j}{\sqrt{2}} = 1 * e^{+j(3\pi/4)}$$

Quad-mode Analysis (contd.)

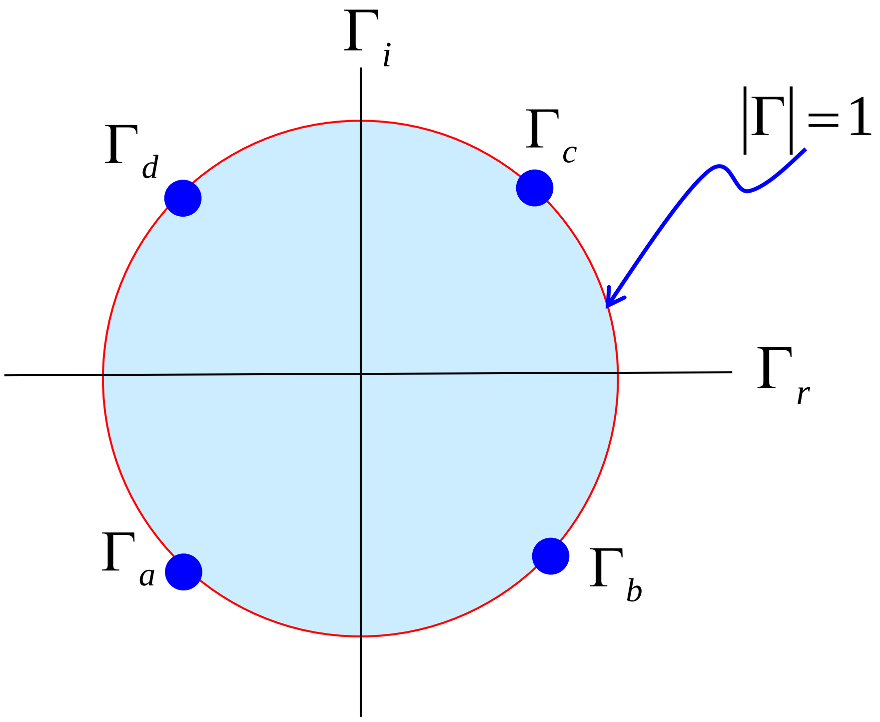
- Therefore the reflected wave at port 1 is:

$$V_{1d}^{-} = V_{1d}^{+}\Gamma_d = \frac{V_s}{8}e^{+j(3\pi/4)}$$

- And so from the even symmetry of mode D we find:

$$V_{1d}^{-} = -V_{2d}^{-} = V_{3d}^{-} = -V_{4d}^{-} = \frac{V_s}{8}e^{+j(3\pi/4)}$$

- Not surprisingly, the symmetry of the quadrature hybrid has resulted in four modal solutions that possess precisely the same symmetry when plotted on the complex Γ-plane.



Quad-mode Analysis (contd.)

- Since our circuit is linear, we can determine the solution to our original circuit as a superposition of our four modal solutions:

$$V_1^+ = V_{1a}^+ + V_{1b}^+ + V_{1c}^+ + V_{1d}^+ = \frac{V_s}{8} + \frac{V_s}{8} + \frac{V_s}{8} + \frac{V_s}{8} = \frac{V_s}{2}$$

$$V_1^- = V_{1a}^- + V_{1b}^- + V_{1c}^- + V_{1d}^- = \frac{V_s}{8}\left(e^{-j(3\pi/4)} + e^{-j(\pi/4)} + e^{+j(\pi/4)} + e^{+j(3\pi/4)}\right) = 0$$

- Similarly:

$$V_2^- = V_{2a}^- + V_{2b}^- + V_{2c}^- + V_{2d}^- = -j\frac{V_s}{2\sqrt{2}}$$

$$V_3^- = V_{3a}^- + V_{3b}^- + V_{3c}^- + V_{3d}^- = -\frac{V_s}{2\sqrt{2}}$$

$$V_4^- = V_{4a}^- + V_{4b}^- + V_{4c}^- + V_{4d}^- = 0$$

Quad-mode Analysis (contd.)

