Lecture – 13

Date: 16.09.2014

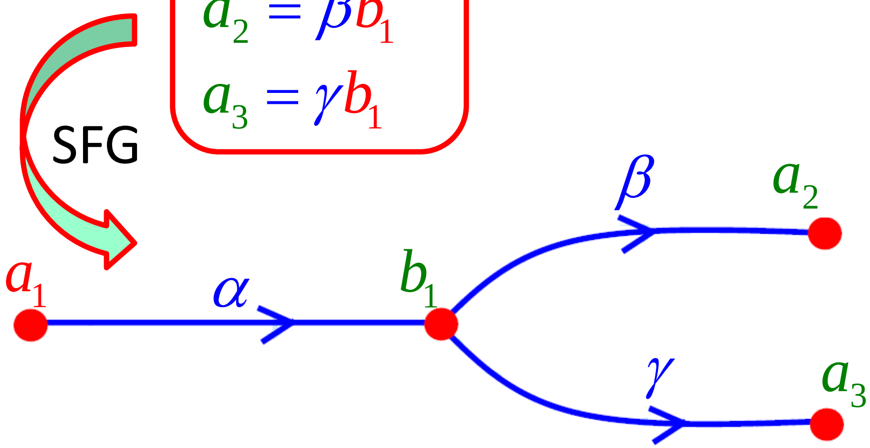
- The Signal Flow Graph (Contd.)
- Impedance Matching and Tuning
- L – Type Matching Network
- Example

Signal Flow Graph (contd.)

Splitting Rule

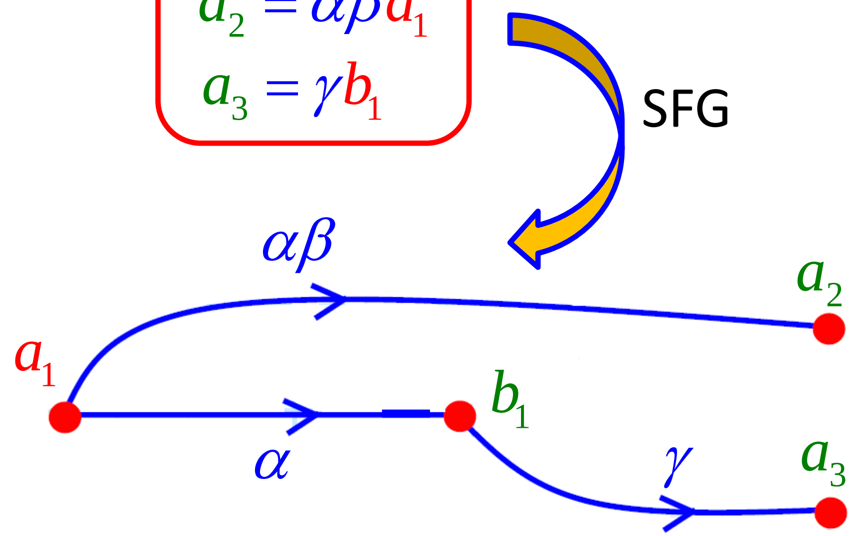
- Now consider the three equations

$$b_1 = \alpha a_1$$
$$a_2 = \beta b_1$$
$$a_3 = \gamma b_1$$



- These equations can be equivalently written as

$$b_1 = \alpha a_1$$
$$a_2 = \alpha\beta a_1$$
$$a_3 = \gamma b_1$$

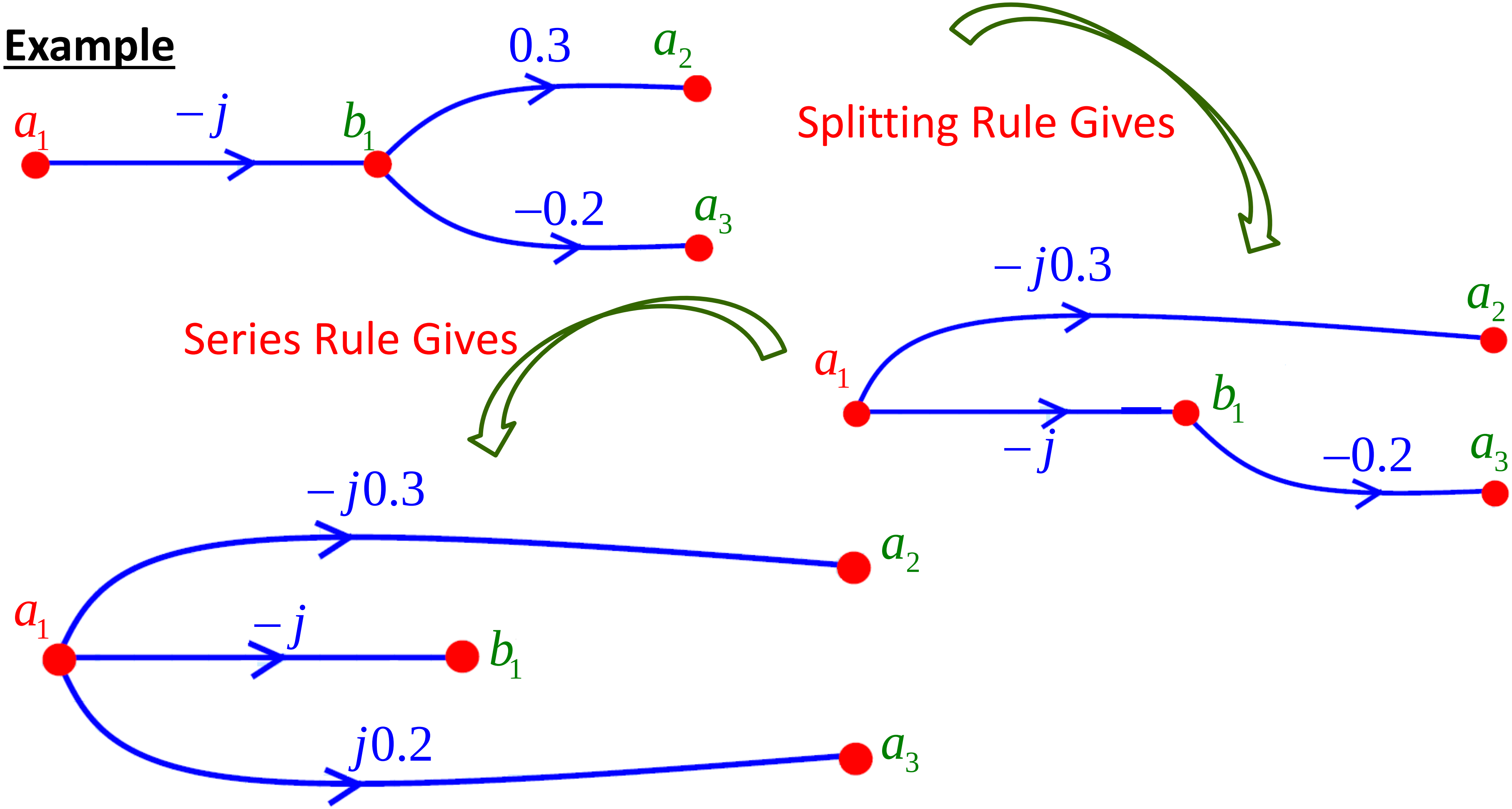


Rule 4 – Splitting Rule

If a node has one (and only one!) incoming branch, and one (or more) exiting branches, the incoming branch can be "split", and directly combined with each of the exiting branches.

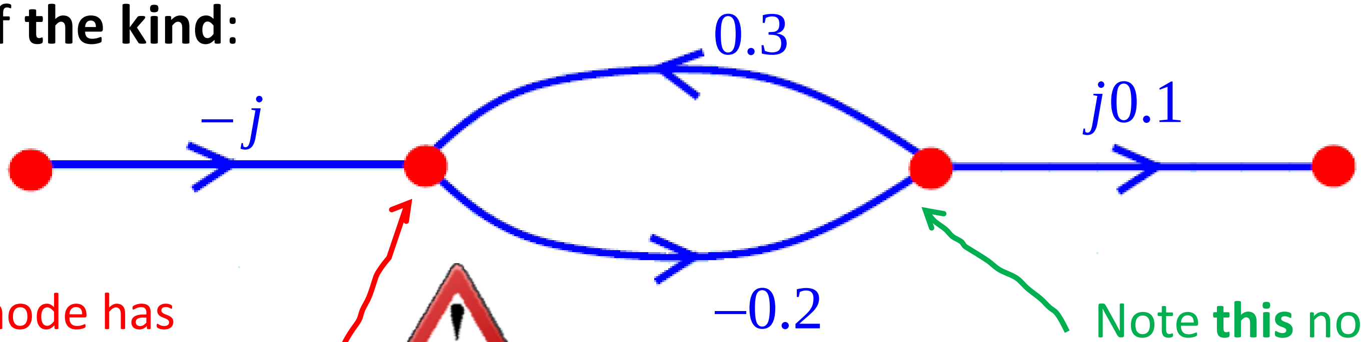
Signal Flow Graph (contd.)

Example



Signal Flow Graph (contd.)

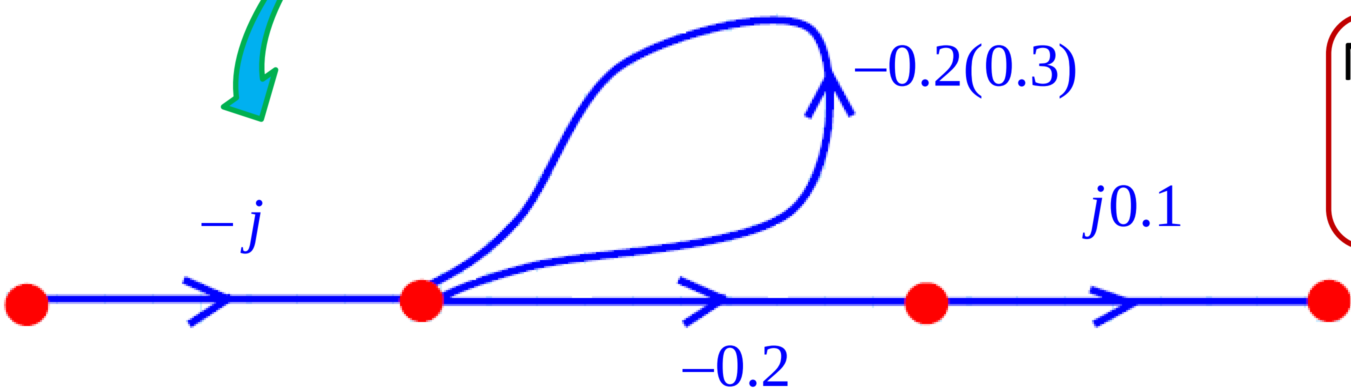
- The splitting rule is **particularly useful** when we encounter signal flow graphs of **the kind**:



Note this node has **two** incoming branches !!

Note **this** node has only **one** incoming branch !!

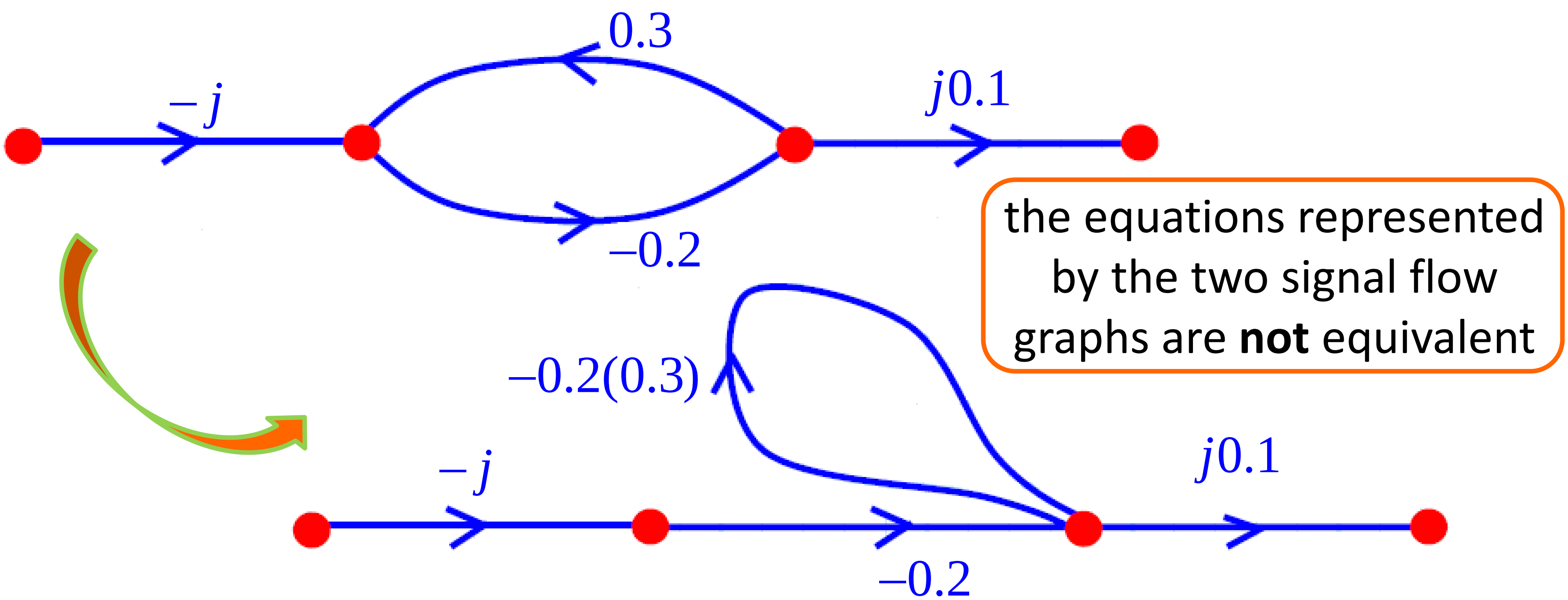
We can **split** the -0.2 branch, and rewrite the graph as:



Now apply **self-loop** rule to simplify it further

Signal Flow Graph (contd.)

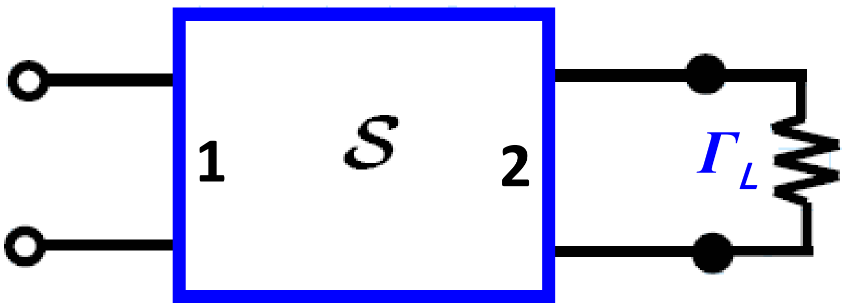
Q: Can we split the **other** branch of the loop?

