Lecture – 9

Date: 01.02.2017

- Even- and Odd-mode Analysis
- Generalized S-Parameters
- Example
- T-Parameters

Odd/Even Mode Analysis

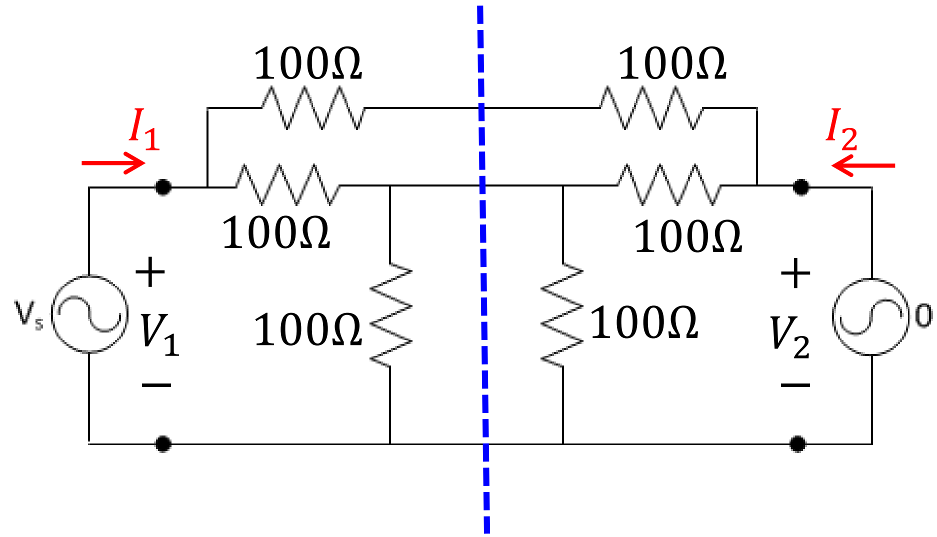
Q: Although symmetric **circuits** appear to be plentiful in microwave engineering, it seems **unlikely** that we would often encounter symmetric **sources** . Do virtual shorts and opens typically ever occur?

A: One word—**superposition**!

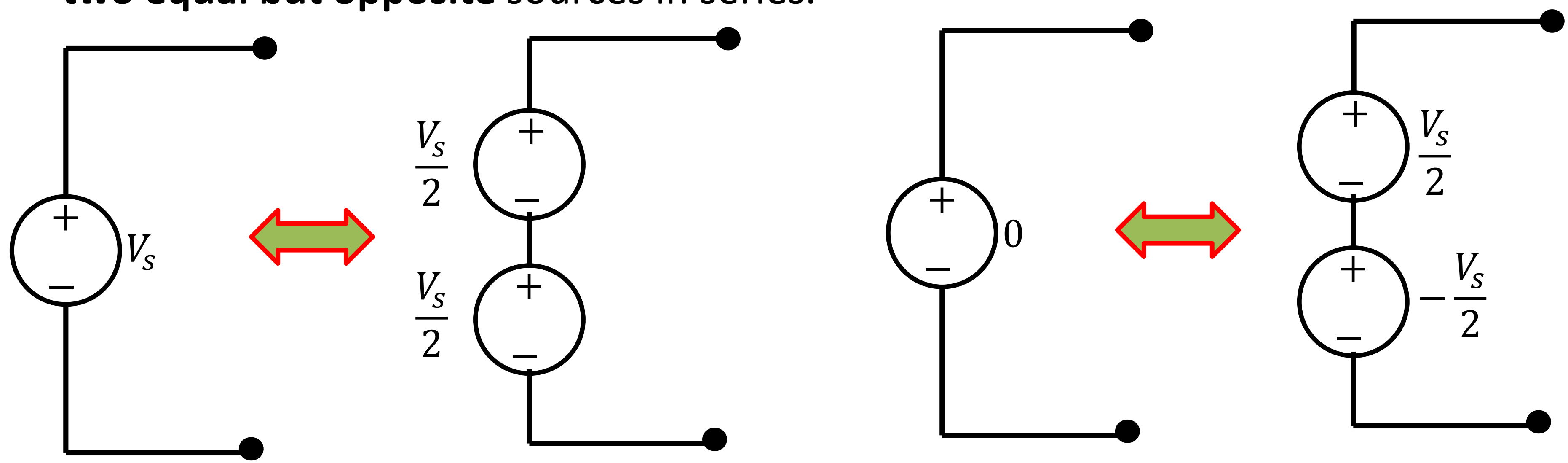
If the elements of our circuit are **independent** and **linear**, we can apply superposition to analyze **symmetric circuits** when **non-symmetric** sources are attached.

Odd/Even Mode Analysis (contd.)

- For example, we wish to determine the **admittance matrix** of this circuit. We would place a **voltage source at port 1**, and a **short circuit at port 2**—a set of **asymmetric** sources if there ever was one!

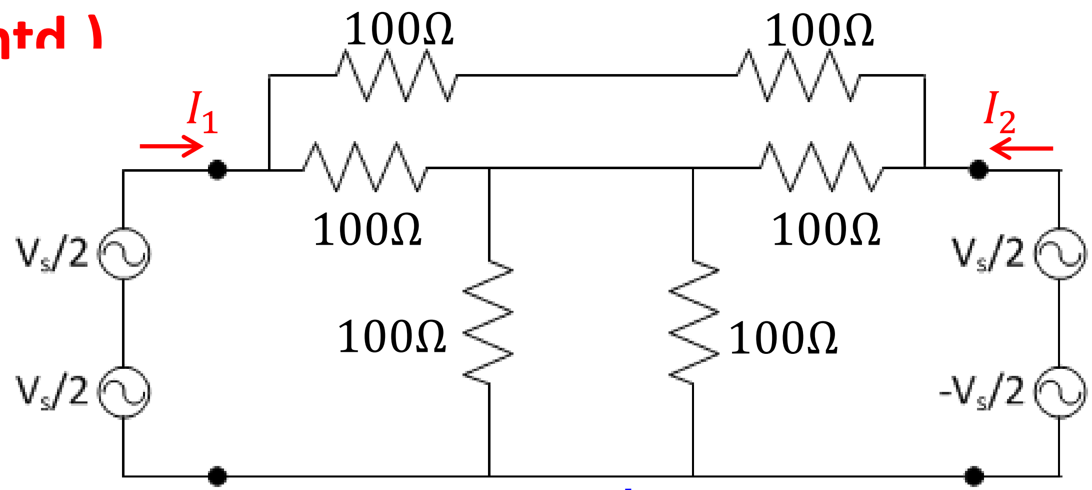


- Here's the really **neat** part. Actually the source on port 1 can be modelled as **two equal** voltage sources in series, whereas the source at port 2 can be modelled as **two equal but opposite** sources in series.

