Lecture – 22

Date: 06.04.2017

- Stability
- Transistor Amplifier Design
  - Maximum Gain Amplifier
  - The Ideal Gain Element
  - Design for Specified Gain

Introduction

Q: So all there is to making a good RF/microwave amplifier is the design of proper **matching networks**?

A: There is one other problem that confronts the RF/microwave amplifier designer. That problem is **stability** (of the amplifier, not the designer).

An unstable amplifier is also known as an **oscillator**—a source of RF/microwave energy!

Q: Under what **conditions** will an amplifier oscillate?

A: An amplifier will go **unstable** if **either** of these two conditions are true:

$$|\Gamma_{out}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \right| > 1.0$$

$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| > 1.0$$

In other words, the amplifier will oscillate if either the input or output reflection coefficient of the gain element has a **magnitude greater than one**.

Introduction

Q: Hey wait! I thought we learnt that the **maximum** value of any reflection coefficient magnitude was 1 (i.e., $\Gamma \leq 1.0$) → this defined the **validity region** of our Smith Chart!

A: Remember, the inequality $\Gamma \leq 1.0$ is true for any **passive** load or device. Our gain element is an **active** device → it must have a DC source of power → As a result, we find that $\Gamma > 1.0$ is quite **possible**!

Q: But, we learnt that the region outside the $\Gamma = 1.0$ circle on the Smith Chart corresponded to loads with negative **values of resistance**. Does this mean that Z_{in} or Z_{out} could have real (i.e. resistive) components that are **negative**?

A: That's **exactly** what it means!

Q: What **is** a negative resistor exactly?

A: Ohm's law still applies. However, the current through a negative resistor is 180^o **out-of-phase** with the voltage across it.

> The resistor current is at its minimum value when the voltage across it is at it maximum —and **vice versa**!

Stability

- This behavior (the occurrence of negative resistance) drives our amplifier circuit a little **wacky**, and it begins to oscillate!
- Then the question is – how to avoid such an unfortunate occurrence?
- Remember, the amplifier instability occurs when:

$$|\Gamma_{out}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \right| > 1.0 \qquad |\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| > 1.0$$

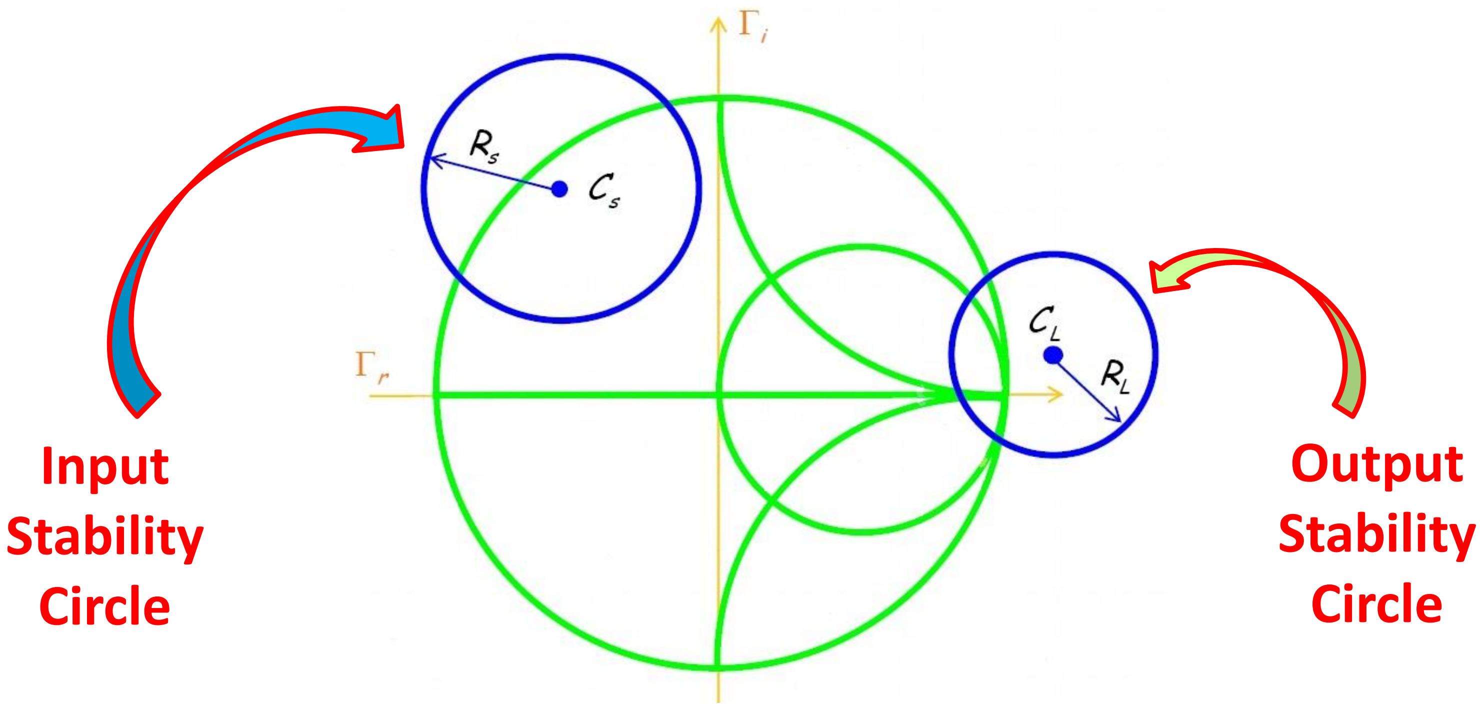
Thus, for a **given** gain element (i.e., S_{11}, S_{21}, S_{22}, S_{12}), the amplifier stability is determined by the value of Γ_L and Γ_s

We can **solve** these equations to determine the specific range of values of Γ_L and Γ_s that will **induce oscillation**.

We find that these unstable values—when plotted on the **complex** Γ **plane**—form a circle. These circles are known as a **stability circles**.

Stability (contd.)

