Lecture – 4

Date: 16.01.2017

- Reflection Coefficient Transformation
- Power Considerations on a TL
  - Return Loss and Insertion Loss
  - Standing Wave and SWR
- Sourced and Loaded TL
- Lossy Transmission Line

Reflection Coefficient Transformation

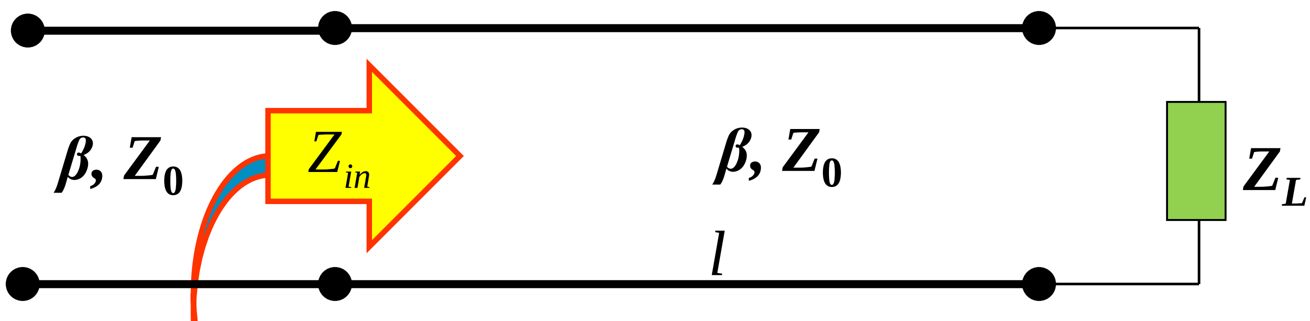
- We know that the **load** at the end of some length of a transmission line (with characteristic impedance Z_0) can be specified in terms of its impedance Z_L **or** its reflection coefficient Γ_0.

- Note **both** values are complex, and **either one** completely specifies the load—if you know **one**, you know the **other**!

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_L = Z_0\left(\frac{1+\Gamma_0}{1-\Gamma_0}\right)$$

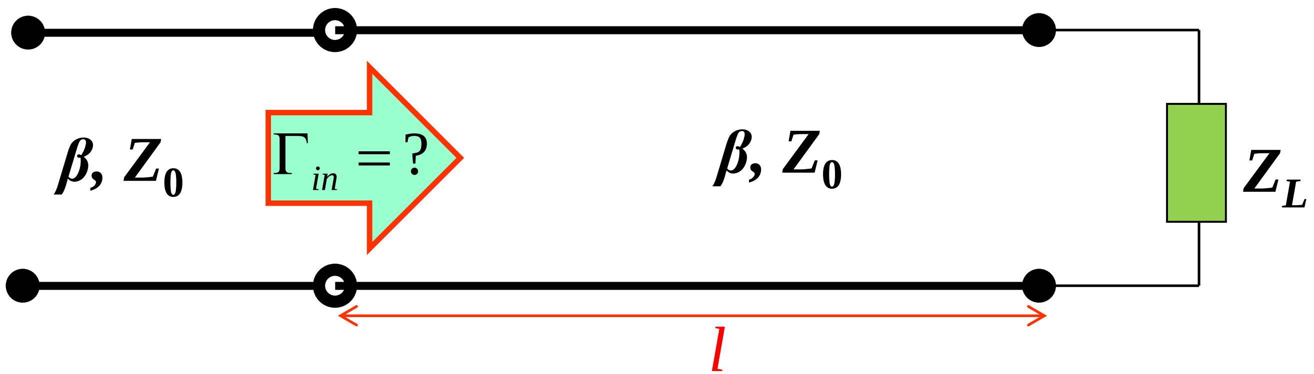
- Recall that we determined how a length of transmission line **transformed** the load **impedance** into an input **impedance** of a (generally) different value:



$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

Reflection Coefficient Transformation (contd.)

Q: Say we know the load in terms of its **reflection coefficient**. How can we express the **input** impedance in terms its **reflection coefficient** (call this Γ_{in})?



A: Well, we **could** execute these **three** steps:

1. Convert Γ_0 to Z_L:

$$Z_L = Z_0\left(\frac{1+\Gamma_0}{1-\Gamma_0}\right)$$

2. Transform Z_L down the line to Z_{in}:

$$Z_{in} = Z_0\frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}$$

3. Convert Z_{in} to Γ_{in}:

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

Q: Yikes! This is a **ton** of complex arithmetic—isn't there an **easier** way?

A: Actually, there **is**!

Reflection Coefficient Transformation (contd.)

- Recall, the input impedance of a TL of length l, terminated with a load Γ_0, is:

Directly insert this into:

$$Z_{in} = Z(z=-l) = \frac{V(z=-l)}{I(z=-l)} = Z_0\left(\frac{e^{j\beta l} + \Gamma_0 e^{-j\beta l}}{e^{j\beta l} - \Gamma_0 e^{-j\beta l}}\right)$$

Note this **directly** relates Γ_0 to Z_{in} (steps 1 and 2 combined!).

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

directly relates Γ_0 to Γ_{in}.

$$\Gamma_{in} = \Gamma_0 e^{-j2\beta l}$$

Q: Hey! This result looks **familiar**.

A: Absolutely! Recall that we found the reflection coefficient **function** $\Gamma(z)$:

$$\Gamma_{in} = \Gamma_0 e^{-j2\beta l} \quad \Rightarrow \quad \Gamma(z=-l) = \Gamma_0 e^{-j2\beta l}$$

$$\Gamma_{in} = \Gamma_0 e^{-j2\beta l}$$

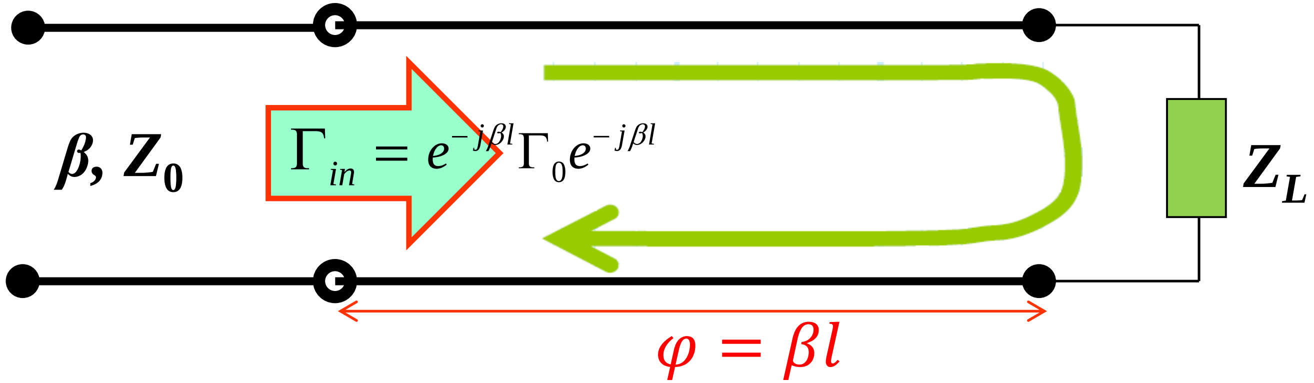
the **magnitude** of Γ_{in} is the **same** as the magnitude of Γ_0!

$$|\Gamma_{in}| = |\Gamma_0 e^{-j2\beta l}| = |\Gamma_0|$$

The reflection coefficient at the input is simply related to Γ_0 by a **phase shift** of $2\beta l$.

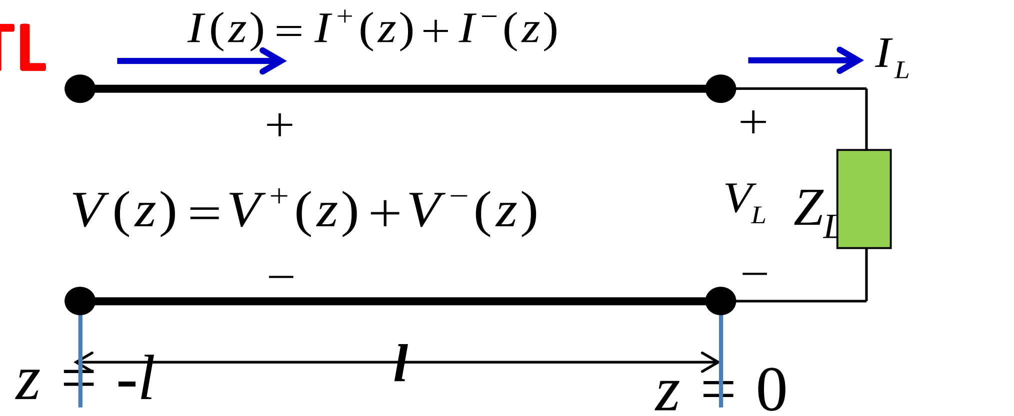
Reflection Coefficient Transformation (contd.)