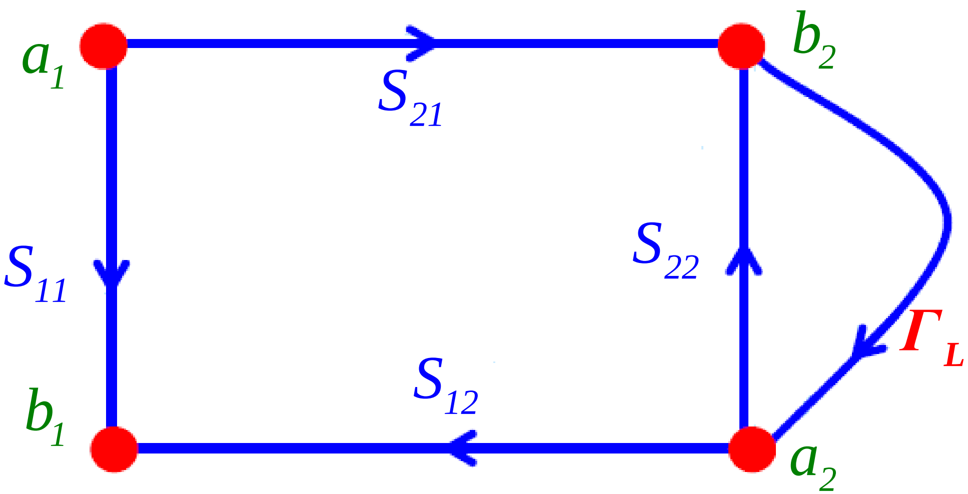


A: NO!! Do not make this mistake! We **cannot** split the 0.3 branch because it terminates in a node with **two** incoming branches (i.e., -j and 0.3). This is a **violation** of rule 4.

Example-1

Consider the basic 2-port network, terminated with load Γ_L:





determine the value: $\Gamma_1 = \frac{b_1}{a_1}$

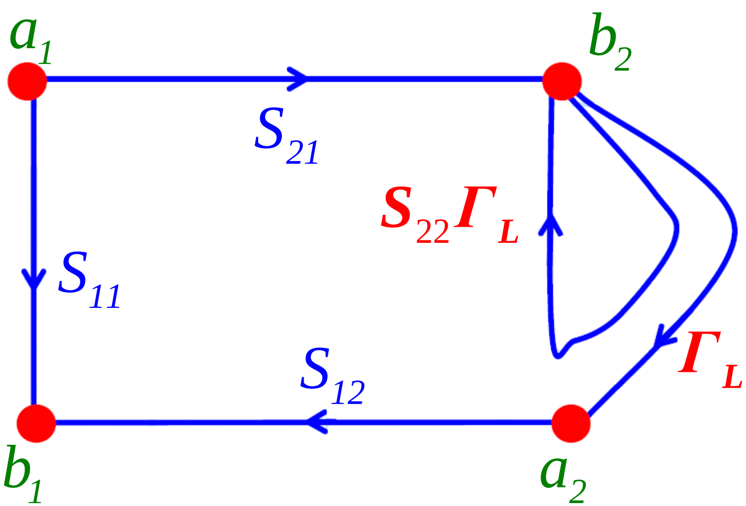
Solution:

- Isn't this simply S_{11} ?
- Only if $\Gamma_L = 0$ (and in this situation it is not!)

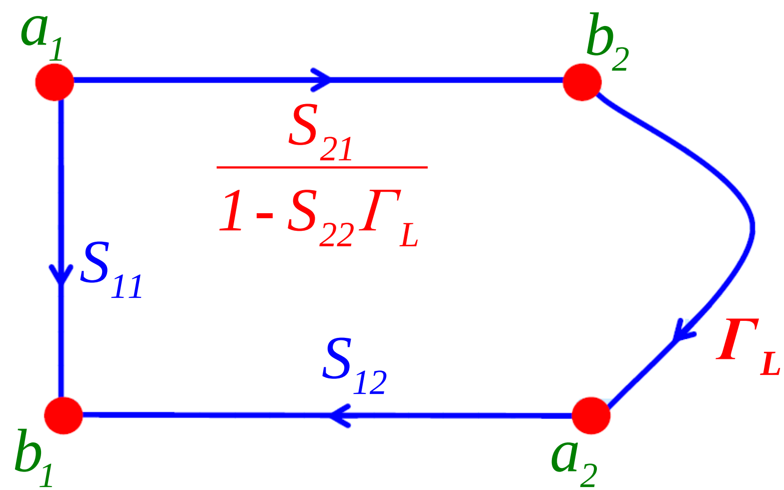
Example-1 (contd.)

- let's decompose (simplify) the signal flow graph and find out!

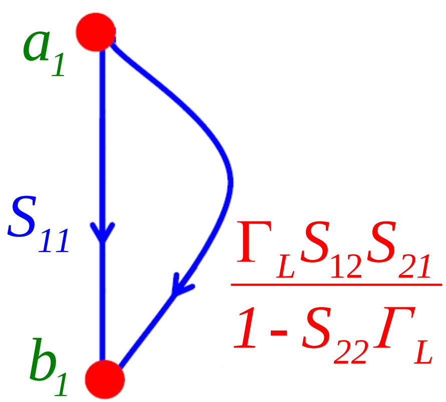
Step-1: splitting rule on node a_2



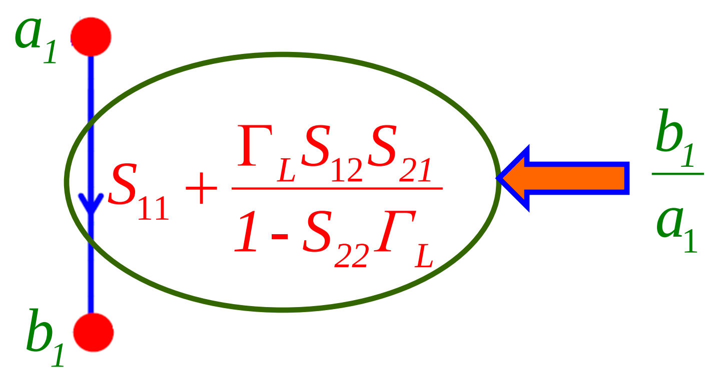
Step-2: self-loop rule on node b_2



Step-3: series rule gives

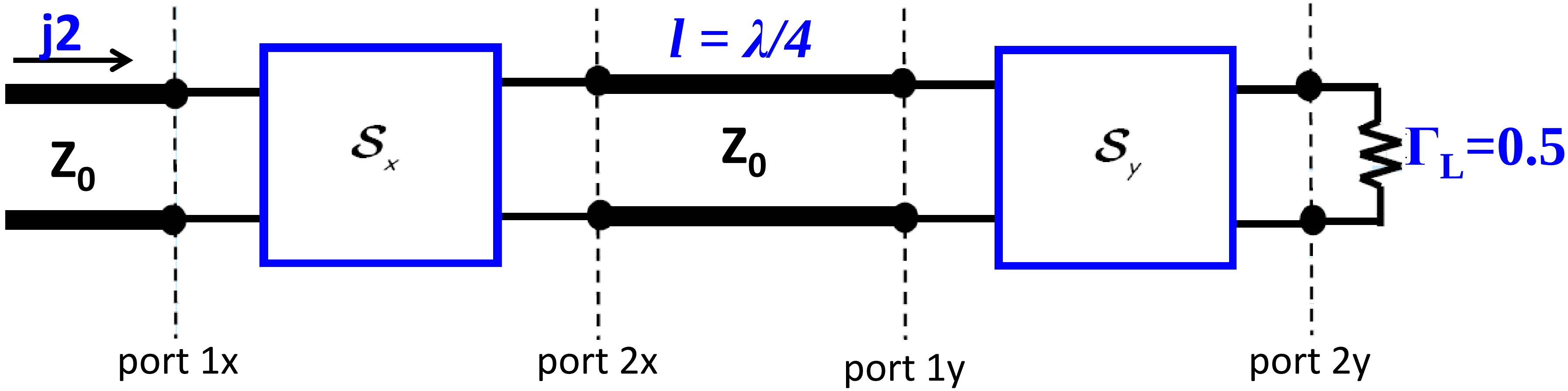


Step-4: parallel rule gives



Example – 2

Below is a **single**-port device (with **input** at port $1x$) constructed with two two-port devices (**S**$_x$ and **S**$_y$), a quarter wavelength transmission line, and a load impedance.



Given

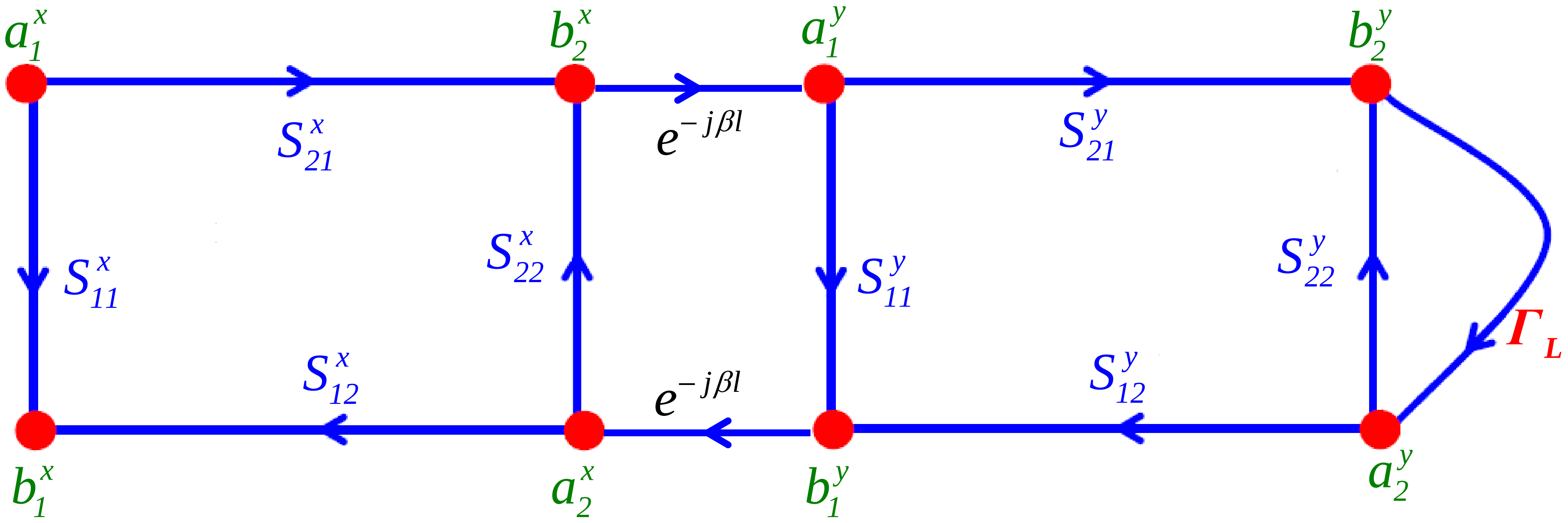
$Z_0 = 50\Omega$

$$S_x = \begin{bmatrix} 0.35 & 0.5 \\ 0.5 & 0 \end{bmatrix} \qquad S_y = \begin{bmatrix} 0 & 0.8 \\ 0.8 & 0.4 \end{bmatrix}$$

Draw the complete **signal flow graph** of this circuit, and then reduce the graph to determine: **a)** The total current through load Γ_L; **b)** The power delivered to (i.e., absorbed by) port 1x.

Example – 2 (contd.)

- The signal flow graph describing this network is:

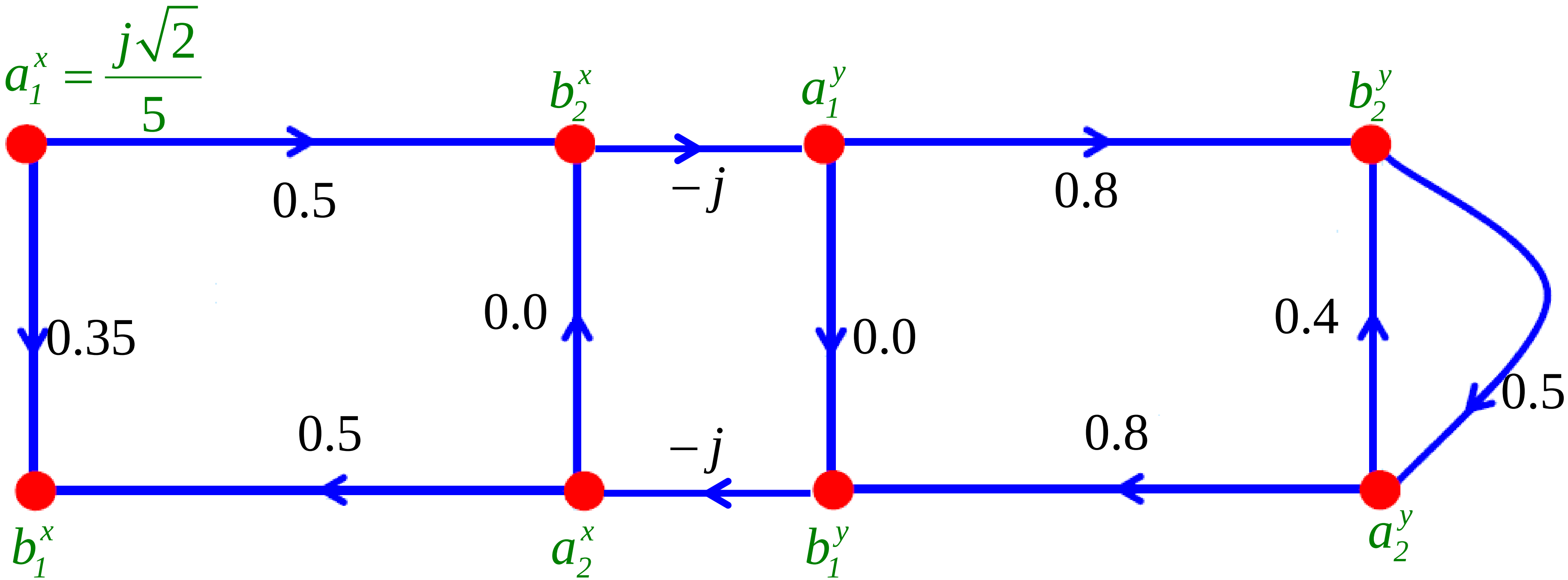


- We know that the value of the wave **incident** on port 1 of device $\mathbf{S}_x$ is:

$$a_1^x = \frac{V_{1x}^+\left(z_{1x} = z_{1xP}\right)}{\sqrt{Z_0}} = \frac{j2}{50} = \frac{j\sqrt{2}}{5}$$

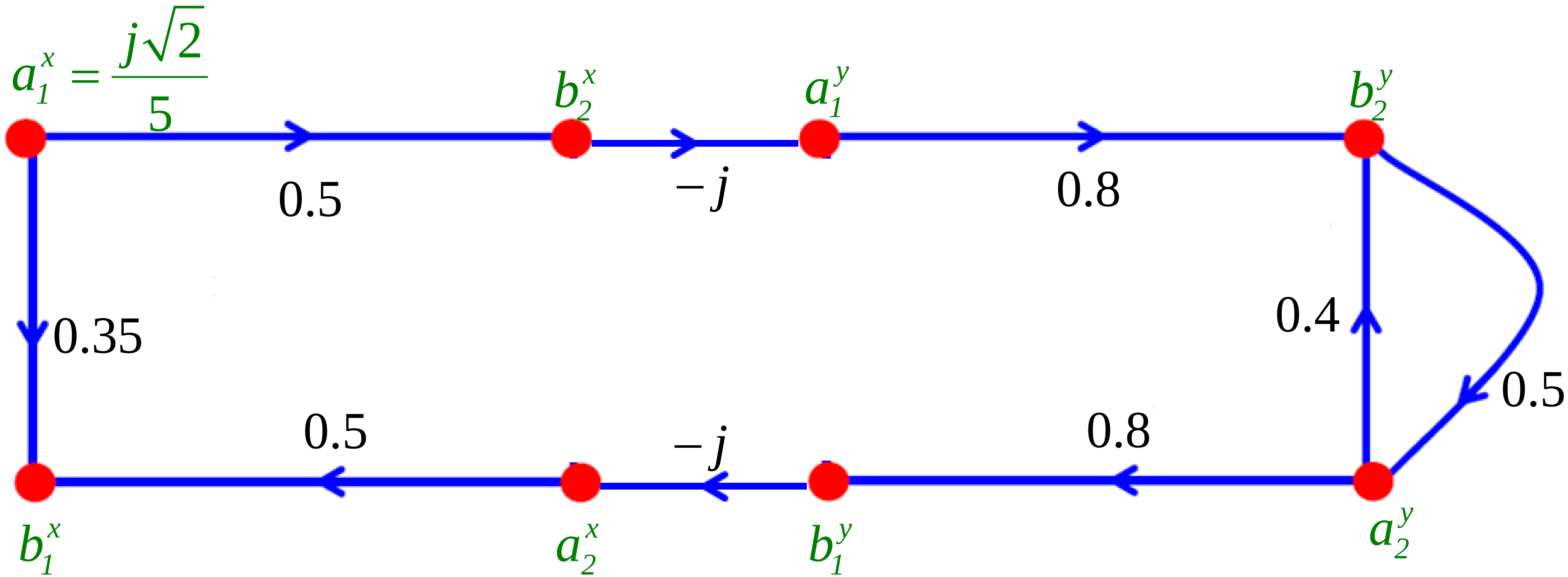
Example – 2 (contd.)