- From these results we can determine the scattering parameters S_{11}, S_{21}, S_{31}, S_{41}.

$$S_{11} = \frac{V_1^-}{V_1^+} = \left(\frac{2}{V_s}\right) * 0 = 0$$

$$S_{21} = \frac{V_2^-}{V_1^+} = \left(\frac{2}{V_s}\right) * \left(-j\frac{V_s}{2\sqrt{2}}\right) = \frac{-j}{\sqrt{2}}$$

$$S_{31} = \frac{V_3^-}{V_1^+} = \left(\frac{2}{V_s}\right) * \left(-\frac{V_s}{2\sqrt{2}}\right) = \frac{-1}{\sqrt{2}}$$

$$S_{41} = \frac{V_4^-}{V_1^+} = \left(\frac{2}{V_s}\right) * 0 = 0$$

- Given the symmetry of the device, we can extend these four results to determine the entire scattering matrix:

$$\boldsymbol{S} = \begin{bmatrix} 0 & -j/\sqrt{2} & -1/\sqrt{2} & 0 \\ -j/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\ -1/\sqrt{2} & 0 & 0 & -j/\sqrt{2} \\ 0 & -1/\sqrt{2} & -j/\sqrt{2} & 0 \end{bmatrix}$$

The 180°- hybrid Coupler

- Recall there are **two** different types of ideal **4-port** 3dB couplers: the **symmetric** solution and the **anti-symmetric** solution.
  - We now know that the symmetric solution is the **Quadrature Hybrid.**
  - The anti-symmetric solution is called the **180 Degree Hybrid** (aka, ring hybrid, rat-race hybrid, Magic-T).

- Therefore the 180° **Hybrid Coupler** (sometimes known as the "ring", "rat-race", or "Magic-T" hybrid) is a lossless, matched and reciprocal 4-port device, with a scattering matrix of the **anti-symmetric** form.

$$\boldsymbol{S} = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix} \quad \xrightarrow{\alpha = \beta = 1/\sqrt{2}} \quad \boldsymbol{S} = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

The 180^o- hybrid Coupler (contd.)

- This coupler is also a **3dB coupler**—the power into a given port (with all other ports matched) is equally divided between two of the three output ports.
- The relative **phase** between the outputs, however, is **dependent** on which port is the input.
- For example, if the **input** is port 1 or port 3, the two signals will be **in phase**—no difference in their relative phase!
- However, if the input is port 2 or port 4, the output signals will be 180^o **out of phase** ($e^{j\pi} = -1$)!
- An interesting application of this coupler can be seen if we place **two input signals** into the device, at ports 2 and 3 (with ports 1 and 4 connected to matched impedance).
- Note the signal out of port 1 would therefore be:

$$V_1^-(z) = S_{12}V_2^+(z) + S_{13}V_3^+(z) = \frac{1}{\sqrt{2}}\left(V_3^+(z) + V_2^+(z)\right)$$

- While the signal out of port 4 is:

$$V_4^-(z) = S_{42}V_2^+(z) + S_{43}V_3^+(z) = \frac{1}{\sqrt{2}}\left(V_3^+(z) - V_2^+(z)\right)$$

The 180^o- hybrid Coupler (contd.)

$$V_1^-(z) = S_{12}V_2^+(z) + S_{13}V_3^+(z) = \frac{1}{\sqrt{2}}\left(V_3^+(z) + V_2^+(z)\right)$$