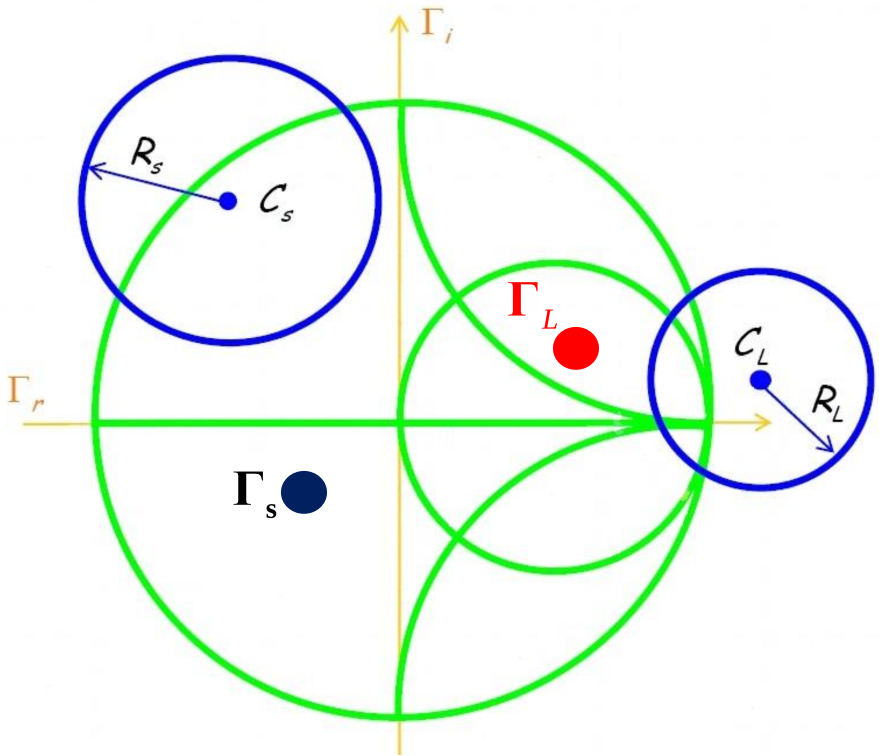
- These circles are defined in terms of a complex value C, which specifies the location of the stability circle **center** on the complex Γ plane, and a real value R, which specifies the **radius** of the stability circle.
- There is **one** stability circle for Γ_L (i.e., C_L and R_L) and **another** for Γ_s (i.e., C_s and R_s). Typically, the Γ values that lie **inside** the stability circle will create amplifier oscillation.



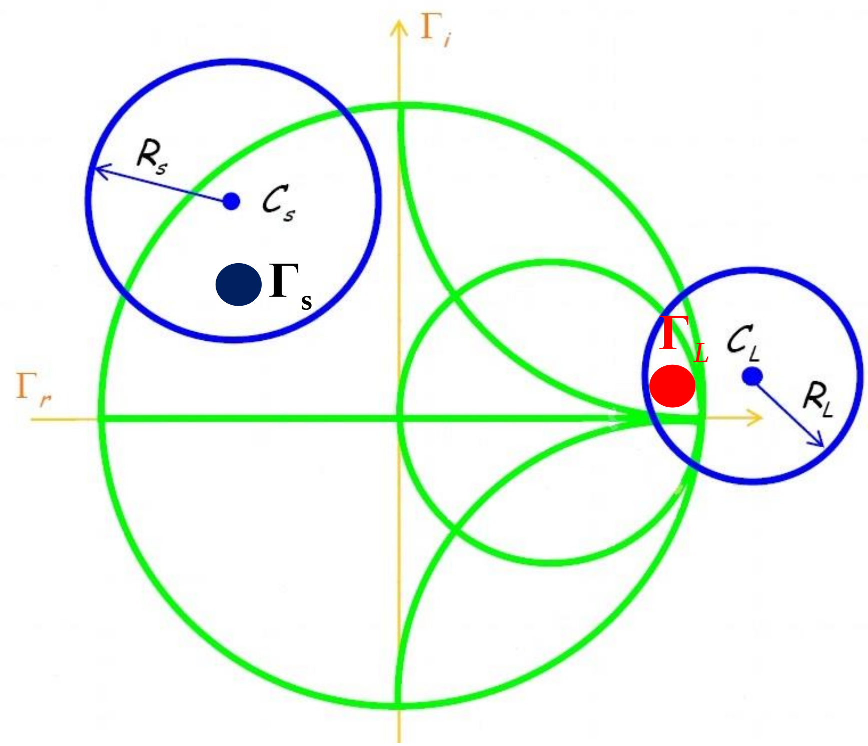
Stability (contd.)

Q: So what do we **use** these stability circles for?

A: As an amplifier designer, we must make sure that our **design values** Γ_L and Γ_s lie **outside** these circles—otherwise, our well-designed amplifiers will **oscillate**!



A Stable Design



An Unstable Design

Stability (contd.)

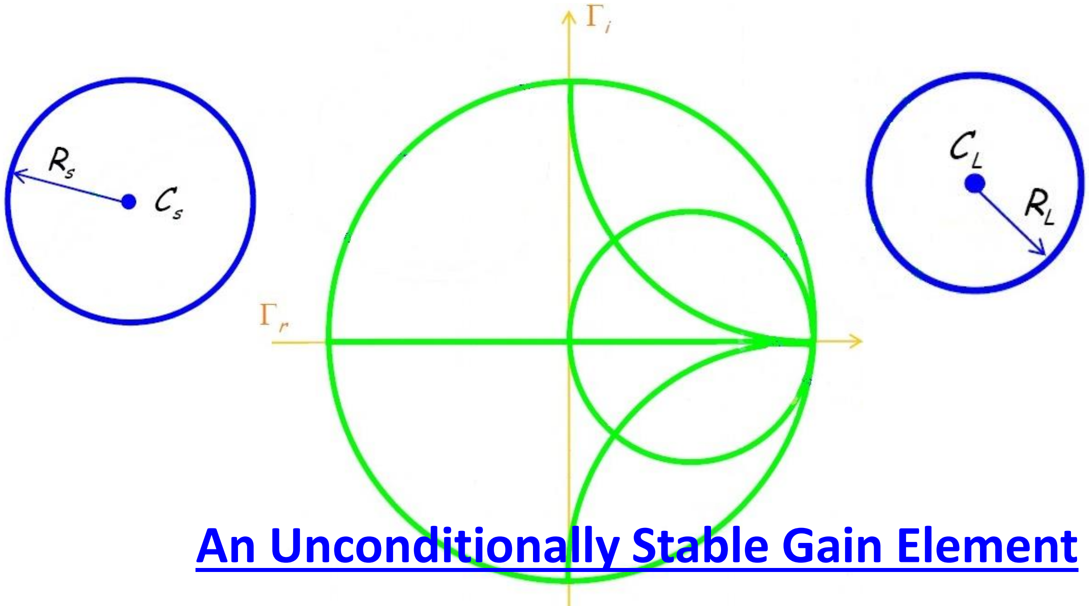
Q: Yikes! Must we **always** determine these circles and check our design for instability?