- Let us place the given numeric values of branches on this SFG:



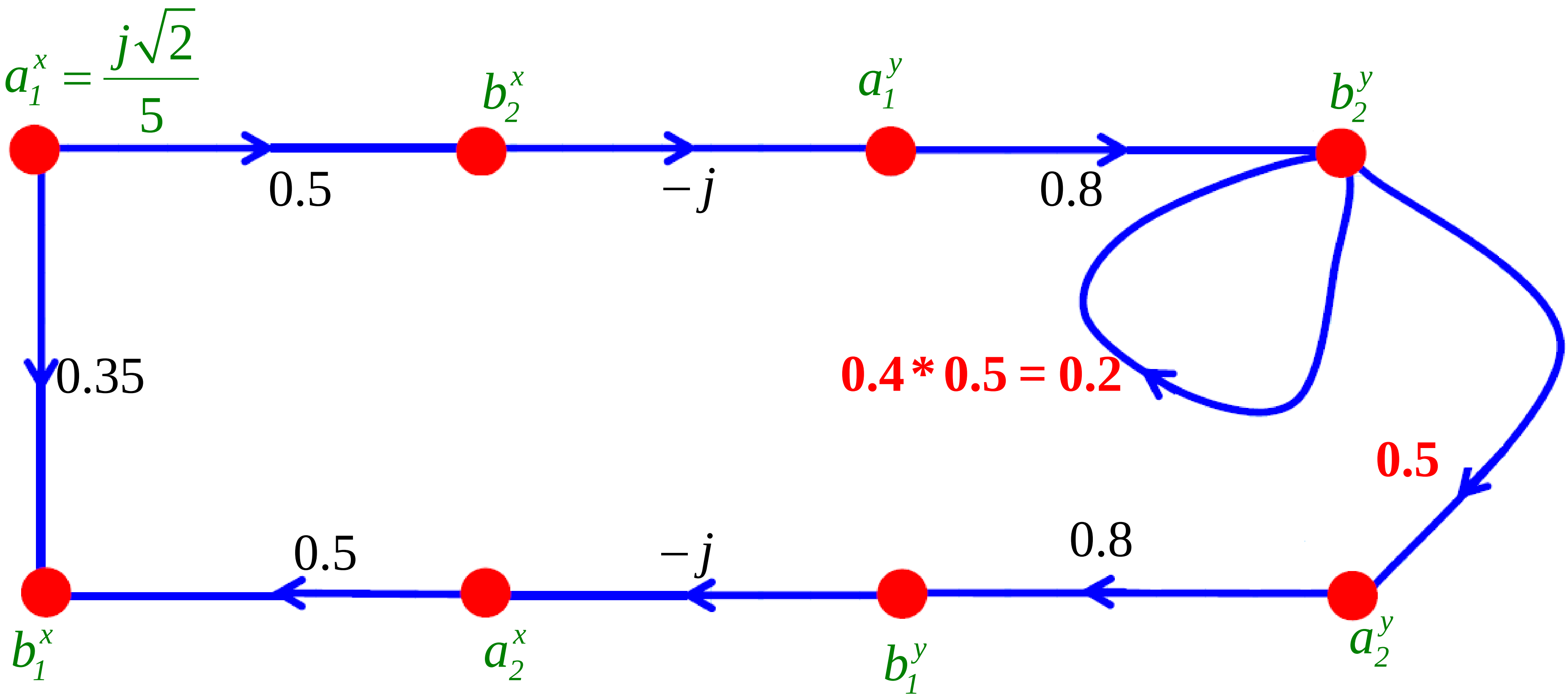
Example – 2 (contd.)

- Remove the zero valued branches:



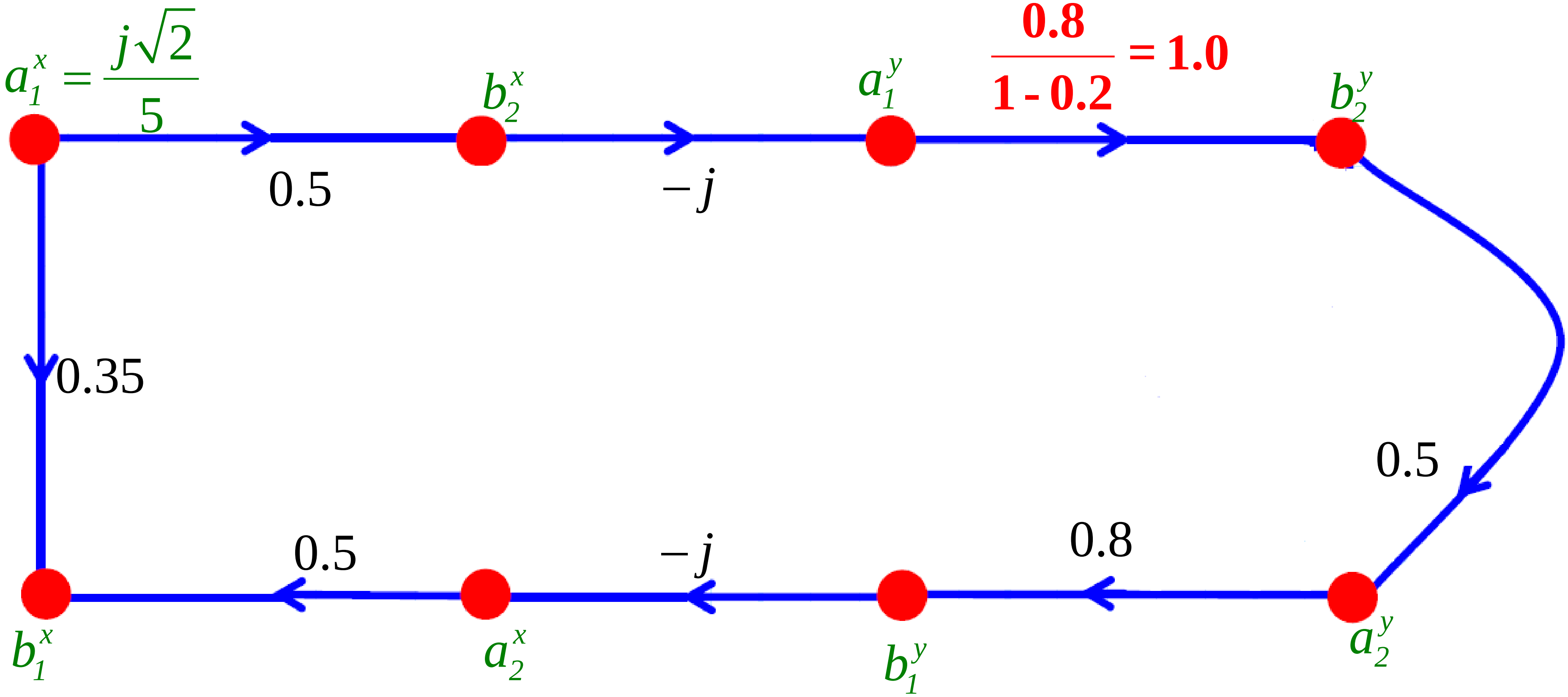
Example – 2 (contd.)

- Now apply "splitting" rule at node a_{2y}



Example – 2 (contd.)

- Then apply "self-loop" rule at node b_{2y}



Example – 2 (contd.)

- let's use this simplified signal flow graph to find the solutions to our questions!

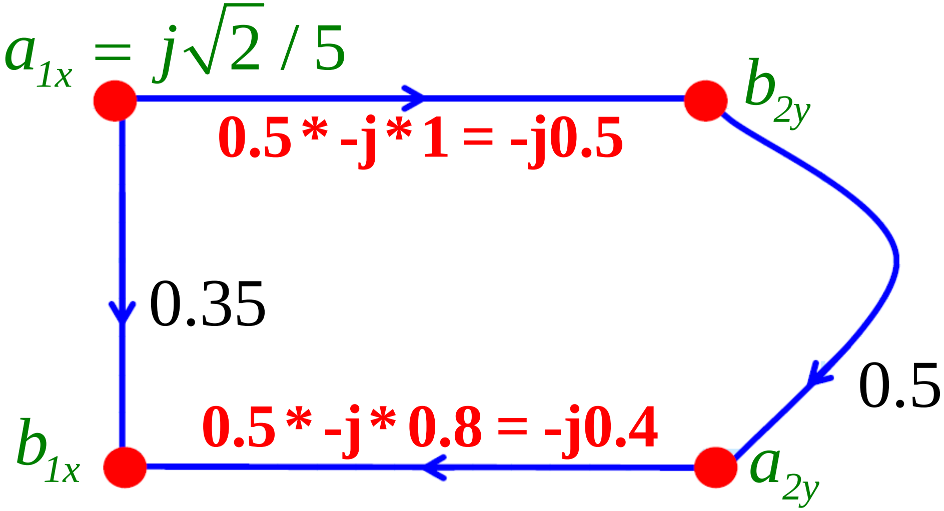
a) The total current through load Γ_L

$$I_L = -I\left(z_{2y} = z_{2yP}\right) = -\frac{V_{2y}^{+}\left(z_{2y} = z_{2yP}\right) - V_{2y}^{-}\left(z_{2y} = z_{2yP}\right)}{Z_0}$$

$$\Rightarrow I_L = -\frac{a_{2y} - b_{2y}}{\sqrt{Z_0}} = \frac{b_{2y} - a_{2y}}{\sqrt{50}}$$

Thus, we need to determine the value of nodes a_{2y} and b_{2y}

- **Using the "series" rule on the SFG gives**



Therefore,

$$b_{2y} = -j0.5 * a_1^x = -j0.5 * \frac{j\sqrt{2}}{5} = 0.1\sqrt{2}$$

$$a_{2y} = 0.5 * b_{2y} = 0.05\sqrt{2}$$

Example – 2 (contd.)

- **Thus the total current through Γ_L is:**

$$I_L = \frac{b_{2y} - a_{2y}}{\sqrt{50}} = \frac{(0.1 - 0.05)\sqrt{2}}{\sqrt{50}} = \frac{0.05}{5} = 10mA$$

b) The **power** delivered to (i.e., absorbed by) port 1x is:

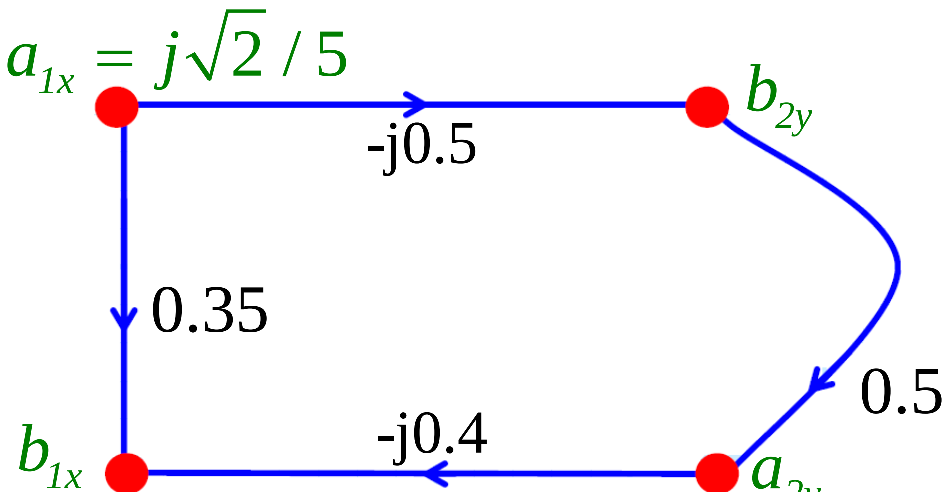
$$P_{abs} = P^+ - P^- = \frac{\left|V_{1x}^+\left(z_{1x} = z_{1xP}\right)\right|^2}{2Z_0} - \frac{\left|V_{1x}^-\left(z_{1x} = z_{1xP}\right)\right|^2}{2Z_0}$$

$$\Rightarrow P_{abs} = \frac{|a_{1x}|^2 - |b_{1x}|^2}{2}$$

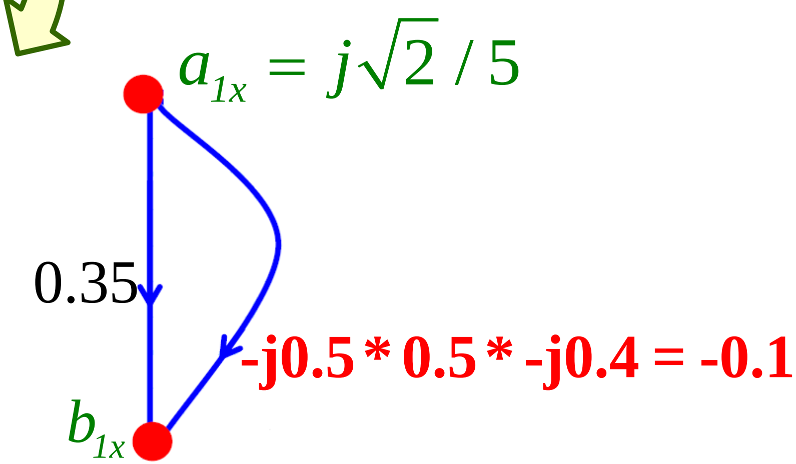
Requires knowledge of nodes a_{1x} and b_{1x}

Example – 2 (contd.)

