Lecture – 7 Date: 28.01.2017

- Admittance Smith Chart
- High Frequency Network Analysis (intro)
- Impedance, Admittance and Scattering Matrix
- Matched, Lossless, and Reciprocal Networks

Admittance Transformation

- RF/Microwave network, similar to any electrical network, has impedance elements in series and parallel
- Impedance Smith chart is well suited while working with series configurations while admittance Smith chart is more useful for parallel configurations
- The impedance Smith chart can easily be used as an admittance calculator

$$z_{in}(z)=\frac{1+\Gamma(z)}{1-\Gamma(z)} \Leftarrow y_{in}(z)=\frac{Y_{in}(z)}{Y_0}=\frac{1/Z_{in}(z)}{1/Z_0}=\frac{1}{Z_{in}(z)/Z_0}=\frac{1}{z_{in}(z)}$$

- Hence, $y_{in}(z)=\dfrac{1-\Gamma(z)}{1+\Gamma(z)} \Rightarrow y_{in}(z)=\dfrac{1+e^{-j\pi}\Gamma(z)}{1-e^{-j\pi}\Gamma(z)}$

It means, to obtain normalized admittance → take the normalized impedance and multiply associated reflection coefficient by -1 = $e^{-j\pi}$ → it is equivalent to a 180^o rotation of the reflection coefficient in complex Γ-plane

Example – 1

- Convert the following normalized input impedance z_{in}' into normalized input admittance y_{in}' using the Smith chart:

$$z_{in}' = 1 + j1 = \sqrt{2}e^{j(\pi/4)}$$

First approach: The normalized admittance can be found by direct inversion as:

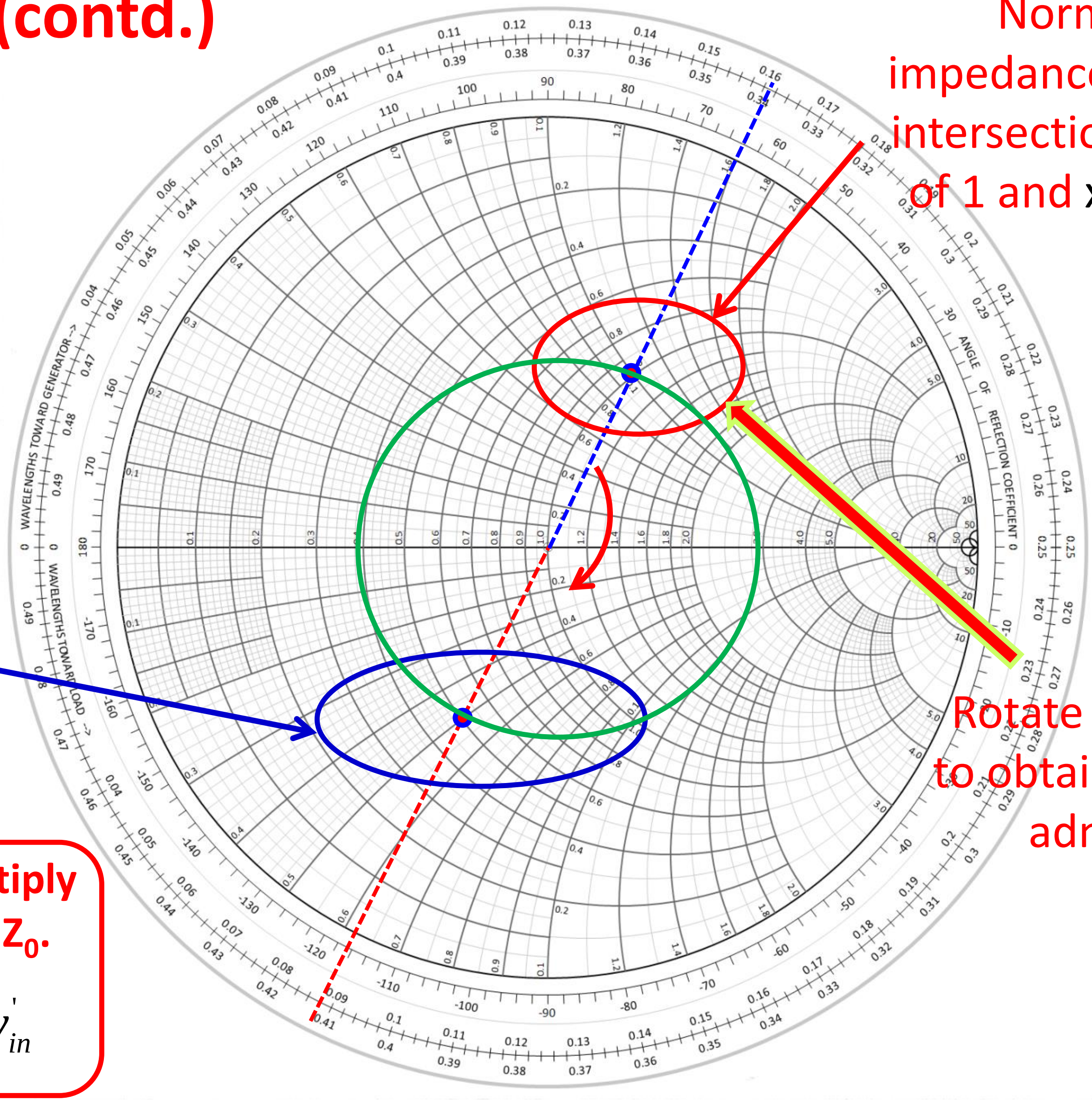
$$y_{in}' = \frac{1}{z_{in}'} = \frac{1}{1+j1} = \frac{1}{\sqrt{2}}e^{-j(\pi/4)} = \frac{1}{2} - j\frac{1}{2}$$

Alternative approach:

- Mark the normalized impedance on Smith chart
- Identify phase angle and magnitude of the associated reflection coefficient
- Rotate the reflection coefficient by 180^o
- Identify the x-circle and r-circle intersection of the rotated reflection coefficient

Example – 1 (contd.)

Quick investigation shows that the normalized impedance (y_{in}') is the intersection of r-circle of 1/2 and x-circle of -1/2



Normalized impedance (z_{in}') is the intersection of r-circle of 1 and x-circle of 1

Rotate this by 180° to obtain normalized admittance

To denormalize, multiply with the inverse of Z_0.

$$Y_{in} = y_{in}^{'} \frac{1}{Z_0} = Y_0 y_{in}^{'}$$

Example – 2

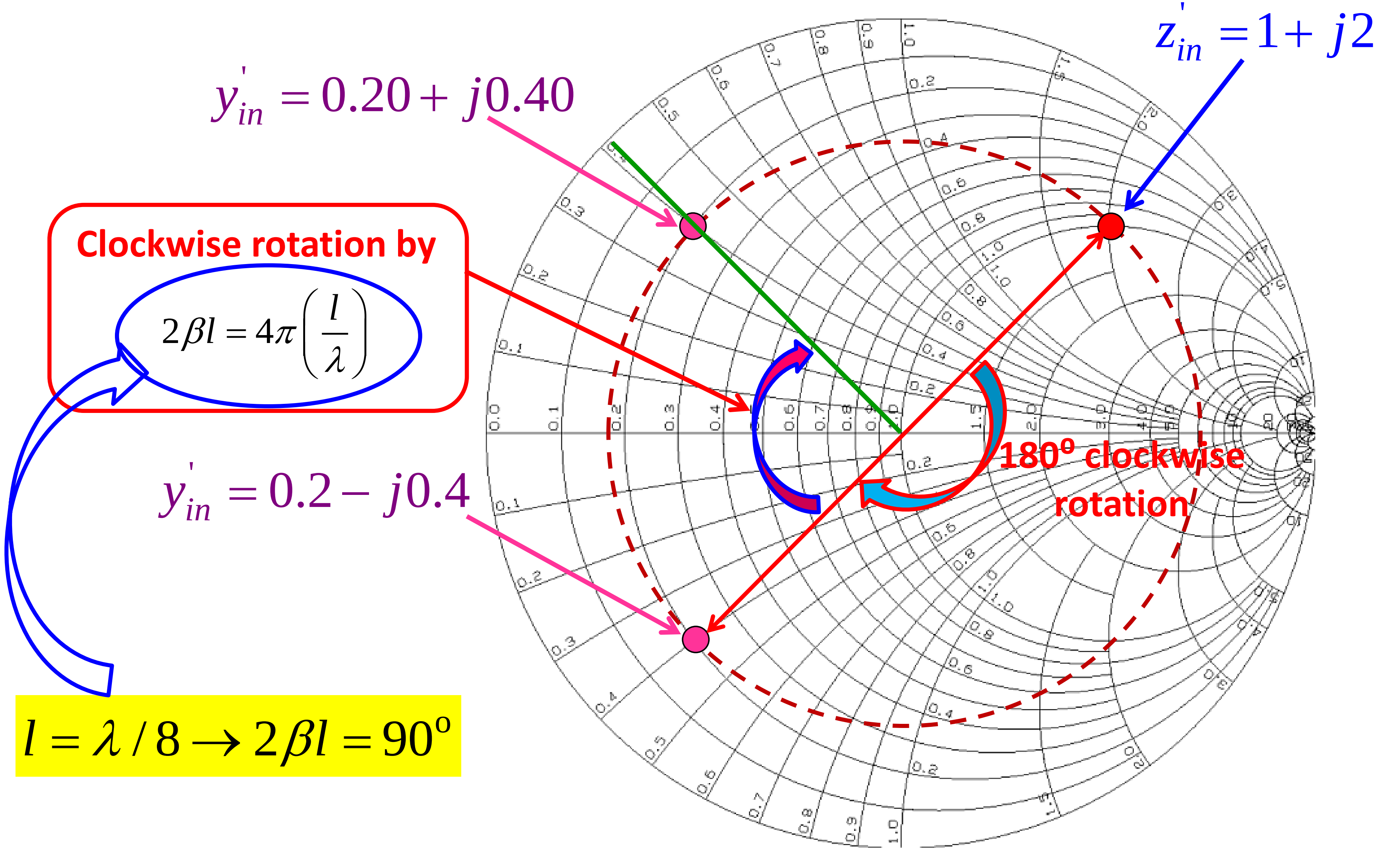
Given: $z_{in}^{'} = 1 + j2$

- Find the normalized admittance $\lambda/8$ away from the load

Steps:

1. Mark the normalized impedance on Smith Chart
2. Clockwise rotate it by 180^o
3. Identify the normalized admittance and the phase angle of the associated reflection coefficient
4. Clockwise rotate the reflection coefficient (associated with the normalized admittance) by $2\beta l$ (here $l = \lambda/8$)
5. The new location gives the required normalized admittance

Example – 2 (contd.)