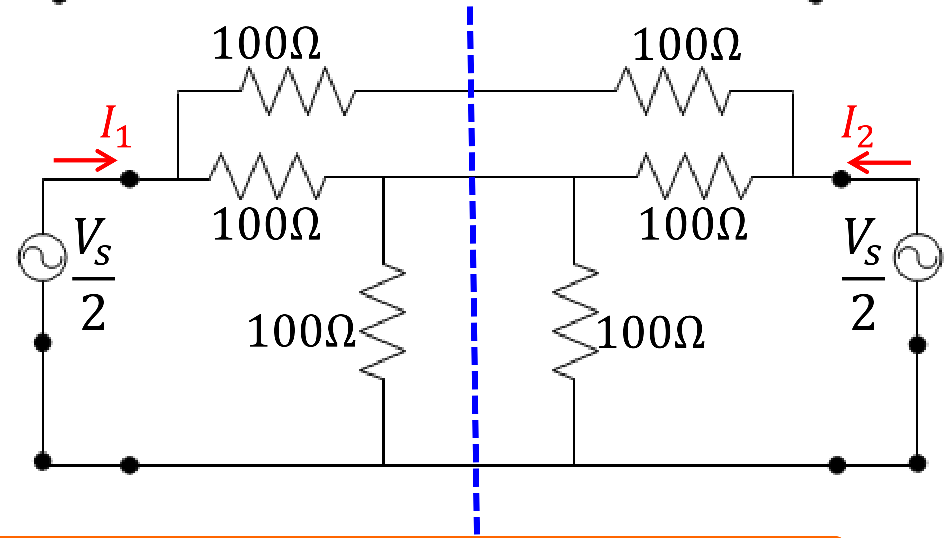


Odd/Even Mode Analysis (contd.)

- Therefore an **equivalent** circuit is:



- the **above** circuit (due to the sources) is obviously **asymmetric**—**no** virtual ground, **nor** virtual short is present. But, let's say we **turn off** (i.e., set to V =0) the **bottom** source on **each side** of the circuit:



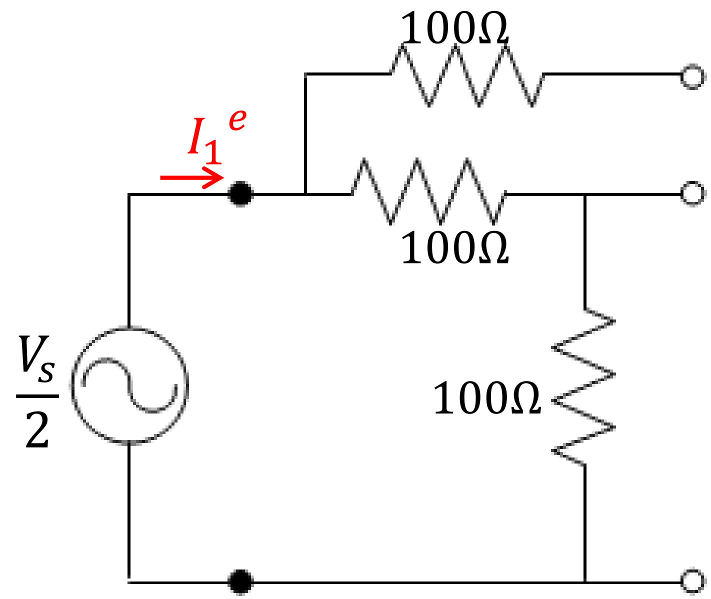
Our **symmetry** has been **restored**! The symmetry plane is a **virtual open**.

- The circuit is referred to as its **even mode**, and analysis of it is known as the **even mode analysis**. The solutions are known as the even mode **currents** and **voltages**!

Odd/Even Mode Analysis (contd.)

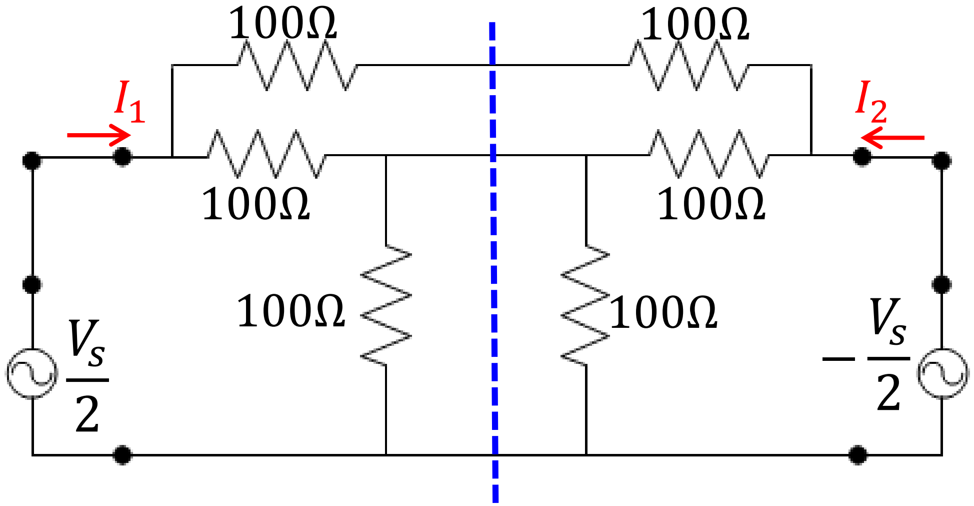
- Evaluating the resulting **even mode** half circuit we find:

$$I_1^e = \frac{V_s}{2}\frac{1}{200} = \frac{V_s}{400} = I_2^e$$

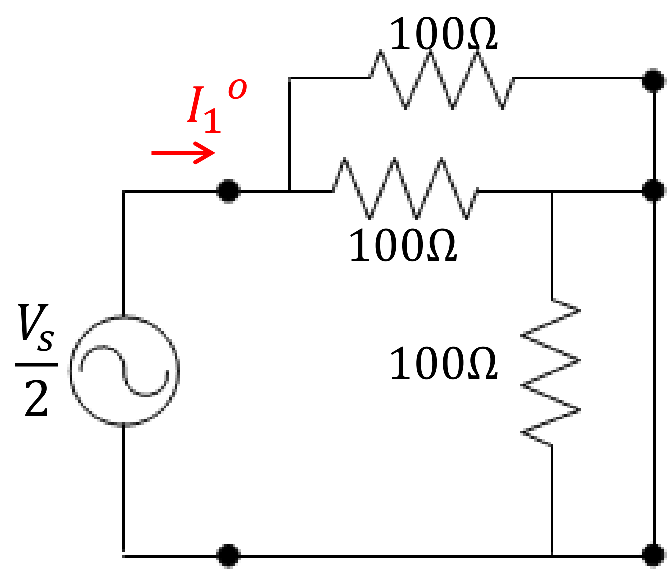


- Now, let's turn the bottom sources **back on**—but turn **off** the **top two**!
- We now have a circuit with **odd symmetry**—the symmetry plane is a **virtual short**!

This circuit is referred to as its **odd mode**, and analysis of it is known as the **odd mode analysis**.



Odd/Even Mode Analysis (contd.)



- Evaluating the resulting **odd mode** half circuit we find:

$$\boxed{I_1^o = \frac{V_s}{2}\frac{1}{50} = \frac{V_s}{100} = -I_2^o}$$

Q: But what good is this "even mode" and "odd mode" analysis? After all, the source on port 1 is $V_{s1} = V_s$, and the source on port 2 is $V_{s2} = 0$. What are the currents $I_1 = I_2$ for **these** sources?

