Lecture – 2

Date: 05.01.2017

- Definition of Some TL Parameters
- Examples of Transmission Lines

Transmission Lines (contd.)

- For a lossless transmission line the second order differential equation for phasors are:

$$\beta = \omega\sqrt{LC}$$

$$\frac{d^2\,\mathrm{V(z)}}{dz^2} + \beta^2 V(z) = 0$$

$$\frac{d^2\,I(\mathrm{z})}{dz^2} + \beta^2 I(z) = 0$$

General Solution For Voltage Phasor

V_0^+ and V_0^- are complex constants

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

- Similarly the current phasor for a lossless line can be described:

$$I(z) = -\frac{1}{j\omega L}\frac{dV(z)}{dz} = -\frac{1}{j\omega L}\frac{d}{dz}\left[V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}\right]$$

$$\Rightarrow I(z) = \frac{\beta}{\omega L}\left[V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}\right]$$

Gives the Definition of Characteristic Impedance

Transmission Lines (contd.)

$$Z_0 = \frac{\omega L}{\beta} = \frac{\omega L}{\omega\sqrt{LC}} = \sqrt{\frac{L}{C}}$$

Completely Dependent on L and C

Characteristic Impedance for a Lossless Line is Real

$$\therefore I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$$

Opposite Signs in these Terms Gives a Clue about Current Flow in Two Different Directions

- The time dependent form of the voltage and current along the transmission line can be derived from phasors as:

$$v(z,t) = \mathrm{Re}\left[V(z)e^{j\omega t}\right] = \mathrm{Re}\left[V_0^+ e^{-j(\beta z-\omega t)} + V_0^- e^{j(\beta z+\omega t)}\right]$$

$$i(z,t) = \mathrm{Re}\left[I(z)e^{j\omega t}\right] = \mathrm{Re}\left[\frac{V_0^+}{Z_0} e^{-j(\beta z-\omega t)} - \frac{V_0^-}{Z_0} e^{j(\beta z+\omega t)}\right]$$

Transmission Lines (contd.)

- For the simple case of $\boldsymbol{V_0}^+$ **and** $\boldsymbol{V_0}^-$ being real, the voltage and current along the transmission line can be expressed as:

$$\boxed{\begin{aligned} v(z,t) &= V_0^+\cos(\omega t-\beta z)+V_0^-\cos(\omega t+\beta z) \\ i(z,t) &= \frac{V_0^+}{Z_0}\cos(\omega t-\beta z)-\frac{V_0^-}{Z_0}\cos(\omega t+\beta z) \end{aligned}}$$

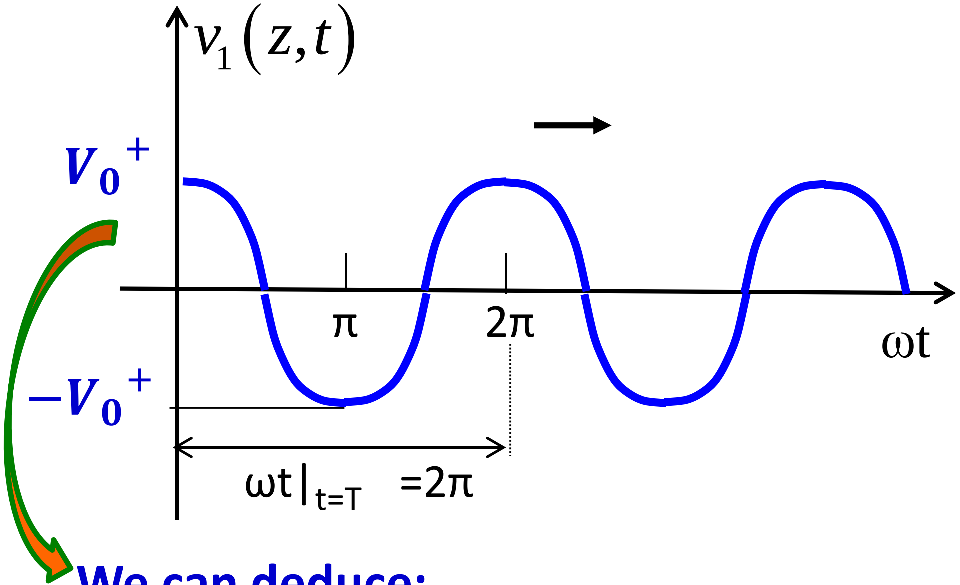
$$V_0^+\cos(\omega t-\beta z) \qquad V_0^-\cos(\omega t+\beta z)$$

Wave Functions

- Let us examine the wave characteristics of $\boxed{v_1(z,t)=V_0^+\cos(\omega t-\beta z)}$

Transmission Lines (contd.)

- **For fixed position z and variable t:**



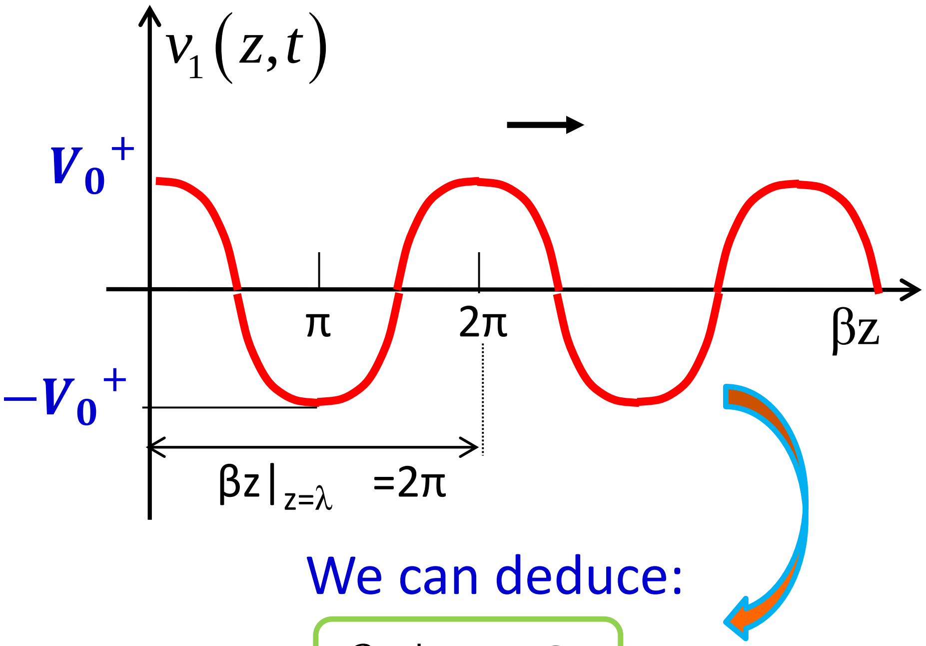
We can deduce:

$$\boxed{\omega t|_{t=T} = 2\pi}$$

Time Period of Wave

$$\boxed{\Rightarrow T = \frac{2\pi}{\omega} = \frac{1}{f}}$$

- **For fixed time t and variable position z**



We can deduce:

$$\boxed{\beta z|_{z=\lambda} = 2\pi}$$

$$\boxed{\Rightarrow \lambda = \frac{2\pi}{\beta}}$$

Wavelength

Transmission Lines (contd.)

- **What is the physical meaning of β**

ϕ

Let us consider once again: $\boxed{V_0^+ \cos(\omega t) - \beta z)}$

Apparently β represents the relative phase of this wave function in space (ie, function of transmission line position)