Admittance Smith chart

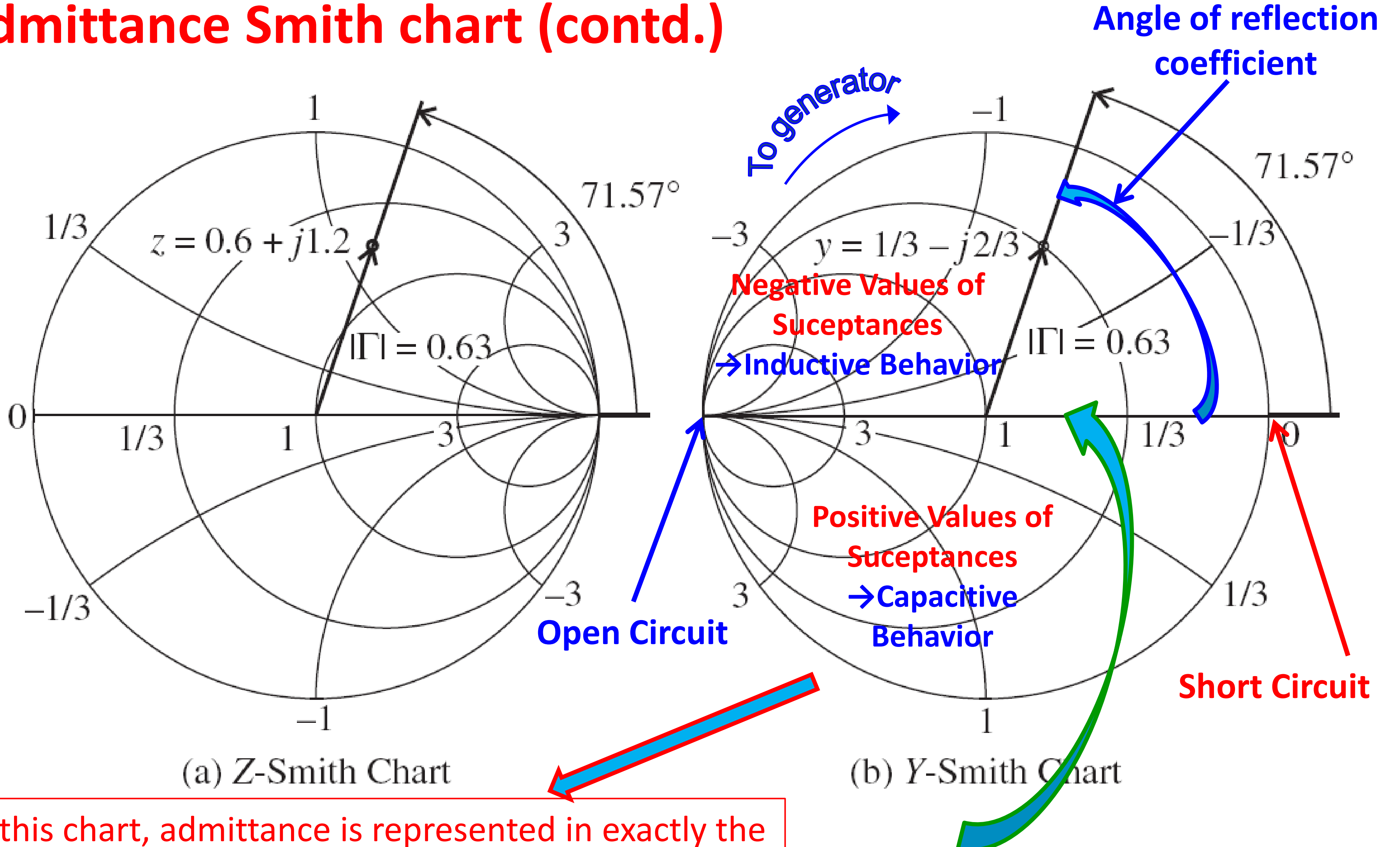
- Alternative approach to solve parallel network elements is through 180° rotated Smith chart
- This rotated Smith chart is called **admittance Smith chart or Y-Smith chart**
- The corresponding normalized resistances become normalized conductances & normalized reactances become normalized suceptances

$$r = \frac{R}{Z_0} \Rightarrow g = \frac{G}{Y_0} = Z_0 G$$

$$x = \frac{X}{Z_0} \Rightarrow b = \frac{b}{Y_0} = Z_0 B$$

- **The Y-Smith chart preserves:**
  - The direction in which the angle of the reflection coefficient is measured
  - The direction of rotation (either toward or away from the generator)

Admittance Smith chart (contd.)



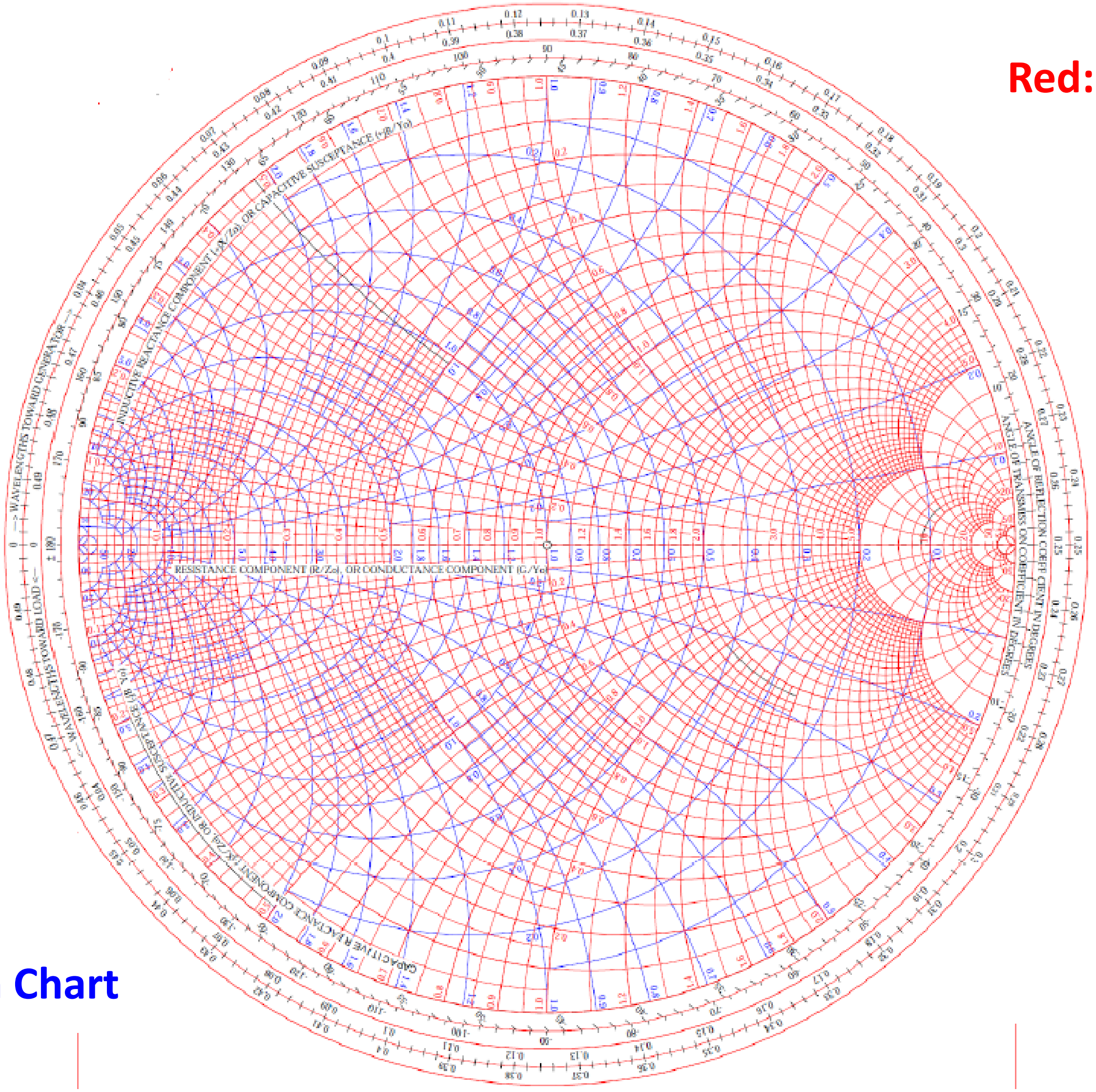
(a) Z-Smith Chart

(b) Y-Smith Chart

In this chart, admittance is represented in exactly the same manner as the impedance in the Z-smith Chart → without 180^{0} rotation