The **phase shift** associated with transforming Γ_0 down a transmission line can be attributed to the phase shift associated with the wave propagating a length l down the line, reflecting from load Z_L, and then propagating a length l back up the line.



Power Considerations on a TL

- We have discovered that **two waves propagate** along a transmission line, one in each direction ($V^+(z)$ *and* $V^-(z)$).



The result is that electromagnetic energy flows along the transmission line at a given rate (i.e., **power**).

Q: How much power flows along a transmission line, and where does that power go?

A: We can answer by determining the power **absorbed** by the **load**!

$$P_{abs} = \frac{1}{2}\mathrm{Re}\left(V_L I_L^*\right) = \frac{1}{2}\mathrm{Re}\left(V(0)I(0)^*\right) = \frac{\left|V_0^+\right|^2}{2Z_0}\left(1 - \left|\Gamma_0\right|^2\right)$$

$$P_{abs} = \frac{\left|V_0^+\right|^2}{2Z_0} - \frac{\left|V_0^+\Gamma_0\right|^2}{2Z_0} = \frac{\left|V_0^+\right|^2}{2Z_0} - \frac{\left|V_0^-\right|^2}{2Z_0}$$

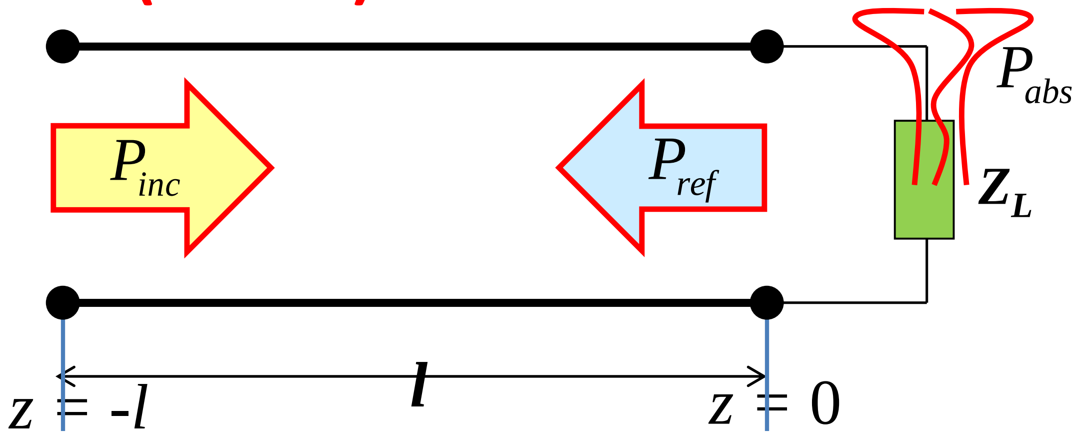
$$P_{ref} = \frac{\left|V_0^+\Gamma_0\right|^2}{2Z_0} = \left|\Gamma_0\right|^2 P_{inc}$$

Incident Power, P_{inc}

Reflected Power, P_{ref}

Power Considerations on a TL (contd.)

- It is thus apparent that the power flowing towards the load (P_{inc}) is either absorbed by the load (P_{abs}) or reflected back from the load (P_{ref})

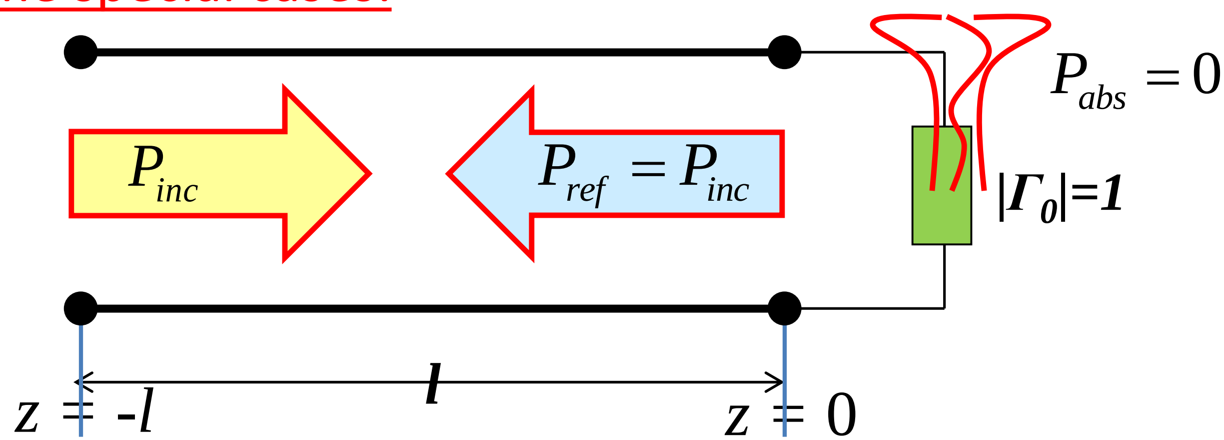


Now let us consider some special cases:

1. $|\Gamma_0| = 1$

$$P_{ref} = |\Gamma_0|^2 P_{inc} = P_{inc}$$

$$\Rightarrow P_{abs} = 0$$



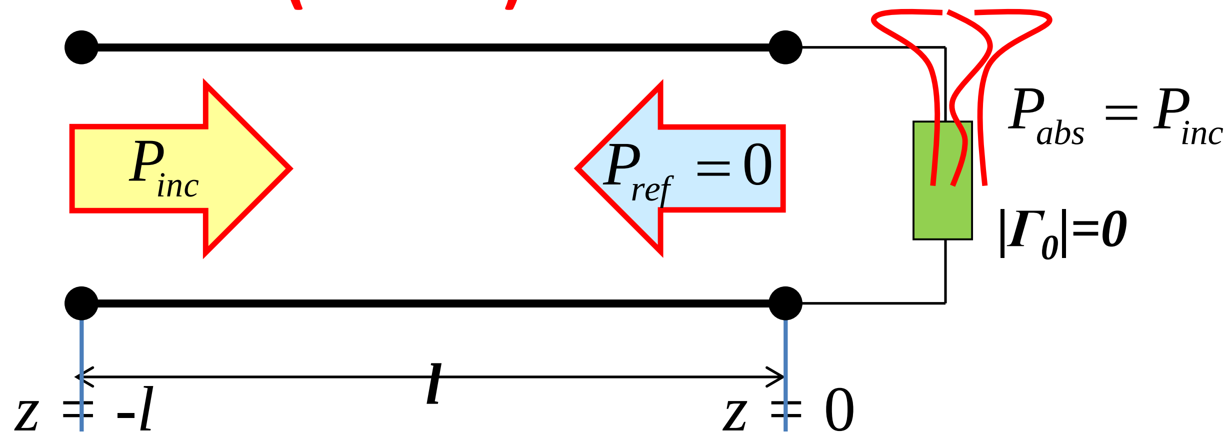
There is no power absorbed by the load → all the incident power is reflected

Power Considerations on a TL (contd.)

2. $\left|\Gamma_0\right| = 0$

$$P_{ref} = \left|\Gamma_0\right|^2 P_{inc} = 0$$

$$\Rightarrow P_{abs} = P_{inc}$$



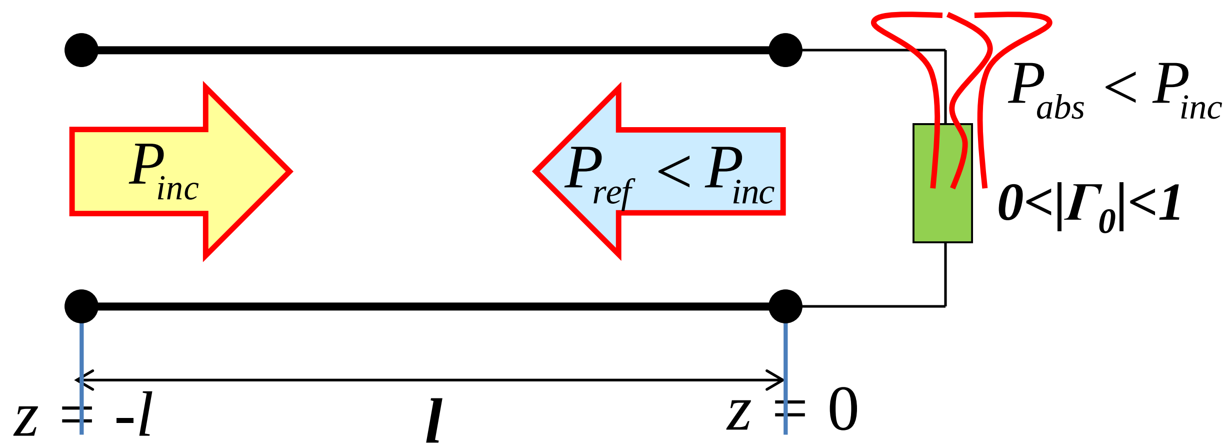
all the incident power is absorbed by the load

None of the incident power is **reflected**

3. $0 < \left|\Gamma_0\right| < 1$

$$0 < P_{ref} = \left|\Gamma_0\right|^2 P_{inc} < 0$$

$$\Rightarrow 0 < P_{abs} = P_{inc}\left(1 - \left|\Gamma_0\right|^2\right) < P_{inc}$$



Power Considerations on a TL (contd.)