A: Not necessarily! Some gain elements are **unconditionally stable**. As the name suggests, these gain elements result in stable amplifiers for any and all **realizable** values of Γ_L and Γ_S.

Q: So an unconditionally stable gain element has stability circles with **zero radius** (i.e., R = 0)?

A: **Could** be, but all that is required for a gain element to be unconditionally stable is for its stability circles to lie completely **outside** the Γ = 1 circle (i.e., the Smith chart)

Stability (contd.)



An Unconditionally Stable Gain Element

Obviously, it is always assumed that the loads and sources attached to amplifier will **always** have **positive** resistances, such that $\Gamma_L < 1$ and $\Gamma_s < 1$.

For this condition, we find that the values of Γ_L and Γ_s that result in an unstable amplifier must have a magnitude **greater than 1** (i.e., $\Gamma_L > 1$ or $\Gamma_s > 1$)

An **amplifier** constructed with an unconditionally stable gain element will be **unconditionally stable**!

Stability (contd.)

Q: How will I **recognize** an unconditionally stable gain element if I see one? Must I determine and plot the stability circles?

A: There are three **tests** that we can apply—using the scattering parameters S_{11}, S_{21}, S_{22}, S_{12} — to **directly** determine if a gain element is unconditionally stable.

Test-1

Check the S-matrix of the gain element and check the following condition. The **necessary conditions** for a gain element to be unconditionally stable are:

$$|S_{11}| < 1.0 \qquad |S_{22}| < 1.0$$

However, this gives unconditional stability to only **unilateral** gain element (i.e., $S_{12} = 0$ or approx. $|S_{12}| << |S_{21}|$)

Otherwise there will be always situations when $\Gamma_{in} > 1$ and $\Gamma_{out} > 1$, leading to instability as can be seen in the following expressions:

$$|\Gamma_{out}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \right| > 1.0 \qquad |\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| > 1.0$$

Therefore, (for $S_{12} \neq 0$) we find that our gain element must pass **two more tests**

Stability (contd.)

Test-2

Alternatively, it can be shown that the amplifier will be unconditionally stable if the following conditions are met:

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} > 1.0 \quad \textbf{and} \quad \Delta < 1.0$$

Where, **K** is called the **Rollett Factor** and **Δ** is the determinant of the S-matrix

Test-3

Furthermore, the amplifier can be shown to be unconditionally stable if the following condition holds true:

$$\mu = \frac{1 - |S_{11}|^2}{|S_{22} - S_{11}^* \Delta| + |S_{12}S_{21}|} > 1.0$$

Larger values of μ imply greater stability

Ref: M. L. Edwards and J. H. Sinksy, "A New Criteria for Linear 2-Port Stability Using a Single Geometrically Derived Parameter," IEEE Trans. Microwave Theory and Tech., Vol. 40, Dec. 1992

Example

$$S_{11} = 0.869\angle{-159^\circ},$$
$$S_{12} = 0.031\angle{-9^\circ},$$
$$S_{21} = 4.250\angle{61^\circ},$$
$$S_{22} = 0.507\angle{-117^\circ}.$$

$$|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| = 0.336,$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 0.383$$

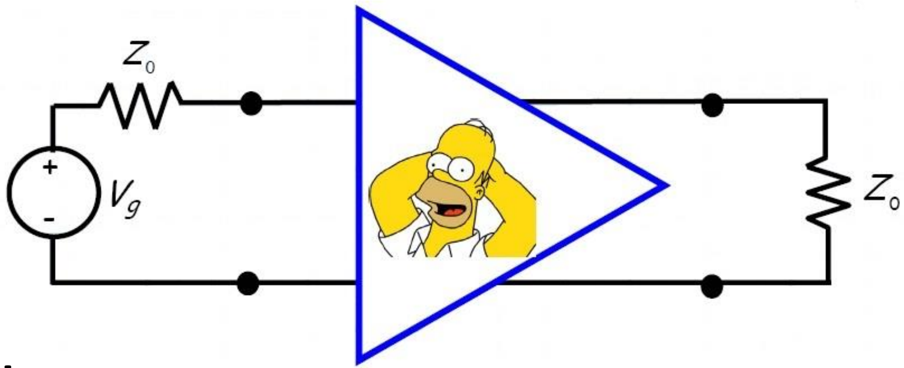
Potentially Unstable !

$$\mu = 0.678$$

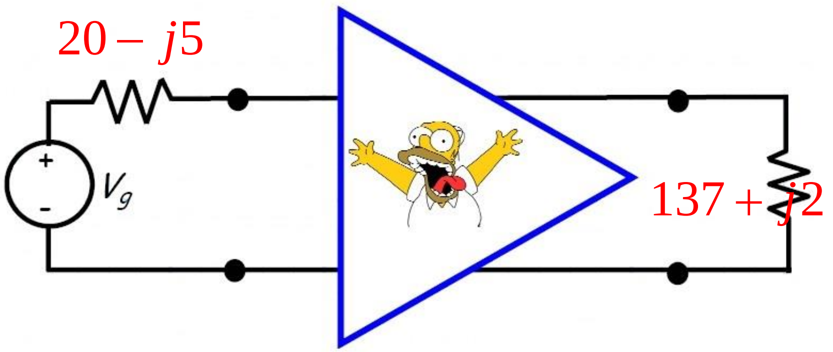
Stability (contd.)