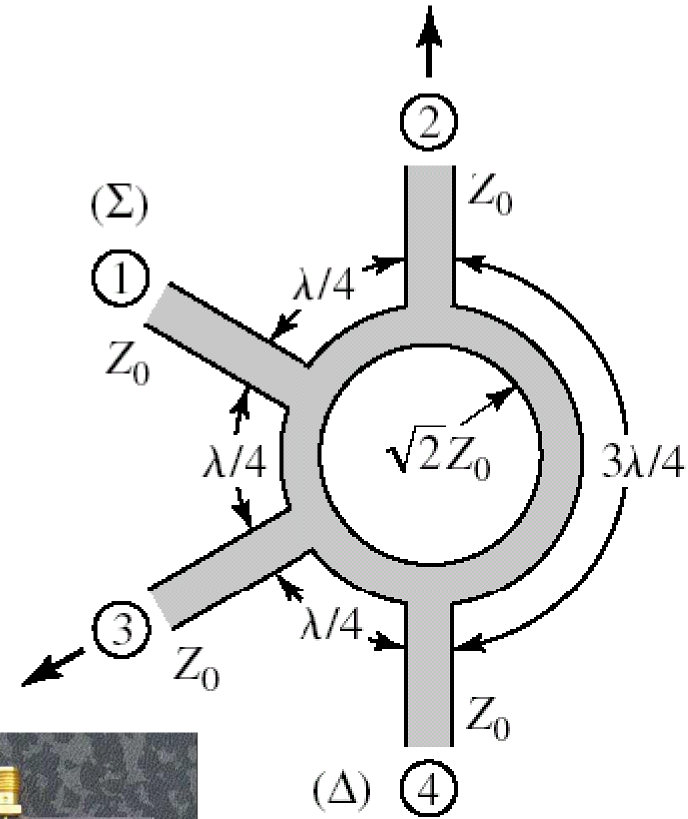
$$V_4^-(z) = S_{42}V_2^+(z) + S_{43}V_3^+(z) = \frac{1}{\sqrt{2}}\left(V_3^+(z) - V_2^+(z)\right)$$

- Note that the output of port 1 is proportional to the **sum** of the two inputs. Port 1 of a 180^o Hybrid Coupler is therefore often referred to as the **sum** (Σ) port.
- Likewise, port 4 is proportional to the **difference** between the two inputs. Port 4 of a 180^o Hybrid Coupler is therefore often referred to as the **delta** (Δ) port.
- There are **many** applications where we wish to take the sum and/or difference between two signals!
- The 180^o Hybrid Coupler can likewise be used in the **opposite** manner. If we have **both** the sum and difference of two signals available, we can use this device to separate the signals into their separate components!

The 180°- hybrid Coupler (contd.)

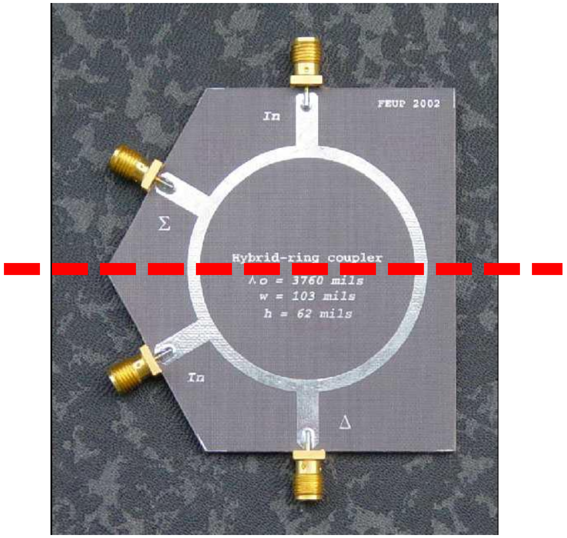
Q: How is this hybrid coupler constructed?

A: Like the quadrature hybrid, it is simply made of **lengths** of transmission lines. However, unlike the quadrature hybrid, the characteristic impedance of each line is **identical** $\sqrt{2Z_0}$, but the lengths of the lines are dissimilar.