Power Absorbed is Negative

4. $|\Gamma_0| > 1$

$$P_{ref} = |\Gamma_0|^2 P_{inc} > P_{inc} \Rightarrow P_{abs} = P_{inc}\left(1 - |\Gamma_0|^2\right) < 0$$

What type of load it could be?

Alternatively, we can say that the load creates extra power → i.e, acts as a power source and not a sink!

Definitely not a passive load → A passive device can't produce power

Therefore: $|\Gamma_0| \leq 1$ **For all passive loads**

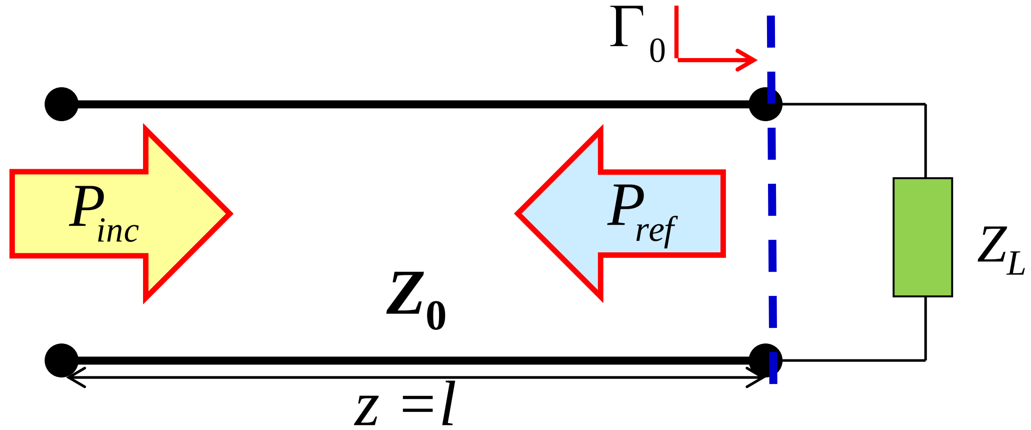
Q: Can Γ_0 every be **greater** than one?

A: Sure, if the "load" is an **active** device. In other words, the load must have some **external power** source connected to it.

Q: What about the case where $|\Gamma_0| < 0$, shouldn't we examine **that** situation as well?

A: That would be just plain **silly**; do **you** see why?

Return Loss



- The **ratio** of the reflected power from a load, to the incident power on that load, is known as **return loss**. Typically, return loss is expressed in **dB:**

Return Loss (R.L.): $$RL[dB] = -10\log\left(\frac{P_{ref}}{P_{inc}}\right) = -10\log\left(|\Gamma_0|^2\right)$$

- The return loss tells us the percentage of the incident power reflected at the point of mismatch
- For example, if the return loss is **10dB**, then **10%** of the power is **reflected** while the **90%** is **absorbed/transmitted** → i.e, we lose 10% of the incident power
- For the **return loss of 30dB**, the **reflected power is 0.1%** of the incident power → **we lose only 0.1% of the incident power**
- A larger numeric value of return loss actually indicates smaller lost power → **An ideal return loss would be ∞** → **matched condition**

Return Loss (contd.)

- A **return loss** of **0dB** indicates that **reflection coefficient** is **ONE** → reactive termination
- Return Loss **(RL)** is very helpful as it provides **real-valued** measures of mismatch (unlike the **complex-valued** Z_L and Γ_0)

A match is good if the return loss is high. A high return loss is desirable and results in a lower insertion loss.

Insertion Loss

- This is another parameter to address the mismatch problem and is defined as:

$$IL[dB] = -10\log\left(\frac{P_{transmitted}}{P_{incident}}\right) = -10\log\left(\frac{P_{incident} - P_{reflected}}{P_{incident}}\right) - 10\log\left(1 - \left|\Gamma_{in}\right|^2\right)$$

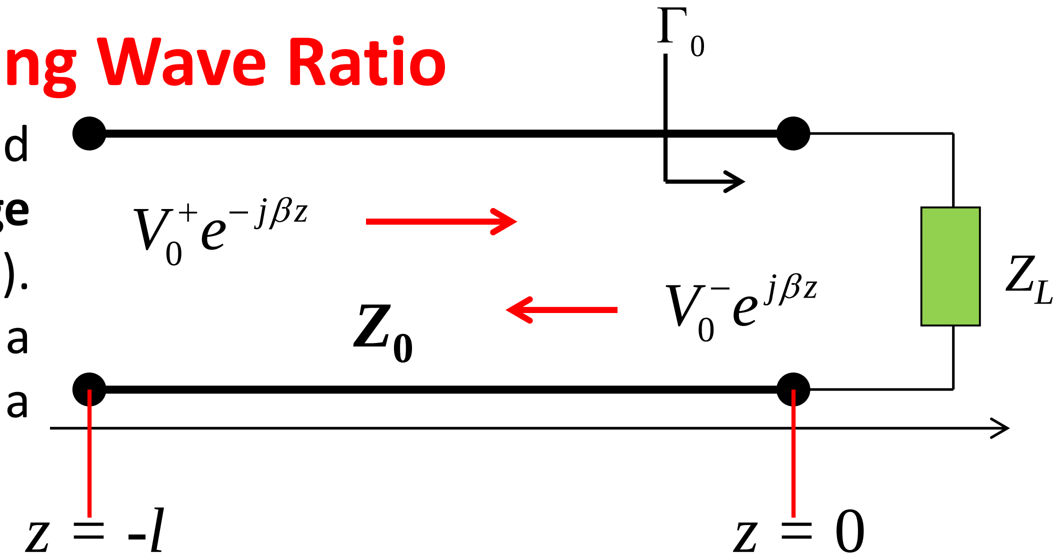
For open- and short-circuit conditions: $IL \rightarrow \infty$

For perfectly matched conditions: $IL = 0$

insertion loss signifies the loss of signal power resulting from the insertion of a device in a transmission line.

Standing Wave and Standing Wave Ratio

- Another traditional real-valued measure of load match is **Voltage Standing Wave Ratio** (VSWR). Consider again the **voltage** along a terminated transmission line, as a function of **position** z.



$$V(z) = V_0^+ \left[e^{-j\beta z} + \Gamma_0 e^{+j\beta z} \right] \Rightarrow V(-l) = V_0^+ \left[e^{j\beta l} + \Gamma_0 e^{-j\beta l} \right]$$

- **For a short circuited line: $\Gamma_0 = -1$**

$$\Rightarrow V(-l) = V_0^+ \left(e^{+j\beta l} - e^{-j\beta l} \right)$$

2jsin(β*l*)

$$\Rightarrow v(-l,t) = \mathrm{Re}\left(V(-l) e^{j\omega t} \right) = \mathrm{Re}\left(2jV_0^+(z) \sin(\beta l) e^{j\omega t} \right)$$

Standing Wave and Standing Wave Ratio (contd.)

$$\therefore v(-l,t) = 2V_0^+ \sin(\beta l)\cos(\omega t + (\pi/2))$$

Definitely not a traveling wave!!

Always zero for $-l=0$ i.e., the point of short-circuit

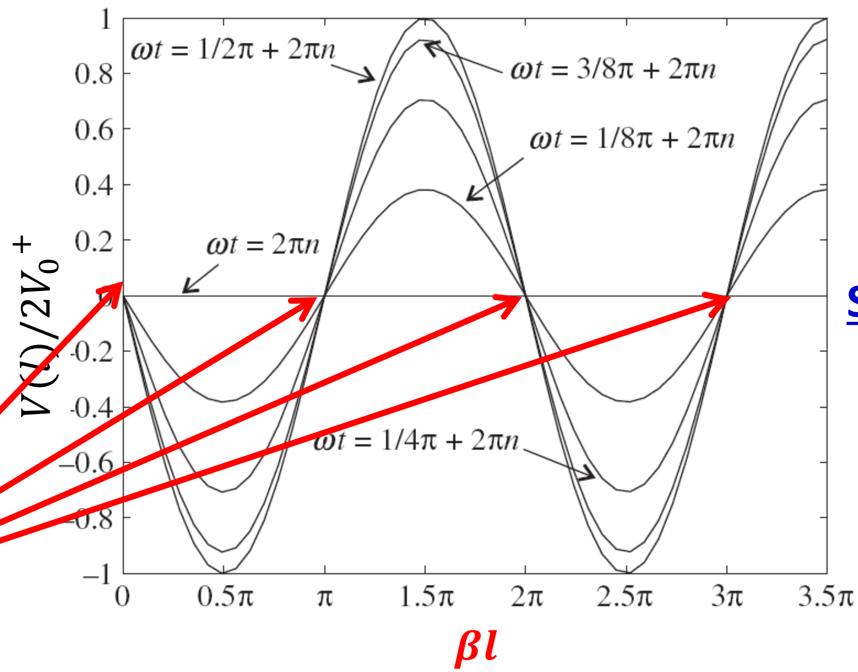
Where has the traveling wave V(z) gone?