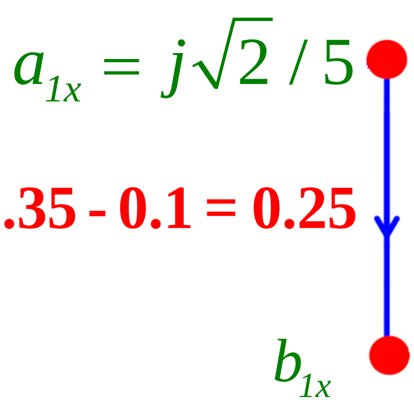
- **We have:**



apply “series” rule



apply “parallel” rule



Therefore:

$$b_{1x} = 0.25 a_{1x} = \left(0.25 * j\sqrt{2}/5\right) = j0.05\sqrt{2}$$

Example – 2 (contd.)

- Therefore, the power delivered to (i.e., absorbed by) port 1x is:

$$\Rightarrow P_{abs} = \frac{\left|j\sqrt{2}/5\right|^2 - \left|j0.05\sqrt{2}\right|^2}{2} = \frac{0.08 - 0.005}{2} = 37.5mW$$

Introduction – Impedance Transformation

One of the most important and fundamental two-port networks that microwave engineers design is a **lossless matching network** (otherwise known as an **impedance transformer**).

Q: In high frequency circuits, a source and load are connected by a TL. Can we implement matching networks in transmission line circuits?

Furthermore, these matching networks seem too good to be true—can we **really** design and construct them to provide a **perfect** match?

A: We can **easily** provide a **near** perfect match at **precisely one frequency**

But, since lossless matching and transmission lines are made of entirely **reactive elements** (not to mention the reactive components of source and load impedance), we find that **changing** the frequency will typically "**unmatch**" our circuit!

Introduction (contd.)

Therefore, a difficult challenge for any RF/microwave design engineer is to design a **wideband** matching network—a matching network that provides an **"adequate"** match over a wide range of frequencies.

- Generally speaking, matching network design requires a **tradeoff** between following parameters for desirable attributes:

  1. Bandwidth
  2. Complexity
  3. Implementation
  4. Adjustability

Matching with Lumped Components

- let's begin to examine how matching networks are **built**!
- we begin with the **simplest** solution: An **L-network**, consisting of a **single capacitor** and a **single inductor.**

Q: Just **two** elements! That seems simple enough. Do we **always** use these L-networks when constructing lossless matching networks?

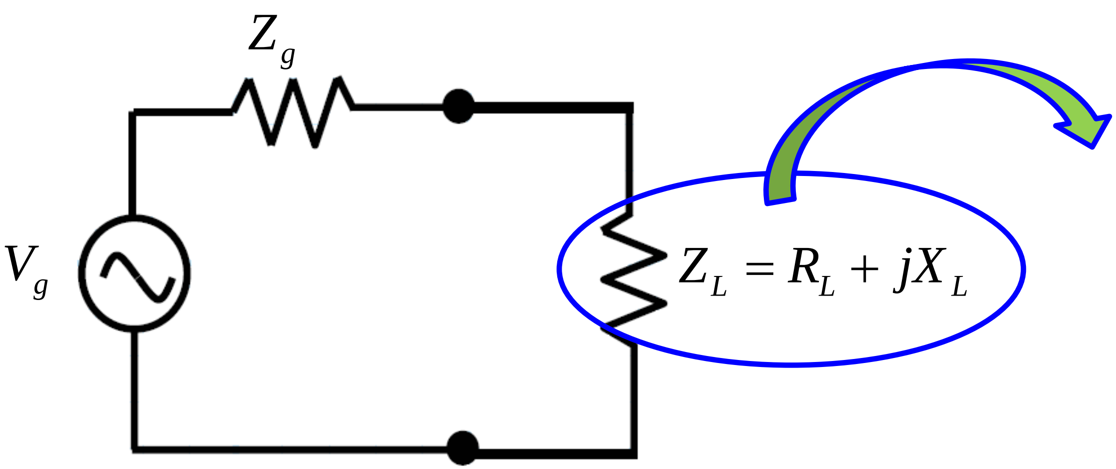
A: Nope. L-networks have **two** major drawbacks:

1. They are **narrow-band**.
2. Capacitors and inductors are **difficult to make** at microwave frequencies!

Soon we will see how these L-networks actually **work**

Matching Network Analysis

- Consider following circuit where a **passive load** is attached to an **active source**:



It will **absorb power** — power that is **delivered** to it by the **source** → given by expression

$$P_L = \frac{1}{2}\{V_L I_L^*\}$$

$$\Rightarrow P_L = \frac{1}{2}\mathrm{Re}\left\{\left(V_g \frac{Z_L}{Z_g + Z_L}\right)\left(\frac{V_g}{Z_g + Z_L}\right)^*\right\} \Rightarrow = \frac{1}{2}|V_g|^2 \frac{\mathrm{Re}\{Z_L\}}{|Z_g + Z_L|^2} = \frac{1}{2}|V_g|^2 \frac{R_L}{|Z_g + Z_L|^2}$$

Recall that the power delivered to the load will be **maximized** (for a given V_g and Z_g) **if** the load impedance is equal to the **complex conjugate** of the source impedance ($Z_L = Z_g^*$)

Matching Network Analysis (contd.)

- We call the maximized power, the **available power** P_{avl} **from the source** → it is, after all, the **largest** amount of power that the source can **ever** deliver!

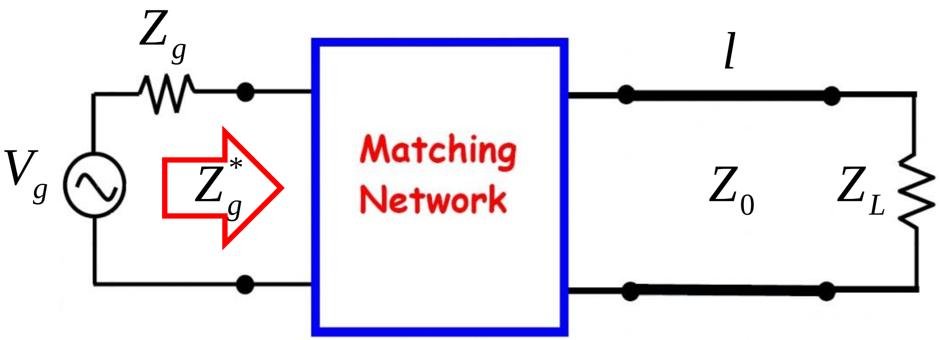
$$P_L^{\max} \doteq P_{avl} = \frac{1}{2}\left|V_g\right|^2 \frac{R_g}{\left|Z_g + Z_g^*\right|^2} = \frac{1}{2}\left|V_g\right|^2 \frac{R_g}{\left|2R_g\right|^2} = \frac{\left|V_g\right|^2}{8R_g}$$

- Note the available power of the **source** is dependent on **source** parameters **only** (i.e., V_g and R_g). This makes sense! Do **you** see why?
- Thus, we can say that to "take full advantage" of all the available power of the source, we must make the load impedance the complex conjugate of the source impedance.
- Otherwise, the power delivered to the load will be less than power made available by the source! In other "words":

$$P_L \leq P_{avl}$$

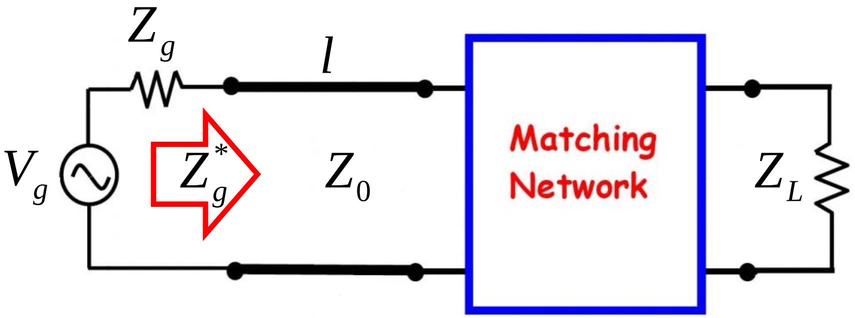
Matching Networks and Transmission Lines

Note: we can construct a network to transform the **input impedance** of the transmission line into the complex conjugate of **the source impedance**:



Q: But, do we **have** to place the matching network between the source and the transmission line?

A: Nope! We could **also** place a (different) matching network between the transmission line and the load.

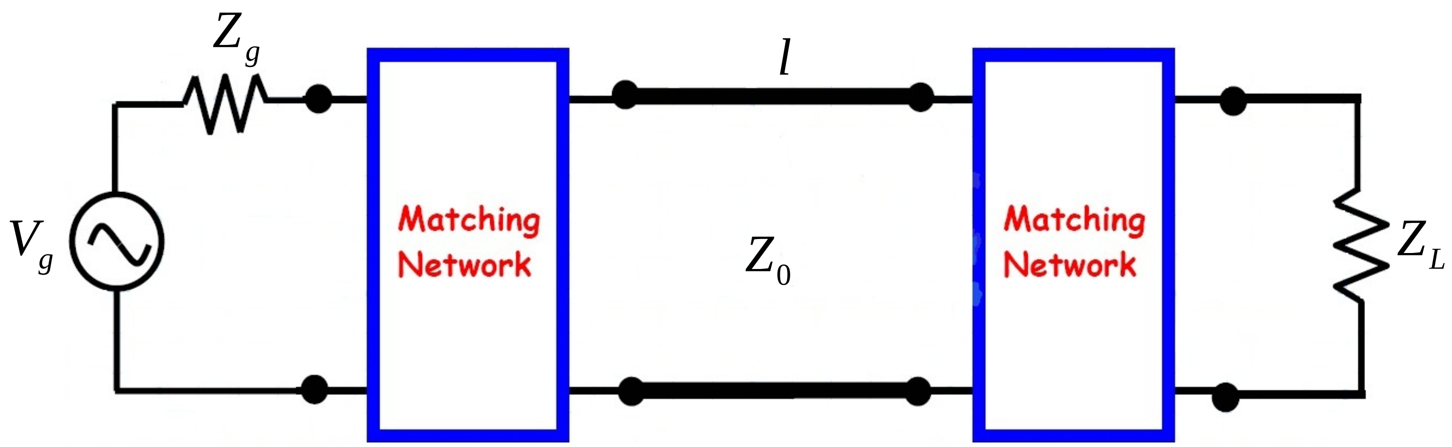


Matching Networks and Transmission Lines (contd.)

Q: So **which** method should we choose? Do engineers typically place the matching network between the source and the transmission line, **or** place it between the transmission line and the load?

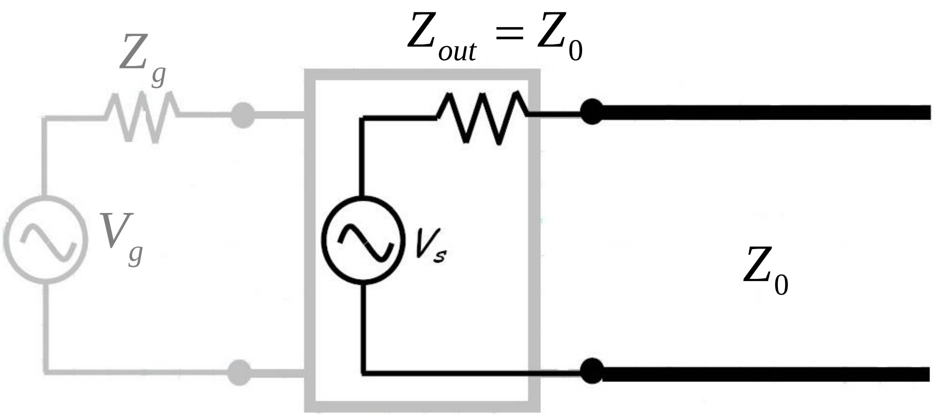
A: Actually, the typical solution is to do **both**!

- We find that often there is a matching network between the a source and the transmission line, **and** between the line and the load.

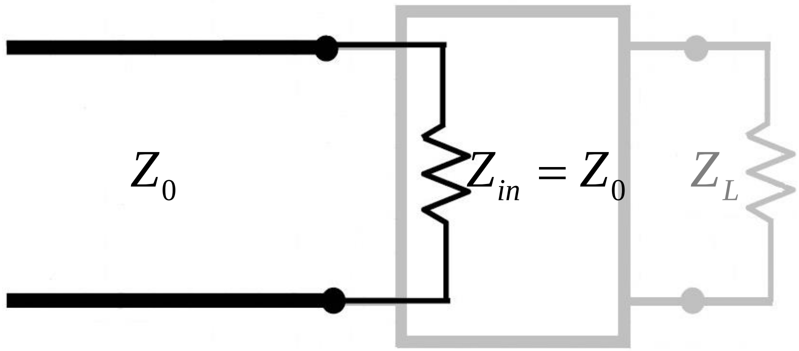


Matching Networks and Transmission Lines (contd.)

- The first network matches the **source** to the **transmission line**—in other words, it transforms the **output impedance** of the equivalent source to a value numerically equal to **characteristic impedance** Z_0:



- The second network matches the **load** to the **transmission line**—in other words it transforms the **load impedance** to a value numerically equal to **characteristic impedance** Z_0:



Q: Yikes! Why would we want to build **two** separate matching networks, instead of just **one**?

Matching Networks and Transmission Lines (contd.)

A: By using two separate matching networks, we can **decouple** the design problem. Recall again that the design of a **single** matching network solution would depend on four separate parameters:

1. the source impedance Z_g
2. load impedance Z_L
3. the TL characteristic impedance Z_0
4. the TL length l

- Alternatively, the design of the network matching the **source** and **transmission line** depends on **only**:

1. the source impedance Z_g
2. the transmission line characteristic impedance Z_0

- In addition, the design of the network matching the **load** and **transmission line** depends on **only**:

1. the source impedance Z_L
2. the transmission line characteristic impedance Z_0

Note that **neither** design depends on the transmission line **Length** l!

Q: **How is that possible?**

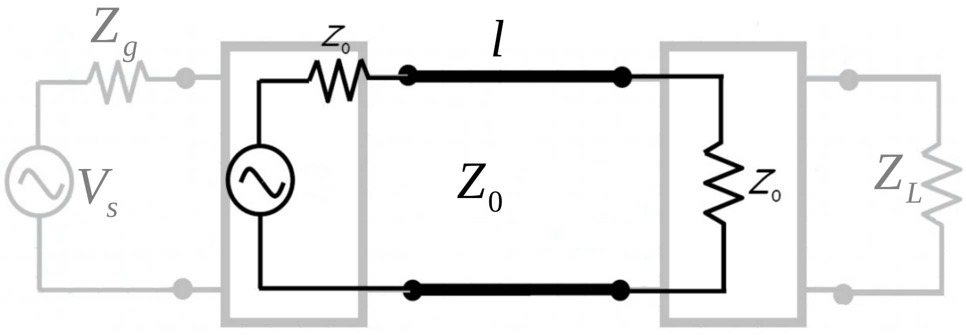
A: **Remember the case where $Z_g = Z_0 = Z_L$. For that special case, we found that a conjugate match was the result—regardless of the transmission line length.**

Matching Networks and Transmission Lines (contd.)