A: Recall that these sources are the **sum** of the even and odd mode sources:

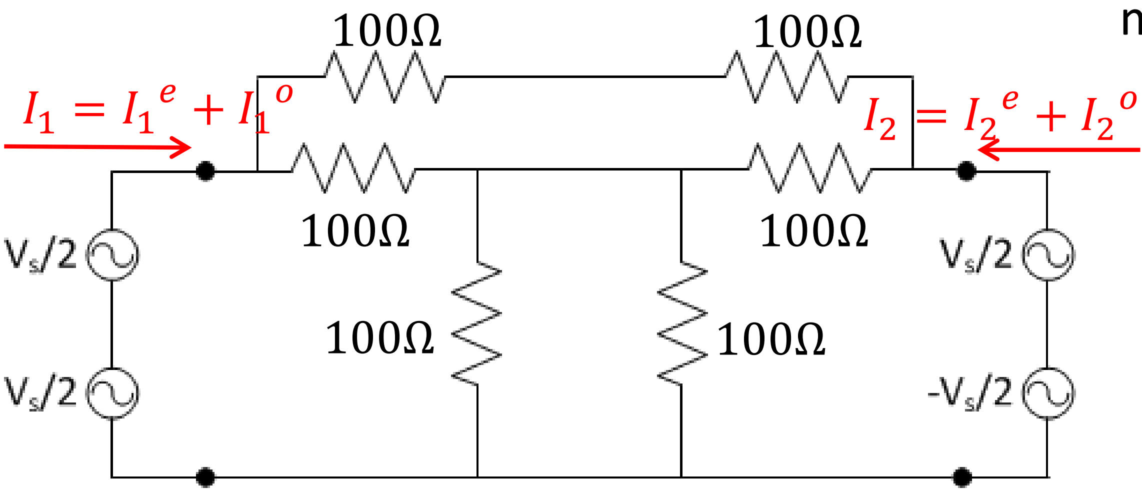
First Source: $\boxed{V_s = \frac{V_s}{2} + \frac{V_s}{2}}$ Second Source: $\boxed{V_s = \frac{V_s}{2} - \frac{V_s}{2}}$

- and thus—since all the devices in the circuit are **linear**—we know from superposition that the currents I_1 and I_2 are simply the **sum** of the **odd** and **even** mode currents !

$$\boxed{I_1 = I_1^e + I_1^o}$$

$$\boxed{I_2 = I_2^e + I_2^o}$$

Odd/Even Mode Analysis (contd.)



- Thus, **adding** the odd and even mode analysis results together:

$$I_1 = I_1^e + I_1^o = \frac{V_s}{400} + \frac{V_s}{100} = \frac{V_s}{80}$$

$$I_2 = I_2^e + I_2^o = \frac{V_s}{400} - \frac{V_s}{100} = -\frac{3V_s}{400}$$

- And then the **admittance parameters** for this two port network is:

$$y_{11} = \frac{I_1}{V_{s1}}\bigg|_{V_{s2}=0} = \frac{V_s}{80}\frac{1}{V_s} = \frac{1}{80}$$

$$y_{21} = \frac{I_2}{V_{s1}}\bigg|_{V_{s2}=0} = -\frac{3V_s}{400}\frac{1}{V_s} = \frac{-3}{400}$$

- And from the **symmetry** of the device we know:

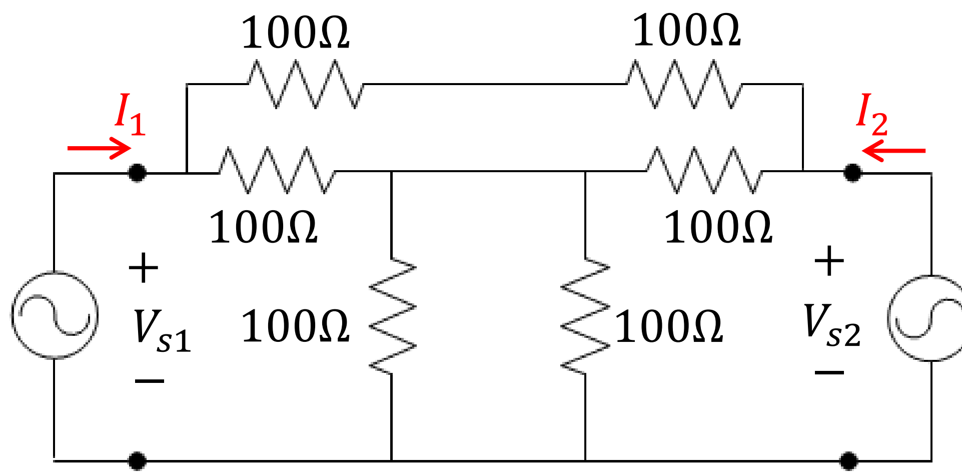
$$y_{22} = y_{11} = \frac{1}{80}$$

$$y_{12} = y_{21} = \frac{-3}{400}$$

- Thus, the full **admittance matrix** is:

$$Y = \begin{bmatrix} 1/80 & -3/400 \\ -3/400 & 1/80 \end{bmatrix}$$

Odd/Even Mode Analysis (contd.)



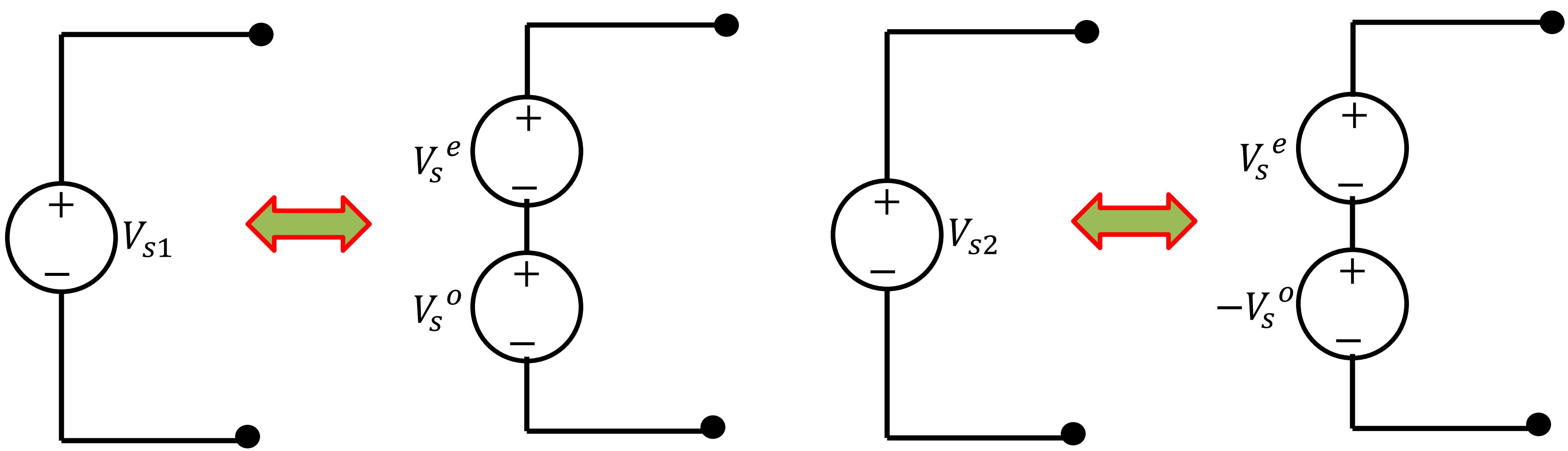
Q: What happens if **both** sources are **non-zero**? Can we use symmetry then?

A: Absolutely! Consider this problem, where **neither** source is equal to zero:

- In this case we can define an even mode and an odd mode source as:

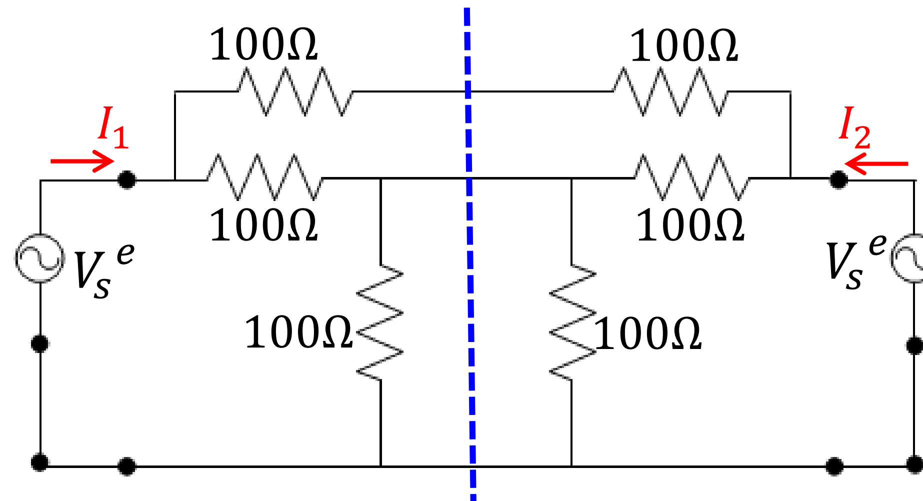
$$V_s^e = \frac{V_{s1} + V_{s2}}{2}$$

$$V_s^o = \frac{V_{s1} - V_{s2}}{2}$$

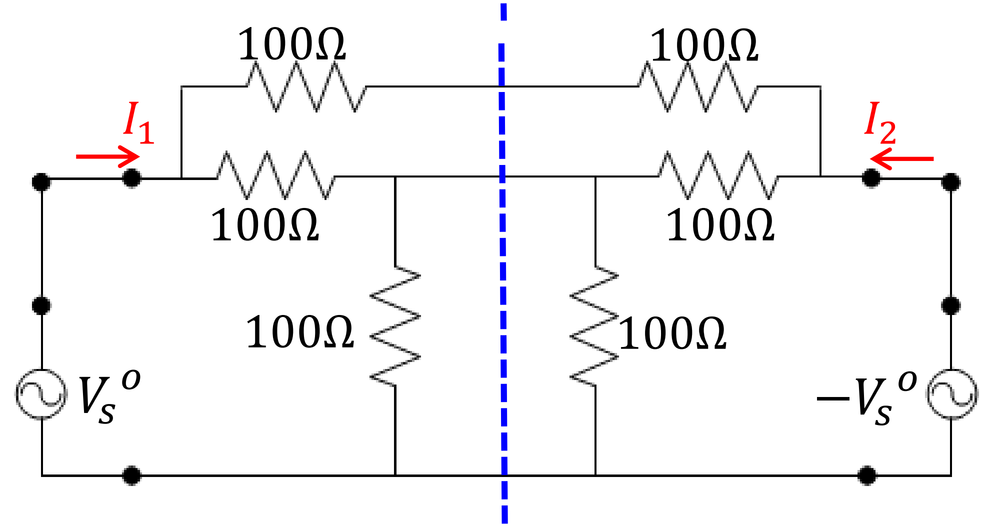


Odd/Even Mode Analysis (contd.)

- We can then analyze the **even mode** circuit:



- And then the **odd mode** circuit:



And then combine these results in a **linear superposition**!

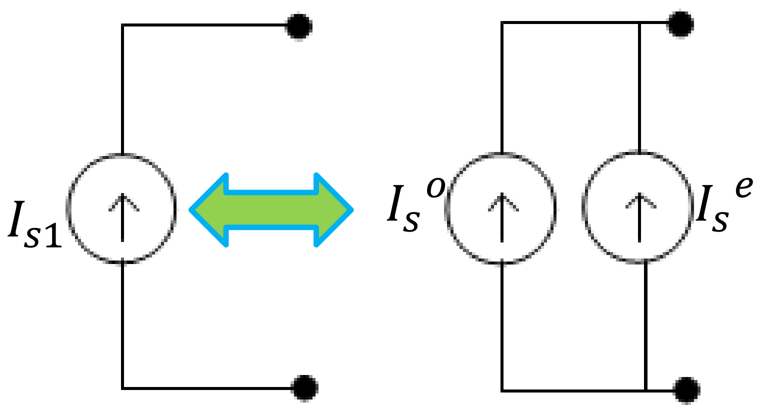
Odd/Even Mode Analysis (contd.)

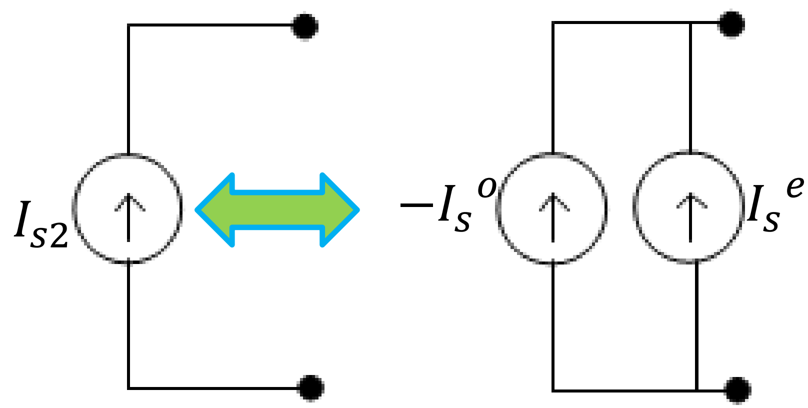
Q: What about **current sources**? Can I likewise consider them to be a **sum** of an odd mode source and an even mode source?

A: Yes, but be **very** careful! The current of two source will add if they are placed in **parallel**—**not** in series! Therefore:

$$I_s^e = \frac{I_{s1} + I_{s2}}{2}$$

$$I_s^o = \frac{I_{s1} - I_{s2}}{2}$$



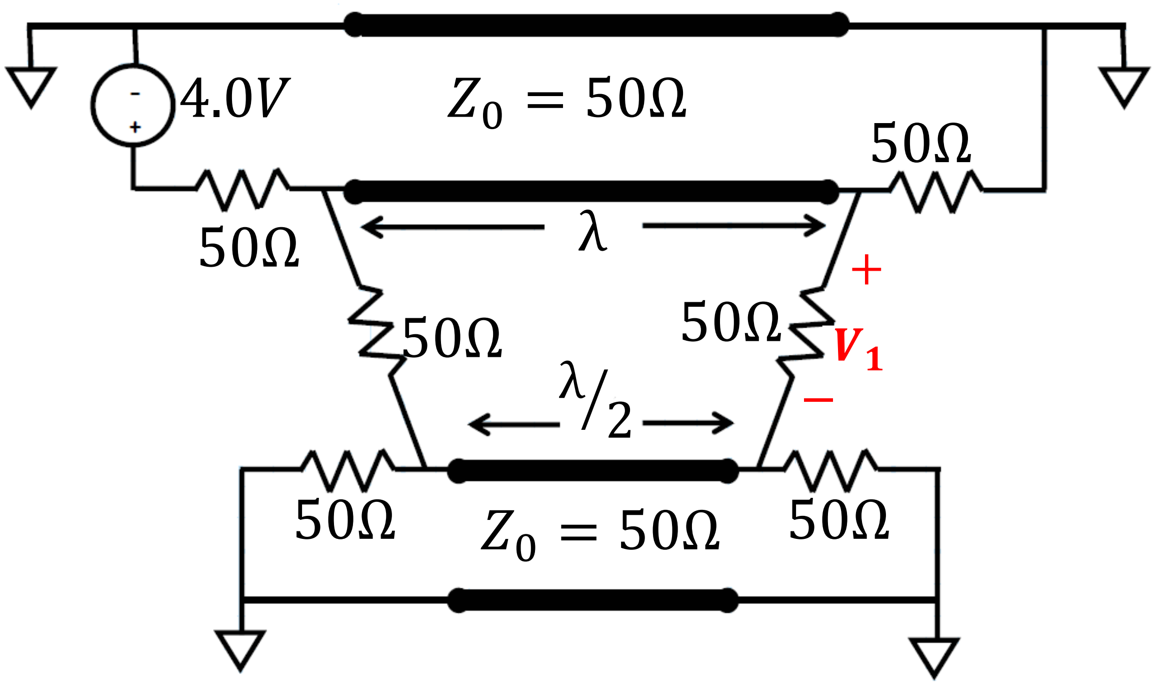


- One **final** word (I promise!) about circuit symmetry and even/odd mode analysis: **precisely** the **same** concept exits in **electronic circuit** design!

Specifically, the **differential** (odd) and **common** (even) **mode** analysis of bilaterally symmetric electronic circuits, such as **differential amplifiers**!

Example – 1

- Carefully (**very** carefully) consider the **symmetric** circuit below:

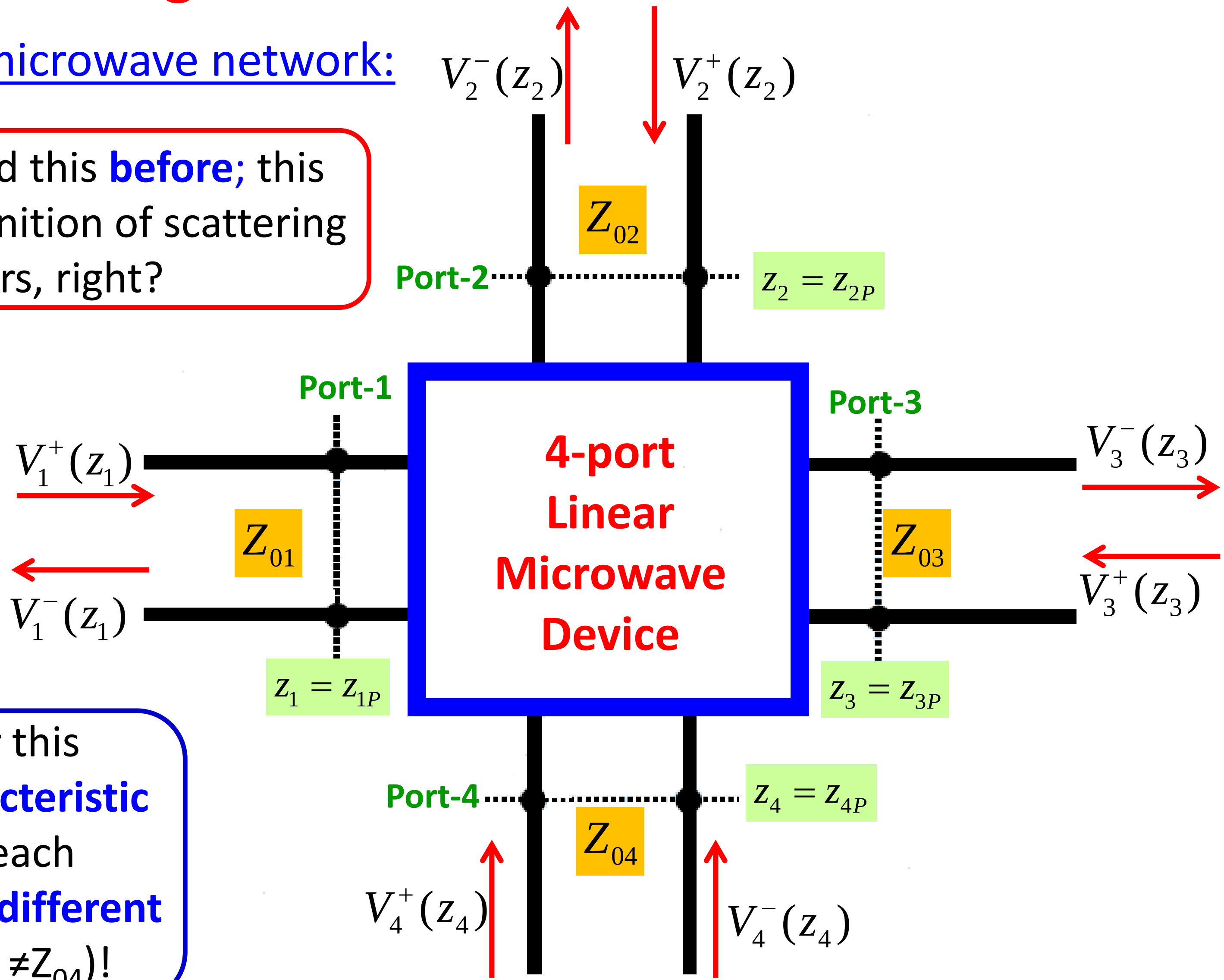


Use **odd-even mode analysis** to determine the value of voltage V_1.

Generalized Scattering Parameters

Consider now **this** microwave network:

Boring! We studied this **before**; this will lead to the definition of scattering parameters, right?



Not exactly. For this network, the **characteristic impedance** of each transmission line is **different** (i.e., $Z_{01} \neq Z_{02} \neq Z_{03} \neq Z_{04}$)!

Generalized Scattering Parameters (contd.)

Yikes! You said the s-parameters are **dependent** on transmission line characteristic impedance Z_0. If these values are **different** for each port, **which** Z_0 do we use?

For this **general** case, we must use **generalized scattering parameters**! First, we define a slightly new form of complex wave amplitudes

$$a_n = \frac{V_n^+}{\sqrt{Z_{0n}}} \qquad b_n = \frac{V_n^-}{\sqrt{Z_{0n}}}$$

- <u>**The key things to note are:**</u>

a

variable a (e.g., a_1,a_2, …) denotes the complex amplitude of an **incident (i.e., plus)** wave.

b

variable b (e.g., a_1,a_2, …) denotes the complex amplitude of an **exiting (i.e., minus)** wave.

<u>We now get to **rewrite** all our transmission line knowledge in terms of these generalized complex amplitudes!</u>

Generalized Scattering Parameters (contd.)

- First, our two propagating wave amplitudes (i.e., plus and minus) are **compactly** written as:

$$V_n^+ = a_n\sqrt{Z_{0n}} \qquad V_n^- = b_n\sqrt{Z_{0n}}$$

- Therefore:

$$V_n^+(z_n) = a_n\sqrt{Z_{0n}}.e^{-j\beta z_n} \qquad V_n^-(z_n) = b_n\sqrt{Z_{0n}}.e^{+j\beta z_n} \qquad \Gamma(z_n) = \frac{b_n}{a_n}e^{+j2\beta z_n}$$

- Similarly, the total voltage, current, and impedance at the **nth** port are:

$$V_n(z_n) = \sqrt{Z_{0n}}\left(a_n.e^{-j\beta z_n} + b_n e^{+j\beta z_n}\right)$$

$$I_n(z_n) = \frac{\left(a_n.e^{-j\beta z_n} - b_n e^{+j\beta z_n}\right)}{\sqrt{Z_{0n}}} \qquad Z(z_n) = \frac{a_n.e^{-j\beta z_n} + b_n e^{+j\beta z_n}}{a_n.e^{-j\beta z_n} - b_n e^{+j\beta z_n}}$$

- Assuming that our port planes are defined with $z_{nP} = 0$, we can determine the total voltage, current, and impedance **at port n** as:

$$V_n \doteq V_n(z_n = 0) = \sqrt{Z_{0n}}\left(a_n + b_n\right)$$

$$I_n \doteq I_n(z_n = 0) = \frac{\left(a_n - b_n\right)}{\sqrt{Z_{0n}}} \qquad Z_n \doteq Z(z_n = 0) = \frac{a_n + b_n}{a_n - b_n}$$

Generalized Scattering Parameters (contd.)

- Similarly, the **power** associated with each wave is:

$$P_n^+ = \frac{|V_n^+|^2}{2Z_{0n}} = \frac{|a_n|^2}{2}$$

$$P_n^- = \frac{|V_n^-|^2}{2Z_{0n}} = \frac{|b_n|^2}{2}$$

- As such, the power **delivered to** port n (i.e., the power **absorbed by** port n) is:

$$P_n = P_n^+ - P_n^- = \frac{|a_n|^2 - |b_n|^2}{2}$$

So what's the **big deal**? This is yet **another way** to express transmission line activity.

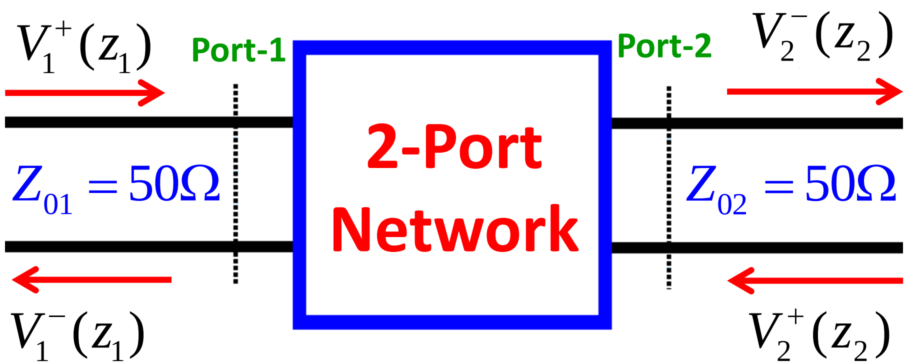
Do we **really** need to know this, or is this simply a strategy for making the next quiz **even harder**?

- You may have noticed that this notation (a_n, b_n) provides descriptions that are a bit **"cleaner"** and more symmetric between current and voltage.
- However, the **main reason** for this notation is for evaluating the **scattering parameters** of a device with **dissimilar** transmission line impedance (e.g., $Z_{01} \neq Z_{02} \neq Z_{03} \neq Z_{04}$).

Generalized Scattering Parameters (contd.)

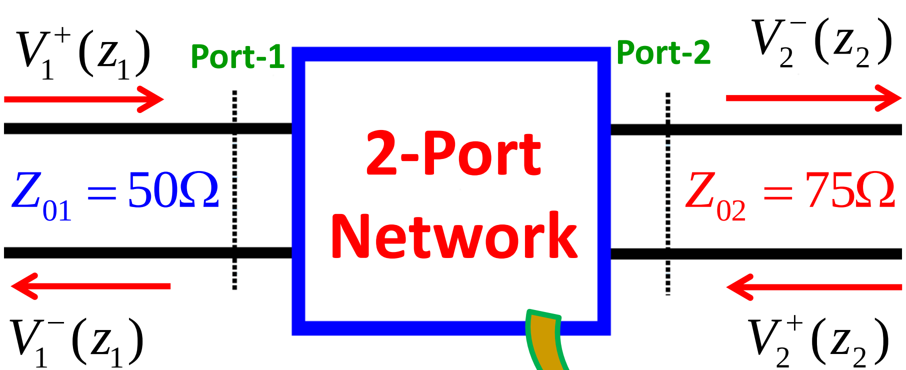
- For these cases we must use **generalized scattering parameters:**

$$S_{mn} = \frac{V_m^-}{V_n^+}\frac{\sqrt{Z_{0n}}}{\sqrt{Z_{0m}}}$$ when $V_k^+(z_k) = 0$ for all $k \neq n$)



Here, $Z_{01} = Z_{02}$

$$S_{21} = \frac{V_2^-}{V_1^+}$$ when, $V_2^+ = 0$



Here, $Z_{01} \neq Z_{02}$

$$S_{21} \neq \frac{V_2^-}{V_1^+}$$ when, $V_2^+ = 0$

Instead

$$S_{21} = \frac{V_2^-}{V_1^+}\frac{\sqrt{50}}{\sqrt{75}}$$ when, $V_2^+ = 0$

Generalized Scattering Parameters (contd.)

- generalized s-parameters can be more **compactly** written in terms of our **new** wave amplitude notation:

$$S_{mn} = \frac{V_m^-}{V_n^+}\frac{\sqrt{Z_{0n}}}{\sqrt{Z_{0m}}} = \frac{b_m}{a_n} \qquad \text{when } a_k = 0 \text{ for all } k \neq n)$$

generalized form of scattering parameter—**always** provides the correct answer, **regardless** of the values of Z_{0m} or Z_{0n}!

- But **why can't** we define the scattering parameter as $S_{mn} = V_m^-/V_n^+$, **regardless** of Z_{0m} or Z_{0n}?? **Who says** we must define it with those **awful** Z_{0n} values in there?

Recall that a lossless device will **always** have a **unitary** scattering matrix. As a result, the scattering parameters of a lossless device will **always** satisfy, for example:

$$\sum_{m=1}^{N} \left|S_{mn}\right|^2 = 1$$

This is true only if generalized scattering parameters are used

The scattering parameters of a lossless device will form a unitary matrix **only** if defined as $S_{mn} = b_m/a_n$. If we use $S_{mn} = V_m^-/V_n^+$, the matrix will be unitary **only** if the connecting transmission lines have the **same** characteristic impedance.

Generalized Scattering Parameters (contd.)

- Do we really **care** if the matrix of a lossless device is unitary or not?

Absolutely! we do!

lossless device $\Leftrightarrow$ unitary scattering matrix

This relationship is a very powerful one. It allows us to **identify** lossless devices, and it allows us to determine **if** specific lossless devices are **even possible**!

Example – 2

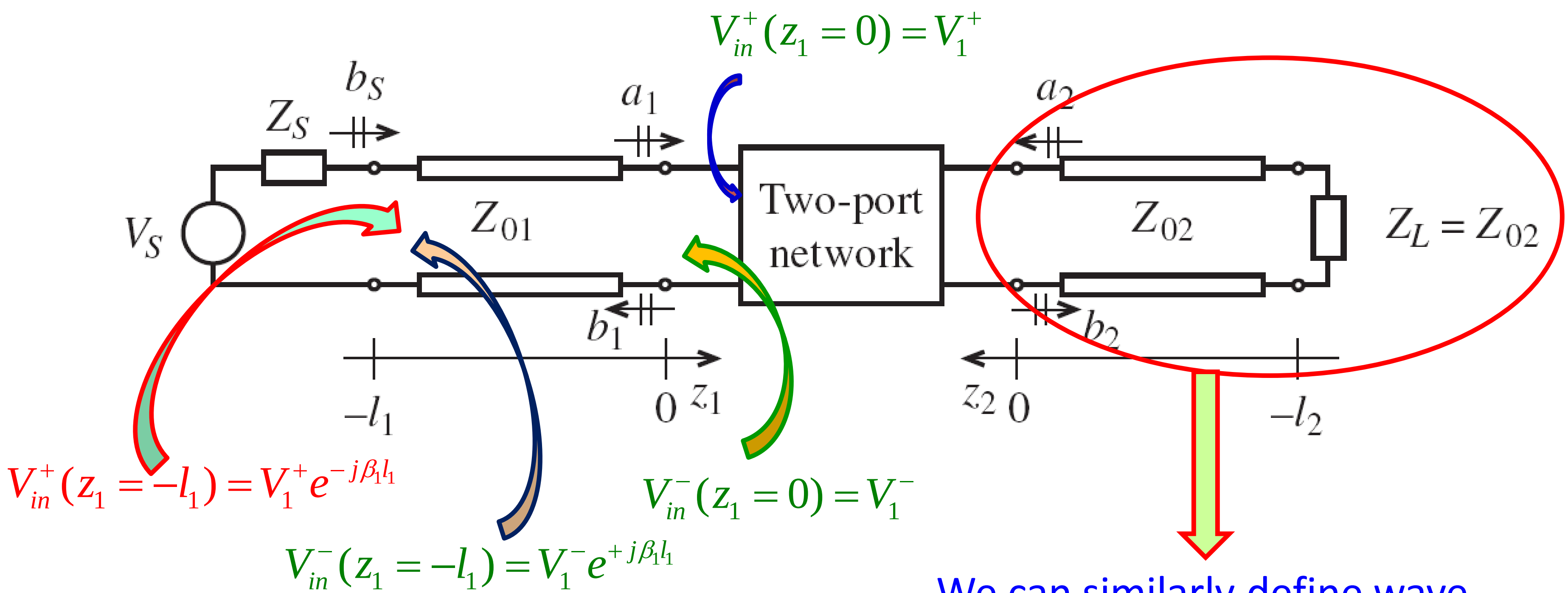
- let's consider a **perfect connector**—an electrically **very small** two-port device that allows us to connect the ends of different transmission lines together.

Determine the S-matrix of this ideal connector:

1. First case: it connects two transmission lines with same characteristic impedance of Z_0.
2. Second case: it connects two transmission lines with characteristic impedances of Z_{01} and Z_{02} respectively.

Shifting of Planes

- It is not often easy or feasible to match network ports for determination of S-parameters→ in such a situation S-parameters are determined through transmission lines of finite length.
- Let us consider a 2-port network to understand these situations.



We can similarly define wave functions on the output

Shifting of Planes (contd.)

- The equations can be combined to form following matrix

$$\begin{Bmatrix} V_{in}^{+}(-l_1) \\ V_{out}^{+}(-l_2) \end{Bmatrix} = \begin{bmatrix} e^{-j\beta_1 l_1} & 0 \\ 0 & e^{-j\beta_2 l_2} \end{bmatrix} \begin{Bmatrix} V_1^{+} \\ V_2^{+} \end{Bmatrix}$$

Links the incident waves at the network ports shifted by TL segments

$$\begin{Bmatrix} V_{in}^{-}(-l_1) \\ V_{out}^{-}(-l_2) \end{Bmatrix} = \begin{bmatrix} e^{+j\beta_1 l_1} & 0 \\ 0 & e^{+j\beta_2 l_2} \end{bmatrix} \begin{Bmatrix} V_1^{-} \\ V_2^{-} \end{Bmatrix}$$

Links the incident waves at the network ports shifted by TL segments

- We can also deduce that S-parameters are linked to the generalized coefficients a_n and b_n (which in turn can be expressed through voltages) through following expressions (if we assume $Z_{01} = Z_{02}$)

$$\begin{Bmatrix} V_1^{-} \\ V_2^{-} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{Bmatrix} V_1^{+} \\ V_2^{+} \end{Bmatrix}$$

- Simplification of these three matrix expression results in:

$$\begin{Bmatrix} V_{in}^{-}(-l_1) \\ V_{out}^{-}(-l_2) \end{Bmatrix} = \underbrace{\begin{bmatrix} e^{+j\beta_1 l_1} & 0 \\ 0 & e^{+j\beta_2 l_2} \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} e^{+j\beta_1 l_1} & 0 \\ 0 & e^{+j\beta_2 l_2} \end{bmatrix}}_{\text{S-parameters of the shifted network } [S]^{SHIFT}} \begin{Bmatrix} V_{in}^{+}(-l_1) \\ V_{out}^{+}(-l_2) \end{Bmatrix}$$

Shifting of Planes (contd.)