- As the time and space are decoupled → No wave propagation takes place
- The incident wave is 180° out of phase with the reflected wave → gives rise to zero crossings of the wave at 0, $\lambda/2$, λ, $3\lambda/2$, and so on → standing wave pattern!!!

Standing Wave and Standing Wave Ratio (contd.)

Corresponding Electrical Length (βl): $0, \pi, 2\pi, 3\pi$

Spatial Location: $0, \lambda/2, \lambda, 3\lambda/2$



Standing Wave Pattern for Various Instances of Time

- for arbitrarily terminated line: $$V(-l) = V_0^+\left(e^{+j\beta l} + \Gamma_0 e^{-j\beta l}\right) = V_0^+ e^{+j\beta l}\left(1 + \Gamma_0 e^{-j2\beta l}\right)$$

$$\Rightarrow V(-l) = A(-l)\left(1 + \Gamma(-l)\right)$$

Valid anywhere on the line

Similarly: $$I(-l) = \frac{A(-l)}{Z_0}\left(1 - \Gamma(-l)\right)$$

Valid anywhere on the line

Standing Wave and Standing Wave Ratio (contd.)

- Under the matched condition, $\Gamma_0 = 0$ and therefore $\Gamma(-l) = 0$ → as expected, only positive traveling wave exists.
- For other arbitrary impedance loads: Standing Wave Ratio (SWR) or Voltage Standing Wave Ratio (VSWR) is the measure of mismatch.
- SWR is defined as the ratio of maximum voltage (or current) amplitude and the minimum voltage (or current) amplitude along a line → therefore, for an arbitrarily terminated line:

$$VSWR = ISWR = SWR = \left|\frac{V(-l)_{\max}}{V(-l)_{\min}}\right| = \left|\frac{I(-l)_{\max}}{I(-l)_{\min}}\right|$$

We have: $V(-l) = V_0^+ e^{+j\beta l}\left(1+\Gamma_0 e^{-j2\beta l}\right)$

- **Two possibilities for extreme values:** $\Gamma_0 e^{-j\beta l} = 1$ $\quad$ $\Gamma_0 e^{-j\beta l} = -1$

Max. voltage: $|\mathrm{V}(-l)|_{\max} = |V_0^+|\left(1+|\Gamma_0|\right)$ **Min. voltage:** $|\mathrm{V}(-l)|_{\min} = |V_0^+|\left(1-|\Gamma_0|\right)$

$$\therefore VSWR = \frac{1+|\Gamma_0|}{1-|\Gamma_0|}$$

Apparently: $0 \le \Gamma_0 \le 1$ → $\therefore 1 \le VSWR < \infty$

Standing Wave and Standing Wave Ratio (contd.)

- Note if $|\Gamma_0| = 0$ (i.e., $Z_L = Z_0$), then VSWR = 1. We find for this case:

$$\left|V(z)\right|_{max} = \left|V(z)\right|_{min} = \left|V_0^+\right|$$

In other words, the voltage magnitude is a **constant** with respect to position z.

- Conversely, if $|\Gamma_0| = 1$ (i.e., $Z_L = Z_0$), then VSWR = ∞. We find for **this** case:

$$\left|V(z)\right|_{max} = 2\left|V_0^+\right|$$

$$\left|V(z)\right|_{min} = 0$$

In other words, the voltage magnitude varies **greatly** with respect to position z.

- **Similarly,** **We have:**

$$I(-l) = \frac{V^+}{Z_0}\left(e^{+j\beta l} + \Gamma_0 e^{-j\beta l}\right)$$

$$\therefore ISWR = \frac{1+\left|\Gamma_0\right|}{1-\left|\Gamma_0\right|}$$

$$\therefore 1 \leq ISWR < \infty$$

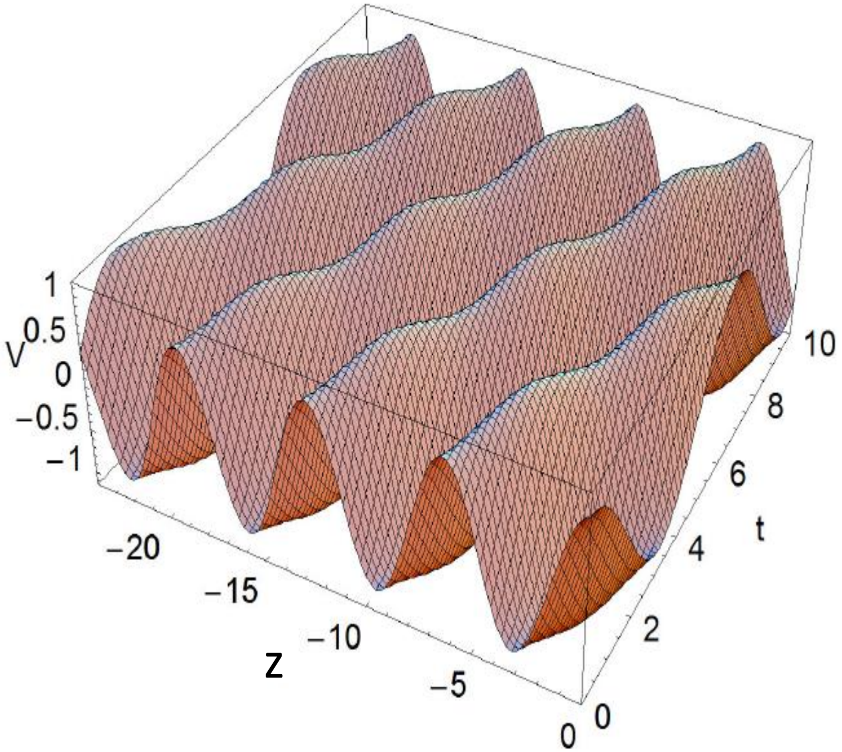
Thus: VSWR=ISWR=SWR

In our course we will mention both as VSWR

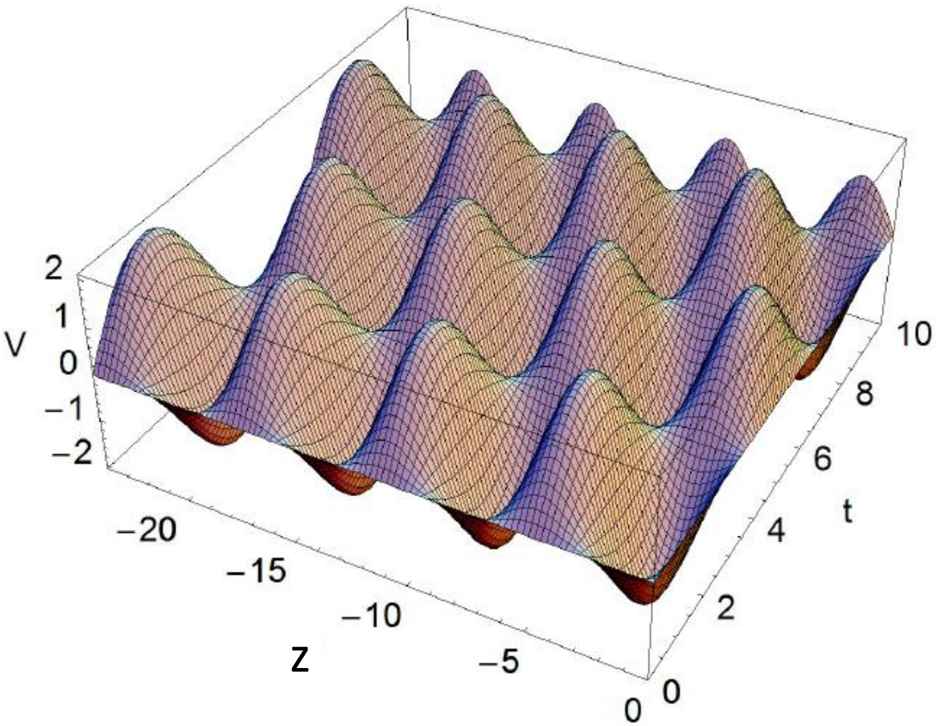
As with **return loss**, VSWR is dependent on the **magnitude** of $|\Gamma_0|$ (i.e, $|\Gamma_0|$) **only** !

<u>In practice, SWR can only be defined for lossless line as the SWR equation is not valid for attenuating voltage and current</u>

Standing Wave and Standing Wave Ratio (contd.)