Thus, by matching the source to line impedance Z_0 **and** likewise matching the load to the line impedance, a conjugate match is **assured**—but the **length** of the transmission line does **not** matter!

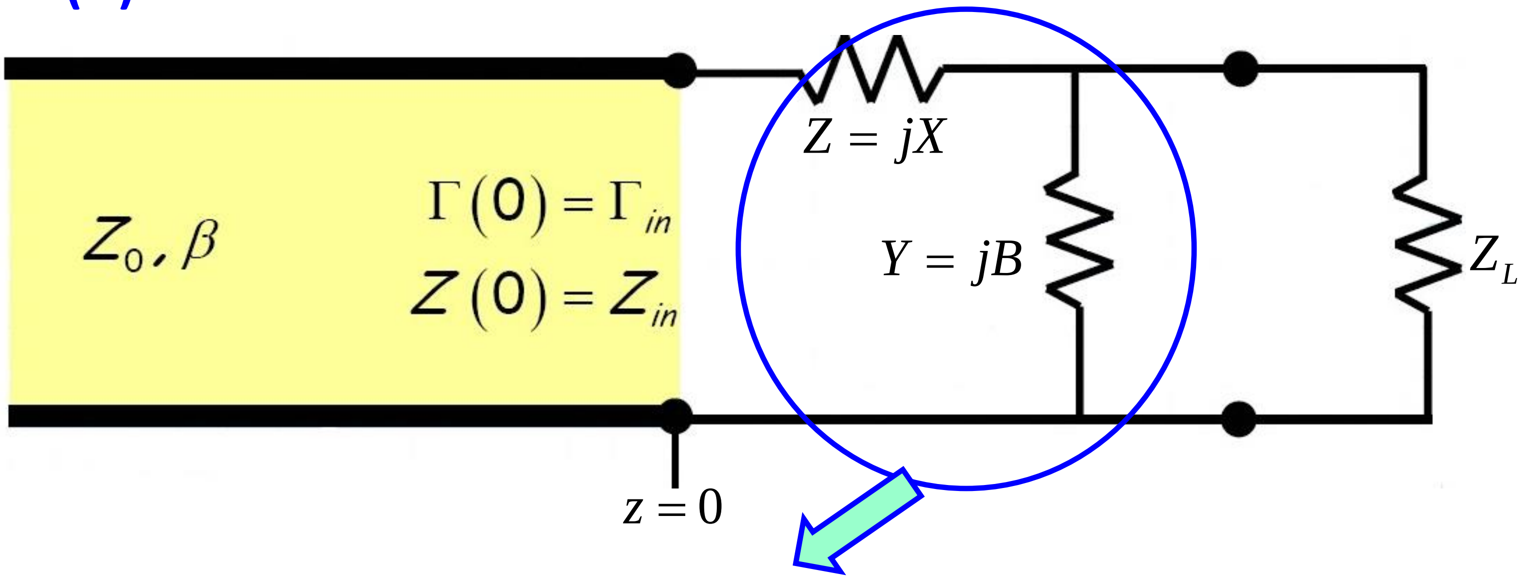
In fact, the typical problem for microwave engineers is to match a load (e.g., device input impedance) to a **standard** transmission line impedance (typically $Z_0 = 50\Omega$); **or** to independently match a source (e.g., device output impedance) to a **standard** line impedance.

A **conjugate match** is thus obtained by connecting the two with a transmission line of **any length**!



L-Network Analysis

- Let us consider the **first** matching L-network, and denote it as matching **network (A)**:



Note that this matching network consists of just **two** lumped elements, which must be **purely reactive** — in other words, a **capacitor** and an **inductor**!

- To make Γ_{in} = 0, the **input impedance** of the network must be:

$$Z_{in} = Z_0$$

L-Network Analysis (contd.)

- Using **basic** circuit analysis we can find that:

$$Z(z=0) = Z_{in} = jX + \frac{(1/jB)Z_L}{(1/jB)+Z_L} = jX + \frac{Z_L}{1+jBZ_L}$$

- Note that a **matched** network, with $Z_{in} = Z_0$, means that:

$$\mathrm{Re}\{Z_{in}\} = Z_0 \qquad \mathrm{Im}\{Z_{in}\} = 0$$

Gives two equations and aids in the determination of two unknowns (X, and B)

Essentially, the L-network matching network can be viewed as consisting of **two distinct parts**, each attempting to satisfy a specific requirement.

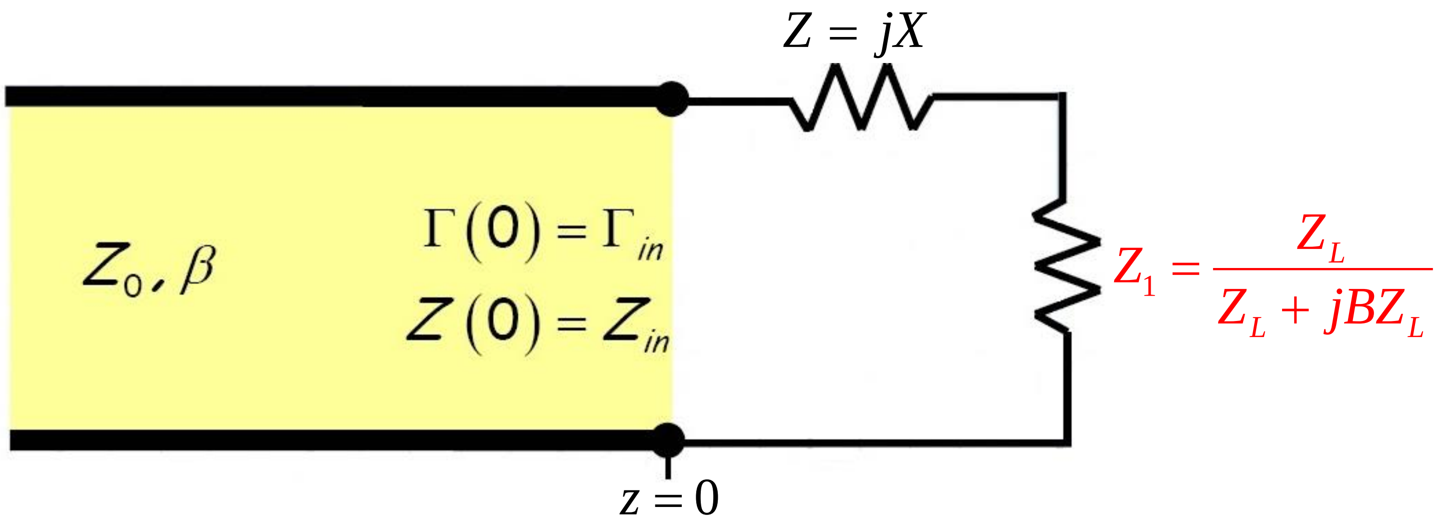
L-Network Analysis (contd.)

Part 1: Selecting Y = jB

- Since the shunt element Y and Z_L are in **parallel**, we can combine them into one element Y_1:

$$Y_1 \doteq Y + \frac{1}{Z_L} = jB + Y_L$$

The impedance of this element is therefore: $Z_1 = \frac{1}{Y_1} = \frac{1}{jB + Y_L} = \frac{Z_L}{Z_L + jBZ_L}$

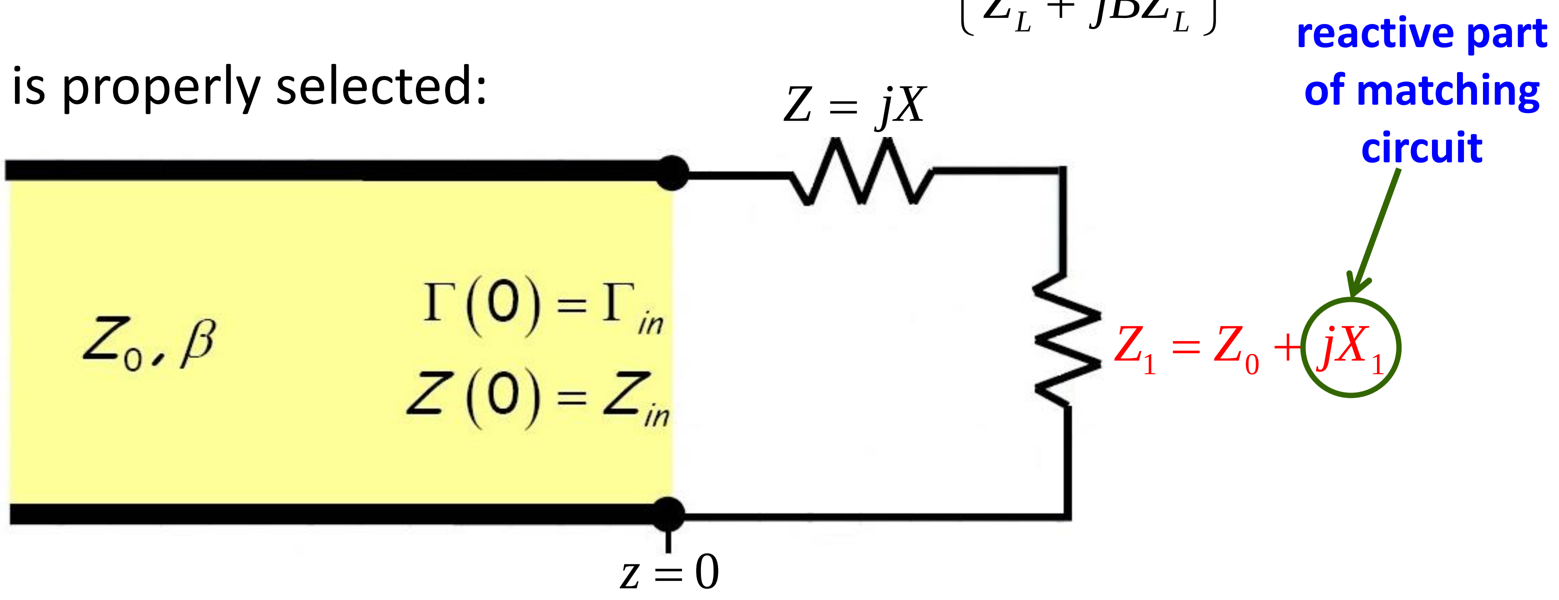


L-Network Analysis (contd.)

- To achieve a perfect match, we must set the value of susceptance B such that:

$$\mathrm{Re}\{Z_{in}\} = \mathrm{Re}\{Z_1\} \qquad \Rightarrow Z_0 = \mathrm{Re}\left\{\frac{Z_L}{Z_L + jBZ_L}\right\}$$

- Thus if B is properly selected:



L-Network Analysis (contd.)

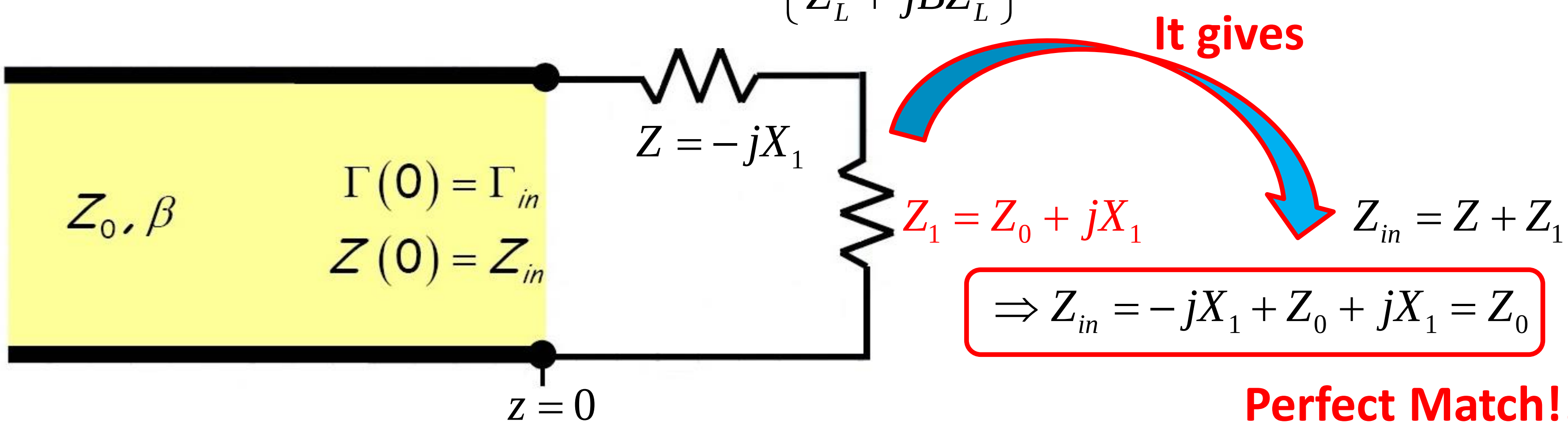
Part 2: Selecting Z =jX

- Note that the impedance $Z_1 = Z_L || (1/jB)$ has the ideal real value of Z_0. However, it also posses an **annoying** imaginary part of:

$$X_1 = \mathrm{Im}\{Z_1\} = \mathrm{Im}\left\{\frac{Z_L}{Z_L + jBZ_L}\right\}$$

- However, this imaginary component can be easily removed by setting the **series** element Z =jX to its equal but **opposite** value!

$$X = -X_1 = -\mathrm{Im}\left\{\frac{Z_L}{Z_L + jBZ_L}\right\}$$



$$\Rightarrow Z_{in} = -jX_1 + Z_0 + jX_1 = Z_0$$

Perfect Match!!!

L-Network Analysis (contd.)