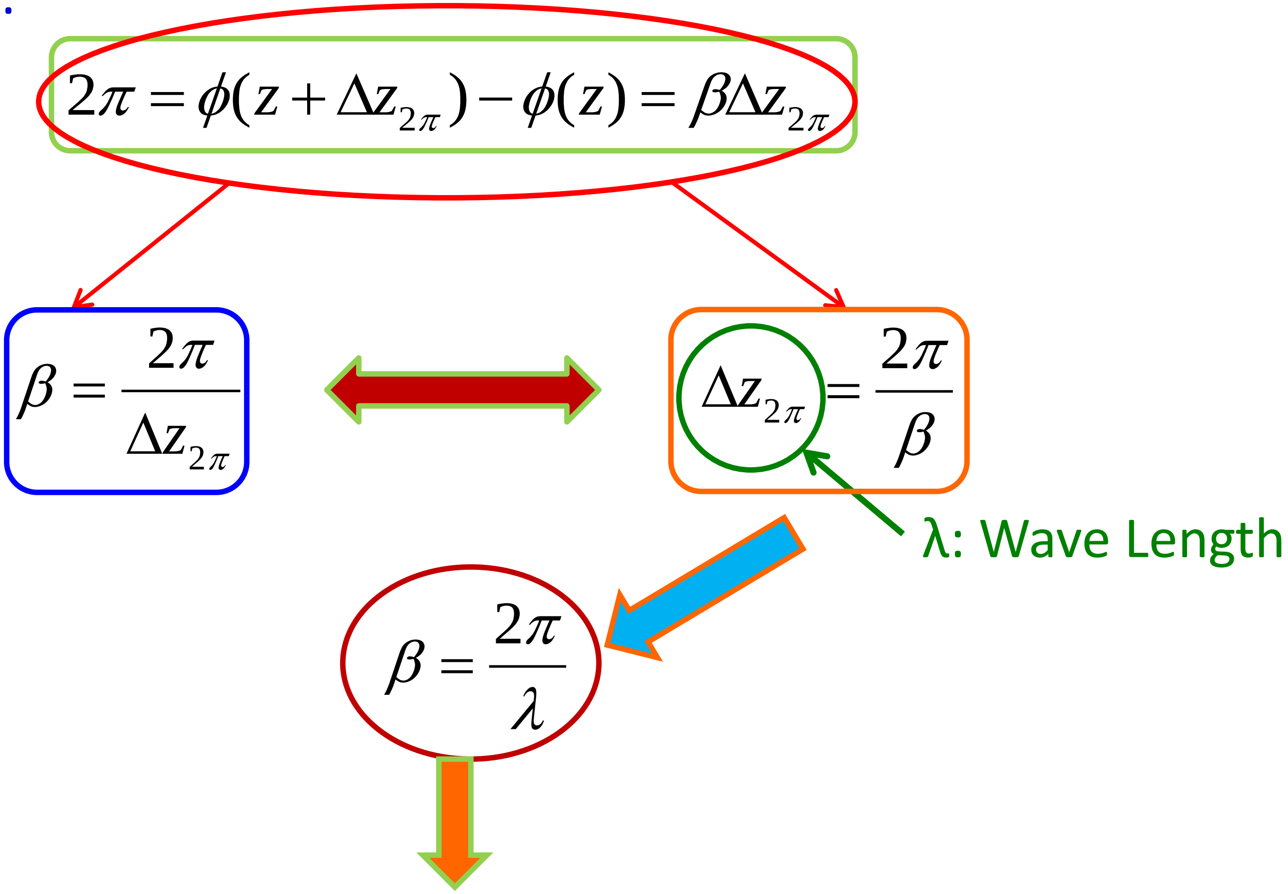
In principle, the value of β must have units of (ϕ/z) ⇒ **Radians/meter**

Therefore, if the values of β is small, we will need to move a significant distance Δz down the transmission line in order to observe a change in the relative phase of the oscillation

Conversely, if the value of β is large, a significant change in relative phase can be observed if traveling a short distance Δz down the transmission line

Transmission Lines (contd.)

- **For example**, in order to observe a change in relative phase of 2π, the distance Δ*z* is:



$$2\pi = \phi(z+\Delta z_{2\pi}) - \phi(z) = \beta\Delta z_{2\pi}$$

$$\beta = \frac{2\pi}{\Delta z_{2\pi}}$$

$$\Delta z_{2\pi} = \frac{2\pi}{\beta}$$

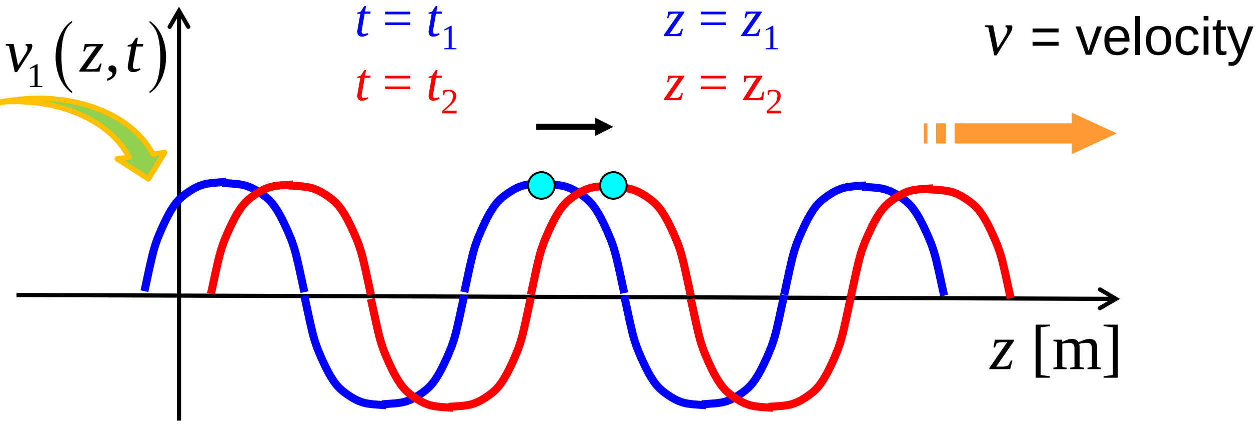
λ: Wave Length

$$\beta = \frac{2\pi}{\lambda}$$

Can’t we call it spatial counterpart?

Transmission Lines (contd.)

- For variable position z and variable time t



- It is apparent that the phase of both these are identical and hence:

$$v_1(z_1,t_1) = v_2(z_2,t_2) \Rightarrow \cos(\beta z_1 - \omega t_1) = \cos(\beta z_2 - \omega t_2)$$

$$\beta z_1 - \omega t_1 = \beta z_2 - \omega t_2$$

Speed of Propagation → $$\boxed{\frac{z_2 - z_1}{t_2 - t_1}} = \frac{\omega}{\beta}$$

Phase Velocity (v_p)

$$\boxed{\Rightarrow v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}}$$

Transmission Lines (contd.)

- Simplified Expression for Wavelength:

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} = \frac{v_p}{f} = v_p T$$

i.e, the wavelength is the distance traveled by the wave in a time interval equal to one period

Let us examine this expression:

$$\frac{z_2 - z_1}{t_2 - t_1} = \frac{\omega}{\beta}$$

- $t_2 > t_1$ and ${}^{\omega}/_{\beta}$ is a positive quantity → this implies that $z_2 - z_1$ must be positive or $z_2 > z_1$
- It ensures that the point of constant phase moves towards right (i.e, toward the load in the transmission line)
- In other words, the wave function $V_0^+ cos(\omega t - \beta z)$ represents a traveling wave moving at a velocity v_p towards the load
- This wave is called outgoing wave when seen from the source and incident wave when viewed from the load

Transmission Lines (contd.)

- Similarly, the analysis of $V_0^{-}\, cos(\omega t + \beta z)$ will show that this function represents a traveling wave at a velocity v_p to the left (i.e, towards the source in a transmission line)
- This wave is called incoming wave when seen from the source and reflected wave when viewed from the load
- $V_0^{+}\, e^{-j\beta z}$ is called incident wave (phasor form) **and** $V_0^{-}\, e^{j\beta z}$ is called reflected wave (phasor form)
- In general, the voltage and current on a transmission line is composed of incident and reflected wave
- The quantity βz is known as electrical length of the line
- Therefore:

$$\boxed{V(z) = V^{+}(z) + V^{-}(z) = V_0^{+} e^{-j\beta z} + V_0^{-} e^{j\beta z}}$$

$$\boxed{I(z) = \frac{V_0^{+}}{Z_0} e^{-j\beta z} - \frac{V_0^{-}}{Z_0} e^{j\beta z} = \frac{V^{+}(z) - V^{-}(z)}{Z_0}}$$

Characteristic Impedance (Z_0)

- The characteristic impedance is defined as :

Z_0 = (incoming voltage wave) / (incoming current wave)

= (outgoing voltage wave) / (outgoing current wave)

- For a generic transmission line:

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

The incoming and outgoing voltage and current waves are position dependent → the ratio of voltage and current waves are independent of position → actually is a constant → an important characteristic of a transmission line → called as Characteristic Impedance

- Z_0 is not an impedance in a conventional circuit sense
- definition is based on the incident and reflected voltage and current waves
- this definition has nothing in common with the total voltage and current expressions used to define a conventional circuit impedance
- Its importance will be apparent during the course of this COURSE!!!

Example – 1

- A plane wave propagating in a lossless dielectric medium has an electric field given as $E_x = E_0\cos(\omega t - \beta z)$ with a frequency of 5.0 GHz and a wavelength of 3.0 cm in the material. Determine the propagation constant, the phase velocity, the relative permittivity of the medium, and the intrinsic impedance of the wave.

The propagation constant:

$$\beta = \frac{2\pi}{\lambda} \Longrightarrow \beta = \frac{2\pi}{0.03} \Longrightarrow \boxed{\therefore \beta = 209.4 m^{-1}}$$

The phase velocity:

$$v_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \lambda f \Longrightarrow \boxed{v_p = 0.03 \times 5 \times 10^9 = 1.5 \times 10^8 m/\sec}$$

Lower than the speed of light in free medium

Example – 1 (contd.)