Combined Z- and Y- Smith Charts

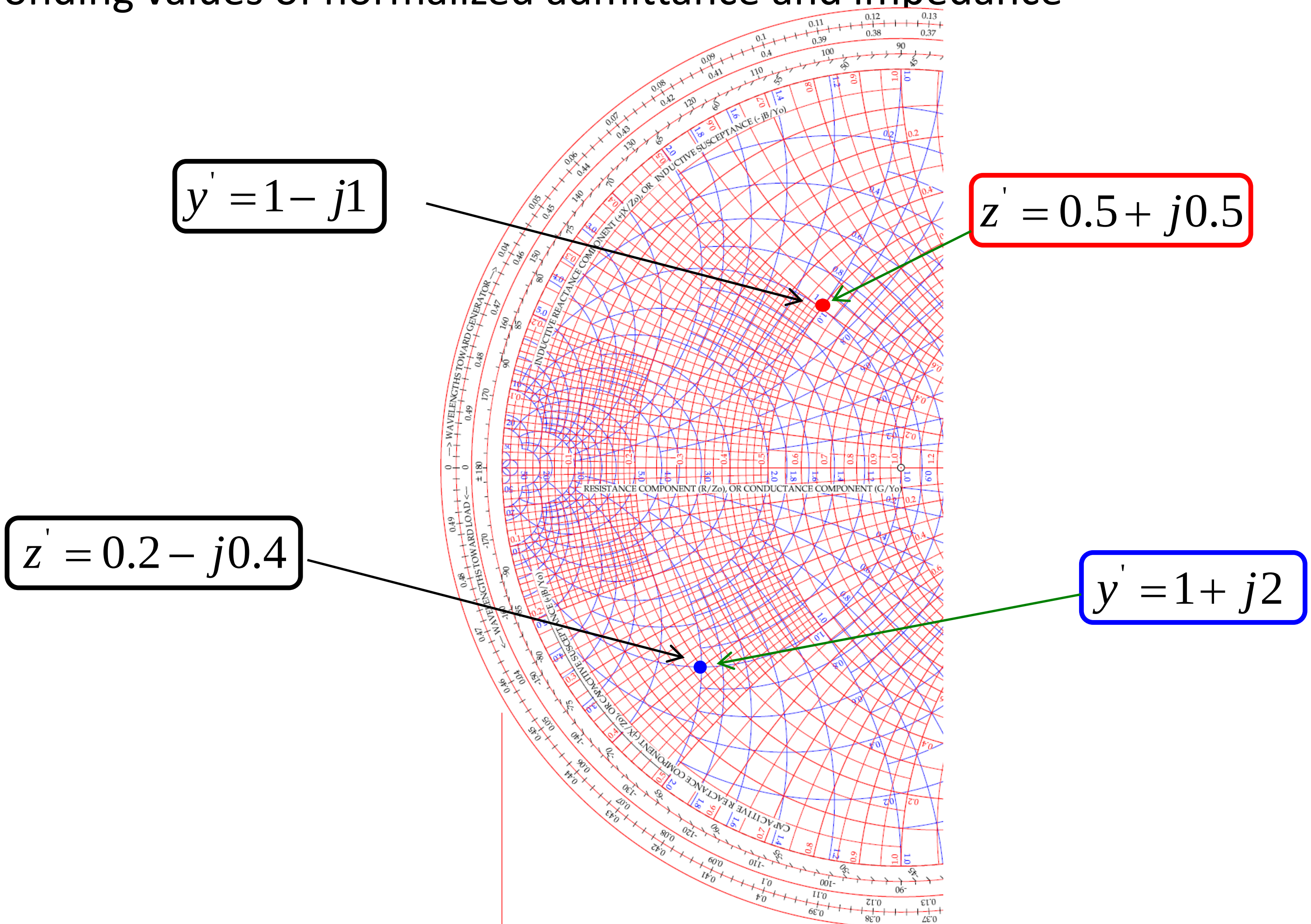
Red: Z – Smith Chart



Blue: Y – Smith Chart

Example – 3

- Identify (a) the normalized impedance z' = 0.5 + j0.5, and (b) the normalized admittance value y' = 1 + j2 in the combined ZY-Smith Chart and find the corresponding values of normalized admittance and impedance



High Frequency Networks

- **Requirement of Matrix Formulation**

Current/Voltage or
Incident/Reflected
Traveling Wave

directional coupler
(more than one port)

Current/Voltage or
Incident/Reflected
Traveling Wave

These are called networks

Can we characterize this using an impedance or admittance!

NO!!

What is the way?

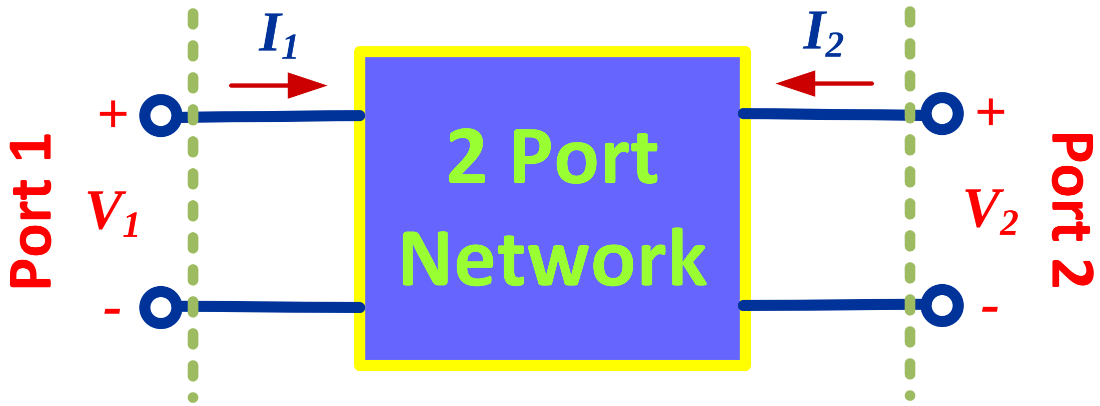
Impedance or Admittance Matrix. Right?

In principle, N by N impedance matrix completely characterizes a linear N-port device. Effectively, the impedance matrix defines a multi-port device the way a Z_L describes a single port device (e.g., a load)

Linear networks can be completely characterized by parameters measured at the network ports without knowing the content of the networks.

Multiport Networks

- Networks can have any number of ports – however, analysis of a 2-port, 3-port or 4-port network is sufficient to explain the theory and the associated concepts



- The ports can be characterized with many parameters (Z, Y, S, ABCD). Each has a specific advantage.
- For 2-port Network, each parameter set is related to 4 variables:
  - 2 independent variables for excitation
  - 2 dependent variables for response

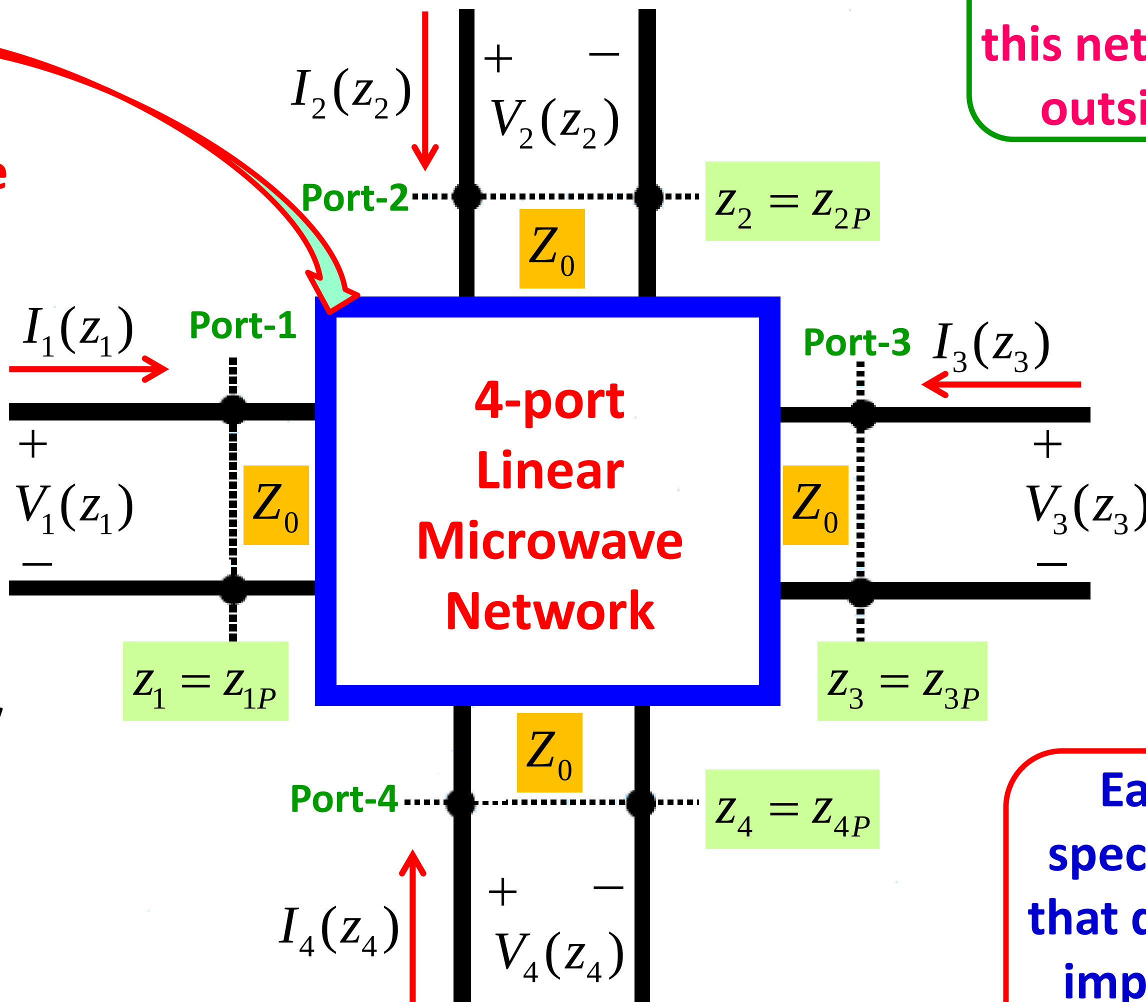
The Impedance Matrix

- Let us consider the following 4-port network:

Four identical TLs used to connect this network to the outside world

This could be a simple linear device or a large/complex linear microwave system

Either way, the network can be fully described by its **impedance matrix**



Each TL has specific location that defines input impedances to the network

The arbitrary locations are known as **ports** of the network

The Impedance Matrix (contd.)