- we can solve the preceding equations for the **required** values X and B to **satisfy** these two equations — to create a **matched** network!

$$B = \frac{X_L \pm \sqrt{R_L / Z_0}\sqrt{R_L^2 + X_L^2 - Z_0 R_L}}{R_L^2 + X_L^2}$$

$$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{BR_L}$$

Where, $Z_L = R_L + jX_L$

Note:

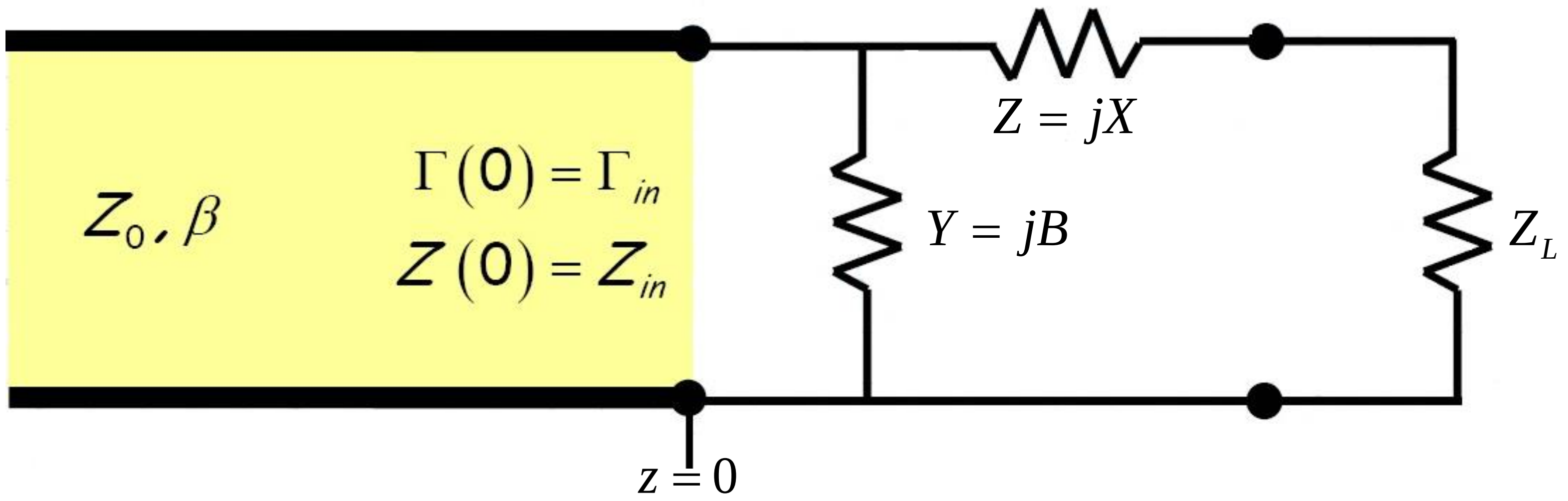
1) Because of the ± , there are **two** solutions for B (and thus X)

2) For jB to be purely imaginary (i.e., reactive), B must be **real** → R_L **must** be greater than Z_0 ($R_L > Z_0$) to insure that B and thus X is real.

In other words, this matching network **(type-A)** can only be used when $R_L > Z_0$. Notice that this condition means that the normalized load z_L' lies **inside** the r = 1 circle on the Smith Chart!

L-Network Analysis (contd.)

- Now let's consider the **second** of the two L-networks, which we call **network (B)**. Note it is **also** formed with just two lumped elements.



- To make Γ_{in} = 0, the **input admittance** of the network must be:

$$Y_{in} = Y_0$$

- From circuit theory we can determine that the input **admittance** for this network is:

$$Y_{in} = jB + \frac{1}{jX + Z_L}$$

L-Network Analysis (contd.)

- We can describe a **matched** network, with $Y_{in} = Y_0$ as:

$$\operatorname{Re}\{Y_{in}\} = Y_0 \qquad \operatorname{Im}\{Y_{in}\} = 0$$

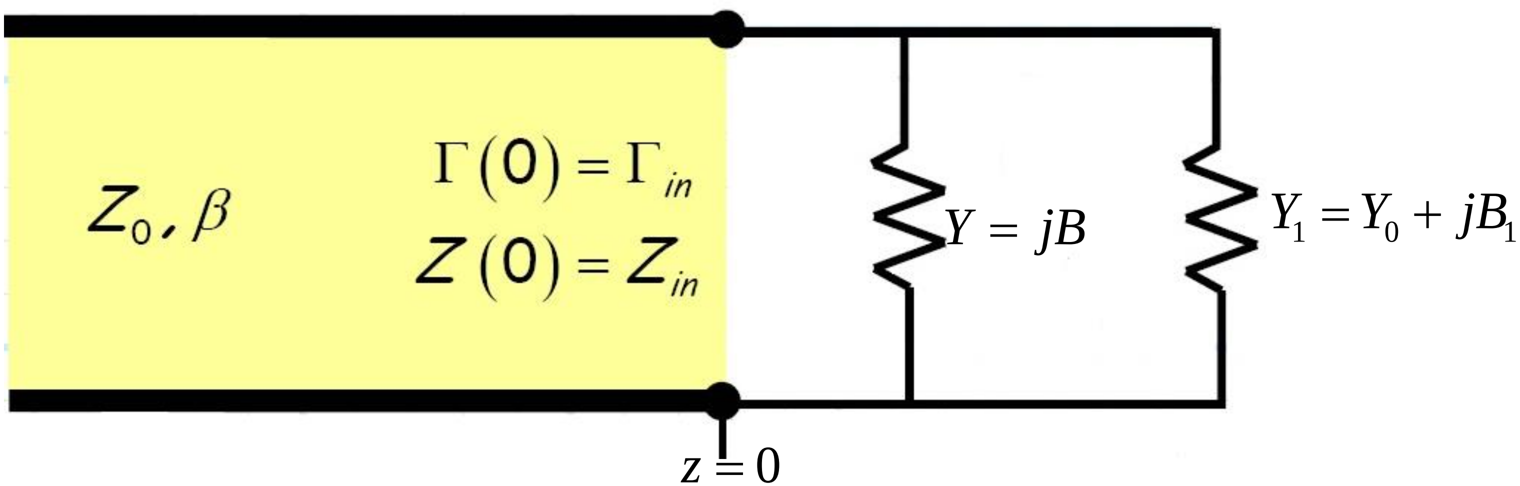
Gives two equations and aids in the determination of two unknowns (X, and B)

- For this design, we set the value of Z =jX such that the admittance Y_1:

$$Y_1 \doteq \frac{1}{Z + Z_L} = \frac{1}{jX + Z_L}$$

Gives real part as

$$Y_0 = \operatorname{Re}\left\{\frac{1}{jX + Z_L}\right\}$$



L-Network Analysis (contd.)

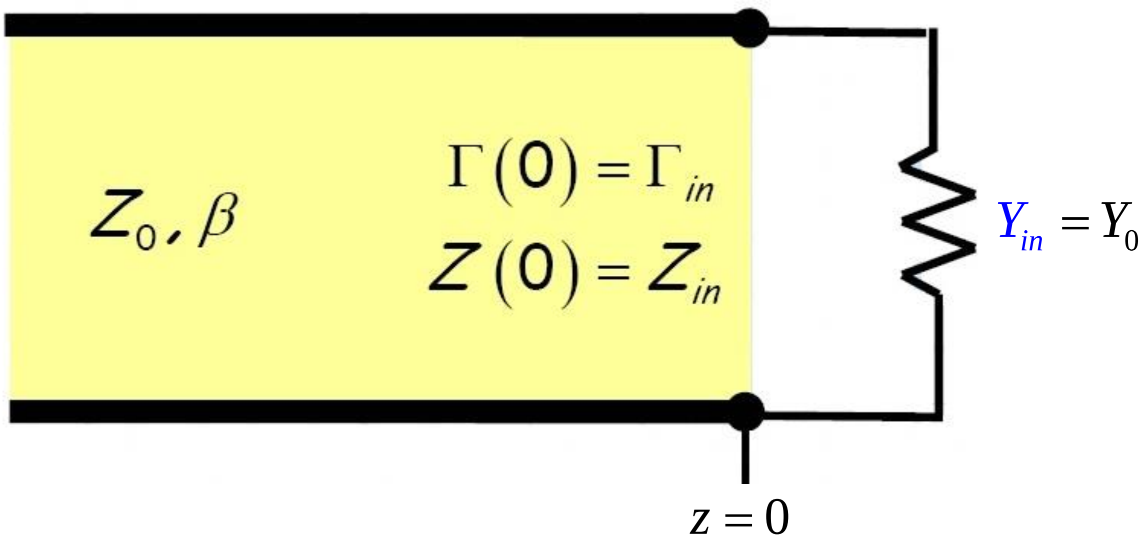
- It is apparent that a perfect match will occur if the shunt element Y = jB is set to "cancel" the reactive component of Y_1:

$$B = -\operatorname{Im}\{Y_1\} = -\operatorname{Im}\left\{\frac{1}{jX + Z_L}\right\}$$

So that:

$$Y_{in} = Y + Y_1 = -jB_1 + (Y_0 + jB_1) = Y_0$$

Perfect Match!!!



L-Network Analysis (contd.)

- we can solve the preceding equations for the **required** values X and B to **satisfy** these two equations — to create a **matched** network!

$$X = \pm\sqrt{R_L(Z_0 - R_L)} - X_L$$

$$B = \pm\frac{\sqrt{(Z_0 - R_L)/R_L}}{Z_0}$$

Where, $Z_L = R_L + jX_L$

Note:

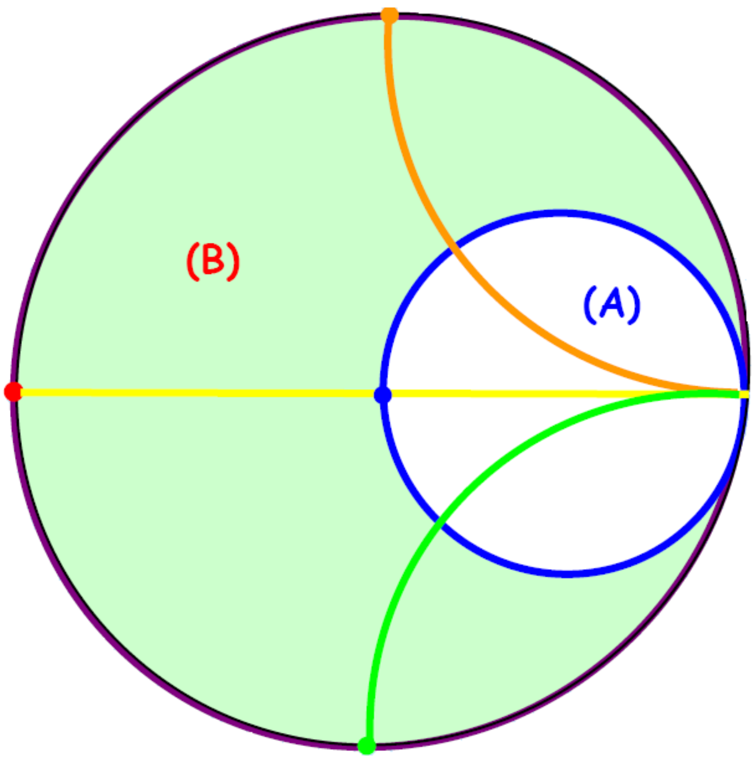
1) Because of the ± , there are **two** solutions for B (and thus X)

2) For jB and jX to be purely imaginary (i.e., reactive), B and X must be **real**

→ R_L **must** be less than Z_0 ($R_L < Z_0$) to insure that B and X are real.

In other words, this matching network **(type-B)** can only be used when $R_L < Z_0$. Notice that this condition means that the normalized load z_L' lies **outside** the r = 1 circle on the Smith Chart!

L-Network Analysis (contd.)



Smith Chart Regions Depicting the Validity Regions for Networks A and B

L – Type Matching Network (contd.)

- Once the values of X and B are found, we can determine the required values of inductance L and/or capacitance C, **for the signal frequency ω_0 !**

$$X = \begin{cases} \omega_0 L & \text{If} \quad X > 0 \\ -\dfrac{1}{\omega_0 C} & \text{If} \quad X < 0 \end{cases}$$

and

$$B = \begin{cases} \omega_0 C & \text{If} \quad B > 0 \\ -\dfrac{1}{\omega_0 L} & \text{If} \quad B < 0 \end{cases}$$

Make sure that **you see and know why these equations are true**

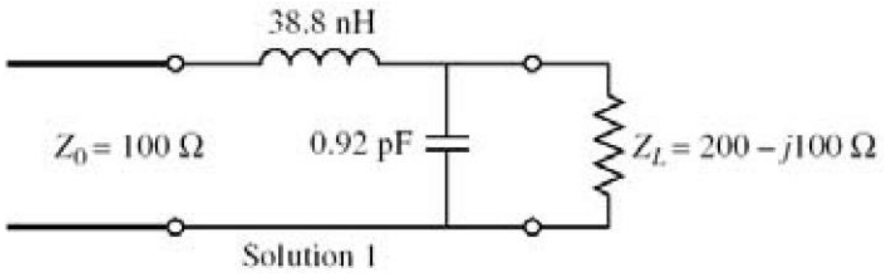
As a result, we see that the reactance or susceptance of the elements of our L-network will have the proper values for matching at precisely **one and only one frequency!**

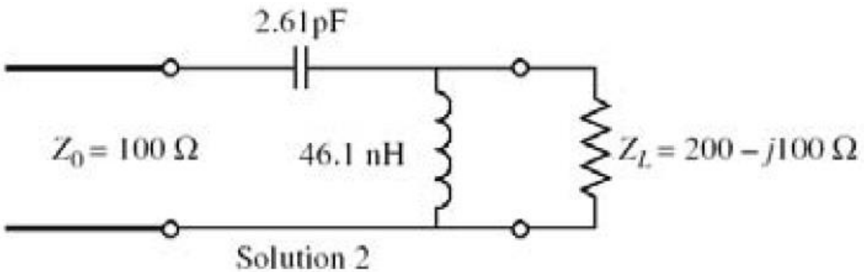
And this frequency **better be the signal frequency ω_0 !**

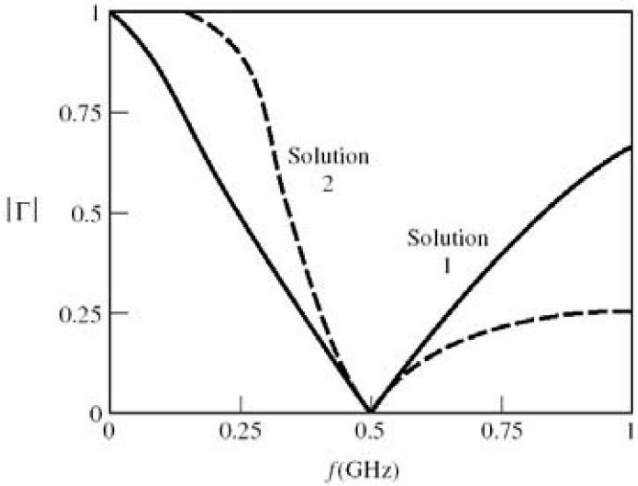
L – Type Matching Network (contd.)