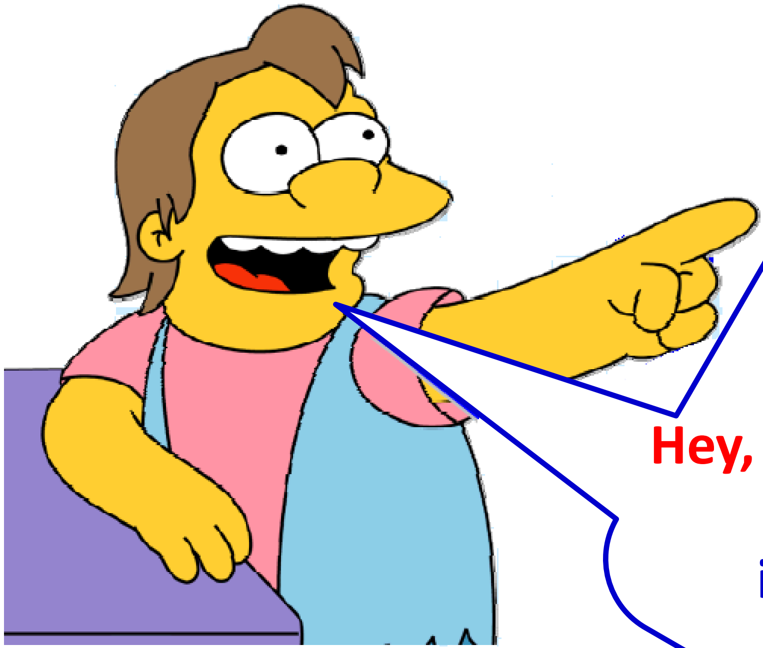
Relative permittivity of the medium:

$$v_p = \frac{c}{\sqrt{\varepsilon_r}} \Rightarrow \varepsilon_r = \left(\frac{c}{v_p}\right)^2 \Rightarrow \boxed{\varepsilon_r = \left(\frac{3\times10^8}{1.5\times10^8}\right)^2 = 4.0}$$

Characteristic impedance of the wave:

$$\eta_{wave} = \frac{\eta_0}{\sqrt{\varepsilon_r}} \Rightarrow \boxed{\eta_{wave} = \frac{377}{\sqrt{4}} = 188.5\Omega}$$

Line Impedance (Z)



Hey, I know what this is!
The ratio of
incoming voltage to
incoming current
wave. Right?

NO!

Line Impedance (Z) – contd.

- Actually, line impedance is the ratio of total complex voltage (incoming + outgoing) wave to the total complex current voltage wave.

$$\boxed{Z(z)=\frac{V(z)}{I(z)}} \Rightarrow \boxed{=\frac{V^{+}(z)+V^{-}(z)}{\left(\left(V^{+}(z)-V^{-}(z)\right)\big/ Z_0\right)}} \Rightarrow \boxed{\neq Z_0}$$

In most of the cases

- However, the line and characteristic impedance can be equal if either the incoming or outgoing voltage wave equals ZERO!

- Say, if $V^{-}(z)=0$ then:

$$\boxed{Z(z)=\frac{V^{+}(z)+V^{-}(z)}{\left(\left(V^{+}(z)-V^{-}(z)\right)\big/ Z_0\right)}=Z_0}$$

Line Impedance (Z) – contd.

It appears to me that Z_0 is a transmission line parameter, depending only on the transmission line values R, L, C and G.

Whereas, $Z(z)$ depends on the magnitude and the phase of the two propagating waves $V^+(z)$ and $V^-(z)$ → values that depend not only on the transmission line, but also on the two things attached to either end of the transmission line.

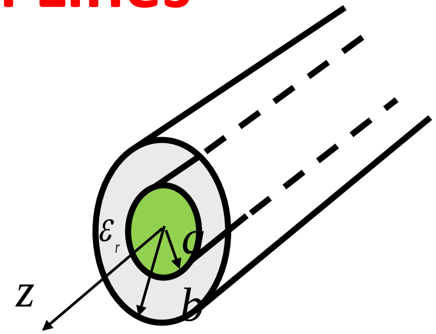
Right?

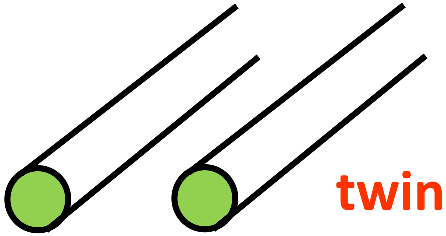
Exactly!!!

Example of Transmission Lines

Two common examples:

coaxial cable

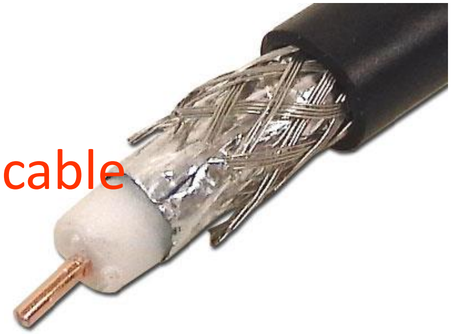




twin line

A transmission line is normally used in the balanced mode, meaning equal and opposite currents (and charges) on the two conductors.

Here’s what they look like in real-life:



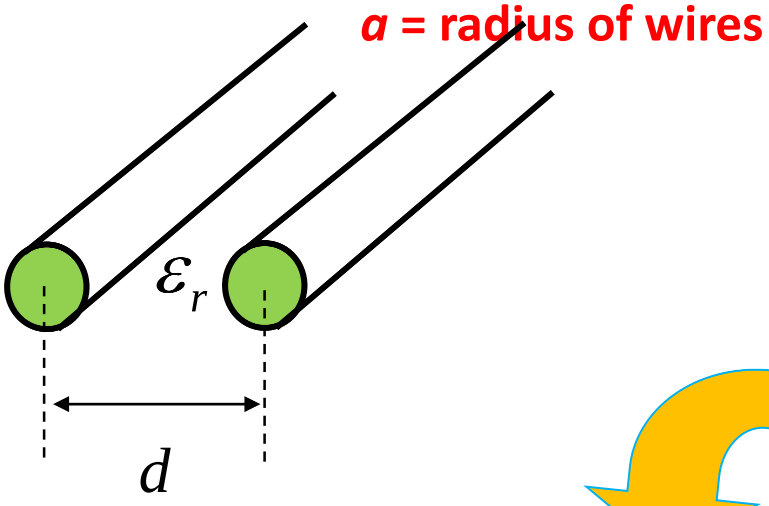
coaxial cable

twin line

coax to twin line matching section

Example of Transmission Lines (contd.)

Twin Line



$$C = \frac{\pi\varepsilon_0\varepsilon_r}{\cosh^{-1}\left(\frac{d}{2a}\right)} \quad [\mathrm{F/m}]$$

$$L = \frac{\mu_0}{\pi}\cosh^{-1}\left(\frac{d}{2a}\right) \quad [\mathrm{H/m}]$$

$$Z_0 = \frac{1}{\pi}\eta_0\frac{1}{\sqrt{\varepsilon_r}}\cosh^{-1}\left(\frac{d}{2a}\right) \quad [\Omega]$$

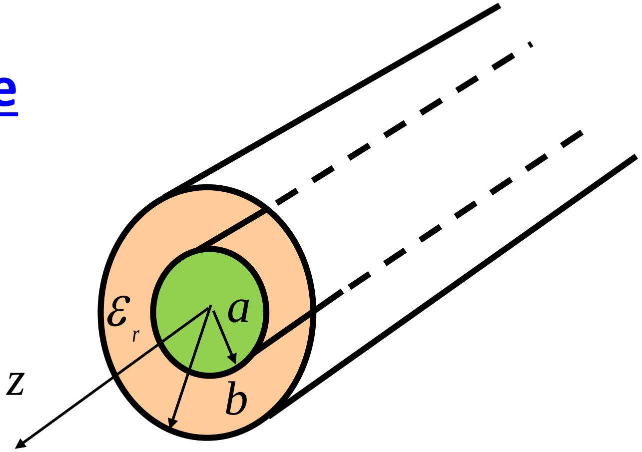
$$\cosh^{-1}x = \ln\left(x+\sqrt{x^2-1}\right) \xrightarrow{x\to\infty} \ln 2x$$

$$Z_0 \approx \frac{1}{\pi}\eta_0\frac{1}{\sqrt{\varepsilon_r}}\ln\left(\frac{d}{a}\right) \quad [\Omega]$$

$$a \ll d$$

Example of Transmission Lines (contd.)