Standing Wave Pattern at Γ_0=0.1



Standing Wave Pattern at Γ_0=1

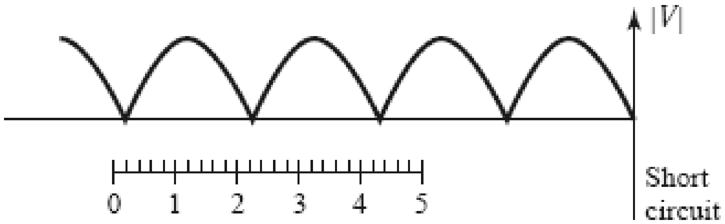
- It is apparent that the maximum and minimum repeats periodically and its values can be used to identify the degree of mismatch by calculating the SWR

HW#1

Example – 1

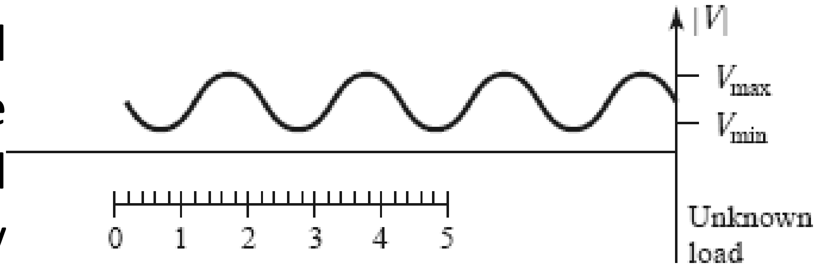
- The following two-step procedure has been carried out with a 50Ω coaxial slotted line to determine an unknown load impedance:

1. short circuit is placed at the load plane, resulting in a standing wave on the line with infinite SWR and sharply defined voltage minima, as shown in Figure.



On the arbitrarily positioned scale on the slotted line, voltage minima are recorded at: $z = 0.2cm, \quad 2.2cm, \quad 4.2cm$

2. The short circuit is removed and replaced with the unknown load. The standing wave ratio is measured as SWR = 1.5, and voltage minima, which are not as sharply defined as those in step 1, are recorded at:



$$z = 0.72cm, \quad 2.72cm, \quad 4.72cm$$

Find the load impedance.

Example – 1 (contd.)

- Knowing that voltage minima repeat every $\lambda/2$, we have from the data of step 1 that λ = 4.0 cm.
- In addition, because the reflection coefficient and input impedance also repeat every $\lambda/2$, we can consider the load terminals to be effectively located at any of the voltage minima locations listed in step 1.
- Thus, if we say the load is at $4.2cm$, then the data from step 2 show that the next voltage minimum away from the load occurs at $2.72cm$.
- It gives: $l_{min} = 4.2 - 2.72 = 1.48cm = 0.37\lambda$

$$\theta_\Gamma = \pi + 2\beta l_{\min}$$

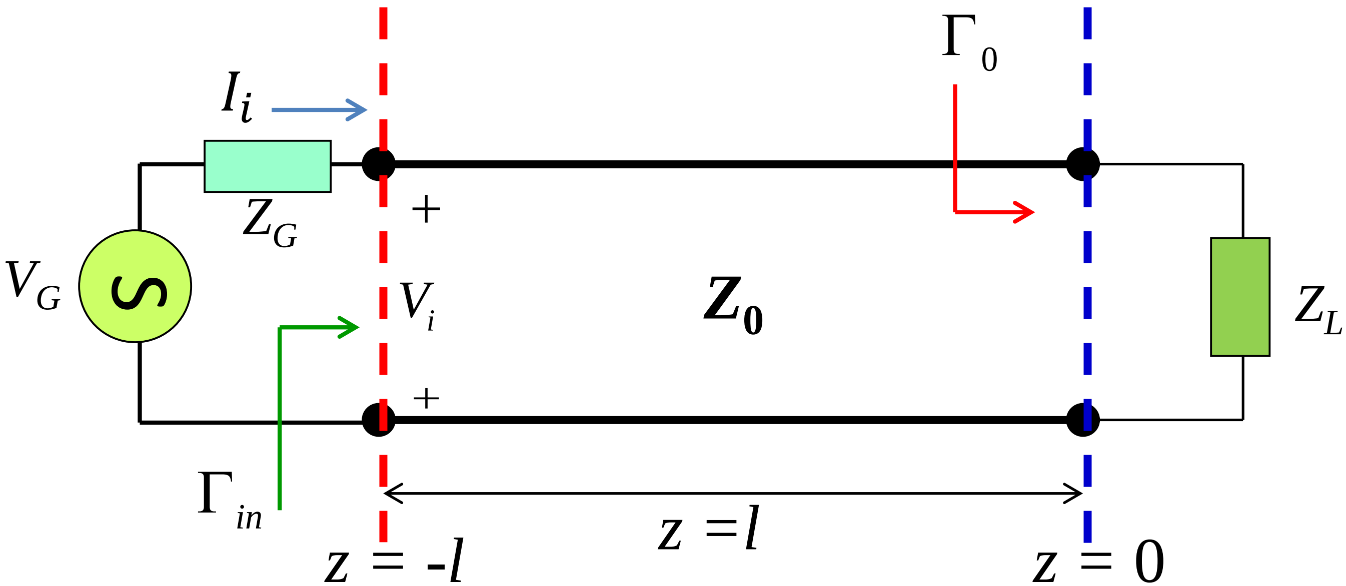
$$\theta_\Gamma = \pi + \left(2 \times \frac{2\pi}{\lambda} l_{\min}\right) = 86.4^\circ$$

- Now: $|\Gamma_0| = \frac{SWR-1}{SWR+1}$ → $|\Gamma_0| = \frac{1.5-1}{1.5+1} = 0.2$
- Therefore: $\Gamma_0 = 0.2e^{j86.4^\circ} = 0.0126 + j0.1996$
- The unknown impedance is then:

$$Z_L = Z_0\left(\frac{1+\Gamma_0}{1-\Gamma_0}\right) \rightarrow Z_L = 50\left(\frac{1+\Gamma_0}{1-\Gamma_0}\right) = 47.3 + j19.7\Omega$$

Sourced and Loaded Transmission Line

- Thus far, we have discussed a TL with terminated load impedance → Let us now consider a TL with terminated load impedance and a source at the input (with line-to-source mismatch)



- At $z = 0$

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

- The current and voltage along the TL is:

$$V(z) = V_0^+\left[e^{-j\beta z} + \Gamma_0 e^{+j\beta z}\right]$$

$$I(z) = \frac{V_0^+}{Z_0}\left[e^{-j\beta z} - \Gamma_0 e^{+j\beta z}\right]$$

V_0^+ depends on the signal **source**! To determine its exact **value**, we must now apply **boundary conditions** at $z = -l$.

Sourced and Loaded Transmission Line (contd.)

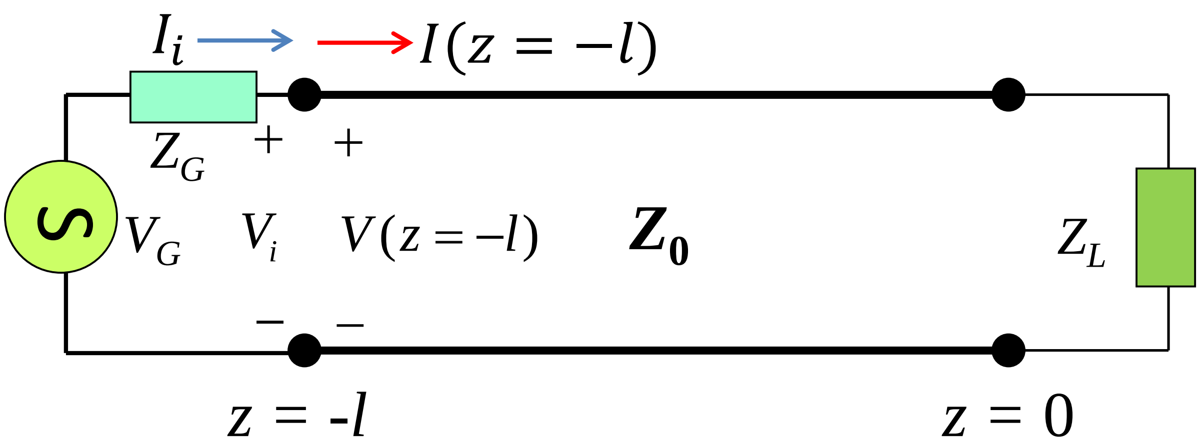
- At the **beginning** of the transmission line:

$$V(z=-l)=V_0^+\left[e^{+j\beta l}+\Gamma_0 e^{-j\beta l}\right]$$

$$I(z=-l)=\frac{V_0^+}{Z_0}\left[e^{+j\beta l}-\Gamma_0 e^{-j\beta l}\right]$$

- Likewise, we know that the **source** must satisfy:

$$V_G=V_i+Z_G I_i$$



- From **KVL** we find:

$$V_i=V(z=-l)$$

- From **KCL** we find:

$$I_i=I(z=-l)$$

- **Combining** these equations, we find:

$$V_G=V_0^+\left[e^{+j\beta l}+\Gamma_0 e^{-j\beta l}\right]+Z_G\frac{V_0^+}{Z_0}\left[e^{+j\beta l}-\Gamma_0 e^{-j\beta l}\right]$$

One equation → **one** unknown (V_0^+)!!