Q: Do we really need to **care** if our design is unconditionally stable? Aren't we really **just** concerned with whether our design values Γ_L and Γ_s lie inside the stability circle?

A: Remember, the values Γ_L and Γ_s are determined for the **specific** values of source and load impedances connected to the amplifier (**presumably** Z_0).



- But what if the resulting amplifier is **not** connected to these ideal impedances? The ideal source or load impedance Z_0 is **never** achieved with **perfection,** and often achieved **not at all** (consider all the **narrow-band** devices we have studied!).



We **do not specifically** know what source and load impedances our amplifier might encounter, we have to generally design an amplifier that is stable for them **all—**one that's **unconditionally stable**!

Stability (contd.)

Q: Anything **else** we need to know about amplifier stability?

A: One last **very** important thing → Recall that amplifiers, like all microwave devices, are **dependent on frequency**. Thus, all of the important values involved in our design (e.g., S_{11}, S_{21}, S_{22}, S_{12}, Γ_L and Γ_s) will **change** as a function of frequency!

Q: I see, amplifier performance, most notably **gain**, will change as a function of frequency, and so maximum power transfer will occur at just our **design frequency** → We've seen this kind of thing **before**!

A: True, but for amplifiers there is also a **new** twist → The amplifier stability conditions (i.e., stability tests) must be satisfied at **any and all frequencies**!

- If for **even one frequency** we find that either:

$$|\Gamma_{out}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \right| > 1.0$$

or

$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| > 1.0$$

→ our amplifier will **oscillate**—**even** if that frequency is **not** our "design frequency"!

This makes amplifier stability a much more **significant** and **difficult** problem than you might otherwise think.

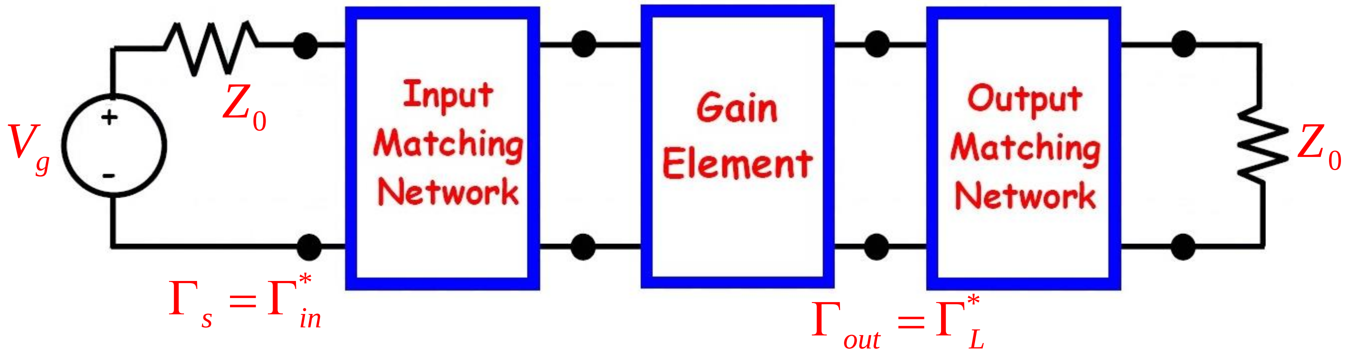
An unconditionally stable amplifier must be unconditionally stable at **all frequencies**!

Transistor Amplifier Design

Q: What happens if we don't like the resulting transducer gain? How can we identify a more suitable gain element?

Q: Since we are using lossless matching networks, won't our resulting device be relatively narrow band? How can we increase the bandwidth of our design?

Maximum Gain Amplifiers



Q: If we design our amplifier such that the source is **matched** to the input of the gain element, and the output of the gain element is **matched** to the load, what **is** the resulting gain?

- If the amplifier is a **unilateral** amplifier ($S_{12} << S_{21}$), where: $\Gamma_{in} = S_{11}$ $\quad \Gamma_{out} = S_{22}$
- Then the transducer gain is called unilateral transducer gain:

$$G_{TU} = \frac{\left(1-|\Gamma_s|^2\right)|S_{21}|^2\left(1-|\Gamma_L|^2\right)}{\left(|1-\Gamma_s S_{11}|^2\right)\left(|1-\Gamma_L S_{22}|^2\right)}$$

Maximum Gain Amplifiers (contd.)

- Inserting the **matched conditions** in the transducer gain expressions we get:

$$G_{TU\,\max} = \frac{1}{1-|\Gamma_s|^2}|S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

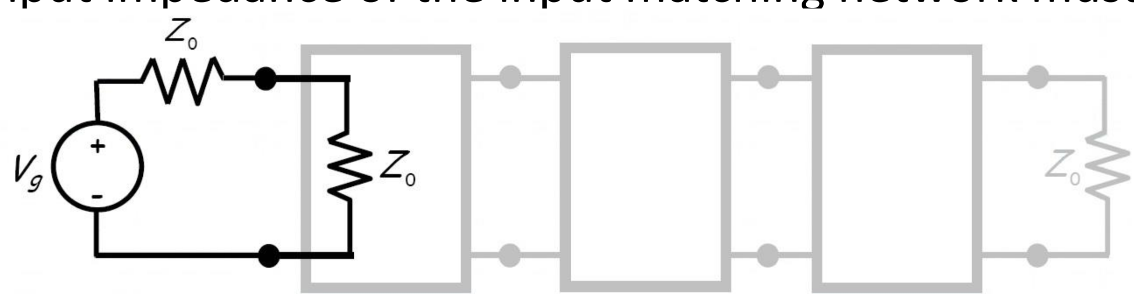
$$G_{TU\,\max} = \frac{1}{1-|\Gamma_s|^2}|S_{21}|^2 \frac{1}{1-|\Gamma_L|^2} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2 \frac{1}{1-|S_{22}|^2}$$

These of course are the **maximum** transducer gain possible, **given** a specific gain element, and a source and load impedance of Z_0

Q: What about the **scattering matrix** of the **amplifier**? Can we determine the scattering parameters of the resulting amplifier?

A: We can certainly determine their **magnitude**!