Coaxial Cable



σ_d = conductivity of dielectric [S/m].

σ_m = conductivity of metal [S/m].

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)} \quad [\mathrm{F/m}]$$

$$G = \frac{2\pi\sigma_d}{\ln\left(\frac{b}{a}\right)} \quad [\mathrm{S/m}]$$

$$L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right) \quad [\mathrm{H/m}]$$

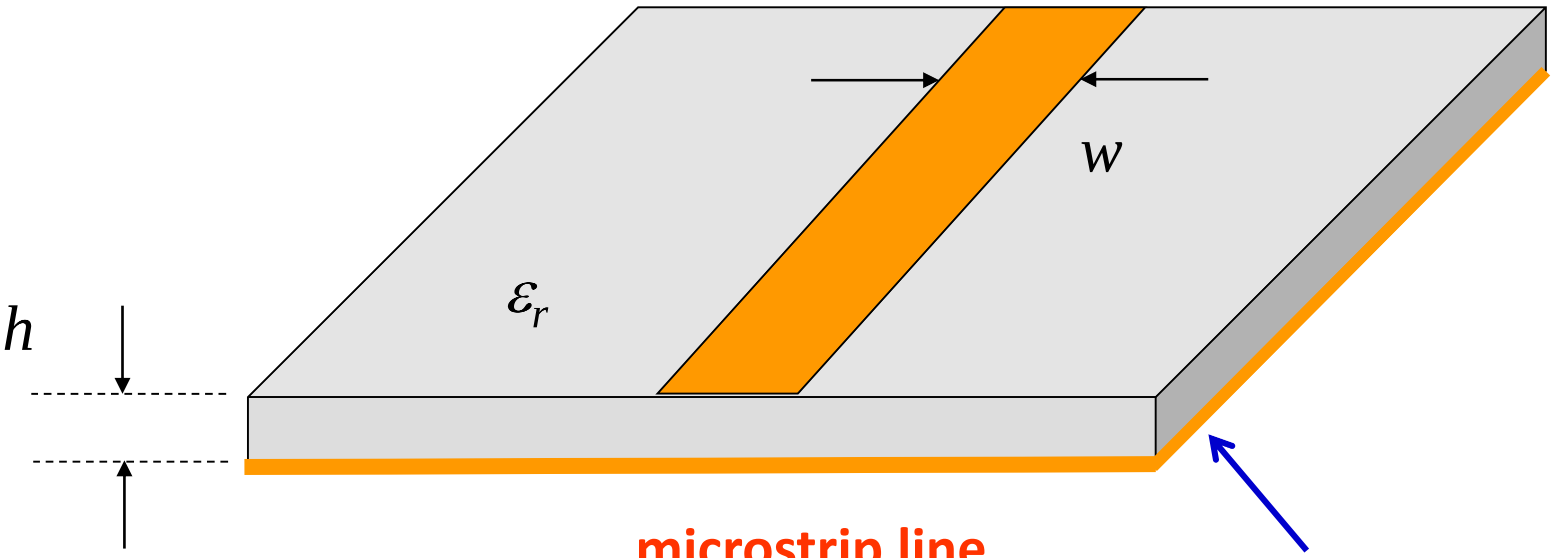
$$R = \frac{1}{\sigma_m\delta}\left(\frac{1}{2\pi a} + \frac{1}{2\pi b}\right) \quad [\Omega/\mathrm{m}]$$

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma_m}}$$

(skin depth of metal)

Example of Transmission Lines (contd.)

Another common example (for printed circuit boards):

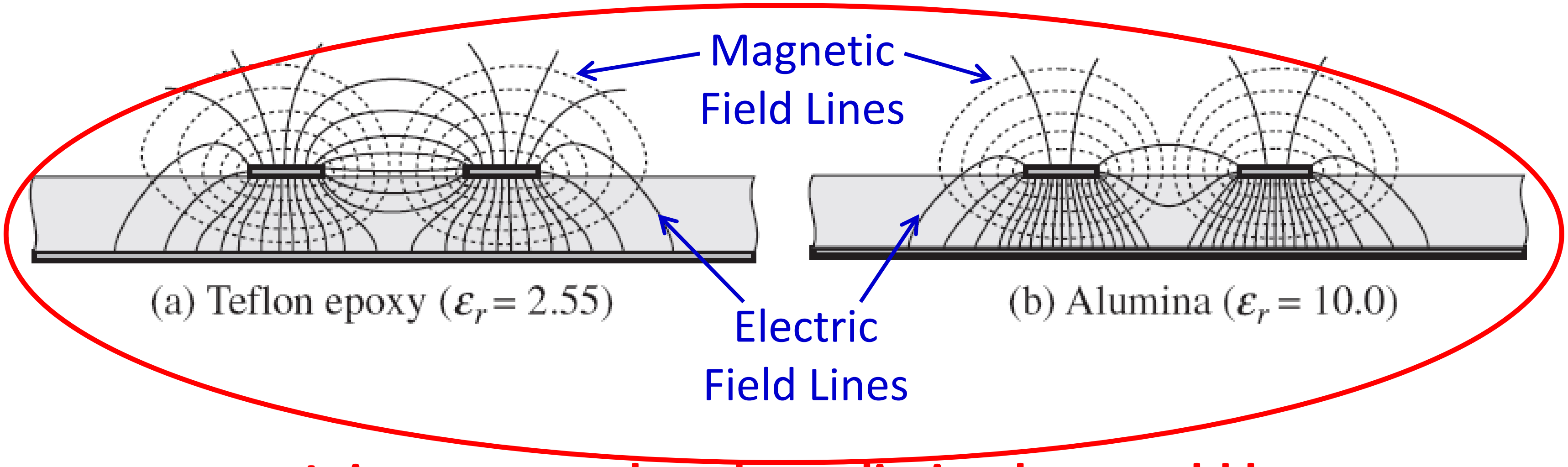


microstrip line

Ground plane helps in preventing the field leakage and thus reduces the radiation loss

Microstrip Line (contd.)

- The severity of field leakage also depends on the relative dielectric constants (ε_r).

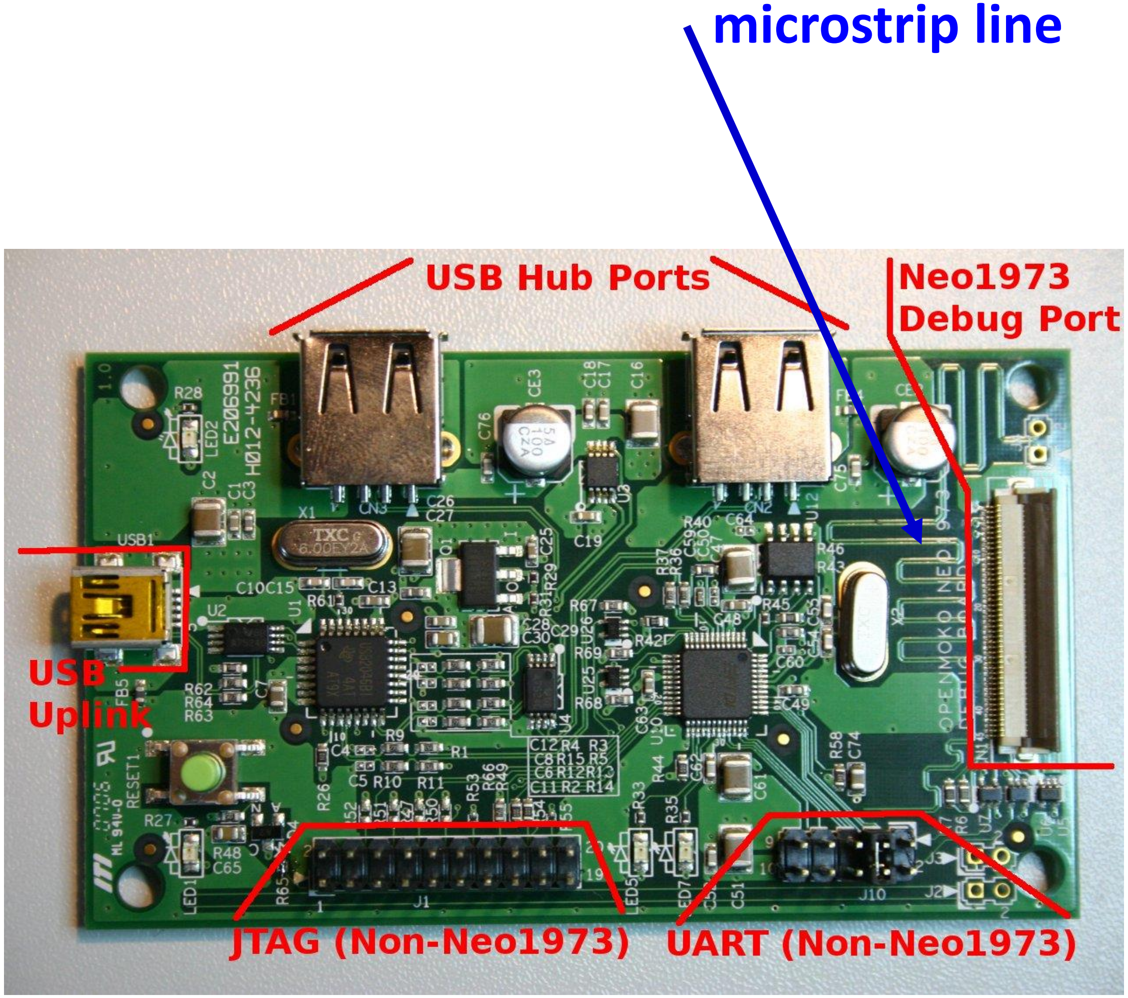


(a) Teflon epoxy ($\varepsilon_r = 2.55$) (b) Alumina ($\varepsilon_r = 10.0$)