- In principle, the current and voltages at the port-n of networks are given as:

$$V_n(z_n = z_{nP}) \qquad I_n(z_n = z_{nP})$$

- However, the simplified formulations are:

$$V_n = V_n(z_n = z_{nP}) \qquad I_n = I_n(z_n = z_{nP})$$

- If we want to say that there exists a non-zero current at port-1 and zero current at all other ports then we can write as:

$$I_1 \neq 0 \qquad I_2 = I_3 = I_4 = 0$$

- In order to define the elements of impedance matrix, there will be need to measure/determine the associated voltages and currents at the respective ports. Suppose, if we measure/determine current at port-1 and then voltage at port-2 then we can define:

$$Z_{21} = \frac{V_2}{I_1}$$

Trans-impedance

- Similarly, the trans-impedance parameters Z_{31} and Z_{41} are:

$$Z_{31} = \frac{V_3}{I_1} \qquad Z_{41} = \frac{V_4}{I_1}$$

- We can also define other trans-impedance parameters such as Z_{34} as the ratio between the complex values I_4 (the current into port-4) and V_3 (the voltage at port-3), given that the currents at all other ports (1, 2, and 3) are zero.

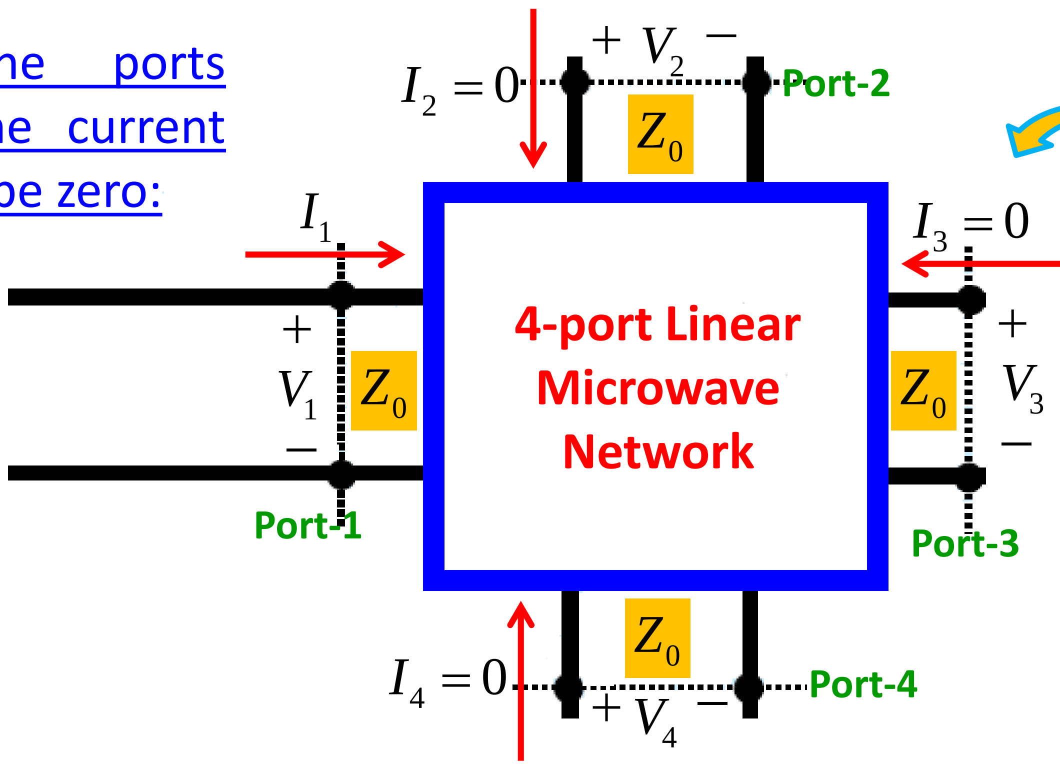
The Impedance Matrix (contd.)

- Therefore, the more generic form of trans-impedance is:

$$Z_{mn} = \frac{V_m}{I_n}$$ **(given that $I_k = 0$ for all $k \neq n$)**

How do we ensure that all but **one port** current is zero?

- Open the ports where the current needs to be zero:



The ports should be opened! not the TL connected to the ports

- then define the respective trans-impedances as:

$$Z_{mn} = \frac{V_m}{I_n}$$ **(given that all ports $k \neq n$ are open)**

The Impedance Matrix (contd.)

- Once we have defined the trans-impedance terms by opening various ports, it is time to formulate the impedance matrix
- Since the network is **linear**, the voltage at any port due to all the port currents is simply the coherent **sum** of the voltage at that port due to **each** of the currents
- For example, the voltage at **port-3** is: $V_3 = Z_{34}I_4 + Z_{33}I_3 + Z_{32}I_2 + Z_{31}I_1$
- Therefore we can generalize the voltage for **N-port** network as: $V_m = \sum_{n=1}^{N} Z_{mn} I_n \quad \Rightarrow \mathbf{V} = \mathbf{ZI}$
- Where **I** and **V** are vectors given as: $\mathbf{V} = [\mathrm{V}_1, \mathrm{V}_2, \mathrm{V}_3, ..., \mathrm{V}_\mathrm{N}]^T \qquad \mathbf{I} = [\mathrm{I}_1, \mathrm{I}_2, \mathrm{I}_3, ..., \mathrm{I}_\mathrm{N}]^T$
- The term **Z** is matrix given by:

$$\mathbf{Z} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & & & \vdots \\ \vdots & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mn} \end{bmatrix}$$

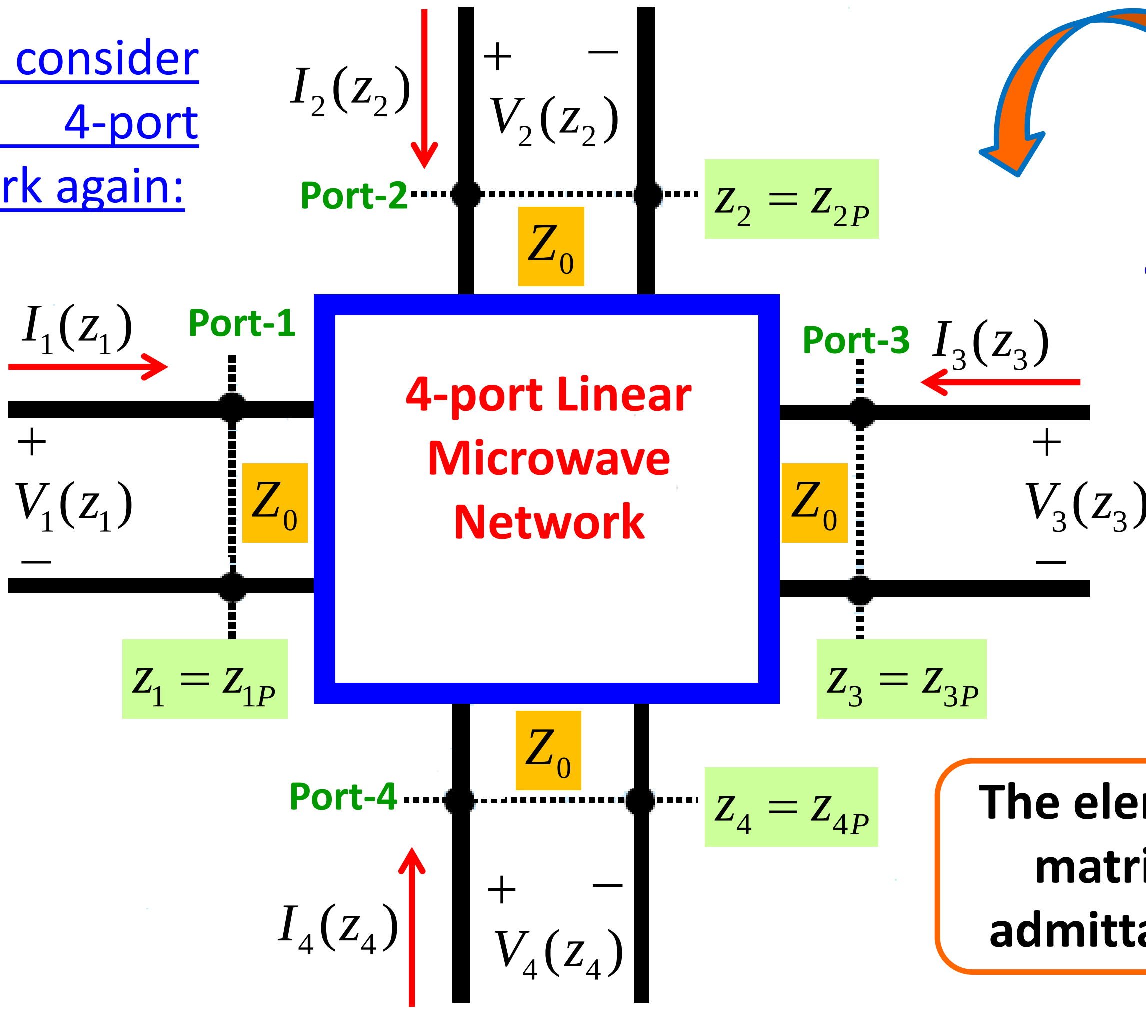
Impedance Matrix

- The values of elements in the impedance matrix are frequency dependents and often it is advisable to describe impedance matrix as:

$$\mathbf{Z}(\omega) = \begin{bmatrix} Z_{11}(\omega) & Z_{12}(\omega) & \cdots & Z_{1n}(\omega) \\ Z_{21}(\omega) & & & \vdots \\ \vdots & & & \\ Z_{m1}(\omega) & Z_{m2}(\omega) & \cdots & Z_{mn}(\omega) \end{bmatrix}$$

The Admittance Matrix

- Let us consider the 4-port network again:



This can be characterized using admittance matrix – if currents are taken as dependent variables instead of voltages