- First of all, remember that if a matching network establishes a match at its **output**, then a match is likewise present at its **input**.
- As a result, the input impedance of the input matching network must be Z_0:

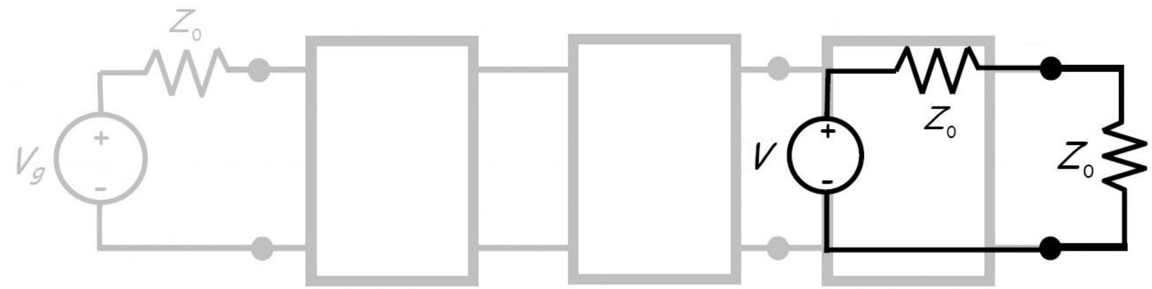


Maximum Gain Amplifiers (contd.)

- Meaning that the scattering parameter S_{11} of the matched amplifier is **zero**!

$$S_{11}^{amp} = 0$$

- Likewise, the o/p impedance of the output matching network must also be Z_0:



- As a result, the scattering parameter S_{22} of the matched amplifier is also **zero**!

$$S_{22}^{amp} = 0$$

- Now, since both ports of the amplifier are matched, we can find the magnitude of the **amplifier** scattering parameter S_{21} to be simply the transducer gain G_{Tmax}.

S-parameters are those of gain element

$$\left|S_{21}^{amp}\right| = G_{TU\max} = \frac{1}{1-|\Gamma_s|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

- Similarly, we can conclude the remaining scattering parameter:

$$\left|S_{12}^{amp}\right| = \frac{1}{1-|\Gamma_L|^2}|S_{12}|^2\frac{1-|\Gamma_s|^2}{|1-\Gamma_s S_{11}|^2}$$

Note that **if** the gain element is **unilateral**, then so too will be the **amplifier**!

The Ideal Gain Element

- Recall that the maximum possible transducer gain, **given a specific gain element**, and a source and load impedance of Z_0 is:

$$G_{T\max} = \frac{1}{1-|\Gamma_s|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

This is achieved by properly constructing input and output matching networks → it's the largest value that we can get for **that particular gain element**

Q: But what if this gain is **insufficient**?

A: In that case we must **change the gain element**, but what should we change the gain element to? What are the characteristics of an **ideal gain element**?

- The answers to these questions are best determined by examining the maximum **unilateral** transducer gain:

$$G_{TU\max} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2}$$

- Recall that for most gain elements, S_{12} **is** small (i.e., approximately unilateral), and in fact $S_{12} = 0$ is **one** ideal characteristic of an **ideal gain element**.
- From the maximum unilateral gain expression, we can determine the remaining **ideal characteristics** of a gain element. Some of these results are rather **self evident**, but others are a bit **surprising**!

The Ideal Gain Element (contd.)

- For example, it is clear that **gain** is increased as S_{21} is **maximized**—no surprise here. What might catch you off guard are the conclusions we reach when we observe the **denominator** of G_{TUmax}

$$G_{TU\max} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2}$$

It appears that the gain will go to **infinity** if $|S_{11}|$ = 1 and/or $|S_{22}|$ = 1!

Q: But that would mean the input and/or output impedance of the gain element is **purely reactive** (e.g. and open or a short). Is this conclusion **accurate**?

A: Yes **and** No.

Remember, this maximum gain is achieved when we establish a **conjugate match**. The equation above says that this maximum gain will increase to infinity if we match to a **reactive** input/output impedance.

And **that's** the catch.

It is **impossible** to match Z_0 to load that is purely reactive!

The Ideal Gain Element (contd.)

- Even otherwise, for maximum power absorption we can only match to an impedance that has a **non-zero resistive component** (i.e., $\Gamma < 1$); else, there's no way for the available power to be **absorbed!**
- Still, it is quite evident that—all other things being equal—a **gain element** with **larger values** of $|S_{11}|$ and $|S_{22}|$ will produce **more gain** than **gain elements** with **smaller values** $|S_{11}|$ and $|S_{22}|$

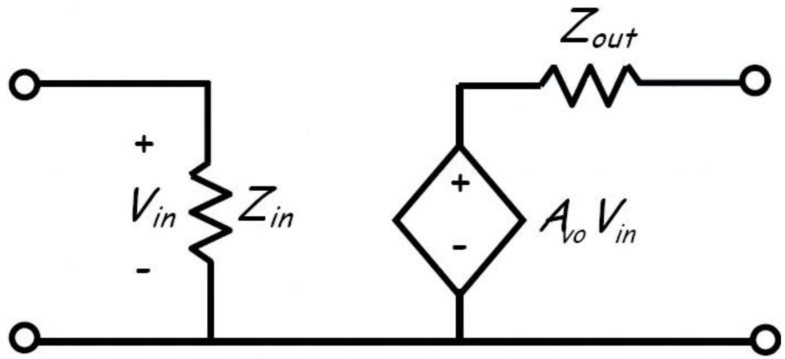
Q: This seems very **counter intuitive**; I would think that an inherently **better-matched** gain element (e.g., $|S_{11}| \approx 0$ and $|S_{22}| \approx 0$) would provide **more gain**.

A: Recall you studied **four types** of amplifier (gain element) models in Analog Circuits: voltage gain, current gain, trans-impedance, and transconductance. Each of these amplifiers were likewise characterized in terms of **input impedance** and **output impedance**.

- Recall also that for each of these models, the **ideal** values of input/output impedance was **always** either zero (a short) or infinity (an open)!

The Ideal Gain Element (contd.)

- In other words, **ideal** amplifiers (**gain elements**) always need to have $|S_{11}| = |S_{22}| = 0$!