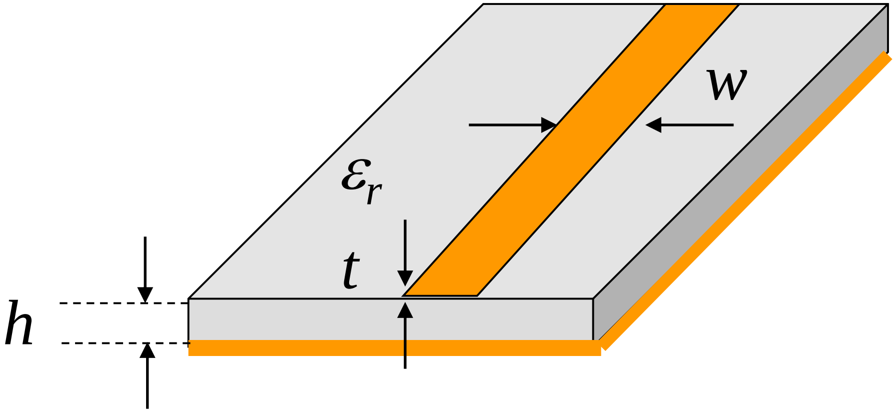
It is apparent that the radiation loss could be minimized by using substrates with high dielectric constants

Alternative approaches to reduce radiation loss and interference are shielded microstrip line and multi-layer boards

Microstrip Line (contd.)



Microstrip Transmission Lines Design



- Simple parallel plate model can not accurately define this structure.
- Because, if the substrate thickness increases or the conductor width decreases then fringing field become more prominent (and therefore need to be incorporated in the model).

Case-I: thickness (t) of the line is negligible

- **For narrow microstrips ($^{w}/_{h} \leq 1$):** $Z_0 = \dfrac{Z_f}{2\pi\sqrt{\varepsilon_{eff}}}\ln\left(8\dfrac{h}{w}+\dfrac{w}{4h}\right)$

Where, $Z_f = \sqrt{\mu_0 / \varepsilon_0} = 377\Omega$ **wave impedance in free space**

$$\varepsilon_{eff} = \frac{\varepsilon_r+1}{2}+\frac{\varepsilon_r-1}{2}\left[\left(1+12\frac{h}{w}\right)^{-1/2}+0.004\left(1-\frac{w}{h}\right)^{2}\right]$$

Effective Dielectric Constant

Microstrip Transmission Lines Design (contd.)

- For wide microstrips $\left({}^{w}/_{h} \geq 1\right)$:

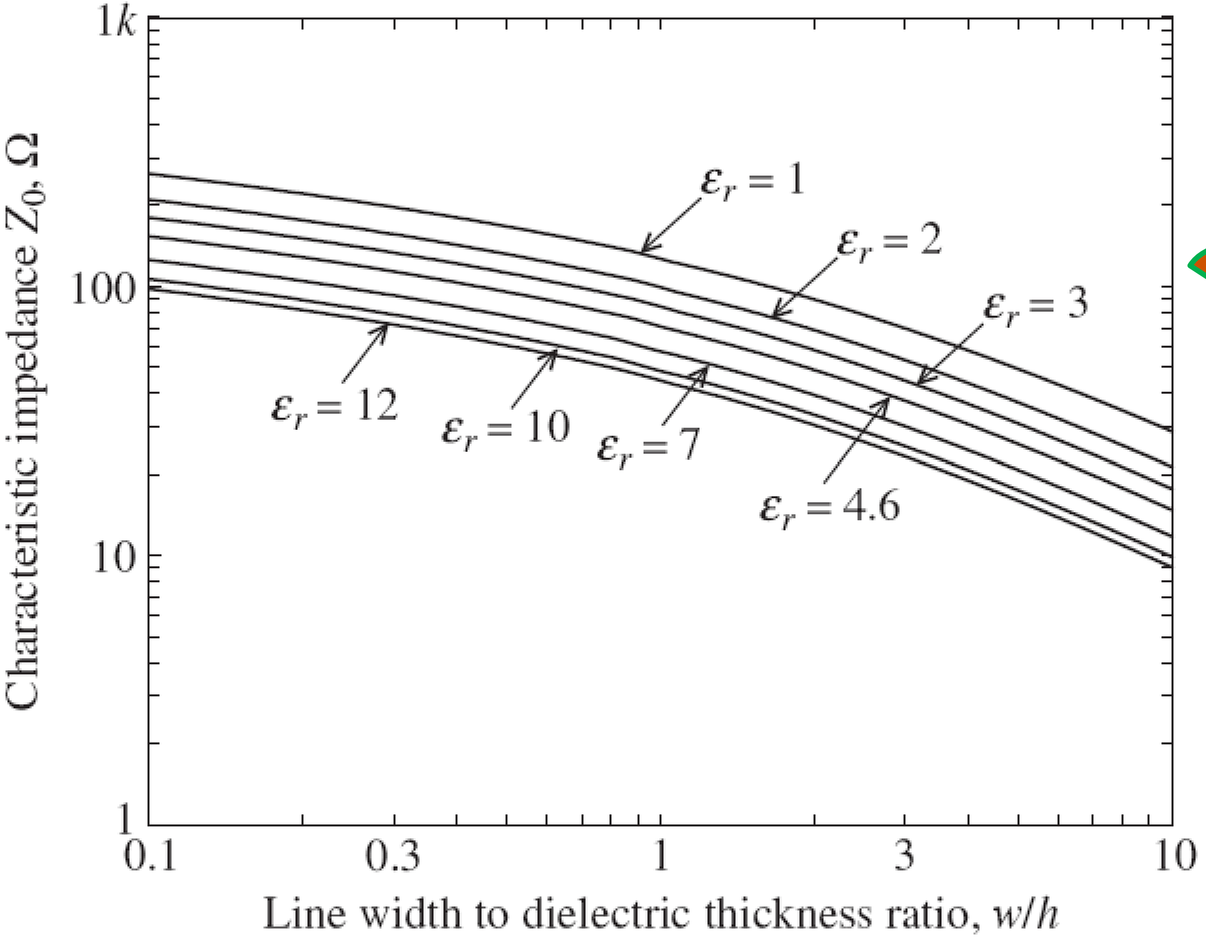
$$Z_0 = \frac{Z_f}{\sqrt{\varepsilon_{eff}}\left(1.393 + \frac{w}{h} + \frac{2}{3}\ln\left(\frac{w}{h} + 1.444\right)\right)}$$

- Where the effective dielectric constant is expressed as:

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + 12\frac{h}{w}\right)^{-1/2}$$

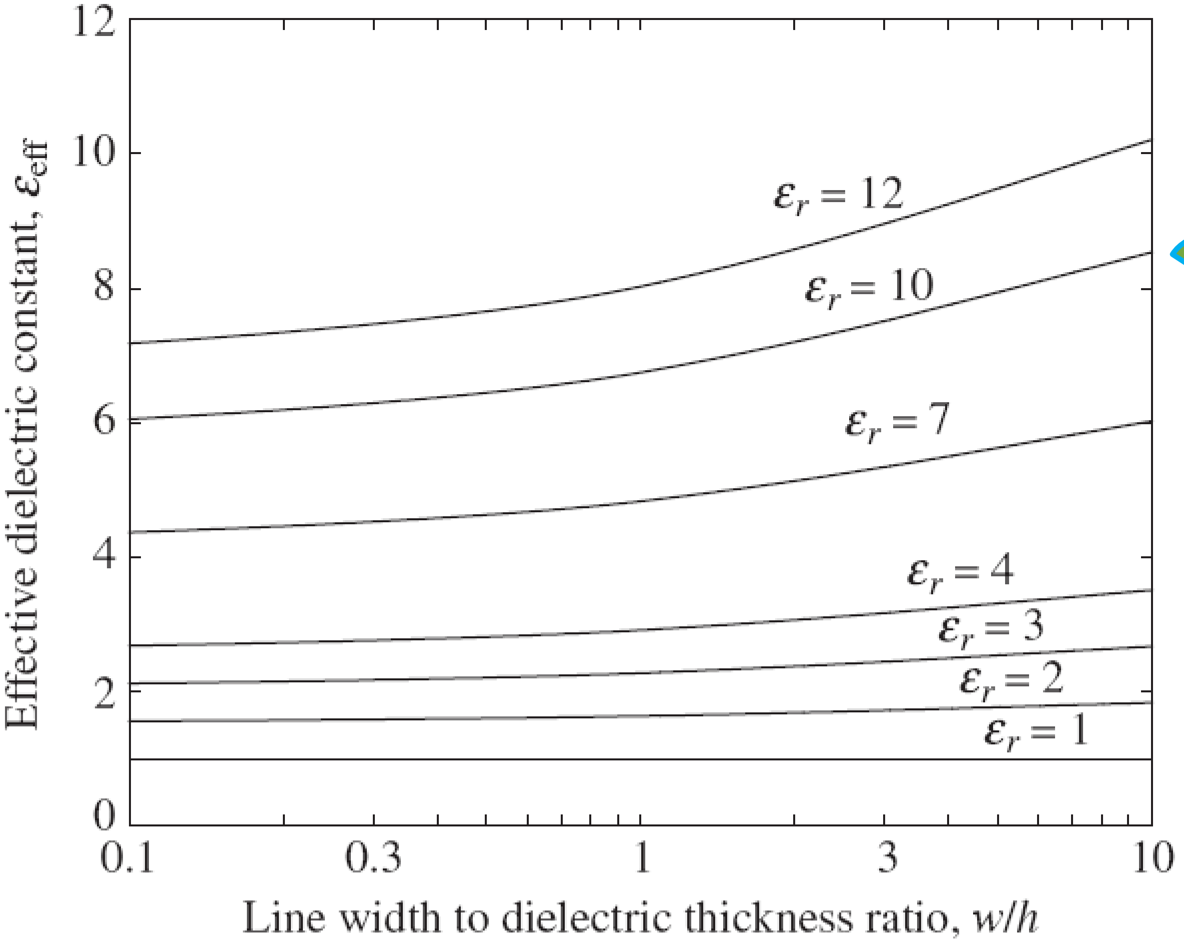
The two distinct expressions give approximate values of characteristic impedance and effective dielectric constant for narrow and wide strip microstrip lines → these can be used to plot Z_0 and ε_{eff} as a function of ${}^{w}/_{h}$.

Microstrip Transmission Lines Design (contd.)



For a desired characteristic impedance using known substrate, the dimension w/h can be identified from this curve

Microstrip Transmission Lines Design (contd.)



Once the line dimensions are known, the effective dielectric constant can be identified

Microstrip Transmission Lines Design (contd.)

- The effective dielectric constant (ε_{eff}) is viewed as the dielectric constant of a homogenous material that fills the entire space around the line. Therefore:

Free Space Wavelength

$$\lambda = \frac{v_p}{f} = \frac{c}{f\sqrt{\varepsilon_{eff}}} = \frac{\lambda_0}{\sqrt{\varepsilon_{eff}}}$$

Speed of Light

- The wavelength in the microstrip line for $W/_h \geq 0.6$ is:

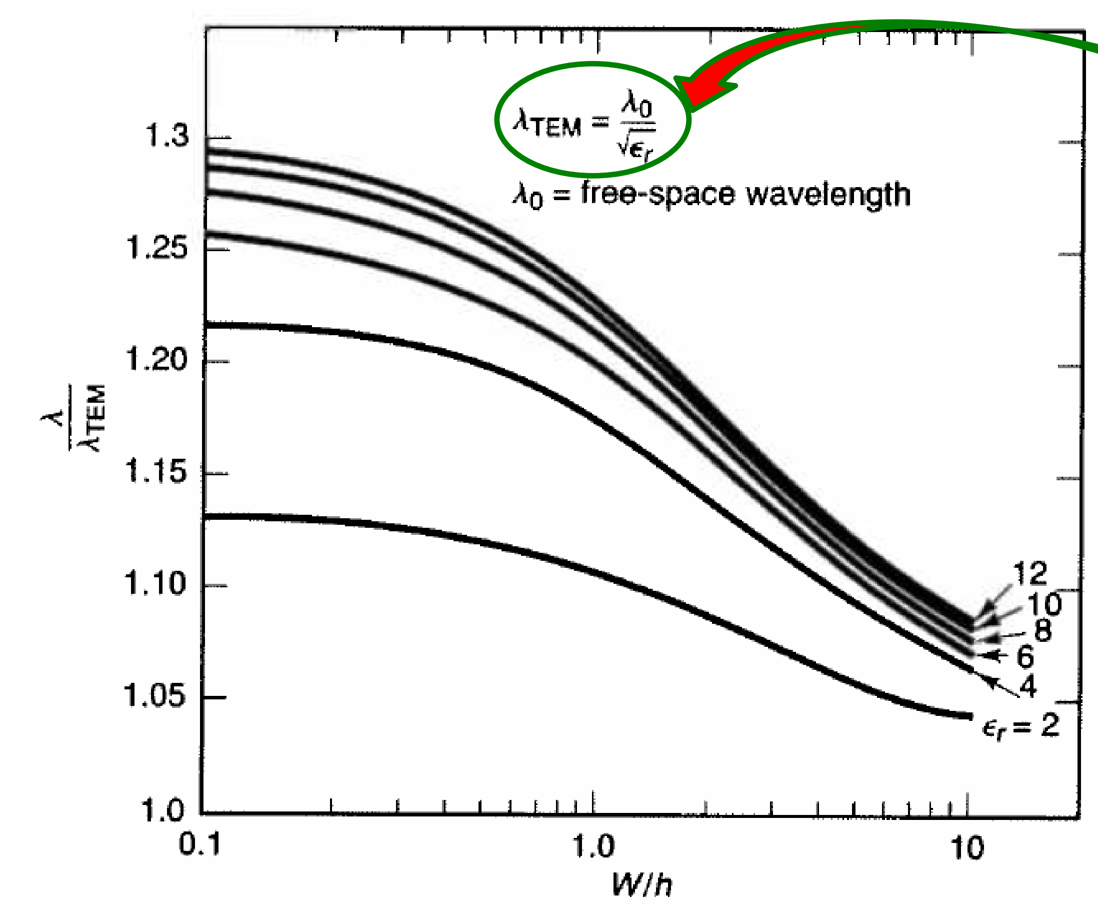
$$\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_r}}\left[\frac{\varepsilon_r}{1+0.63(\varepsilon_r-1)\left(W/h\right)^{0.1255}}\right]^{1/2}$$

- The wavelength in the microstrip line for $W/_h \leq 0.6$ is:

$$\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_r}}\left[\frac{\varepsilon_r}{1+0.6(\varepsilon_r-1)\left(W/h\right)^{0.0297}}\right]^{1/2}$$

Microstrip Transmission Lines Design (contd.)

- In some specifications, wavelength is known. In that case following curve can be used to identify the *w/h* ratio.



It is a good approximation at lower microwave frequencies. However, at higher microwave frequencies this assumption is no more valid.

Microstrip Transmission Lines Design (contd.)

- If Z_0 and ε_r is specified or known, following expression can be used to determine w/h:

For w/h≤2:

$$\frac{w}{h}=\frac{8e^{A}}{e^{2A}-2}$$

Where: $$A=2\pi\frac{Z_0}{Z_f}\sqrt{\frac{\varepsilon_r+1}{2}}+\frac{\varepsilon_r-1}{\varepsilon_r+1}\left(0.23+\frac{0.11}{\varepsilon_r}\right)$$

For w/h≥2:

$$\frac{w}{h}=\frac{2}{\pi}\left(B-1-\ln(2B-1)+\frac{\varepsilon_r-1}{2\varepsilon_r}\left[\ln(B-1)+0.39-\frac{0.61}{\varepsilon_r}\right]\right)$$

Where: $$B=\frac{Z_f\pi}{2Z_0\sqrt{\varepsilon_r}}$$

Case-II: thickness (t) of the line is not negligible → in this scenario all the formulas are valid with the assumption that the effective width of the line increases as:

$$w_{eff}=w+\frac{t}{\pi}\left(1+\ln\frac{2x}{t}\right)$$

Where $x = h$ if $w > {}^{h}/_{2\pi}$ **or** $x = 2\pi w$ if ${}^{h}/_{2\pi} > w > 2t$

Example – 2

A microstrip material with $\varepsilon_r = 10$ and $h = 1.016\ mm$ is used to build a narrow transmission line. Determine the width for the microstrip transmission line to have a characteristic impedance of 50Ω. Also determine the wavelength and the effective relative dielectric constant of the microstrip line.

Using the Formulas:

Let us consider the first formula: $\frac{w}{h} = \frac{8e^{A}}{e^{2A} - 2}$

$$A = 2\pi \frac{Z_0}{Z_f}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right) = 2\pi \frac{50}{377}\sqrt{\frac{10+1}{2}} + \frac{10-1}{10+1}\left(0.23 + \frac{0.11}{10}\right)$$

$$\boxed{\Rightarrow A = 2.1515}$$

Therefore: $\frac{w}{h} = \frac{8e^{2.1515}}{e^{2(2.1515)} - 2} = 0.9563$

Now: h = 1.016 mm = 0.1016 cm = 0.1016(1000/2.54) mils = 40 mils

$$\boxed{\therefore w = 0.9563 * 40 mils = 38.2 mils}$$

Example – 2 (contd.)

$$\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_r}}\left[\frac{\varepsilon_r}{1+0.63(\varepsilon_r-1)(w/h)^{0.1255}}\right]^{1/2}$$

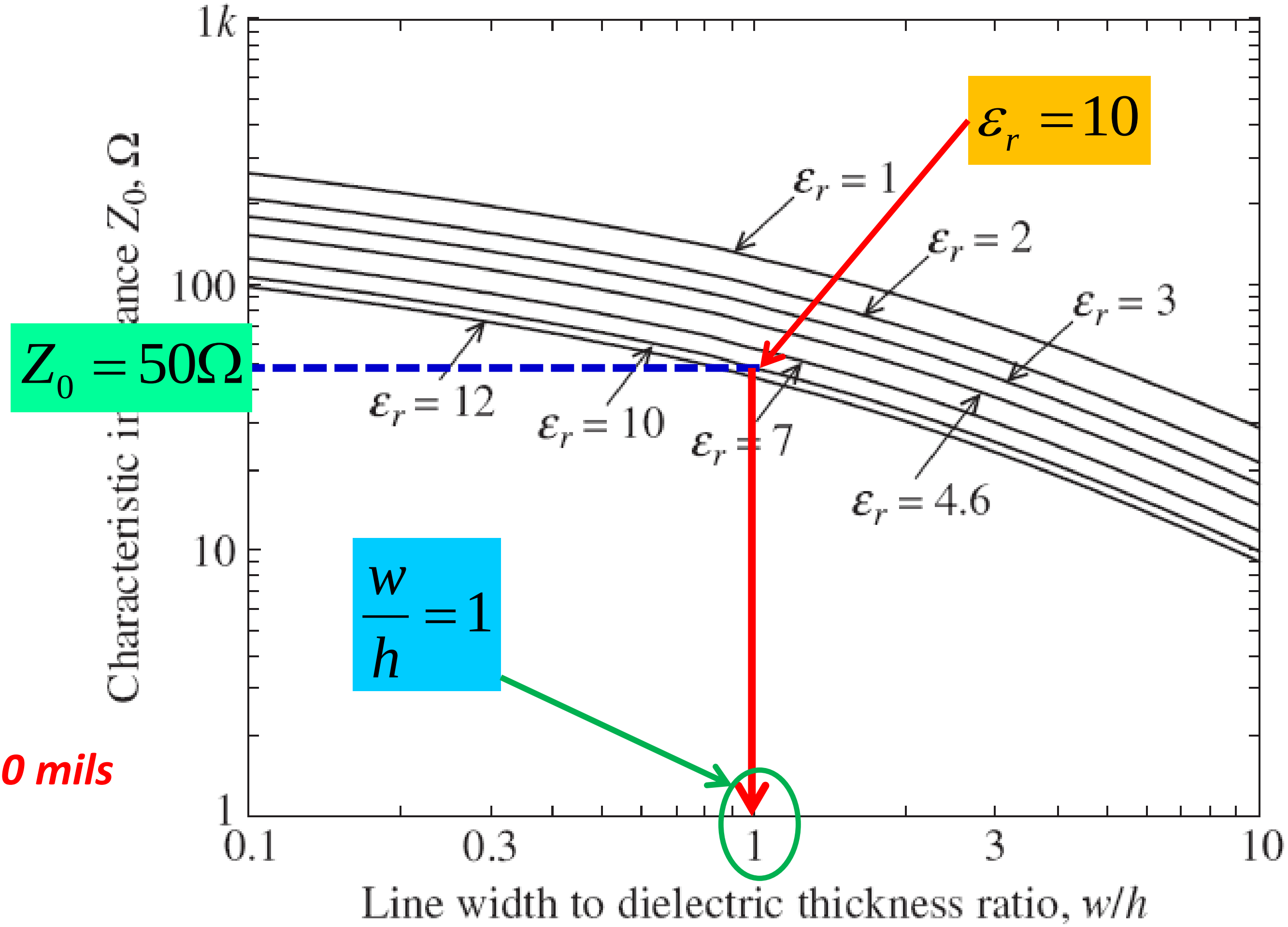
$$\boxed{\therefore \lambda = \frac{\lambda_0}{\sqrt{10}}\left[\frac{10}{1+0.63(10-1)(0.9563)^{0.1255}}\right]^{1/2} = 0.387\lambda_0}$$

$$\lambda = \frac{v_p}{f} = \frac{c}{f\sqrt{\varepsilon_{eff}}} = \frac{\lambda_0}{\sqrt{\varepsilon_{eff}}} \qquad \Rightarrow \varepsilon_{eff} = \left(\frac{\lambda_0}{\lambda}\right)^2$$

$$\boxed{\therefore \varepsilon_{eff} = \left(\frac{1}{0.387}\right)^2 = 6.68}$$

Example – 2 (contd.)

Using the Design Curves



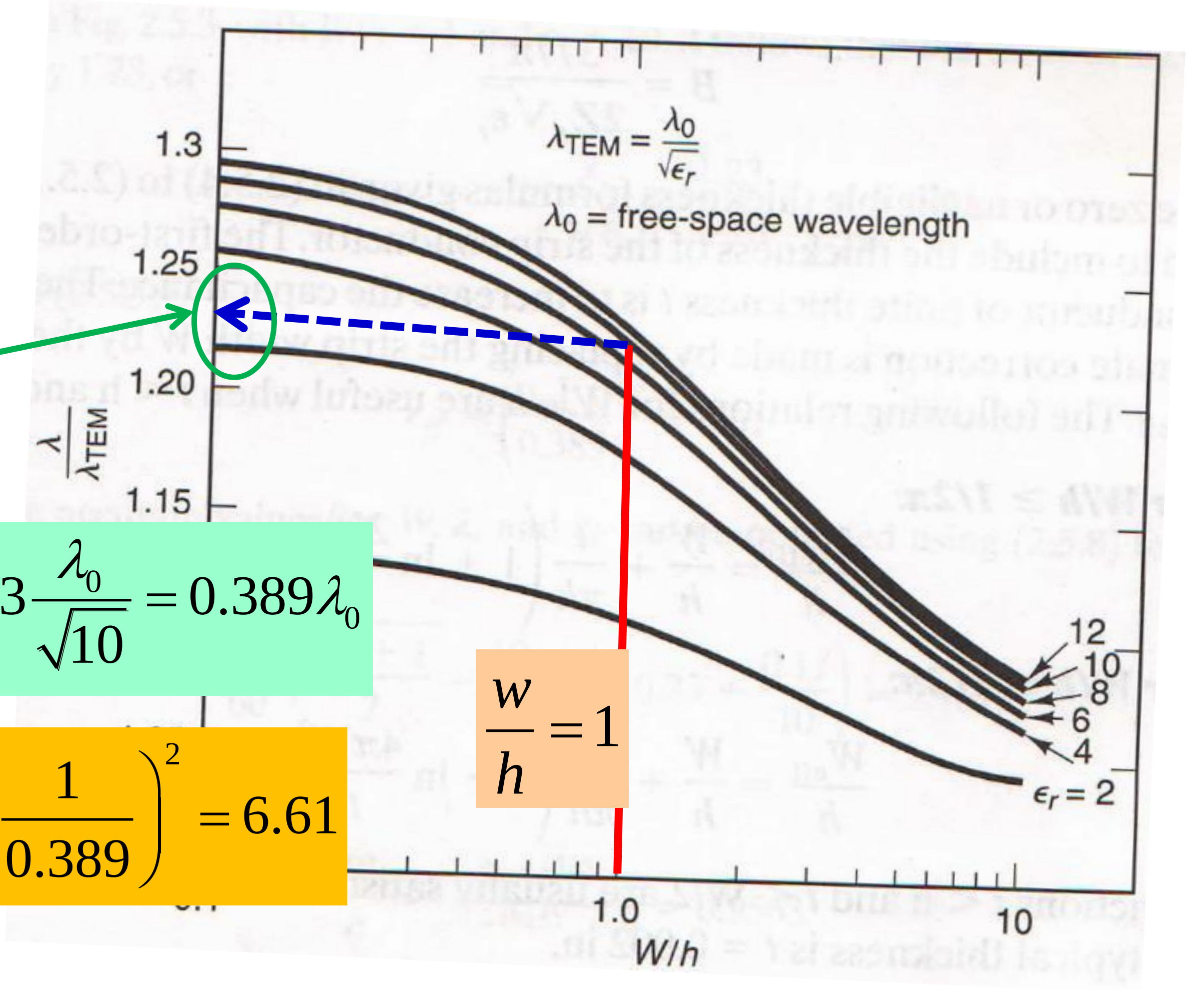
h = 1.016 mm = 40 mils

=> w = 40 mils

Example – 2 (contd.)

Using the Design Curves

$$\frac{\lambda}{\lambda_{TEM}} = 1.23$$



$$\frac{w}{h} = 1$$

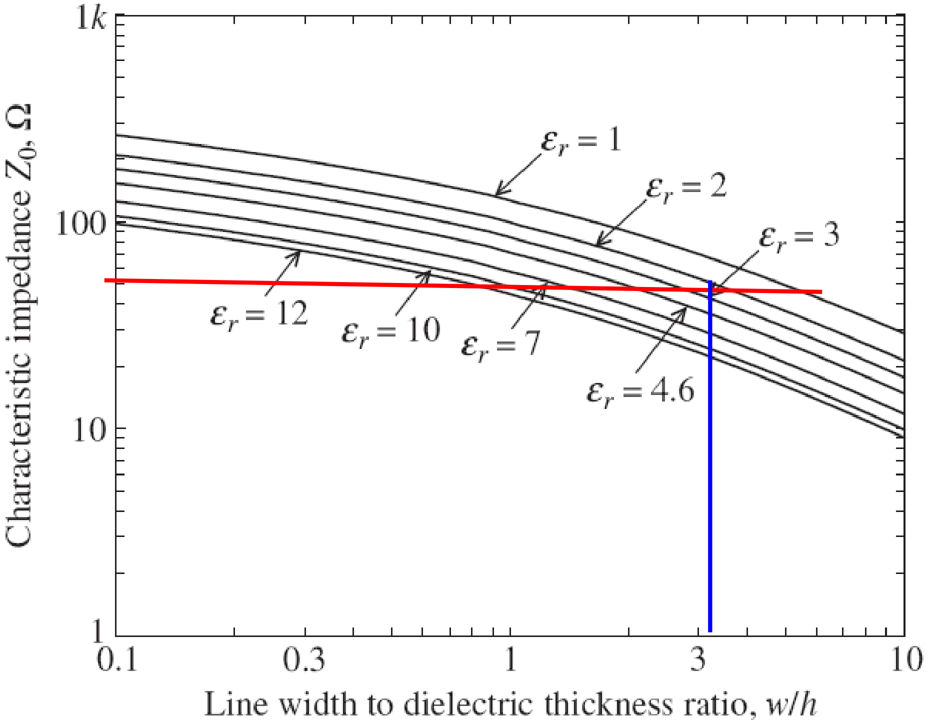
$$\lambda = 1.23\frac{\lambda_0}{\sqrt{\varepsilon_r}} = 1.23\frac{\lambda_0}{\sqrt{10}} = 0.389\lambda_0$$

$$\Rightarrow \varepsilon_{eff} = \left(\frac{\lambda_0}{\lambda}\right)^2 = \left(\frac{1}{0.389}\right)^2 = 6.61$$

Example – 3

a. Using the design curves, calculate W, λ, and ε_{eff} for a characteristic impedance of 50Ω using RT/Duroid with $\varepsilon_r = 2.23$ and $h = 0.7874\,mm$.

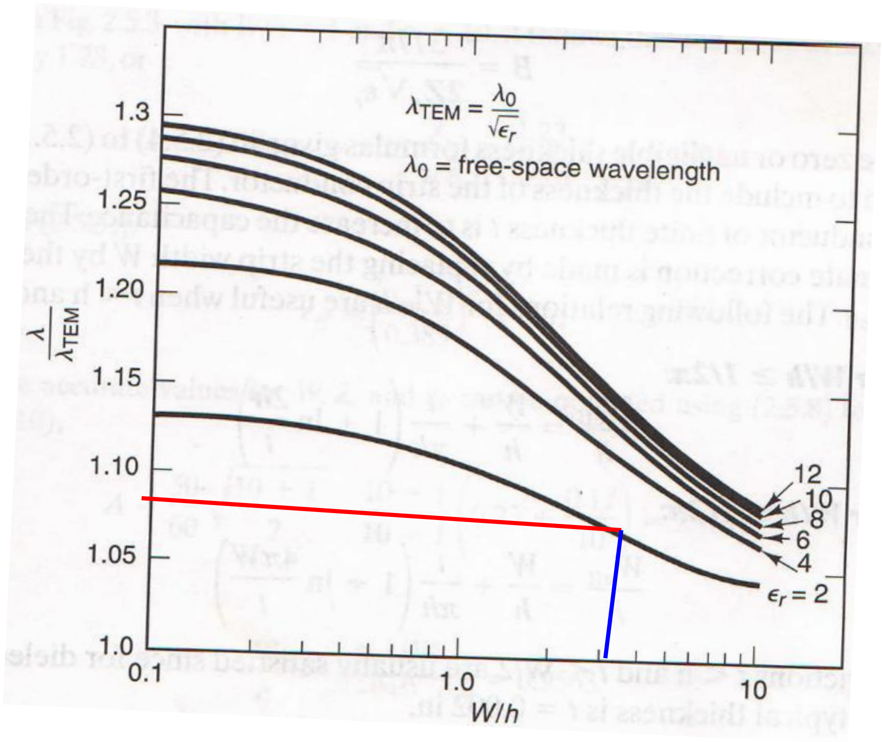
b. Use design equations to show that for RT/Duroid with $\varepsilon_r = 2.23$ and $h = 0.7874\,mm$, a 50Ω-characteristic impedance is obtained with ${}^{W}/_{h} = 3.073$. Also show, $\varepsilon_{eff} = 1.91$ and $\lambda = 0.7236\lambda_0$.



$$\frac{W}{h} \approx 3.1$$

W=$3.1 \times h = 3.1 \times 0.784 = 2.44mm$

Example – 3 (contd.)



For $\frac{W}{h} \approx 3.1$ and $\varepsilon_r = 2.23$

$$\frac{\lambda}{\lambda_{TEM}} = 1.08 \Rightarrow \lambda = 1.08\lambda_{TEM}$$

We know: $\lambda_{TEM} = \frac{\lambda_0}{\sqrt{\varepsilon_r}}$

$$\therefore \lambda = 0.723\lambda_0$$

Also: $\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_{eff}}} \Rightarrow \varepsilon_{eff} = 1.91$

Example – 3 (contd.)

For w/h≥2: $\frac{w}{h}=\frac{2}{\pi}\left(B-1-\ln(2B-1)+\frac{\varepsilon_r-1}{2\varepsilon_r}\left[\ln(B-1)+0.39-\frac{0.61}{\varepsilon_r}\right]\right)$

Where: $B=\frac{Z_f\pi}{2Z_0\sqrt{\varepsilon_r}}$

Therefore: $\frac{w}{h}=\frac{2}{\pi}\left(B-1-\ln(2B-1)+\frac{2.23-1}{2\times 2.23}\left[\ln(B-1)+0.39-\frac{0.61}{2.23}\right]\right)$

Where: $B=\frac{377\pi}{2\times 50\times\sqrt{2.23}}=7.931$

$\therefore\frac{w}{h}=3.073$

- **For $^{W}/_{h}\geq 0.6$:** $\lambda=\frac{\lambda_0}{\sqrt{\varepsilon_r}}\left[\frac{\varepsilon_r}{1+0.63(\varepsilon_r-1)\left(W/h\right)^{0.1255}}\right]^{1/2}$

$$\therefore\lambda=\frac{\lambda_0}{\sqrt{2.23}}\left[\frac{2.23}{1+0.63(2.23-1)\left(3.073\right)^{0.1255}}\right]^{1/2}=0.724\lambda_0$$