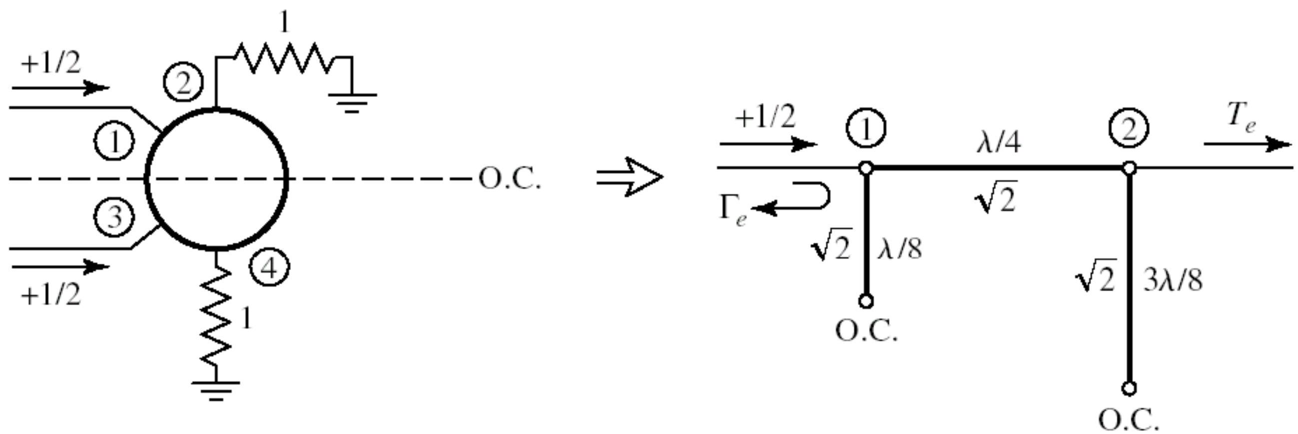
Q: How can we possibly analyze this mess?

A: Note there **is** one plane of bilateral **symmetry** in this circuit—we can use even/odd mode analysis!

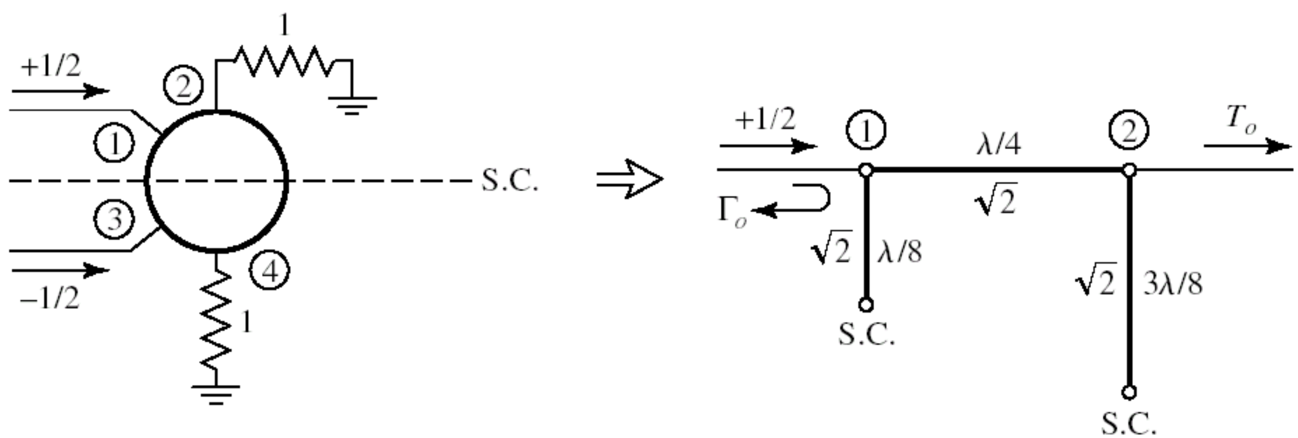


The 180^o- hybrid Coupler (contd.)