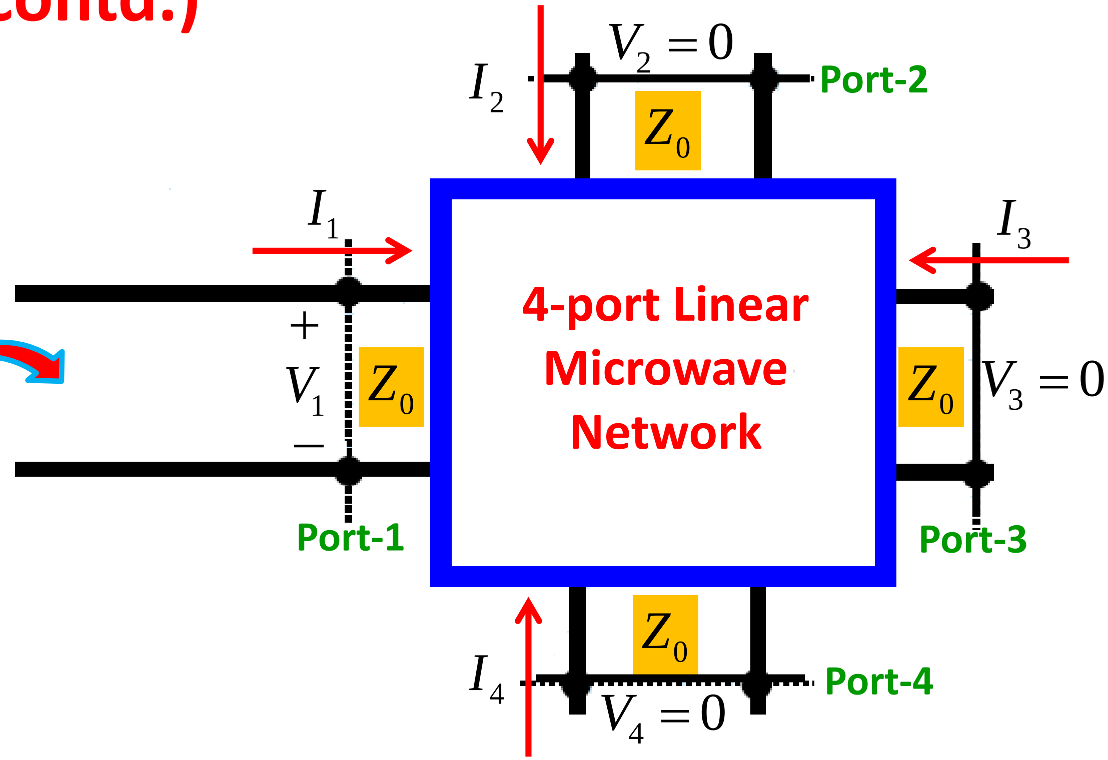
The elements of admittance matrix are called trans-admittance parameters Y_{mn}

- The trans-admittances Y_{mn} are defined as: $Y_{mn} = \frac{I_m}{V_n}$ (given that $V_k = 0$ for all $k \neq n$)

Important: $Y_{mn} \neq \frac{1}{Z_{mn}}$

The Admittance Matrix (contd.)

- the voltage at all but one port must be equal to zero. This can be ensured by short-circuiting the voltage ports.



The ports should be short-circuited! not the TL connected to the ports

- Now, since the network is **linear**, the current at any **one** port due to all the port voltages is simply the coherent **sum** of the currents at that port due to **each** of the port voltages.
- For example, the current at **port-3** is: $$I_3 = Y_{34}V_4 + Y_{33}V_3 + Y_{32}V_2 + Y_{31}V_1$$
- Therefore we can generalize the current for **N-port** network as: $$I_m = \sum_{n=1}^{N} Y_{mn}V_n$$ $$\Rightarrow \mathbf{I = YV}$$

The Admittance Matrix (contd.)

- Where **I** and **V** are vectors given as: $\mathbf{V} = [V_1, V_2, V_3, \ldots, V_N]^T$ $\quad$ $\mathbf{I} = [I_1, I_2, I_3, \ldots, I_N]^T$

- The term **Y** is matrix given by:

$$\boldsymbol{Y} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & & & \vdots \\ \vdots & & & \\ Y_{m1} & Y_{m2} & \cdots & Y_{mn} \end{bmatrix}$$

Admittance Matrix

- The values of elements in the admittance matrix are frequency dependents and often it is advisable to describe admittance matrix as:

$$\boldsymbol{Y}(\omega) = \begin{bmatrix} Y_{11}(\omega) & Y_{12}(\omega) & \cdots & Y_{1n}(\omega) \\ Y_{21}(\omega) & & & \vdots \\ \vdots & & & \\ Y_{m1}(\omega) & Y_{m2}(\omega) & \cdots & Y_{mn}(\omega) \end{bmatrix}$$

You said that: $\boxed{Y_{mn} \neq \dfrac{1}{Z_{mn}}}$

Is there any relationship between admittance and impedance matrix of a given device?

Answer: Let us see if we can figure it out!

The Admittance Matrix (contd.)

- Recall that we can determine the inverse of a matrix. Denoting the matrix inverse of the admittance matrix as $\mathbf{Y}^{-1}$, we find:

$$\mathbf{I} = \mathbf{YV}$$

$$\Rightarrow \mathbf{Y}^{-1}\mathbf{I} = \mathbf{Y}^{-1}(\mathbf{YV}) \quad \Rightarrow \quad \mathbf{Y}^{-1}\mathbf{I} = (\mathbf{Y}^{-1}\mathbf{Y})\mathbf{V} \quad \Rightarrow \quad \mathbf{Y}^{-1}\mathbf{I} = \mathbf{V}$$

- We also know: $\mathbf{V} = \mathbf{ZI}$

$$\mathbf{Z} = \mathbf{Y}^{-1} \quad \textbf{OR} \quad \mathbf{Y} = Z^{-1}$$

Reciprocal and Lossless Networks

- We can **classify** multi-port devices or networks as either **lossless** or **lossy**; **reciprocal** or **non-reciprocal**. Let's look at each classification individually.

Lossless Network

- A **lossless** network/device is simply one that **cannot** absorb power. This does **not** mean that the delivered power at **every port** is zero; rather, it means the total power flowing **into** the **device** must equal the total power **exiting** the **device**.

- A lossless device exhibits an impedance matrix with an interesting **property**. Perhaps not surprisingly, we find for a lossless device that the **elements** of its impedance matrix will be **purely reactive**:

$$\operatorname{Re}(Z_{mn}) = 0$$

For a lossless device

Reciprocal and Lossless Networks

- If the device is lossy, then the elements of the impedance matrix must have **at least** one element with a real (i.e., resistive) component.
- Furthermore, we can similarly say that if the elements of an **admittance** matrix are **all** purely imaginary (i.e., $\mathrm{Re}\{Y_{mn}\} = 0$), then the device is lossless.

Reciprocal Network

- Ideally, most **passive, linear** microwave components will turn out to be **reciprocal**—regardless of whether the designer **intended** it to be or not!
- Reciprocity is a tremendously important characteristic, as it greatly **simplifies** an impedance or admittance matrix!
- Specifically, we find that a reciprocal device will result in a **symmetric** impedance and admittance **matrix**, meaning that:

$$Z_{mn} = Z_{nm} \qquad Y_{mn} = Y_{nm}$$

For a reciprocal device

- For example, we find for a reciprocal device that $Z_{23} = Z_{32}$, and $Y_{12} = Y_{21}$.

Reciprocal and Lossless Networks (contd.)

$$\mathbf{Z}=\begin{bmatrix} j2 & 0.1 & j3 \\ -j & -1 & 1 \\ 4 & -2 & 0.5 \end{bmatrix}$$

neither lossless nor reciprocal

lossless, but not reciprocal

$$\mathbf{Z}=\begin{bmatrix} j2 & j0.1 & j3 \\ -j & -j1 & j1 \\ j4 & -j2 & j0.5 \end{bmatrix}$$

$$\mathbf{Z}=\begin{bmatrix} j2 & -j & 4 \\ -j & -1 & -j2 \\ 4 & -j2 & j0.5 \end{bmatrix}$$

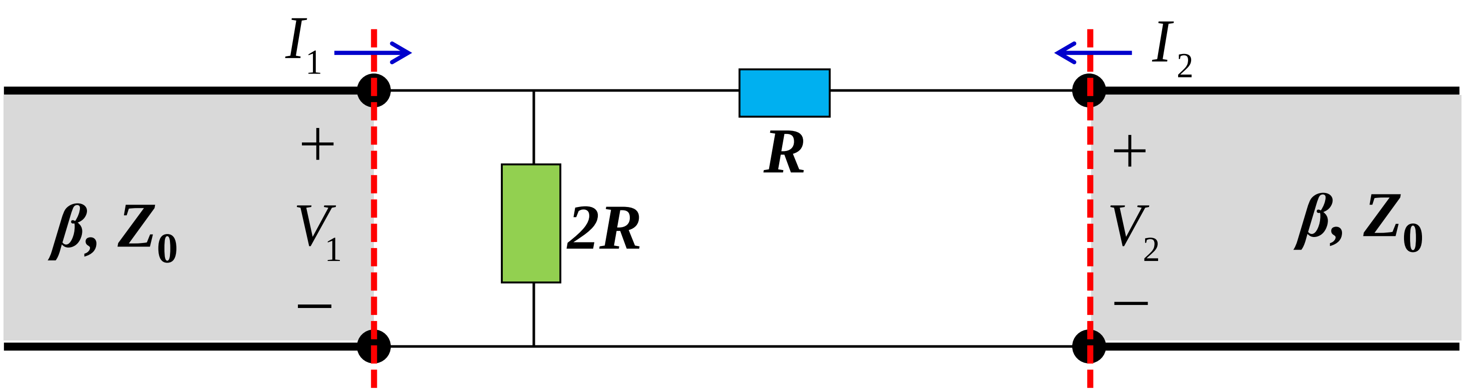
reciprocal, but not lossless

lossless and reciprocal

$$\mathbf{Z}=\begin{bmatrix} j2 & -j & j4 \\ -j & -j & -j2 \\ j4 & -j2 & j0.5 \end{bmatrix}$$

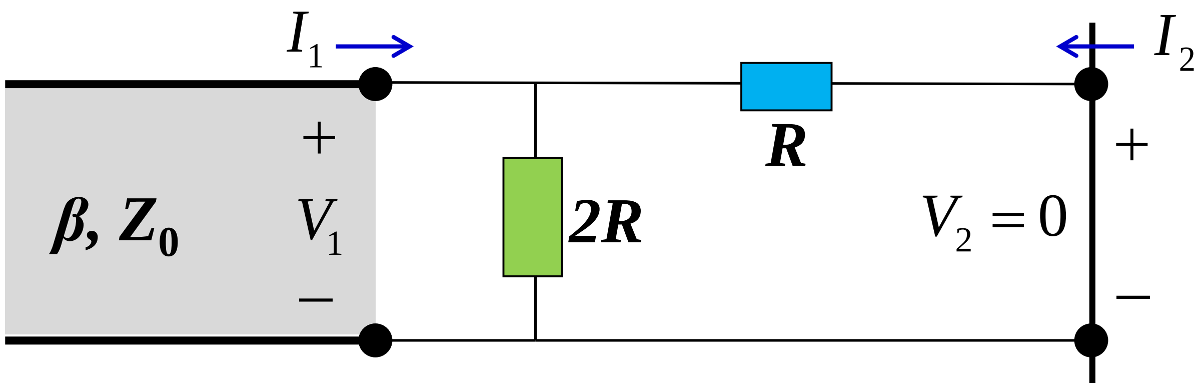
Example – 4

- determine the **Y matrix** of this two-port device.



Example – 4 (contd.)

Step-1: Place a **short** at port 2



Step-2: Determine currents I_1 and I_2

- Note that **after** the short was placed at port 2, both resistors are in **parallel**, with a potential V_1 across each

Therefore current I_1 is

$$\boxed{I_1 = \frac{V_1}{2R} + \frac{V_1}{R} = \frac{3V_1}{2R}}$$

- The current I_2 equals the portion of current I_1 through R but with opposite sign

$$\boxed{I_2 = -\frac{V_1}{R}}$$

Example – 4 (contd.)

Step-3: Determine the trans-admittances Y_{11} and Y_{21}

$$\boxed{Y_{11} = \frac{I_1}{V_1} = \frac{3}{2R}}$$

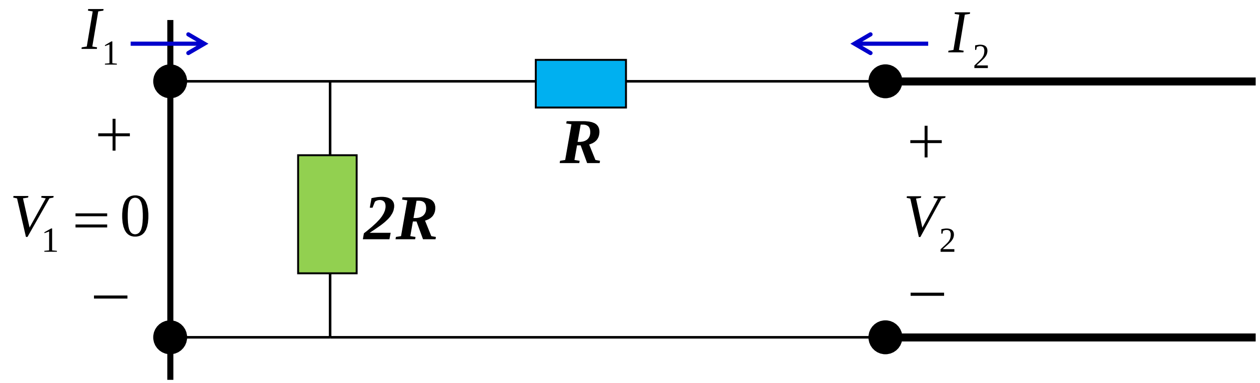
$$\boxed{Y_{21} = \frac{I_2}{V_1} = -\frac{1}{R}}$$

Note that Y_{21} is real and negative

This is **still** a valid physical result, **although** you will find that the **diagonal** terms of an impedance or admittance matrix (e.g., Y_{22} , Z_{11}, Y_{44}) will **always** have a real component that is **positive**

To find the **other two** trans-admittance parameters, we must **move** the short and then **repeat** each of our previous steps!

Example – 4 (contd.)



Step-1:

Place a **short** at port 1

Step-2: Determine currents I_1 and I_2

- Note that **after** a short was placed at port **1**, resistor 2R has **zero** voltage across it—and thus **zero current** through it!

Therefore:

$$\boxed{I_2 = \frac{V_2}{R}} \qquad \boxed{I_1 = -I_2 = -\frac{V_2}{R}}$$

Step-3:

Determine the trans-admittances Y_{12} and Y_{22}

$$\boxed{Y_{12} = \frac{I_1}{V_2} = -\frac{1}{R}} \qquad \boxed{Y_{22} = \frac{I_2}{V_2} = \frac{1}{R}}$$

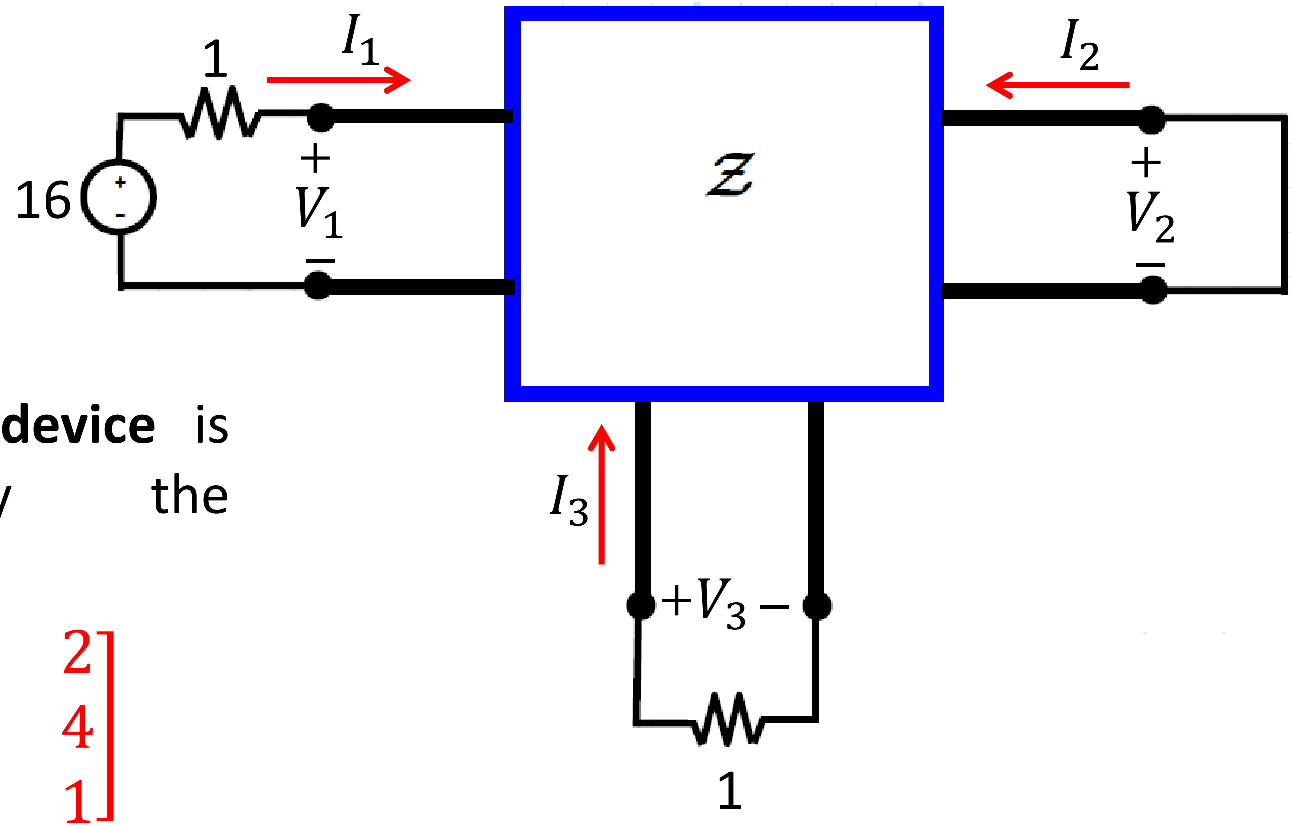
Therefore the admittance matrix is:

$$\mathbf{Y} = \begin{bmatrix} 3/2R & -1/R \\ -1/R & 1/R \end{bmatrix}$$

Is it lossless or reciprocal?