Sourced and Loaded Transmission Line (contd.)

- **Solving**, we find the value of V_0^+:

$$V_0^+ = V_G e^{-j\beta l} \frac{Z_0}{Z_0(1+\Gamma_{in}) + Z_G(1-\Gamma_{in})}$$

$$\Gamma_{in} = \Gamma(z=-l) = \Gamma_0 e^{-j\beta l}$$

- Note this result looks different than the equation in your book (Pozar):

$$V_0^+ = V_G \frac{Z_0}{Z_0 + Z_G} \frac{e^{-j\beta l}}{\left(1-\Gamma_0\Gamma_G e^{-j\beta l}\right)}$$

$$\Gamma_G = \frac{Z_G - Z_0}{Z_G + Z_0}$$

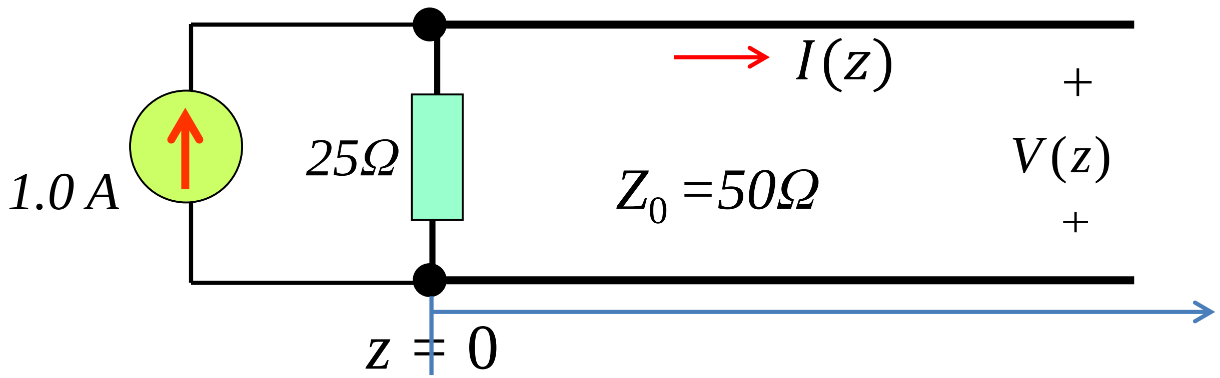
<u>**I like the first expression better.**</u>

Although the two equations are equivalent, **first** expression is explicitly written in terms of $\Gamma_{in} = \Gamma(z=-l)$ (a very **useful**, **precise**, and **unambiguous** value), while the book's expression is written in terms of this **so-called** "source reflection coefficient" Γ_G(a **misleading**, **confusing**, **ambiguous**, and mostly **useless** value).

Specifically, we might be **tempted** to equate Γ_G with the value $\Gamma_{in} = \Gamma(z=-l)$, but it is **not** $\Gamma_G \neq \Gamma(z=-l)$!

Example – 2

- Consider this circuit:



- It is known that the **current** along the transmission line is:

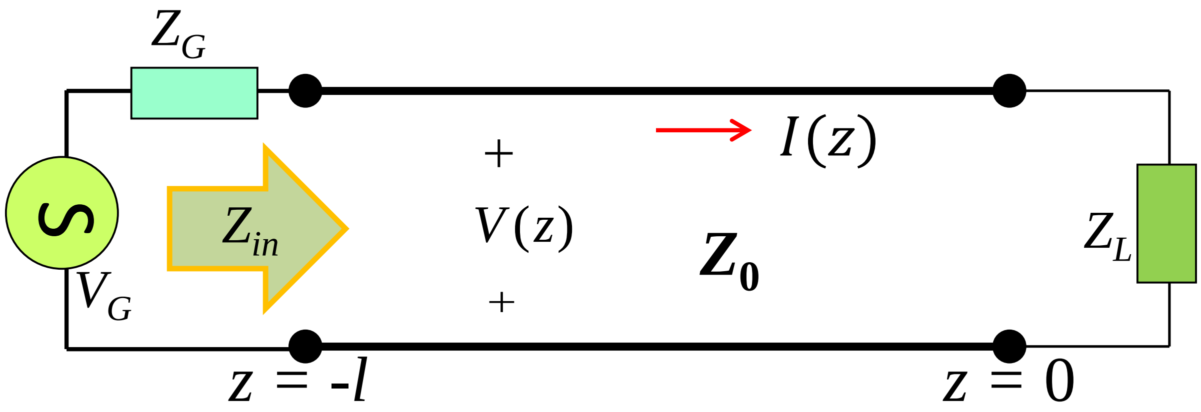
$$I(z) = 0.4e^{-j\beta z} - Be^{+j\beta z} \quad Amp \quad for\ z > 0$$

where B is some unknown complex value.

Determine the value of B.

Sourced and Loaded Transmission Line (contd.)

Q: If the purpose of a transmission line is to transfer **power** from a source to a load, then exactly how much power is **delivered** to Z_L for this circuit??



A: We of course **could** determine V_0^+ and V_0^-, and then determine the power absorbed by the load (P_{abs}) as:

$$P_{abs} = \frac{1}{2}\text{Re}\left\{V(z=0)I^*(z=0)\right\}$$

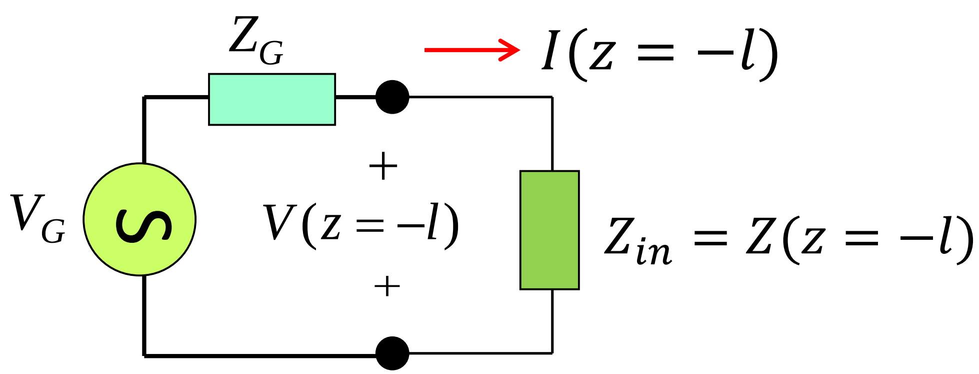
- For **lossless** TL, we know that the power delivered to the load must be **equal** to the power "delivered" to the **input** (P_{in}) of the transmission line:

$$P_{abs} = P_{in} = \frac{1}{2}\text{Re}\left\{V(z=-l)I^*(z=-l)\right\}$$

We can determine this power **without** having to solve for V_0^+ and V_0^- (i.e., V(z) and I(z)). We can simply use our knowledge of **circuit theory**!

Sourced and Loaded Transmission Line (contd.)

- We can **transform** Z_L to the beginning of the transmission line, so that we can replace the transmission line with its **input impedance** Z_{in}:



- by **voltage division** we can determine:

$$V(z=-l) = V_G \frac{Z_{in}}{Z_G + Z_{in}}$$

- from **Ohm's Law** we conclude:

$$I(z=-l) = \frac{V_G}{Z_G + Z_{in}}$$

- Then the **power** P_{in} delivered to Z_{in} (and thus the **power** P_{abs} delivered to Z_L) is:

$$P_{abs} = P_{in} = \frac{1}{2}\mathrm{Re}\left\{V(z=-l)I^*(z=-l)\right\} = \frac{1}{2}\mathrm{Re}\left\{V_G \frac{Z_{in}}{Z_G + Z_{in}} \frac{V_G^*}{(Z_G + Z_{in})^*}\right\}$$

$$\Rightarrow P_{abs} = P_{in} = \frac{1}{2}\frac{|V_G|^2}{|Z_G + Z_{in}|^2}\mathrm{Re}\left\{Z_{in}\right\}$$

Sourced and Loaded Transmission Line (contd.)

- Note that we could **also** determine P_{abs} from our **earlier** expression:

$$P_{abs} = \frac{1}{2}\text{Re}\left\{V(z=0)I^*(z=0)\right\} = \frac{\left|V_0^+\right|^2}{2Z_0}\left(1-\left|\Gamma_0\right|^2\right)$$

we would of course have to **first** determine V_0^+(!):

$$V_0^+ = V_G e^{-j\beta l} \frac{Z_0}{Z_0\left(1+\Gamma_{in}\right)+Z_G\left(1-\Gamma_{in}\right)}$$

- Let's look at **specific cases** of Z_G and Z_L, and see how they affect V_0^+ and P_{abs}.

$Z_G = Z_0$

- For this case, we find that V_0^+ **simplifies** greatly:

$$V_0^+ = \frac{1}{2}V_G e^{-j\beta l}$$

incident wave in this case is **independent** of the load attached at the other end!

Thus, for the **one** case $Z_G = Z_0$, we in fact can consider $V^+(z)$ as being the source wave, and then the reflected wave $V^-(z)$ as being the result of this stimulus.