- However, we must perform **two** separate analysis—one using sources on ports **1 and 3**:



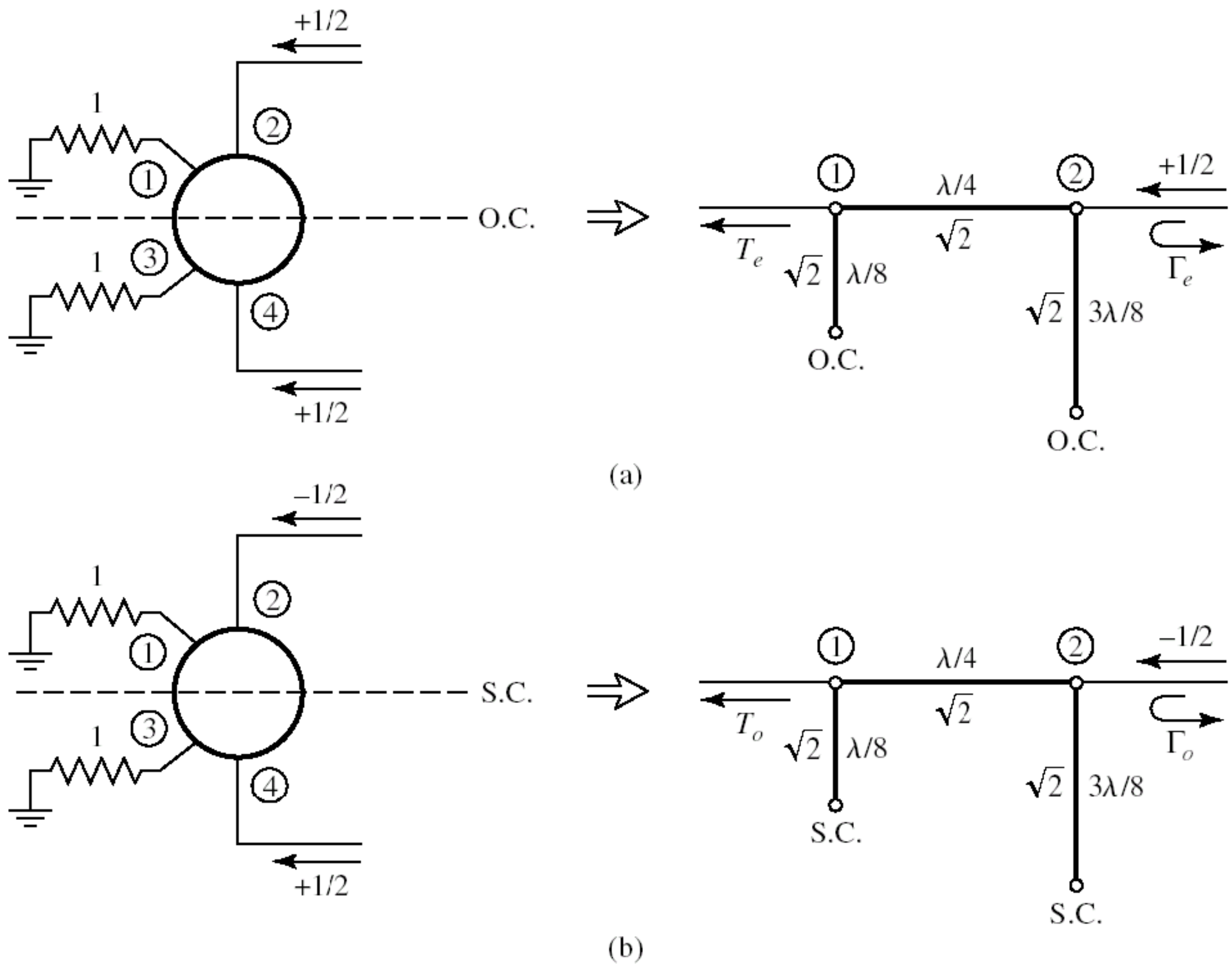
(a)



(b)

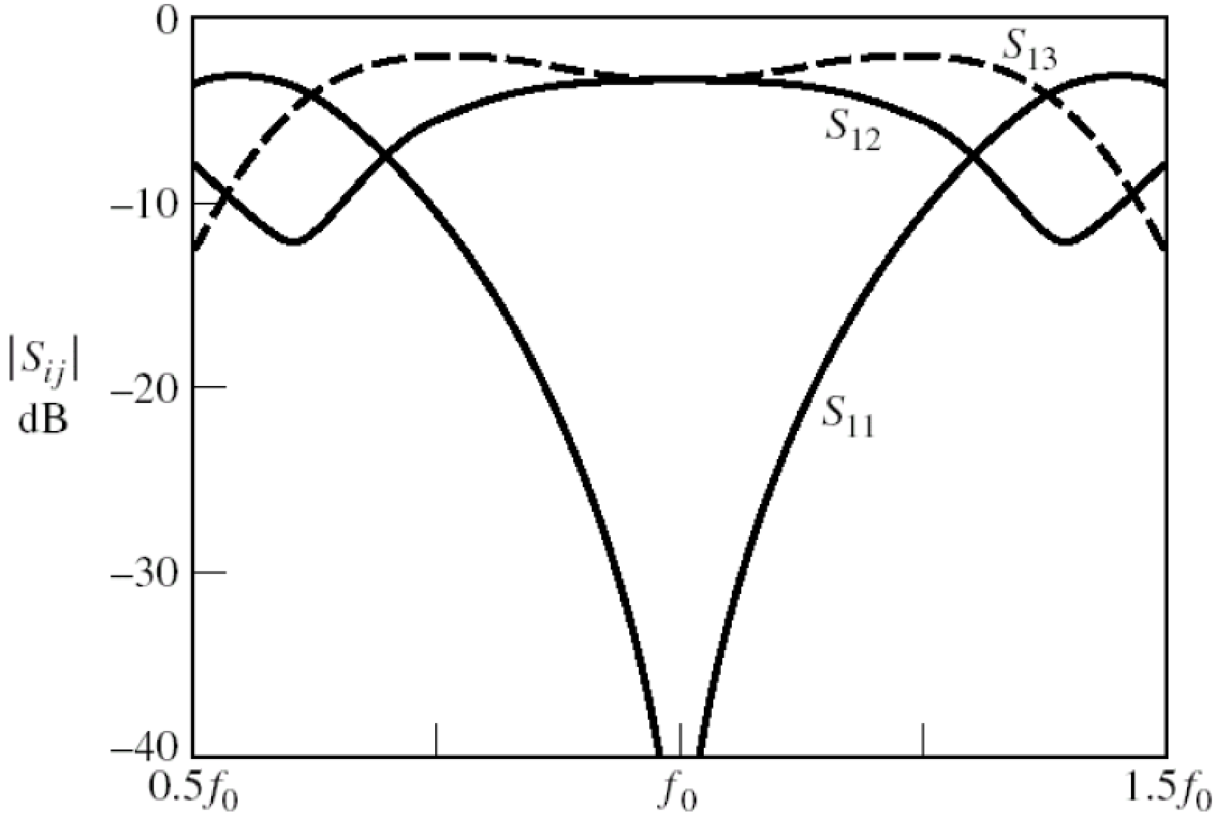
The 180^o- hybrid Coupler (contd.)

- While the **other** uses sources on ports **2 and 4**:



The 180°- hybrid Coupler (contd.)

- Finally, because of the transmission line lengths, we find that the ring hybrid is a **narrow-band** device:



Example

- Using ADS, design a branchline hybrid coupler using 100Ω microstrip on 32-mil RO4003C for a center frequency of 2.5GHz. Include the effects of copper and substrate losses.