Ideally:

$|Z_{in}|$ very small

$|Z_{out}|$ very small

$|A_o|$ very small

- However take example of an **ideal voltage amplifier:** it has a **high input** impedance ($|S_{11}| \approx 1$) and a **low output** impedance ($|S_{22}| \approx 1$). If we construct matching networks on either side of this ideal gain element, the result is an amplifier with **very** high transducer gain !

Q: So how do we "**change**" a gain element to a **more ideal** one?

A: Of course we could always select a **different** transistor, but we could also simply change the **DC bias** of the transistor we are using!

- Recall the **small-signal parameters** (and thus the scattering parameters) of a transistor change as we modify the **DC bias** values. We can select our DC bias such that the value of G_{TUmax} **is maximized.**

Q: Is there any **downside** to this approach?

The Ideal Gain Element (contd.)

A: Absolutely! Recall that we can theoretically match to a very low or very high resistance—at precisely **one frequency**! But we found that the resulting match will typically be **extremely narrowband** for these cases.

Thus, we might consider **reducing** the amplifier gain (i.e., reducing the values $|S_{11}|$ and $|S_{22}|$), in return for achieving a more moderate gain over a **wider frequency bandwidth**!

Additionally, DC bias likewise affects **other** amplifier characteristics, including compression points and noise figure and therefore absolute care is must!

Design for Specified Gain

- The **conjugate matched** design of course **maximizes** the transducer gain of an amplifier. But there are times when wish to design an amplifier with **less** than this maximum possible gain!

Q: Why on Earth would we want to design such a **sub-optimal** amplifier?

A: A general characteristic about amplifiers is that we can always trade **gain** for **bandwidth** (the gain-bandwidth product is an approximate **constant**!). Thus, if we desire a **wider** bandwidth, we must **decrease** the amplifier gain.

Design for Specified Gain (contd.)

Q: Just **how** do we go about doing this?

A: We simply design a "matching" network that is actually **mismatched** to the gain element. We know that the **maximum** transducer gain will be achieved if we design a matching network such that:

$$\Gamma_{in} = \Gamma_{in}^{*} \qquad \Gamma_L = \Gamma_{out}^{*}$$

- Thus, a **reduced gain** (and so wider bandwidth) amplifier must have the characteristic such that:

$$\Gamma_{in} \neq \Gamma_{in}^{*} \qquad \Gamma_L \neq \Gamma_{out}^{*}$$

- Specifically, we should select Γ_s and Γ_L (and then design the matching network) to provide the **desired** transducer gain G_T:

$$G_T = \frac{\left(1-|\Gamma_s|^2\right)|S_{21}|^2\left(1-|\Gamma_L|^2\right)}{\left(|1-\Gamma_s\Gamma_{in}|^2\right)\left(|1-\Gamma_L S_{22}|^2\right)} < G_{T\max}$$

It is apparent that there are **many values** of Γ_s and Γ_L that will provide this sub-optimal gain.

Q: So **which** of these values do we choose?

A: We choose the values of Γ_s and Γ_L that satisfies the above equation, **and** has the **smallest** of all possible magnitudes of $|\Gamma_s|$ and $|\Gamma_L|$

- Remember— **smaller** $|\Gamma|$ leads to **wider** bandwidth!

Design for Specified Gain (contd.)

- This design process is much easier if the gain element is **unilateral**. Recall for that case we find that the transducer gain is:

$$G_{TU} = \frac{\left(1-|\Gamma_s|^2\right)|S_{21}|^2\left(1-|\Gamma_L|^2\right)}{\left(|1-\Gamma_s S_{11}|^2\right)\left(|1-\Gamma_L S_{22}|^2\right)}$$

We can write this as a product of <u>three terms</u>

$$G_{TU} = G_S G_0 G_L$$

<u>Where:</u>

$$G_S = \frac{1-|\Gamma_s|^2}{|1-\Gamma_s S_{11}|^2} \qquad G_0 = |S_{21}|^2 \qquad G_L = \frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

Notice that the value of Γ_s affects G_S **only**, and the value of Γ_L affects G_L **only**. Therefore, the unilateral case again decouples into two **independent** problems

- We can compare these three values with their **maximum** achievable values (when $\Gamma_s = S_{11}^*$ and $\Gamma_L = S_{22}^*$):

$$G_{S\max} = \frac{1}{1-|S_{11}|^2} \qquad G_{L\max} = \frac{1}{1-|S_{22}|^2}$$

$$G_{0\max} = |S_{21}|^2$$

Design for Specified Gain (contd.)

- to increase the BW of amplifier, need to **select** values of G_S and G_L that are **less** (typically by a few dB) than the maximum (i.e., matched) values G_{Smax} and G_{Lmax}.
- Unlike the values G_{Smax} and G_{Lmax}—where there is precisely **one** solution for each ($\Gamma_s = S_{11}^*$ and $\Gamma_L = S_{22}^*$)—there are an **infinite** number of Γ_s (Γ_L) solutions for a specific value of G_S (G_L).

Q: How do we **choose** the respective Γ_s (Γ_L) and eventually G_S (G_L)?

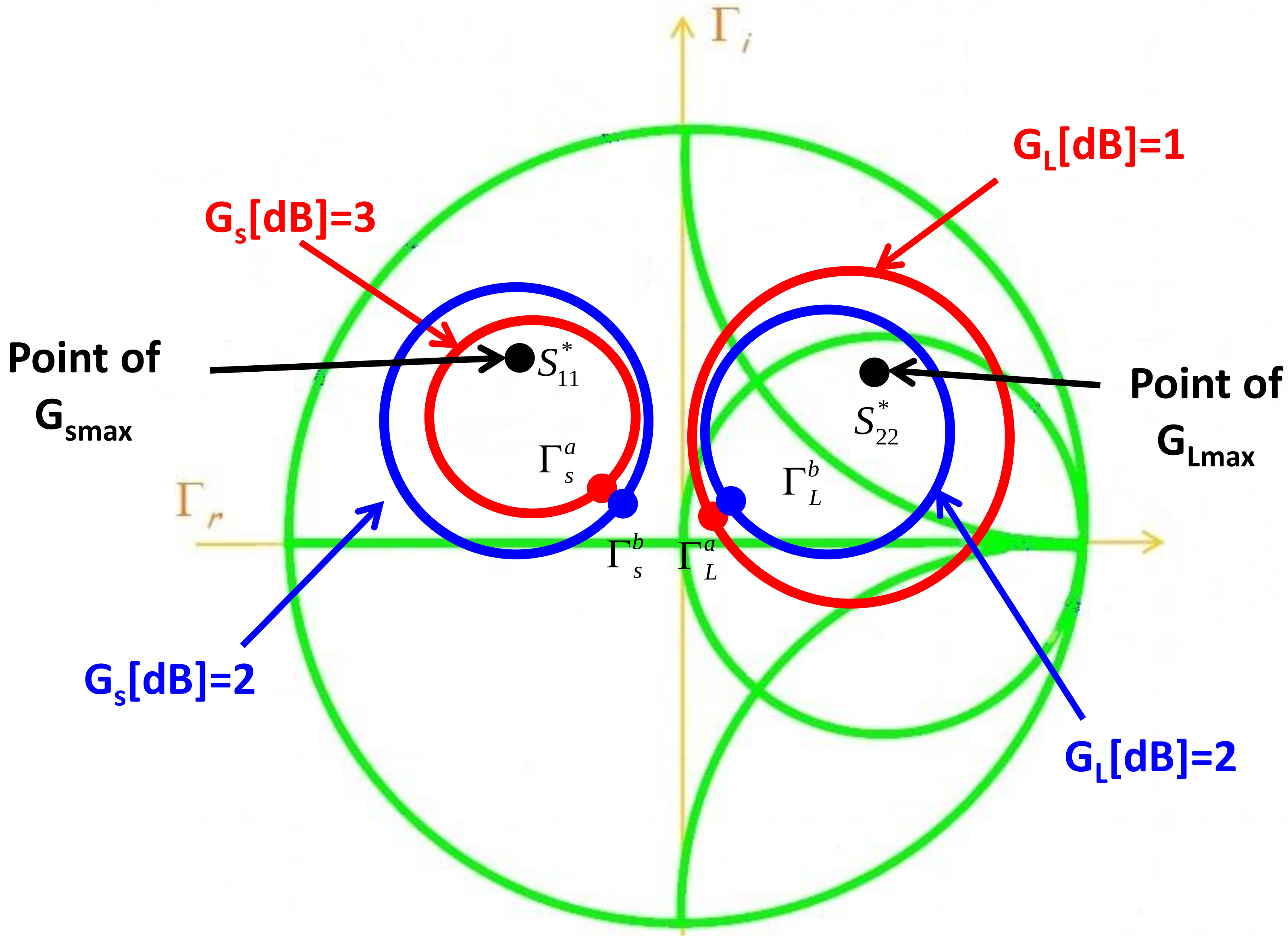
A: We can solve the equations to determine all Γ_s and Γ_L solutions for **specified** design values of G_S and G_L.

$$G_S = \frac{1-|\Gamma_s|^2}{|1-\Gamma_s S_{11}|^2}$$

$$G_L = \frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

- Just as with our stability solutions, the solutions to the equations above form **circles** when plotted on the **complex** Γ-**plane**.
- These circles are known as **constant gain circles**, and are defined by two values: a **complex** value C_s (C_L) that denotes the **center** of the circle on the **complex** Γ-**plane**, and a **real** value R_s (R_L) that specifies the **radius** of that circle.
- Any Γ point **on** (not inside!) a constant gain circle denotes a value of Γ that will provide the requisite gain. To optimize the bandwidth we should choose the point on the circle that is **closest to the center** of the **complex** Γ-**plane**!

Design for Specified Gain (contd.)



Design for Specified Gain (contd.)

- For **example**, say we have an amplifier with:

G_{smax}[dB]=4.0 G_0[dB]=7.0 G_{Lmax}[dB]=3.0

- such that its transducer gain is **14 dB** at its design frequency. To increase the **bandwidth** of this amplifier, we decide to **reduce** the gain to 11 dB.
- Thus, we find that our design goal is: G_s[dB] + GL[dB] =4.0
- From the gain circles on the Smith Chart on the previous slide (assuming they represent the gain circles for this gain element), we find there are **two solutions**; we'll call them solution a and solution b.

Solution a

We determine the Γ_s^a and Γ_L^a from the gain circles:

G_s[dB] = 3.0 G_L[dB] = 1.0

Solution b

We determine the Γ_s^b and Γ_L^b from the gain circles:

G_s[dB] = 2.0 G_L[dB] = 2.0

Design for Specified Gain (contd.)

There are of course an **infinite** number of possible solutions, as there are an infinite number of solutions to $G_S[dB]+G_L[dB]= 4.0$. The two solutions provided here are fairly **representative**.

Q: So **which** solution should we use?

A: That choice is a bit **subjective**.

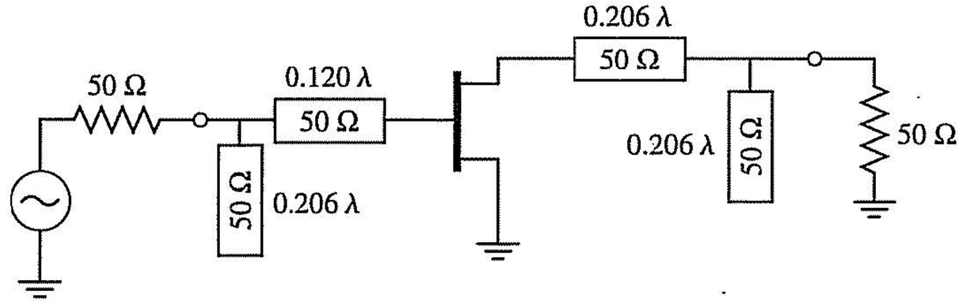
- We note that the point Γ_L^a is **very close** to the center, while the point Γ_s^a pretty **far away** (i.e., Γ_L^a is small and Γ_s^a is large).
- In contrast, both Γ_s^b and Γ_L^b are **fairly close** to the center, although neither is as close as Γ_L^a
- To get the widest bandwidth, I would choose **solution b**, but the only way to know for sure is to design and **analyze both solutions**.
- Often, the design with the widest bandwidth will depend on how you **define** bandwidth!

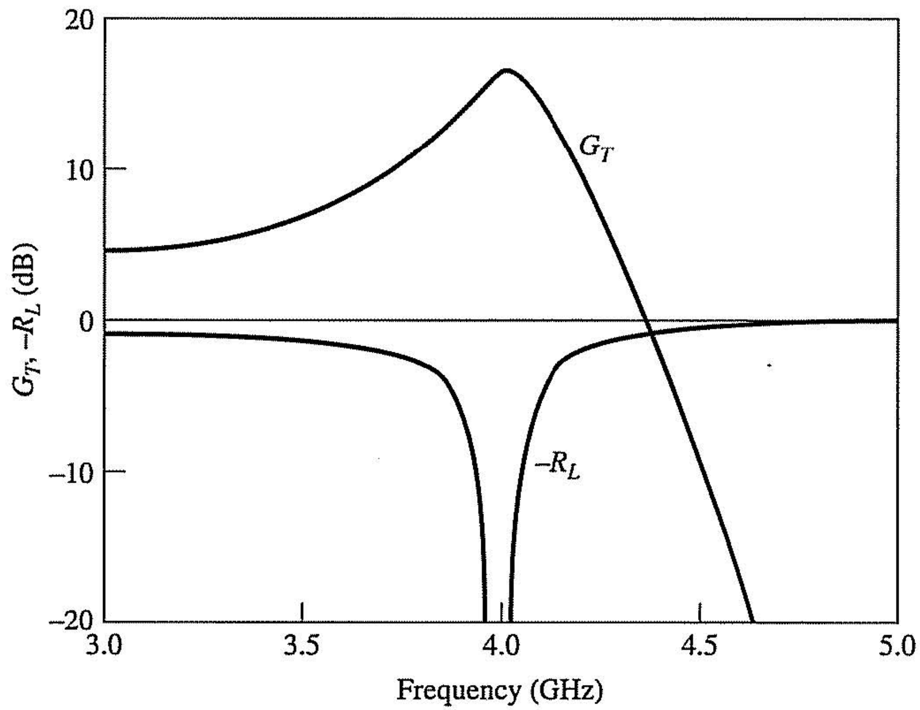
Q: So we reduce the transducer gain by designing and constructing a **mismatched** matching network. Won't that result in **return loss**?

A: Absolutely!

Design for Specified Gain (contd.)

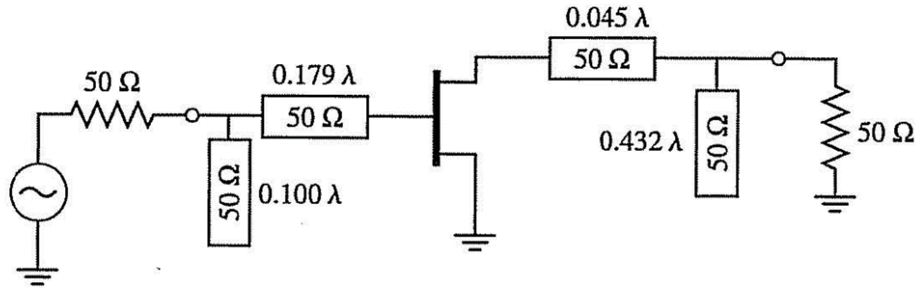
- A conjugate matched amplifier is not only narrow band with regard to gain, it is also **narrow band** with regard to **return loss**. **Only** at the design frequency will the amplifier ports be perfectly matched. As we move away from the design frequency, the return loss **quickly degrades!**



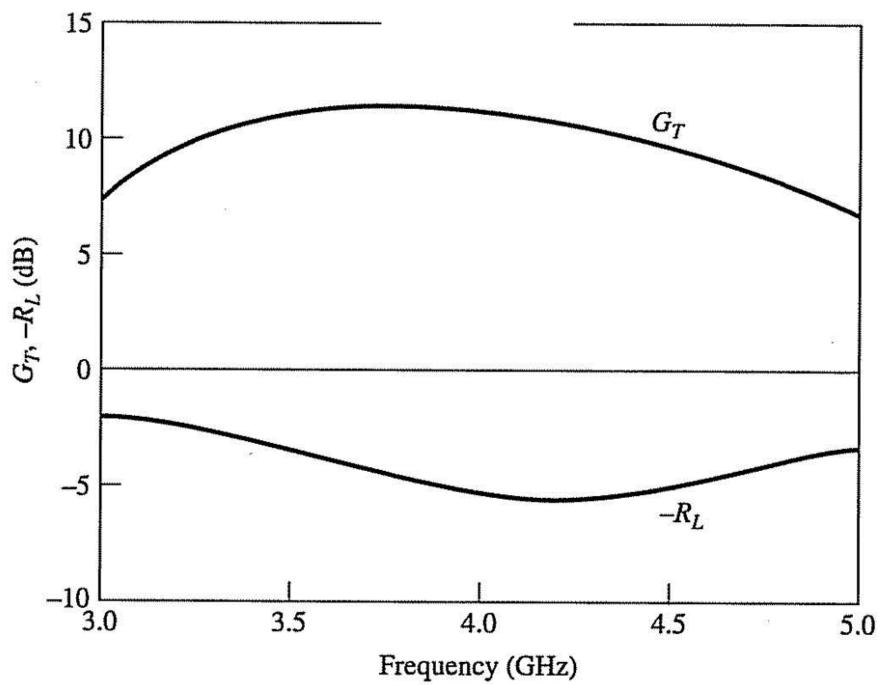


Design for Specified Gain (contd.)

- With the "mismatched" design, we typically find that the return loss is **better** at frequencies away from the design frequency (as compared to the matched design), although at **no frequency** do we achieve a **perfect match** (unlike the matched design).



(b)



Generally speaking, a good (i.e., **acceptable**) return loss over a wide range of frequencies is **better** than a perfect return loss at one frequency and poor return loss everywhere else!

Design for Specified Gain (contd.)

Q: Won't you **ever** stop talking??

A: Lets See!