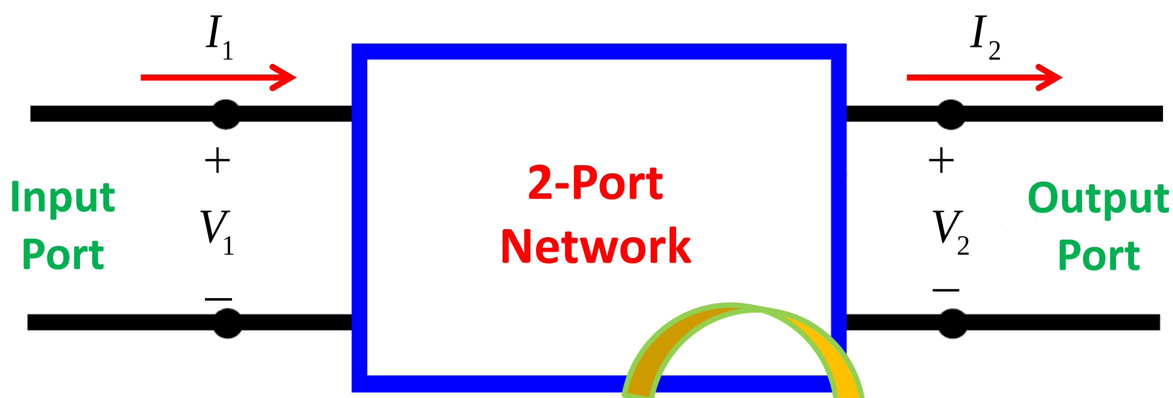
$$[S]^{SHIFT} = \begin{bmatrix} S_{11}e^{+j2\beta_1 l_1} & S_{12}e^{+j(\beta_1 l_1 + \beta_2 l_2)} \\ S_{21}e^{+j(\beta_1 l_1 + \beta_2 l_2)} & S_{22}e^{+j2\beta_2 l_2} \end{bmatrix}$$

Physical Meaning

The first term **(S_{11})** reveals that we have to take into account twice the travel time for the incident voltage to reach port-1 and, upon reflection, return to the end of the TL segment. Similarly for **S_{22}** at port-2. The cross terms **(S_{12} and S_{21})** require additive phase shifts associated with TL segments at port-1 and port-2

The Transmission Matrix

- If a network has **two** ports, then we can **alternatively** define the voltages and currents at each port as:



For such network, we relate the i/p port parameters (I_1 and V_1) and o/p port parameters (I_2 and V_2) using transmission parameters also known as ABCD parameters

The Transmission Matrix (contd.)

$$V_1 = AV_2 + BI_2$$
$$I_1 = CV_2 + DI_2$$

$$\begin{Bmatrix} V_1 \\ I_1 \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} V_2 \\ I_2 \end{Bmatrix}$$

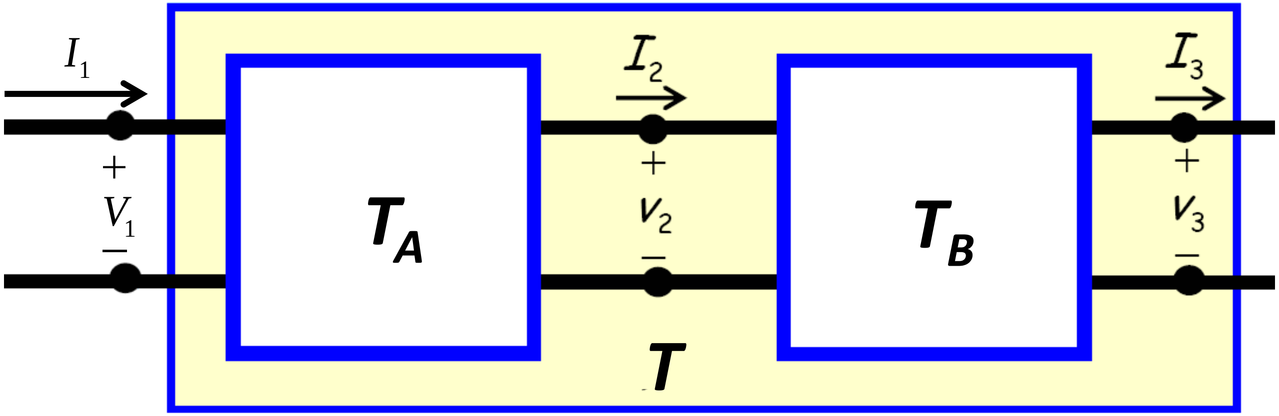
Transmission Matrix 'T'

- Similar to the impedance and admittance matrices, we determine the elements of the transmission matrix using **shorts** and **opens**.
  - Note when $I_2 = 0$ then: $V_1 = AV_2$ ⇒ $A = \frac{V_1}{V_2}$ — **A is unitless (i.e., it is a coefficient)**
  - Note when $V_2 = 0$ then: $V_1 = BI_2$ ⇒ $B = \frac{V_1}{I_2}$ — **B has unit of impedance (i.e., Ohms)**
  - Note when $I_2 = 0$ then: $I_1 = CV_2$ ⇒ $C = \frac{I_1}{V_2}$ — **C has unit of admittance (i.e., mhos)**
  - Note when $V_2 = 0$ then: $I_1 = DI_2$ ⇒ $D = \frac{I_1}{I_2}$ — **D is unitless (i.e., it is a coefficient)**

The Transmission Matrix (contd.)

Crying out loud! We already have the impedance matrix, the scattering matrix, **and** the admittance matrix. **Why do we need the transmission matrix?** Is it somehow **uniquely** useful?

- Let us consider the case where a 2-port network is created by connecting (i.e., cascading) **two** networks:



$$\begin{Bmatrix} V_1 \\ I_1 \end{Bmatrix} = \boldsymbol{T}_A \begin{Bmatrix} V_2 \\ I_2 \end{Bmatrix}$$

$$\begin{Bmatrix} V_2 \\ I_2 \end{Bmatrix} = \boldsymbol{T}_B \begin{Bmatrix} V_3 \\ I_3 \end{Bmatrix}$$

$$\begin{Bmatrix} V_1 \\ I_1 \end{Bmatrix} = \boldsymbol{T} \begin{Bmatrix} V_3 \\ I_3 \end{Bmatrix}$$

The Transmission Matrix (contd.)

- Combining the first two equations we get:

$$\begin{Bmatrix} V_1 \\ I_1 \end{Bmatrix} = \boldsymbol{T}_A \begin{Bmatrix} V_2 \\ I_2 \end{Bmatrix} = \boldsymbol{T}_A \boldsymbol{T}_B \begin{Bmatrix} V_3 \\ I_3 \end{Bmatrix}$$

- Combining this combined relationship to the third we get:

$$\begin{Bmatrix} V_1 \\ I_1 \end{Bmatrix} = \boldsymbol{T}_A \begin{Bmatrix} V_2 \\ I_2 \end{Bmatrix} = \boldsymbol{T}_A \boldsymbol{T}_B \begin{Bmatrix} V_3 \\ I_3 \end{Bmatrix} = \boldsymbol{T} \begin{Bmatrix} V_3 \\ I_3 \end{Bmatrix} \Rightarrow \boxed{\boldsymbol{T} = \boldsymbol{T}_A \boldsymbol{T}_B}$$

- Similarly, for N cascaded networks, the **total** transmission matrix **T** can be determined as the product of all N networks!

$$T = T_1 T_2 T_3 \ldots\ldots\ldots\ldots T_N = \prod_{n=1}^{N} T_n$$

- Note this result is **only** true for the **transmission** matrix **T**. **No** equivalent result exists for **S**, **Z**, **Y** !
- Thus, the transmission matrix can greatly simplify the analysis of complex networks constructed from two-port devices. We find that the T matrix is particularly useful when creating design software for CAD applications.