Example – 5

- Consider this circuit:



- Where the 3-port **device** is characterized by the **impedance matrix**:

$$\mathbf{Z}=\begin{bmatrix}2 & 1 & 2\\ 1 & 1 & 4\\ 2 & 4 & 1\end{bmatrix}$$

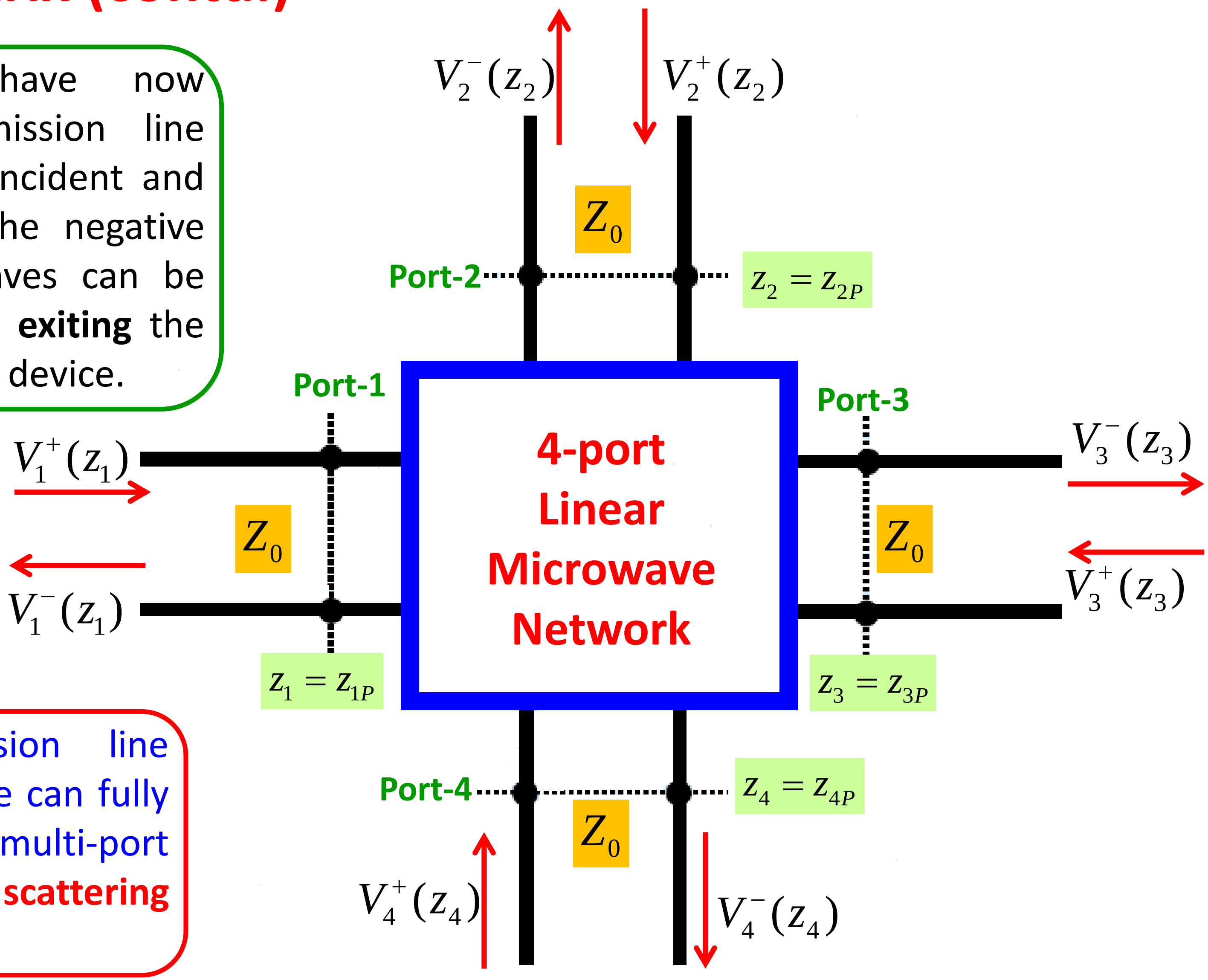
- determine all port **voltages** V_1, V_2, V_3 and all **currents** I_1, I_2, I_3.

Scattering Matrix

- At **"low"** frequencies, a **linear** device or network can be fully characterized using an **impedance or admittance** matrix, which relates the currents and voltages at **each** device terminal to the currents and voltages at **all** other terminals.
- But, at high frequencies, it is **not feasible** to measure total currents and voltages!
- Instead, we can measure the **magnitude** and **phase** of each of the two transmission line **waves** $V^+(z)$ and $V^-(z)$ → enables determination of relationship between the incident and reflected waves at **each** device terminal to the incident and reflected waves at **all** other terminals
- These relationships are completely represented by the **scattering matrix** that **completely** describes the behavior of a linear, multi-port device at a **given frequency** ω, and a given line impedance Z_0

Scattering Matrix (contd.)

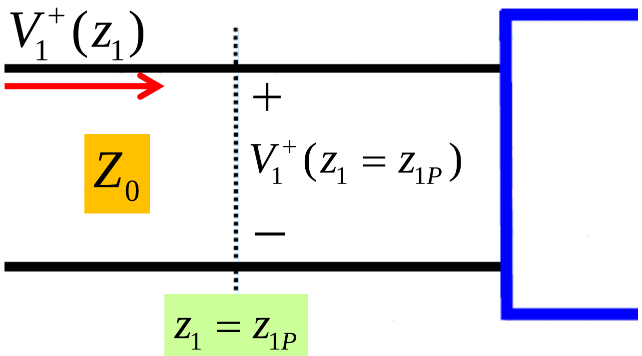
Note that we have now characterized transmission line activity in terms of incident and "reflected" waves. The negative going "reflected" waves can be viewed as the waves **exiting** the multi-port network or device.



Viewing transmission line activity this way, we can fully characterize a multi-port device by its **scattering parameters**!

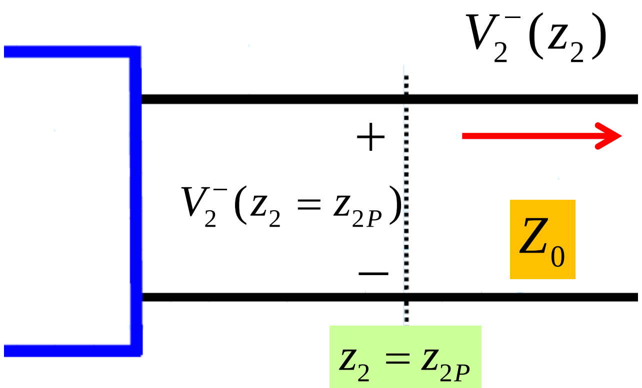
Scattering Matrix (contd.)

- Say there exists an **incident** wave on **port 1** (i.e., $V_1^+(z_1) \neq 0$), while the incident waves on all other ports are known to be **zero** (i.e., $V_2^+(z_2) = V_3^+(z_3) = V_4^+(z_4) = 0$).



Say we measure/determine the voltage of the wave flowing **into port 1,** at the port 1 **plane** (i.e., determine $V_1^+(z_1 = z_{1P})$).

Say we then measure/determine the voltage of the wave flowing **out** of **port 2,** at the port 2 plane (i.e., determine $V_2^-(z_2 = z_{2P})$).



The ratio between $V_1^+(z_1 = z_{1P})$ and $V_2^-(z_2 = z_{2P})$ is known as the **scattering parameter** S_{21}

Therefore:

$$S_{21} = \frac{V_2^-(z_2 = z_{2P})}{V_1^+(z_1 = z_{1P})} = \frac{V_2^- e^{+j\beta z_{2P}}}{V_1^+ e^{-j\beta z_{1P}}} = \frac{V_2^-}{V_1^+} e^{+j\beta(z_{2P} + z_{1P})}$$

Scattering Matrix (contd.)

Similarly:

$$\boxed{S_{31} = \frac{V_3^-(z_3 = z_{3P})}{V_1^+(z_1 = z_{1P})}} \qquad \boxed{S_{41} = \frac{V_4^-(z_4 = z_{4P})}{V_1^+(z_1 = z_{1P})}}$$

- We of course could **also** define, say, scattering parameter S_{34} as the ratio between the complex values $V_3^-(z_3 = z_{3P})$ (the wave **out of** port 3) and $V_4^+(z_4 = z_{4P})$ (the wave **into** port 4), given that the input to all other ports (1,2, and 3) are zero

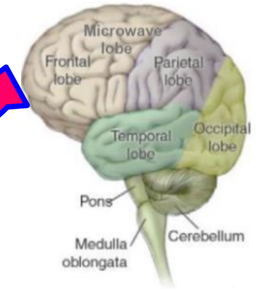
- more **generally**, the ratio of the wave incident on port **n** to the wave emerging from port **m** is:

$$\boxed{S_{mn} = \frac{V_m^-(z_m = z_{mP})}{V_n^+(z_n = z_{nP})} \qquad V_k^+(z_k) = 0 \quad \textbf{for all } \mathbf{k \neq n}}$$

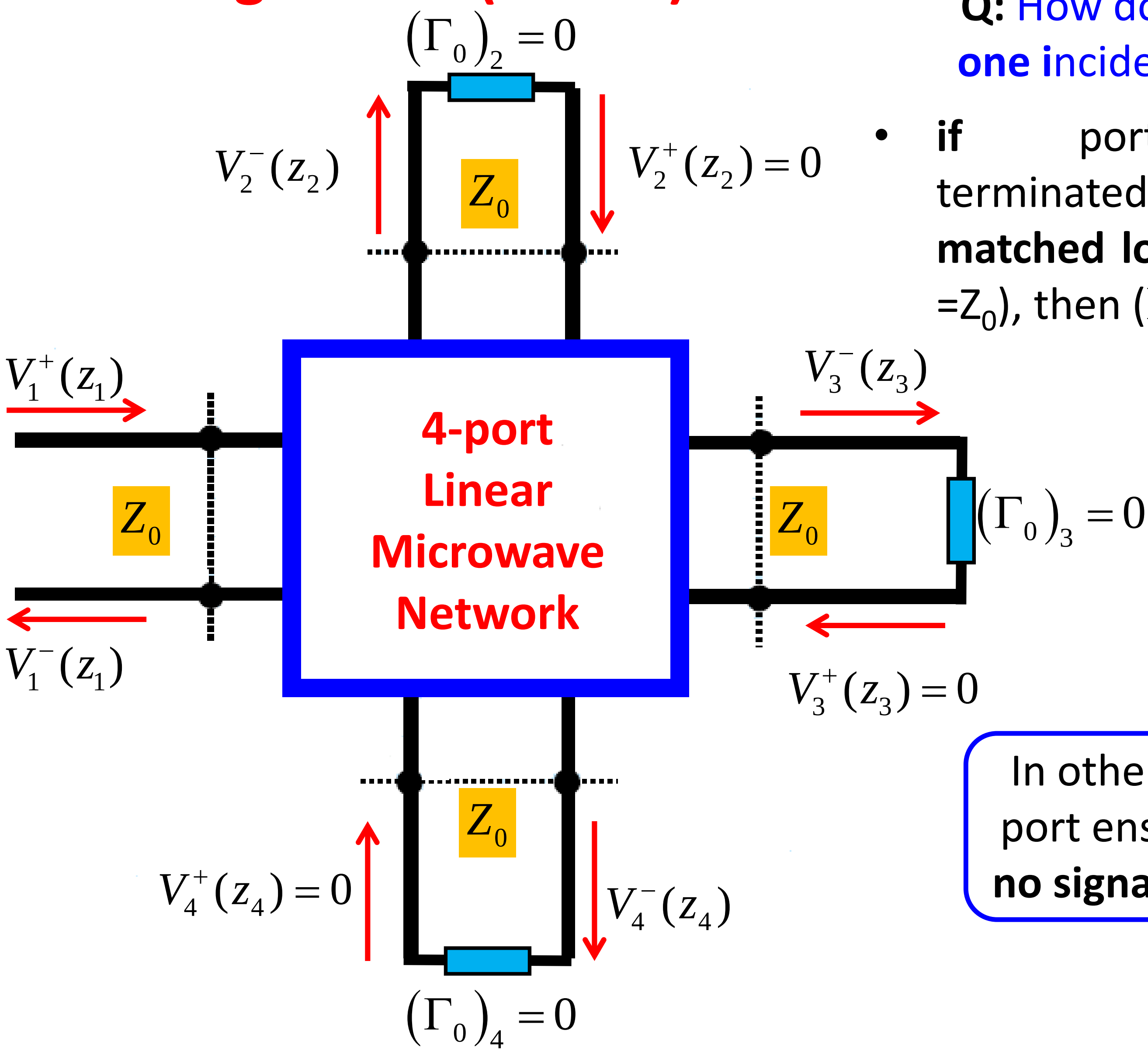
- Note that, frequently the port positions are assigned a **zero** value (e.g., z_{1P}=0, z_{2P}=0). This **simplifies** the scattering parameter calculation:

$$S_{mn} = \frac{V_m^-(z_m = 0)}{V_n^+(z_n = 0)} = \frac{V_m^+ e^{+j\beta 0}}{V_n^- e^{-j\beta 0}} = \frac{V_m^+}{V_n^-}$$

- We will **generally assume** that the port locations are defined as z_{nP}=0, and thus use the **above** notation. But **remember** where this expression came from!



Scattering Matrix (contd.)



Q: How do we ensure that **only one** incident wave is non-zero ?

- **if** ports are terminated in a **matched load** (i.e., $Z_L = Z_0$), then $(\Gamma_0)_n = 0$:

$$\boxed{V_n^+(z_n) = 0}$$

In other words, terminating a port ensures that there will be **no signal** incident on that port!

Scattering Matrix (contd.)

Just between you and me, I think you've messed this up! **In all** previous slides you said that if $\Gamma_0 = 0$, the wave in the **minus** direction would be zero:

$V^{-}(z) = 0$ if $\Gamma_0 = 0$

but just **now** you said that the wave in the **positive** direction would be zero:

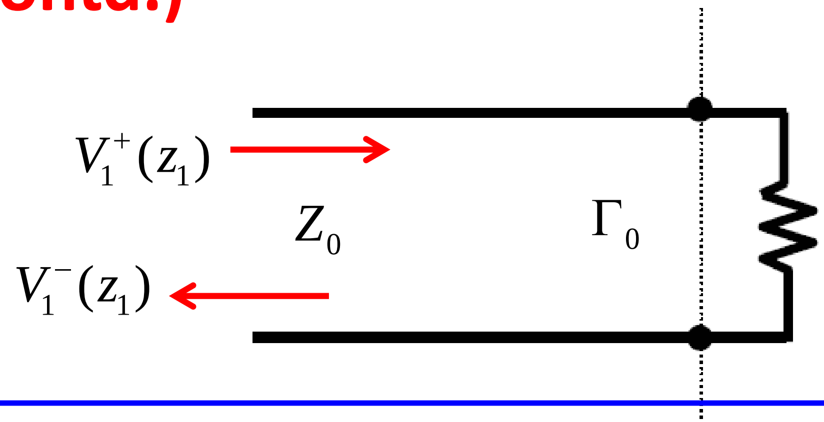
$V^{+}(z) = 0$ if $\Gamma_0 = 0$

Obviously, there is **no way** that **both** statements can be correct!

Actually, **both** statements are correct! You must be careful to understand the **physical definitions** of the plus and minus directions—in other words, the propagation directions of waves $V_n^{+}(z_n)$ and $V_n^{-}(z_n)$!

Scattering Matrix (contd.)

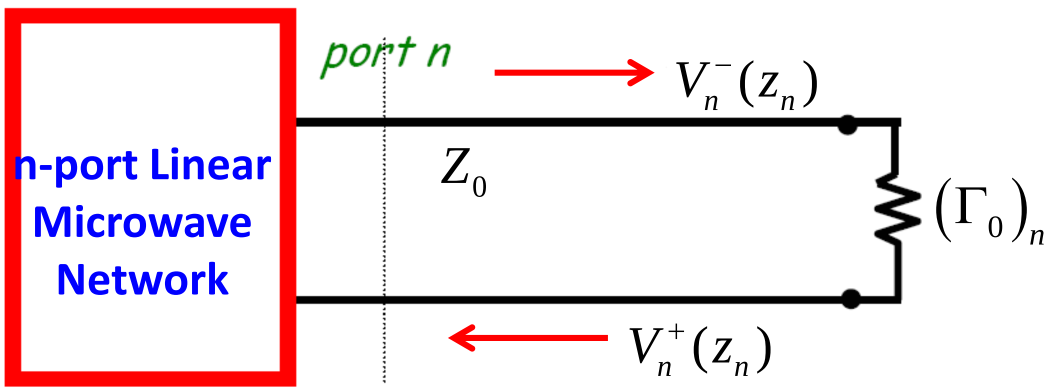
- For example, we **originally** analyzed this case:



$V^-(z) = 0$
if $\Gamma_0 = 0$

In this original case, the wave **incident** on the load is V⁺(z) (**plus** direction), while the **reflected** wave is V⁻(z) (**minus** direction).

- **Contrast** this with the case we are **now** considering:



- For this current case, the situation is **reversed**. The wave incident on the load is **now** denoted as $V_n^-(z_n)$ (coming **out** of port n), while the wave reflected off the load is **now** denoted as $V_n^+(z_n)$ (going **into** port n).

Scattering Matrix (contd.)

- **back** to our discussion of **S-parameters**. We found that **if** $z_{nP} = 0$ for all ports n, the scattering parameters could be directly written in terms of wave **amplitudes** V_n^+ and V_m^-

$$S_{mn} = \frac{V_m^-}{V_n^+} \qquad V_k^+(z_k) = 0$$

for all k ≠ n

- Which we can now **equivalently** state as:

$$S_{mn} = \frac{V_m^-}{V_n^+}$$

(all ports, except port *n*, are terminated in matched loads)

- One more **important** note—notice that for the ports terminated in matched loads (i.e., those ports with **no** incident wave), the voltage of the exiting **wave** is also the **total** voltage!

$$V_m(z_m) = V_m^+ e^{-j\beta z_m} + V_m^- e^{+j\beta z_m} = 0 + V_m^- e^{+j\beta z_m} = V_m^- e^{+j\beta z_m}$$

For all terminated ports!

Scattering Matrix (contd.)

- We can use the scattering matrix to determine the solution for a more **general** circuit—one where the ports are **not** terminated in matched loads!
- Since the device is **linear**, we can apply **superposition**. The output at any port due to **all** the incident waves is simply the coherent **sum** of the output at that port due to **each** wave!
- For example, the **output** wave at port 3 can be determined by (assuming $z_{nP} = 0$):

$$\boxed{V_3^- = S_{34}V_4^+ + S_{33}V_3^+ + S_{32}V_2^+ + S_{31}V_1^+}$$

- More **generally**, the output at port m of an N-port device is:

$$\boxed{V_m^- = \sum_{n=1}^{N} S_{mn}V_n^+ \qquad z_{nP} = 0}$$

- This expression of Scattering parameter can be written in **matrix** form as:

$$V^- = SV^+$$

$$\boxed{V^- = SV^+}$$

Scattering Matrix

$$\boldsymbol{S} = \begin{bmatrix} S_{11} & S_{12} & \dots & S_{1n} \\ S_{21} & & & \vdots \\ \vdots & & & \\ S_{m1} & S_{m2} & \dots & S_{mn} \end{bmatrix}$$

Scattering Matrix (contd.)

- The scattering matrix is N by N matrix that **completely characterizes** a linear, N-port device. Effectively, the scattering matrix describes a multi-port device the way that Γ_0 describes a single-port device (e.g., a load)!
- The values of the scattering matrix for a particular device or network, like Γ_0, are **frequency dependent**! Thus, it may be more instructive to **explicitly** write:

$$\boldsymbol{S}(\omega) = \begin{bmatrix} S_{11}(\omega) & S_{12}(\omega) & \dots & S_{1n}(\omega) \\ S_{21}(\omega) & & & \vdots \\ \vdots & & & \\ S_{m1}(\omega) & S_{m2}(\omega) & \dots & S_{mn}(\omega) \end{bmatrix}$$

- Also realize that—also just like Γ_0—the scattering matrix is dependent on **both** the **device/network** and the Z_0 value of the **TL connected** to it.
- Thus, a device connected to transmission lines with Z_0 =50Ω will have a **completely different scattering matrix** than that same device connected to transmission lines with Z_0 =100Ω
- A device can be **lossless** or **reciprocal**. In addition, we can also classify it as being **matched.**
- Let's examine **each** of these three characteristics, and how they relate to the **scattering matrix.**