If the signal frequency **changes from the design frequency, the** reactance and susceptance of the matching network inductors and capacitors will change → As a result the circuit will **no longer** be matched

Therefore the L-Type matching network has a **narrow bandwidth!**

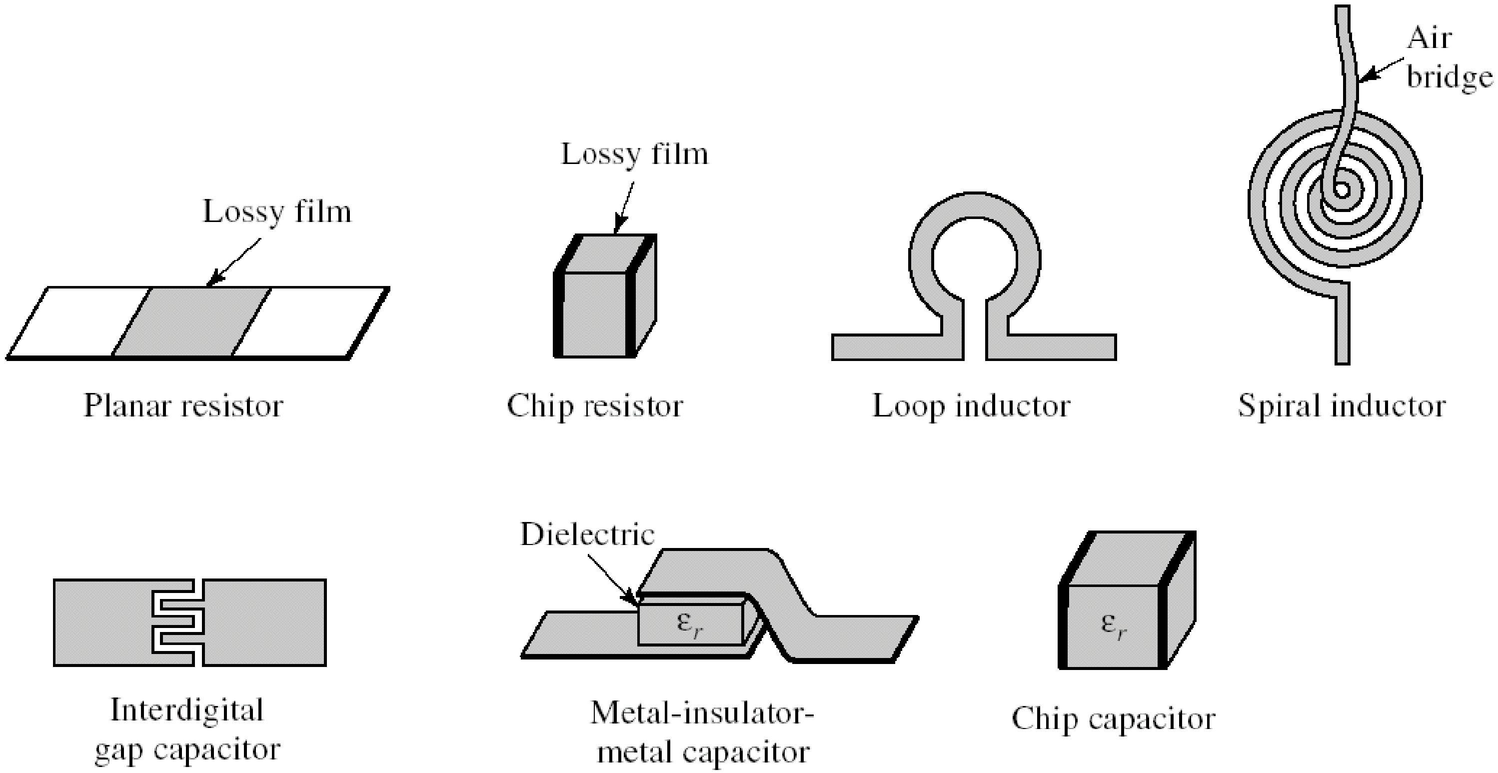






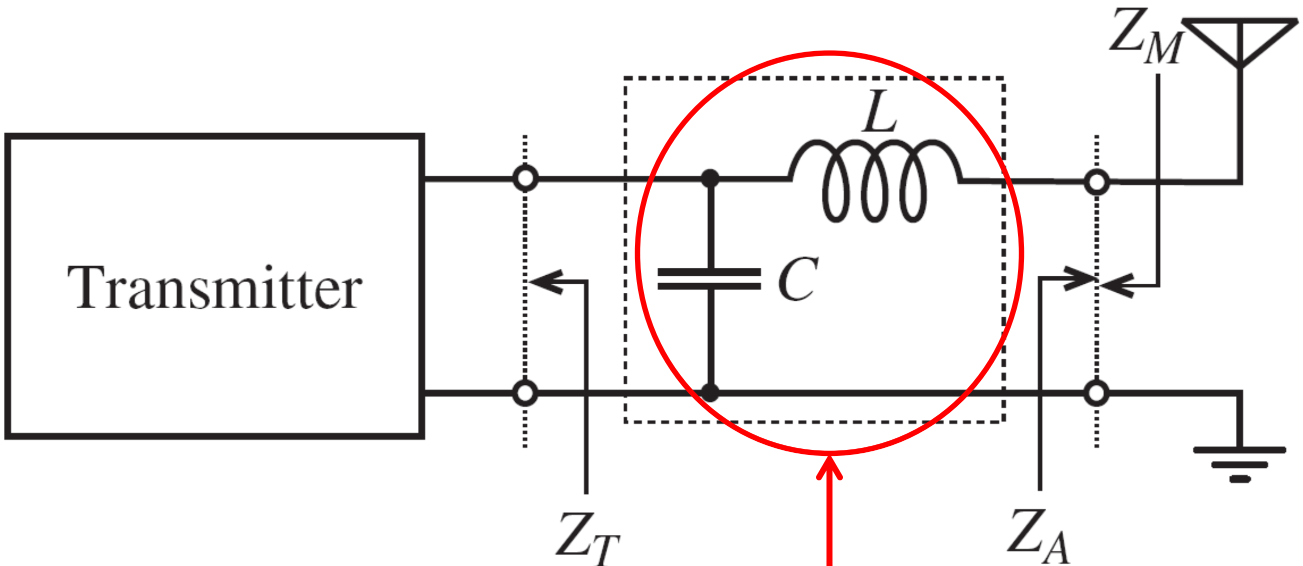
L – Type Matching Network (contd.)

- In addition; it becomes **very** difficult to build quality **lumped** elements with useful values past 1 or 2 GHz. Thus, L-network solutions are generally applicable only in the relatively low **RF region** (i.e., < 2GHz).



Example – 1

The output impedance of a transmitter operating at a frequency of **2GHz** is Z_T = **(150 + j75)Ω**. Design an L–Type matching network, as shown below, such that maximum power is delivered to the antenna whose input impedance is Z_A = **(75 + j15) Ω.**

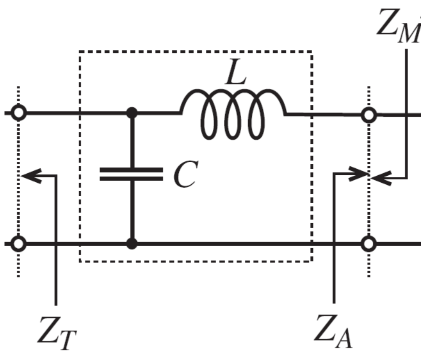


= (150 + j75)Ω

= (75 + j15) Ω.

Matching Circuit – determine C and L

Example – 1 (contd.)



Analytical Approach

The condition of the maximum power transfer implies that:

$$Z_M = Z_A^* = (75 - j15)\,\Omega$$

Series connection of L and an element (of parallel combination of C and Z_T)

$$\Rightarrow Z_M = \frac{1}{Z_T^{-1} + jB_C} + jX_L = Z_A^*$$

$$B_C = \omega C$$

$$X_L = \omega L$$

Let us write:

$$Z_T = R_T + jX_T \qquad Z_A = R_A + jX_A$$

$$\frac{R_T + jX_T}{1 + jB_C(R_T + jX_T)} + jX_L = R_A - jX_A$$

Example – 1 (contd.)

Simplification gives:

$$R_T = R_A(1 - B_C X_T) + (X_A + X_L) B_C R_T$$

$$X_T = R_T R_A B_C - (1 - B_C X_T)(X_A + X_L)$$

$$B_C = \frac{X_T \pm \sqrt{\frac{R_T}{R_A}\left(R_T^2 + X_T^2\right) - R_T^2}}{R_T^2 + X_T^2}$$

In this example, $R_T > R_A$; therefore the square root is +ve and therefore for positive B_C (capacitor) we must choose the plus sign in this expression.

Therefore:

$$X_L = \frac{1}{B_C} - \frac{R_A(1 - B_C X_T)}{B_C R_T} - X_A$$

- **Insert the given values in the obtained expressions to get:**

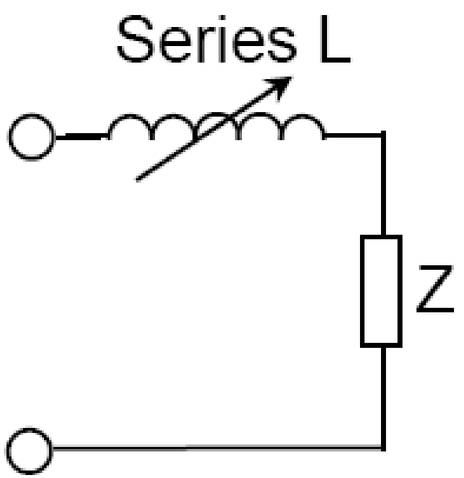
$$B_C = 9.3mS \Rightarrow C = \frac{B_C}{\omega} = 0.73pF \qquad X_L = 76.9\Omega \Rightarrow L = \frac{X_L}{\omega} = 6.1nH$$

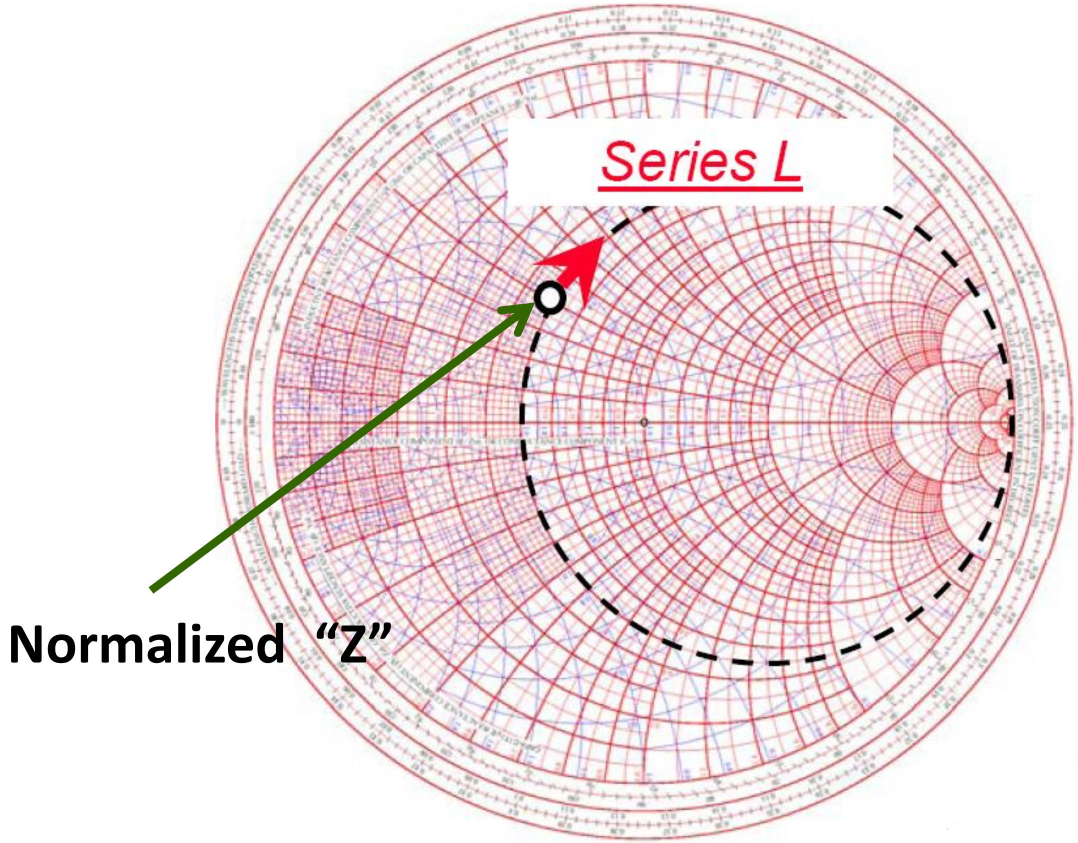
Example – 1 (contd.)

Smith Chart Based Approach

- Before we tread this path, let us have a look on Smith chart navigation when series/shunt reactances are added to any impedance (Z)

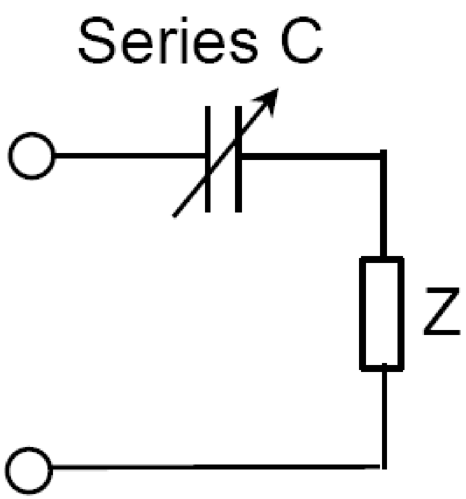
Series Connection of Inductance to a given Impedance

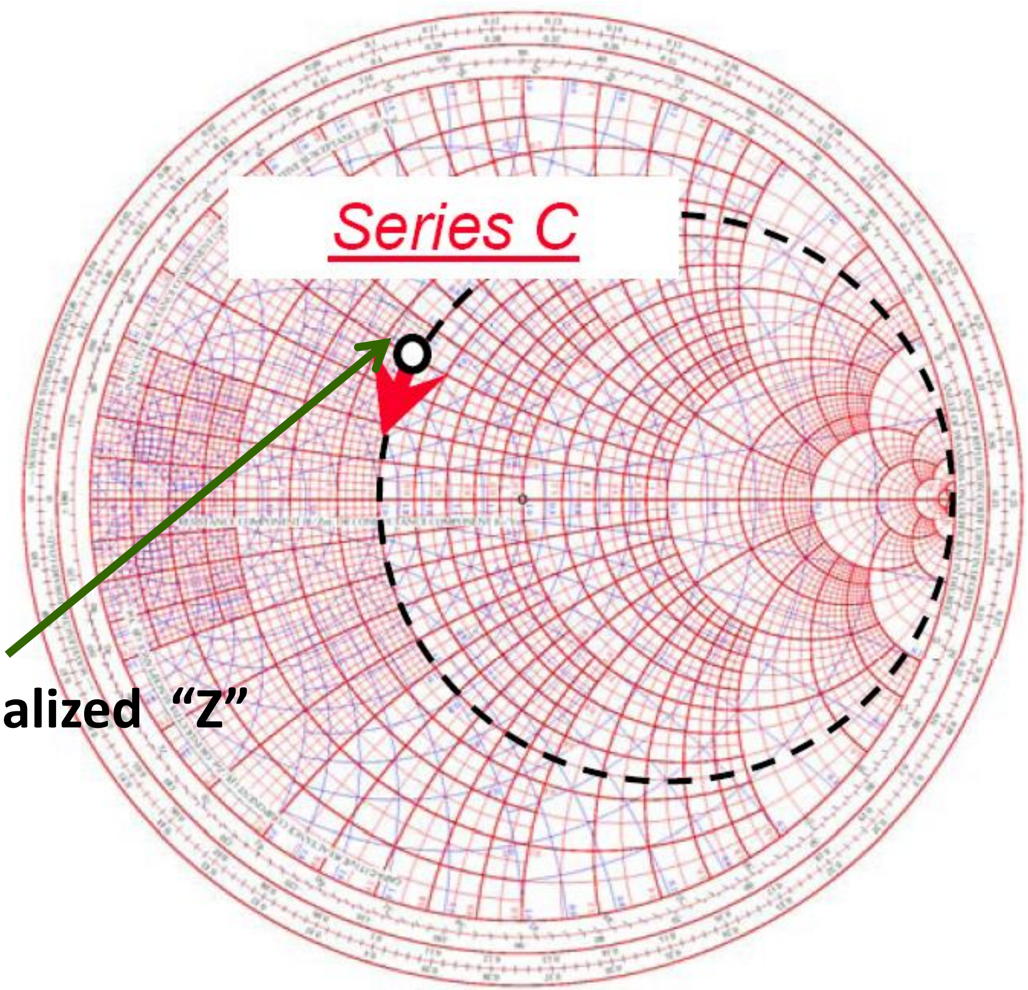




Example – 1 (contd.)