Sourced and Loaded Transmission Line (contd.)

- The complex value V_0^+ is the value of the incident wave evaluated at the end of the transmission line ($V_0^+ = V^+(z=0)$). We can also determine the value of the incident wave at the **beginning** of the transmission line (i.e. $V^+(z=-l)$).

$$V^+(z=-l) = V_0^+ e^{-j\beta(z=-l)} = \left(\frac{1}{2}V_G e^{-j\beta l}\right)e^{+j\beta l} = \frac{V_G}{2}$$

- Likewise, the delivered power for this case can be simply stated as:

$$P_{abs} = \frac{|V_0^+|^2}{2Z_0}\left(1-|\Gamma_0|^2\right) = \frac{|V_G|^2}{8Z_0}\left(1-|\Gamma_0|^2\right)$$

$\boldsymbol{Z_L = Z_0}$

- In this case, we find that $\Gamma_0 = 0$, and thus $\Gamma_{in} = 0$. As a result:

$$V_0^+ = V_G e^{-j\beta l}\frac{Z_0}{Z_0 + Z_G}$$

- Likewise, we find that:

$$P_{abs} = \frac{|V_0^+|^2}{2Z_0}\left(1-|\Gamma_0|^2\right) = \frac{|V_0^+|^2}{2Z_0}$$

Here the delivered power P_{abs} is simply that of the incident wave (P^+), as the matched condition causes the reflected power to be zero ($P^- = 0$)!

Sourced and Loaded Transmission Line (contd.)

- Inserting the value of V_0^+, we find:

$$P_{abs} = \frac{|V_0^+|^2}{2Z_0} = \frac{|V_G|^2}{2}\frac{Z_0}{|Z_0 + Z_G|^2}$$

this result can also be found by recognizing that $Z_{in} = Z_0$ when $Z_L = Z_0$.

$$Z_{in} = Z_G^*$$

For this case, we find Z_L takes on whatever value required to make $Z_{in} = Z_G^*$. This is a **very** important case!

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{Z_G^* - Z_0}{Z_G^* + Z_0}$$

We can show that (trust me!):

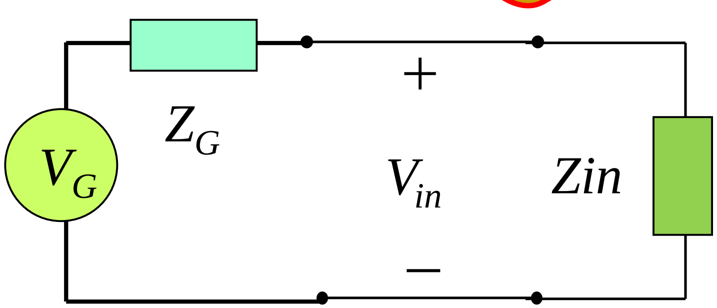
$$V_0^+ = V_G e^{-j\beta l}\frac{Z_G^* + Z_0}{4\,\mathrm{Re}\{Z_G\}}$$

- look at the absorbed power:

$$P_{abs} = \frac{1}{2}\frac{|V_G|^2}{|Z_G + Z_{in}|^2}\mathrm{Re}\{Z_{in}\}$$

It can be shown—for a **given** V_G and Z_G— value of Z_{in} that will absorb the **largest possible** amount of power is the value $Z_{in} = Z_G^*$.

- For this purpose:



Power available for transfer to TL is given by:

$$P_{in} = \frac{1}{2}\mathrm{Re}\left(V_{in}\frac{V_{in}^*}{Z_{in}^*}\right) = \frac{1}{2}\frac{|V_G|^2}{\mathrm{Re}\left(Z_{in}^*\right)}\left|\frac{Z_G}{Z_G + Z_{in}}\right|^2$$

Sourced and Loaded Transmission Line (contd.)

- If $Z_G = R_G + jX_G$ is fixed then for **complex Z_{in}** following conditions must be valid for **maximum P_{in} transferred to TL**

$$\frac{\partial P_{in}}{\partial R_{in}} = \frac{\partial P_{in}}{\partial X_{in}} = 0$$

- Elaboration of these conditions result in:

$$R_G^2 - R_{in}^2 + \left(X_G^2 + 2X_G X_{in} + X_{in}^2\right) = 0$$

$$X_{in}\left(X_G + X_{in}\right) = 0$$

Simplification gives

$$R_{in} = R_G \qquad X_{in} = -X_G$$

$$\Rightarrow Z_{in} = Z_G^*$$

$$P_{abs} = \frac{1}{2}\frac{|V_G|^2}{|Z_G + Z_G^*|^2}\text{Re}\{Z_G^*\}$$

$$\Rightarrow \therefore P_{abs} = \frac{1}{2}|V_G|^2 \frac{1}{4\,\text{Re}\{Z_G^*\}} \doteq P_{avl}$$

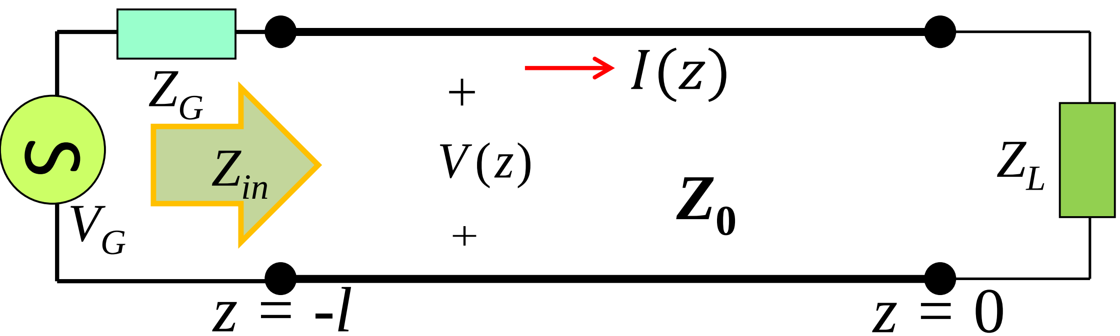
This case is known as the **conjugate match**, and is essentially the goal of every TL problem—to deliver the largest possible power to Z_{in}, and thus to Z_L as well! → This power is known as the **available power** (P_{avl}) of the source.

There are **two** very important things to understand about this result!

Sourced and Loaded Transmission Line (contd.)

Very Important Thing #1

- Consider again:



- if $Z_L = Z_0$, the **reflected** wave will be **zero**, thus:

$$P_{abs} = \frac{|V_G|^2}{2} \frac{Z_0}{|Z_0 + Z_G|^2} \le P_{avl}$$

- But note if $Z_L = Z_0$, the input impedance $Z_{in} = Z_0$ —but then $Z_{in} \neq Z_G^*$ (generally)! In other words, $Z_L = Z_0$ does **not** (generally) result in a **conjugate match**, and thus setting $Z_L = Z_0$ does **not** result in maximum power absorption!

Q: Huh!? This makes **no** sense! A load value of $Z_L = Z_0$ will **minimize** the reflected wave $(P^- = 0)$—**all** of the incident power will be absorbed.

- Any other value of Z_L will result in **some** of the incident wave being reflected—how in the world could this **increase** absorbed power?

Sourced and Loaded Transmission Line (contd.)

- After all, just **look** at the expression for absorbed power:

$$P_{abs} = \frac{|V_0^+|^2}{2Z_0}\left(1-|\Gamma_0|^2\right)$$

Clearly, this value is maximized when $\Gamma_0 = 0$ (i.e., when $\boldsymbol{Z_L} = \boldsymbol{Z_0}$)

A: You are forgetting one very important fact! Although it **is** true that the load impedance $\boldsymbol{Z_L}$ affects the **reflected** wave power P^-, the value of $\boldsymbol{Z_L}$ —as we have shown— **likewise** helps determine the value of the **incident** wave (i.e., the value of P^+) as well.

- Thus, the value of $\boldsymbol{Z_L}$ that minimizes P^- will **not** generally maximize P^+!
- **Likewise** the value of $\boldsymbol{Z_L}$ that maximizes P^+ will not generally minimize P^-.
- Instead, the value of $\boldsymbol{Z_L}$ that maximizes the **absorbed** power P_{abs} is, by definition, the value that maximizes the **difference** $P^+ - P^-$.
- We find that this impedance $\boldsymbol{Z_L}$ is the value that results in the **ideal** case of $\boldsymbol{Z_{in}} = \boldsymbol{Z_G}^*$.

Sourced and Loaded Transmission Line (contd.)

Q: Yes, but what about the case where $Z_G = Z_0$? For **that** case, we determined that the incident wave **is** independent of Z_L. Thus, it would seem that at least for that case, the **delivered** power would be maximized when the **reflected** power was minimized (i.e., $Z_L = Z_0$).

A: True! But think about what the **input** impedance would be in that case— $Z_{in} = Z_0$. Oh by the way, that provides a **conjugate match** ($Z_{in} = Z_0 = Z_G^*$).