Series Connection of Capacitance to a given Impedance

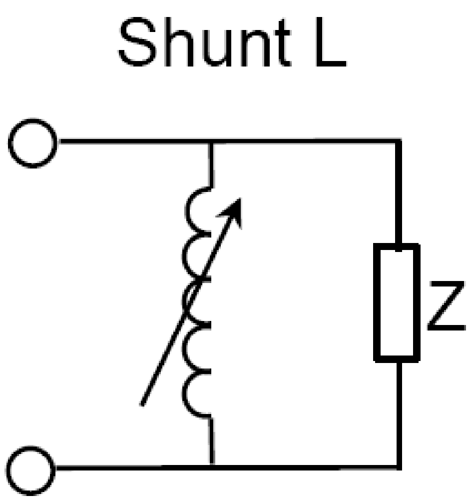


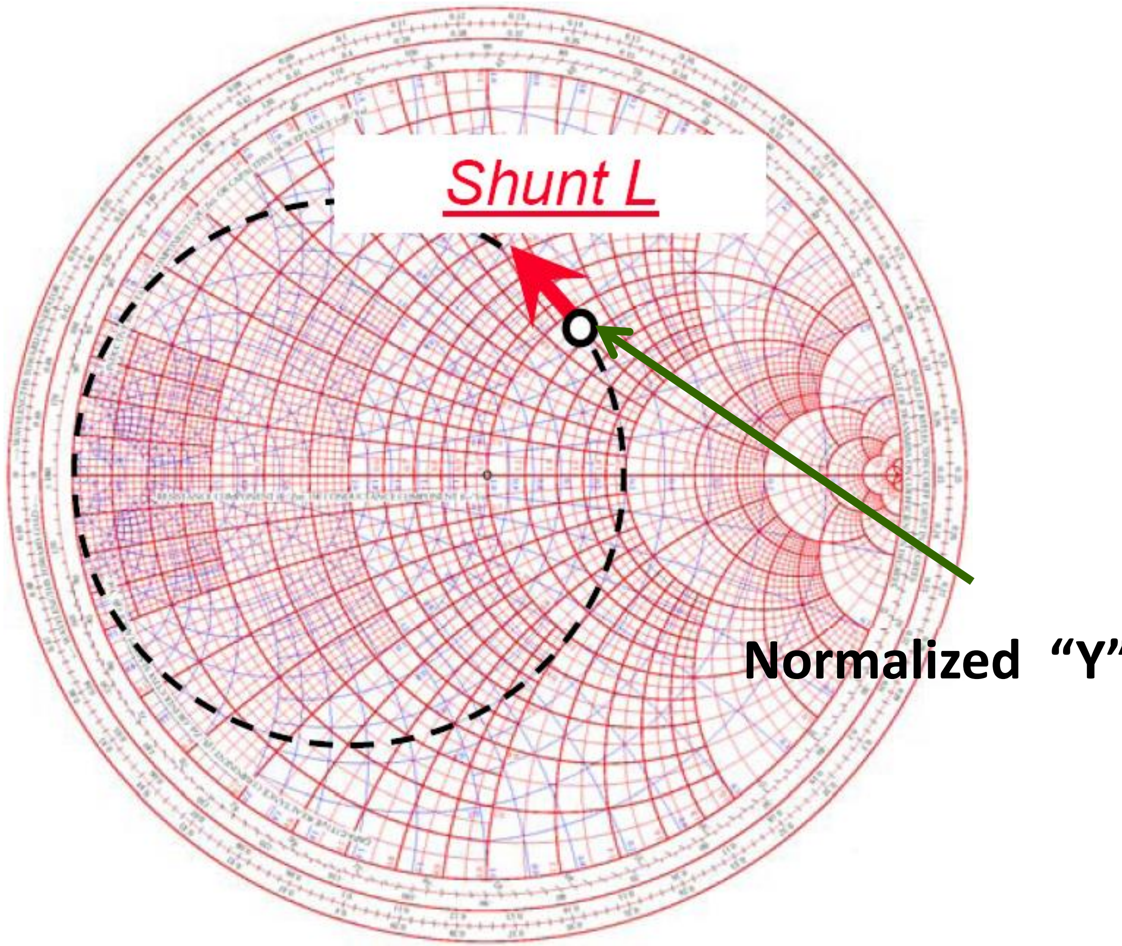


Normalized "Z"

Example – 1 (contd.)

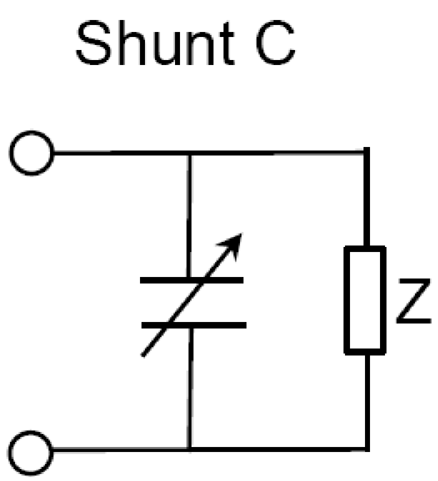
Shunt Connection of Inductance to a given Impedance

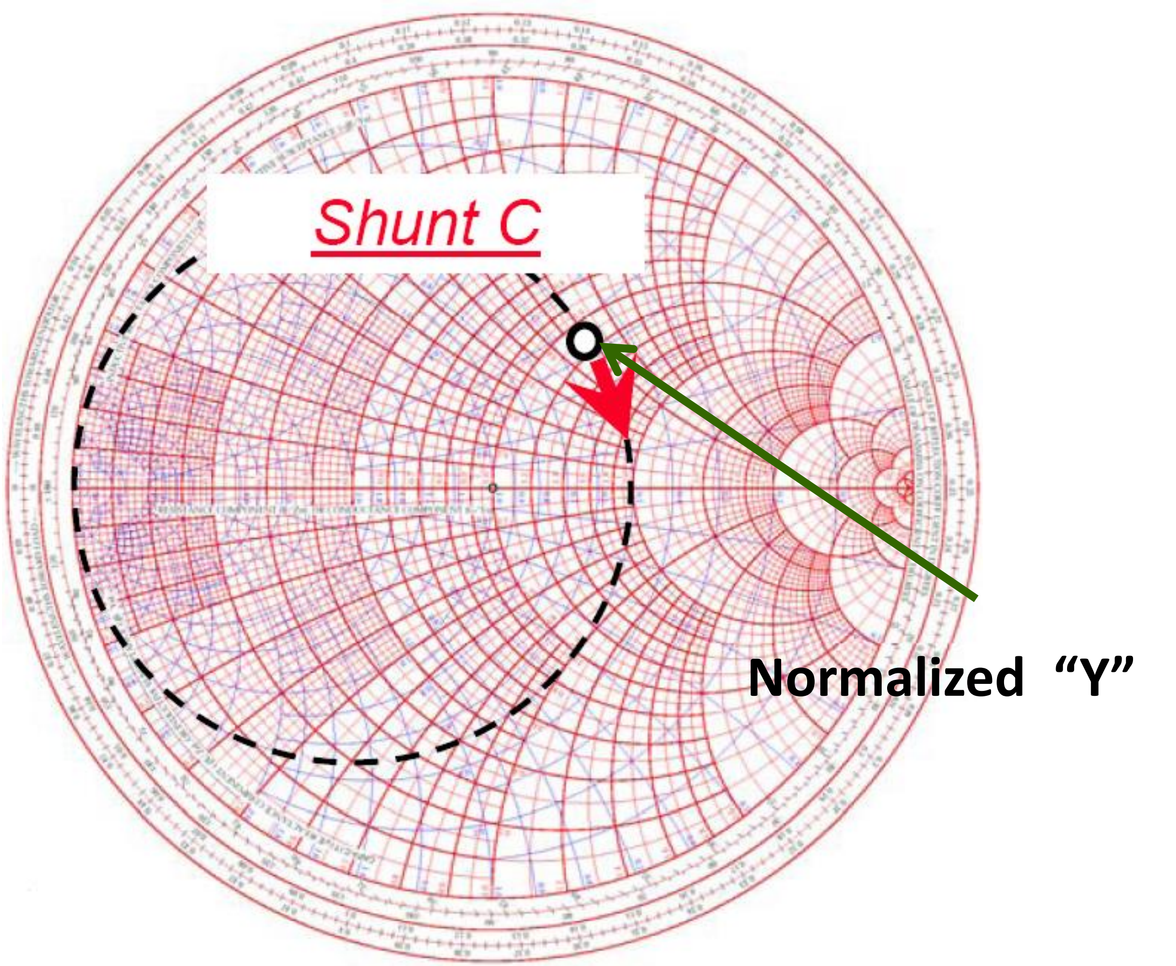




Example – 1 (contd.)

Shunt Connection of Capacitance to a given Impedance



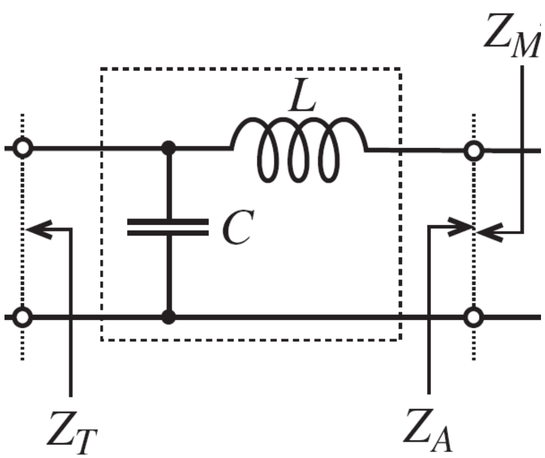


Example – 1 (contd.)

Normalize the transmitter and antenna impedances

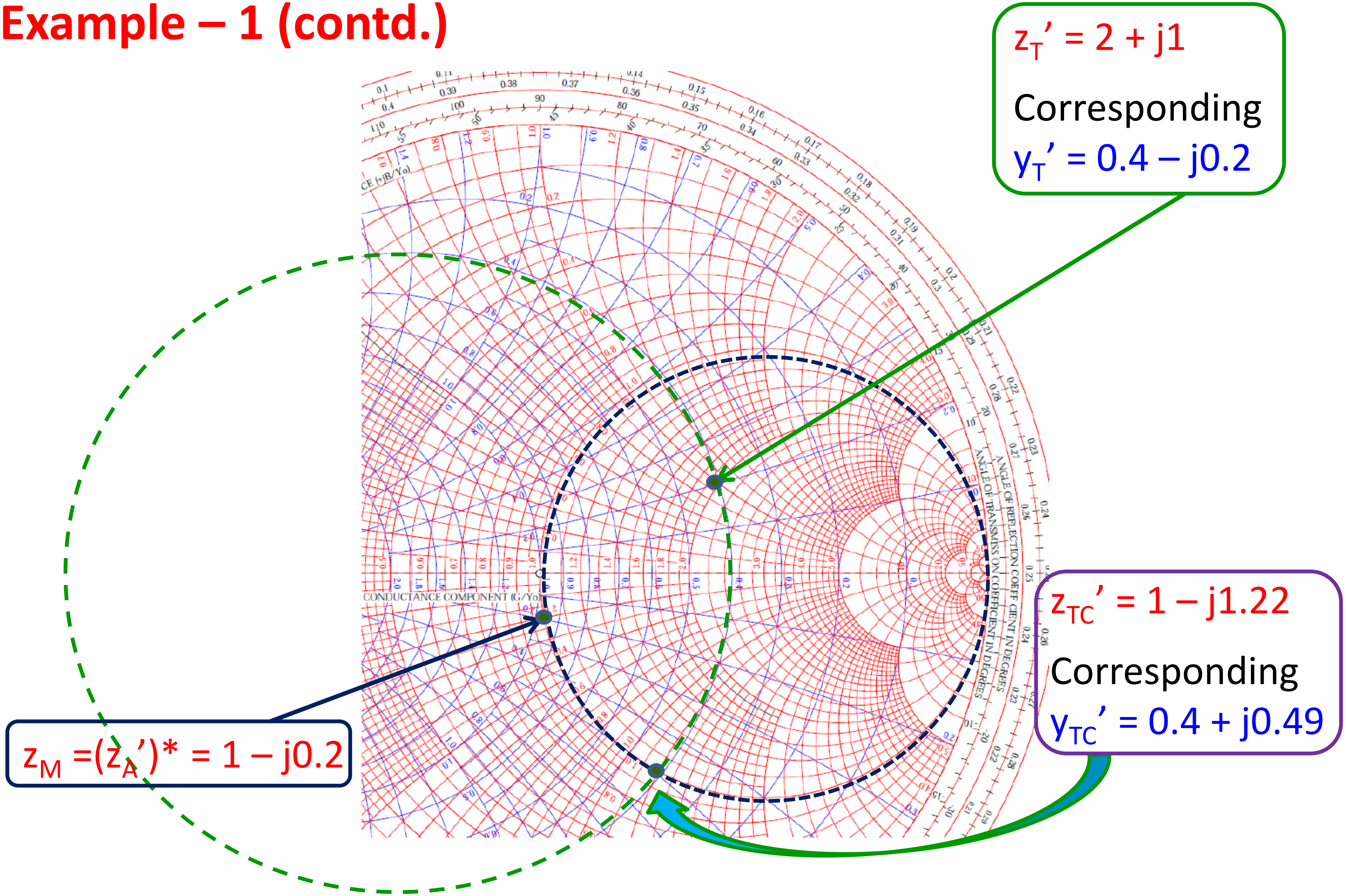
- Since no Z_0 is given, one can choose any. In this example, Z_0 = 75Ω makes simplification easier

$$z_T^{'} = Z_T / Z_0 = 2 + j1 \qquad z_A^{'} = Z_A / Z_0 = 1 + j0.2$$



- C is in shunt with Z_T → the movement will be downward on a constant conductance circle → starting point will be z_T' and the end point will be its intersection with $z_M = 1 - j0.2$ circle
- Then the movement will be upward on a constant resistance circle (from $z_M = (z_A')^* = 1 - j0.2$) → to account for the series inductance L

Example – 1 (contd.)



Example – 1 (contd.)

Therefore, the normalized susceptance jb_c is:

$$jb_c = y'_{TC} - y'_T = 0.4 + j0.49 - (0.4 - j0.2) = j0.69$$

Similarly, the normalized reactance jx_L is:

$$jx_L = z^{*'}_A - z'_{TC} = 1 - j0.2 - (1 - j1.22) = j1.02$$

Finally,

$$L = \frac{x_L Z_0}{\omega} = 6.09nH$$

$$C = \frac{b_c}{\omega Z_0} = 0.73pF$$