- Thus, in some ways, the case $Z_G = Z_0 = Z_L$ (i.e., **both** source and load impedances are numerically equal to Z_0) is **ideal**. A **conjugate match** occurs, the incident wave is **independent** of Z_L, there is **no** reflected wave, and all the math **simplifies** quite nicely:

$$V_0^+ = \frac{1}{2} V_G e^{-j\beta l}$$

$$P_{abs} = P_{avl} = \frac{|V_G|^2}{8Z_0}$$

Sourced and Loaded Transmission Line (contd.)

Very Important Thing #2

- Note the conjugate match criteria **says**: **Given** source impedance $\boldsymbol{Z_G}$, maximum power transfer occurs when the input impedance is set at value $\boldsymbol{Z_{in}} = \boldsymbol{Z_G}^*$.
- It does NOT say: **Given** input impedance $\boldsymbol{Z_{in}}$, maximum power transfer occurs when the source impedance is set at value $\boldsymbol{Z_G} = \boldsymbol{Z_{in}}^*$.

This last statement is in fact false!

- A **factual** statement is this: **Given** input impedance $\boldsymbol{Z_{in}}$, maximum power transfer occurs when the source impedance is set at value $\boldsymbol{Z_G} = 0 - \mathrm{j}X_{in}$(i.e., $\boldsymbol{R_G} = 0$).

Q: Huh??

A: Remember, the value of source impedance $\boldsymbol{Z_G}$ affects the available power P_{avl} of the source. To maximize P_{avl}, the real (resistive) component of the source impedance should be as small as possible (regardless of $\boldsymbol{Z_{in}}$!), a fact that is **evident** when observing the expression for **available power**:

$$P_{avl} = \frac{1}{2}|V_G|^2 \frac{1}{4\,\mathrm{Re}\{Z_G^*\}} = \frac{|V_G|^2}{8R_G}$$

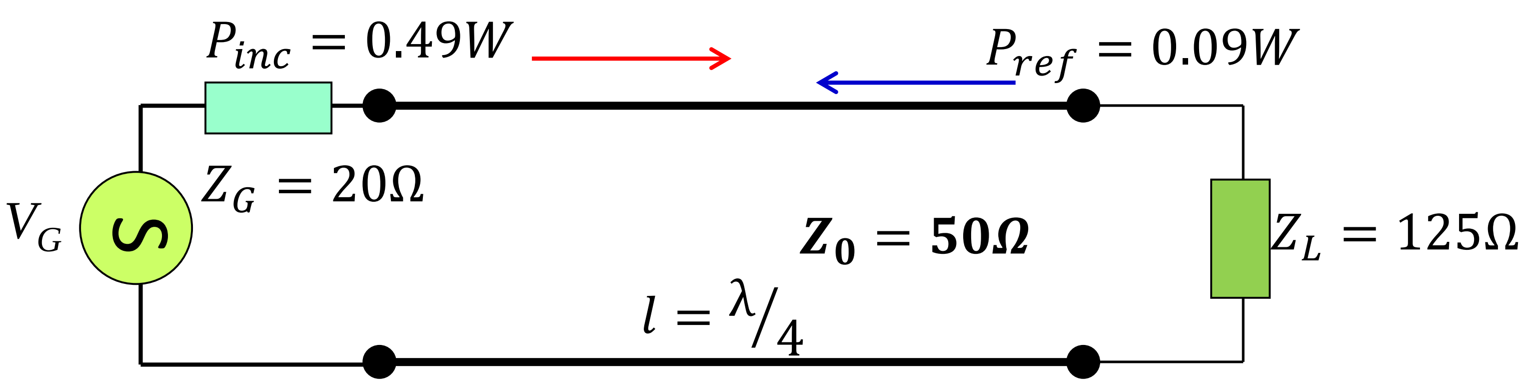
Sourced and Loaded Transmission Line (contd.)

- Thus, **maximizing** the power delivered **to** a load (P_{abs}), **from** a source, has **two** components:

1. Maximize the **power available** (P_{avl}) from a source (e.g., minimize R_G).
2. **Extract** all of this available power by setting the input impedance Z_{in} to a value $Z_{in} = Z_G^*$ (thus $P_{abs} = P_{avl}$).

Example – 3

- Consider this circuit, where the transmission line is **lossless** and has length $l = \lambda/4$:



Determine the magnitude of source voltage V_G (i.e., determine $|V_G|$).

Hint: This is **not** a boundary condition problem. Do **not** attempt to find $V(z)$ and/or $I(z)$!

Lossy Transmission Lines

- Recall that we have been **approximating** low-loss transmission lines as lossless (R =G = 0):

$$\alpha = 0 \qquad \beta = \omega\sqrt{LC}$$

- But, **long** low-loss lines require a **better** approximation:

$$\alpha = \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right) \qquad \beta = \omega\sqrt{LC}$$

- Now, if we have **really long** transmission lines (e.g., long distance communications), we can apply **no** approximations at all:

$$\alpha = \mathrm{Re}\{\gamma\} \qquad \beta = \mathrm{Im}\{\gamma\}$$

For these **very** long transmission lines, $\beta = Im\{\gamma\}$ is a **function** of signal **frequency** ω. This results in an extremely serious problem—signal **dispersion**.

- Recall that the **phase velocity** v_p (i.e., propagation velocity) of a wave in a transmission line is:

$$v_p = \frac{\omega}{\beta}$$

$$\beta = \mathrm{Im}\{\gamma\} = \mathrm{Im}\left\{\sqrt{(R + j\omega L)(G + j\omega C)}\right\}$$

For a lossy line, v_p is a function of frequency ω (i.e., $v_p(\omega)$)—this is **bad**!

Lossy Transmission Lines (contd.)

- Any signal that carries significant **information** must has some non-zero **bandwidth**. In other words, the signal energy (as well as the information it carries) is **spread** across many frequencies.
- If the different frequencies that comprise a signal travel at different velocities, that signal will arrive at the end of a transmission line **distorted**. We call this phenomenon signal **dispersion.**
- Recall for **lossless** lines, however, the phase velocity is **independent** of frequency—**no** dispersion will occur!
- For lossless line:

$$v_p = \frac{1}{\sqrt{LC}}$$

however, a perfectly lossless line is impossible, but we find phase velocity is **approximately** constant if the line is low-loss.

Q: You say "**most** often" not a problem—that phrase seems to imply that dispersion sometimes **is** a problem!

Lossy Transmission Lines (contd.)

A: for low-loss transmission lines, dispersion can be a problem **if** the lines are **very** long—just a small difference in phase velocity can result in significant differences in propagation delay **if** the line is very long!

- examples of long transmission lines include phone lines and cable TV. However, the **original** long transmission line problem occurred with the **telegraph**.
- Early telegraph "engineers" discovered that if they made their telegraph lines **too long**, the dots and dashes characterizing Morse code turned into a muddled, indecipherable **mess**. Although they did not realize it, they had fallen victim to the heinous effects of **dispersion**!
- to send messages over long distances, they were forced to implement a series of intermediate "**repeater**" stations, wherein a human operator received and then **retransmitted** a message on to the next station. This **really** slowed things down!

Q: Is there any way to **prevent** dispersion from occurring?

Lossy Transmission Lines (contd.)

A: You bet! **Oliver Heaviside** figured out how in the **19th** Century!

- Heaviside found that a transmission line would be distortionless (i.e., no dispersion) **if** the line parameters exhibited this **ratio**:

$$\frac{R}{L} = \frac{G}{C}$$

- Let's see **why** this works. Note the complex propagation constant γ can be expressed as:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{LC(R/L + j\omega)(G/C + j\omega)}$$

- **For $\frac{R}{L} = \frac{G}{C}$:**

$$\gamma = \sqrt{LC(R/L + j\omega)(R/L + j\omega)} = (R/L + j\omega)\sqrt{LC} = R\sqrt{\frac{C}{L}} + j\omega\sqrt{LC}$$

- Thus:

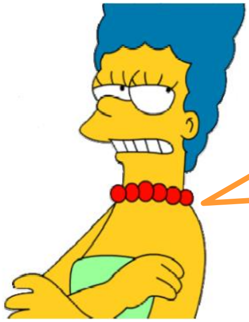
$$\alpha = \text{Re}\{\gamma\} = R\sqrt{\frac{C}{L}} \qquad \beta = \text{Im}\{\gamma\} = \omega\sqrt{LC}$$

- The propagation **velocity** of the wave is thus:

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

The propagation velocity is **independent** of frequency! This lossy transmission line is **not** dispersive!

Lossy Transmission Lines (contd.)



Q: Right. All the transmission lines I use have the property that $R/L > G/C$. I've **never** found a transmission line with this **ideal** property $R/L = G/C$!

A: It is true that typically $R/L > G/C$. But, we can reduce the ratio R/L(until it is equal to G/C) by adding series **inductors** periodically along the transmission line.

This was **Heaviside's** solution—and it worked! **Long** distance transmission lines were made possible.

Q: Why don't we increase G instead?

A: